Taxation, Credit Spreads and Liquidity Traps*

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January 2018

Abstract

We argue that optimal state-contingent variations in asset taxation increase welfare, alter the monetary policy transmission mechanism and insure against liquidity traps. These findings are explained by an endogenous relationship between taxation, the effective rate of return on assets, the inflationary output gap and credit spreads. Such unique link operates via a working-capital cost channel, and affords the policy maker an additional degree of freedom in stabilizing the economy. Optimal policy calls for lowering (increasing) asset taxation following adverse financial (demand) shocks. Severe financial contractions, nonetheless, warrant a more limited tax cut to minimize the occurrence of unintended liquidity traps induced by (otherwise optimal) large fiscal subsidies.

JEL Classification Numbers: E32, E44, E52, E58, E62, E63

Keywords: Asset Taxation; Optimal Policy; Credit Cost Channel; Finance Premium; Zero Lower Bound.

*We are grateful to Tom Holden, Stefano Neri, Raffaele Rossi, Bob Rothschild and John Whittaker for helpful comments and interesting discussions on an earlier draft of this paper. We also thank seminar and conference participants at Lancaster University, Norges Bank, Computing in Economics and Finance (CEF, Fordham University, New York 2017) and Money, Macro and Finance (MMF, King's College, London 2017). An earlier version of this paper was titled: "Unconventional Taxation Policy, Financial Frictions and Liquidity Traps". All remaining errors are our own.

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1 Introduction

A central question in macroeconomics is how (and whether) taxes should vary according to business and financial cycles and in response to various shocks? In the context of a standard neoclassical model with complete markets, Chari, Christiano and Kehoe (1994) established that the short-term tax rate on private assets should fluctuate a great deal in order to absorb most shocks inflicted on the government budget constraint. More recently, during the Great Recession, Barro (2009) and Feldstein (2009) advocated for tax cuts on capital income, with Fernández-Villaverde (2010) showing that lowering such a tax can indeed yield positive multipliers in a model with financial frictions à la Bernanke, Gertler and Gilchrist (1999). In contrast, within a simple New Keynesian model without capital accumulation, Eggertsson (2011) demonstrated that asset tax cuts could actually be contractionary, especially when the economy is stuck in a liquidity trap. Despite the importance of these contributions, the literature remains quiet about the welfare implications of optimal state-contingent asset tax policies following financial and demand shocks, and especially about their possible interactions with conventional monetary policy and the degree of credit frictions. Our paper aims to fill this gap, and to shed new positive and normative insights to the ongoing debate around the role of unconventional fiscal policy in an economy subject to both financial frictions and occasional liquidity traps.

Motivated by the topical relevance of financial market imperfections in explaining business cycle fluctuations, and the various unconventional policy stimulus plans undertaken by many governments and central banks in advanced economies over the last ten years to spur the economy, this paper tackles the following questions: i) what are the macroeconomic and welfare implications of time-varying private asset income taxation in a model with *endogenous* credit spreads?; ii) how should the tax rate on asset income *optimally* adjust in the face of financial and demand shocks that occasionally constrain the policy rate *and* credit spreads to their lower bound?; iii) how do financial frictions and the existence of the lower bound alter the transmission mechanisms of asset income taxation and monetary policy, as well as their possible interactions?

To answer these questions, we present a Dynamic Stochastic General Equilibrium (DSGE) model with nominal price rigidities and a credit cost channel, where firms have to borrow in advance to finance their working-capital needs, as in Ravenna and Walsh (2006).[1] Compared to their model, the loan rate at which firms borrow from the commercial bank not only depends on the nominal policy rate set by the central bank, but also on an endogenous finance premium charged by the lender. We prove that credit default risk, the risk premium and consequently the lending rate are *positively* related to the inflationary marginal production costs proxied by output, which, in turn, is *negatively* linked to the *effective* savings rate due to a standard intertemporal substitution channel. The effective real savings rate, which also represents the effective real rate of return on households financial assets (deposits), is a decreasing function of the tax on interest earnings accrued from savings.[2] Therefore, *endogenous* credit spreads, that depend directly on output, provide an

[1] On the importance of the working-capital (credit) cost channel in explaining business cycle fluctuations, and the 'missing deflation' phenomenon observed during the Great Recession, see Christiano, Eichenbaum and Trabandt (2015).

[2] In this model, deposits serve as the households sole financial asset that is saved at the commercial bank and which is then used to provide working-capital loans to intermediate good firms. Therefore, households private assets / deposits / capital / savings, or (private) asset / deposit / capital / savings taxation, are used interchangeably throughout the text. In this sense, deposits act as household's private capital which facilitates the firm's production via the financial intermediary (the bank). See also Eggertsson (2011) and Fernández-Villaverde (2010), who refer to savings and deposits as capital without explicitly modeling physical capital accumulation by households.

additional policy lever through which the policy maker can influence real economic activity by adjusting capital taxation.[3]

We identify various transmission channels through which capital income taxation can impact the real economy, as well as modify the optimal nominal policy rate setting behaviour. We first show that a capital tax cut acts as a subsidy to the banking system, thereby raising the effective deposit rate faced by households. A higher rate of return on private assets alters the intertemporal consumption-savings decision of the representative household, and triggers a fall in output and in price inflation. Second, we demonstrate that a decline in GDP exerts downward pressure on the cost of borrowing, which, in turn, offsets to a degree the fall in output, while bringing about an additional reduction in the price level via the credit cost channel. The effect of lower capital taxation on inflation is therefore negative, whereas the impact of this policy on output is ambiguous, and depends crucially on the optimal interaction between taxation and standard monetary policy.

Specifically, following negative financial shocks that drive output and inflation in *opposite* directions, as was also observed at the start of the financial crisis (see Gilchrist, Schoenle, Sim and Zakrajsek (2017)), *lowering* asset taxation, all else equal, mitigates the spike in credit spreads and curbs inflationary pressures. This outcome, nonetheless, comes at the expense of a larger fall in output associated with higher effective asset returns. Given the tax cut, optimal monetary policy warrants a slash in the nominal policy rate so the contraction in output is considerably dampened, albeit now at the cost of limited deflationary pressures resulting from the cost channel mechanism. Therefore, subsidizing households asset income together with a sharp nominal policy rate cut is optimal and essential to achieve overall macroeconomic stability when the policy rate is away from the zero lower bound (ZLB). Nevertheless, and *more importantly*, once such an expansionary policy combination is automatically implemented in the face of credit shocks, then a *more severe* disturbance can send the policy rate to the ZLB. Such a scenario prevents the policy maker from further adjusting the policy rate, leading to potentially large output drops, and requiring a relatively *muted* subsidy to capital income to minimize the risk of entering a liquidity trap. As a result, the size of the financial shock and the presence of the lower bound impede upon the merits of an *otherwise optimal* monetary-fiscal policy mix.

Following sizeable adverse demand shocks that push prices and output in the *same* direction, thereby driving the policy rate and credit spreads directly to their lower bound, we find that an optimal *rise* in the capital income tax rate (equivalent to a tax on the banking sector) delivers significant welfare gains. In particular, an optimal automatic stabilizer-type tax policy that lowers the effective savings rate can completely insure against a liquidity trap, and minimize inflation and output volatilities. We argue that the risk premium channel, endogenously driven by output, renders the policy maker an extra degree of freedom to use asset income taxation in order to minimize economic fluctuations, as well as to release the policy rate and credit spreads from the lower bound territory. The recent attempts by the European Central Bank (ECB) to lower deposit rates by paying negative rates on bank reserves, are not inconsistent with the implications of a higher tax on deposits that we advocate for in this model when the ZLB is occasionally binding. Put differently, a tax on households financial wealth serves as a general banking tax tool that is aimed at adjusting the effective return on savings as well as credit spreads against the backdrop of deflationary demand shocks.[4]

[3] We abstract from the fact that there are typically two different agencies, the government and the central bank, in charge of fiscal and monetary policy, respectively. Instead, we assume that the "central bank", also referred to as the "policy maker", chooses both fiscal and monetary policy instruments.

[4] Conti, Neri and Nobili (2016) find that adverse aggregate demand shocks have been the most important contrib-

It is worth mentioning that in our New Keynesian framework, a positive relationship arises between working-capital loans, output (a proxy for the marginal cost) and credit spreads, all else equal. Intuitively, periods of high productivity are associated with an increase in marginal costs, higher levels of debt, and consequently inflated borrowing costs (as measured by the finance premium). Nevertheless, as alluded to above, the response of credit spreads is determined by the nature of the shock hitting the economy. For financial shocks, credit spreads are *countercyclical* with respect to GDP, whereas for demand (preference) shocks, they are *procyclical*.[5] A procyclical reaction of spreads following certain types of shocks is not inconsistent with the models of Carlstrom and Fuerst (1997), Gomes, Yaron and Zhang (2003), and De Fiore and Tristani (2013), among others. Moreover, Christensen and Dib (2008) show that a preference shock produces a positive co-movement between output and the finance premium, while De Graeve (2008) finds that historically the premium tends to rise prior to a recession, with the movement of the premium crucially depending on the source of the shock. That being said, recent evidence suggest an *overall* negative relationship between credit spreads and output (see Gilchrist and Zakrajsek (2012)). Our model, however, can easily generate countercyclical spreads for a combination of large financial, demand and technology shocks. Admittedly, the focus of this theoretical paper is on the optimal response of *state-contingent* asset taxation following *independent* financial and demand shocks as opposed to combined disturbances.

Our model benefits from nesting the three-equation New Keynesian model as a particular case, and from a tractable introduction of asset income taxation, credit risk, an explicit banking sector and occasionally binding lower bound constraints to an otherwise standard Ravenna and Walsh (2006)-type cost channel setup. This stylized framework enables us to provide analytical solutions to inflation and output, and to examine the normative and positive properties of unconventional optimal taxation policies, as well as their interactions with monetary policy rules. Goodfriend and McCallum (2007) and Cúrdia and Woodford (2016), for example, also develop simple, yet insightful, New Keynesian models with financial frictions, but posit a reduced-form intermediation technology to justify the existence of credit spreads. This modeling choice is in contrast to our paper, where borrowing costs are endogenous. More closely related to our paper are those of Demiral (2009) and De Fiore and Tristani (2013), who also derive a micro-founded risk premium, yet focus solely on optimal monetary policy away from liquidity traps. In our paper, we concentrate on the transmission mechanisms of financial and demand shocks accounting for imperfect credit markets and the lower bound, and aim to provide a deeper understanding on how unconventional financial tax policies should react to such disturbances.

Our paper also relates to the literature investigating the effects of various fiscal policies in the presence of financial frictions and the ZLB. Christiano, Eichenbaum and Rebelo (2011), Eggertsson (2011) and Woodford (2011) show that increasing government spending yields a high fiscal multiplier, thus enabling the policy maker to effectively release the economy from a liquidity trap. Carrillo and Poilly (2013) reinforce this point by proving that credit market imperfections considerably magnify the government spending multiplier during a spell in a liquidity trap. Away from the ZLB and in the context of a financial accelerator-type model, Fernández-Villaverde (2010) shows that an exogenous rise in government spending or a capital tax cut produces positive effects on output. In contrast, Uhlig (2010) and Drautzburg and Uhlig (2015) provide a more pessimistic assessment regarding the values of the public spending and tax cut multipliers relative to the pre-

utors to the dis-inflation and the lower real GDP growth experienced in the Eurozone since 2014.

[5] Although not specifically examined, a supply shock also generates *countercyclical* credit spreads in our model.

vious studies. In comparison to these papers, we take a normative stance on optimal asset taxation policies and welfare rather than on the positive aspects of fiscal policy multipliers. We quantify the welfare gains from state-contingent optimal taxation policies following inflationary (deflationary) financial (demand) shocks, and derive analytically the optimal dynamic tax equation that should be set in response to these disturbances. Similar to Mertens and Ravn (2014), we conclude that optimal taxation policies are state-contingent, and depend crucially on the type of shock that drives the economy into a recession in the first place. Unlike their paper, however, our focus is placed on the optimality of asset income taxation in a model with financial frictions and the lower bound, rather than on labour income taxation cut multipliers in a setup featuring expectations driven liquidity traps.

Another strand of literature related to our work nests the analysis of joint optimal monetary and taxation policies in New Keynesian models. Correia, Farhi, Nicolini and Teles (2013) show that distortionary labour and consumption taxes can substitute for adjusting the policy rate, and can circumvent the zero bound problem. Eggertsson and Woodford (2006) also illustrate how consumption taxation can be used to partially offset the adverse effects of the policy rate reaching the ZLB. We also emphasize the need for tax flexibility to neutralize various shocks, although our motivation is different. First, our focus is on the short-run cyclical properties of financial asset taxation as opposed to more conventional labour and consumption taxes. Unlike Correia, Farhi, Nicolini and Teles (2013), we establish that one tax instrument can insulate the economy from the ZLB, as opposed to two instruments that must be jointly determined in their paper. Second, we highlight the interconnection between private asset taxation, financial frictions and credit spreads, a relationship absent from the aforementioned papers and which proves to be imperative in our paper.[6] As private asset taxation in our model can be generalized to any unconventional financial instrument that either taxes or subsidizes the banking sector depending on the nature of the shock, our paper is thus also affiliated with the literature studying joint optimal plans for monetary and macroprudential policies (see Collard, Dellas, Diba and Loisel (2017) and De Paoli and Paustian (2017), among others). However, these papers abstract from the lower bound, implying different state-contingent policy implications in relation to ours. To the best of our knowledge, the welfare and business cycle implications of time-varying optimal asset tax policies, and their interactions with monetary policy rules during normal *and* abnormal times, have not been fully addressed in the literature; especially regarding the impact of this unconventional fiscal policy instrument on the banking sector's multiple interest rates decisions within a simple and tractable three-equation New Keynesian setup.

The remainder of the paper proceeds as follows. Section 2 describes the model and its equilibrium properties. Section 3 details the main transmission mechanisms linking asset income taxation, financial frictions and aggregate macro variables in an analytically tractable way. In Section 4 we explain the parameterization of the model and the solution strategy. Section 5 examines the dynamics and welfare implications of state-contingent optimal policies following financial and demand shocks. Section 6 concludes.

[6] Nakata (2016) examines optimal fiscal and monetary policy in a fully non-linear model with uncertainty and the lower bound imposed on the policy rate. However, his paper does not account for financial frictions as we do here, while his focus is on optimal government spending rather than on optimal asset taxation.

2 The Model

The model economy is populated by households, a final good (FG) firm, a continuum of intermediate good (IG) firms, a competitive commercial bank (the bank), and a benevolent central bank that is responsible for both monetary and fiscal policies. At the beginning of the period and following the realization of aggregate shocks, households lend their deposits (private assets / capital / savings) to the bank, and are paid the nominal deposit rate that also represents the risk-free policy rate. The central bank sets a tax on net deposit returns which serves as an additional stabilization policy instrument in the model. The bank uses households deposits and a central bank cash injection in order to supply working-capital loans to IG firms, and sets the loan rate as an endogenous finance premium over the policy rate. For the given loan rate, monopolistic IG firms decide on the level of employment, prices and the demand for loans. Using a standard Dixit-Stiglitz (1977) technology, the FG firm combines all intermediate goods to produce a homogeneous final good used only for consumption purposes. We now turn to a more detailed exposition of the economic environment and equilibrium properties.

2.1 The Real Economy

Households have identical preferences over consumption (C_t) and labour (H_t). The objective of the representative household is to maximize,

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \vartheta_t \left\{ \frac{C_t^{1-\varsigma}}{1-\varsigma} - \frac{H_t^{1+\gamma}}{1+\gamma} \right\}, \tag{1}$$

where $\mathbb{E}_t$ is the expectations operator, $\beta \in (0,1)$ is the discount factor, ς is the inverse of the intertemporal elasticity of substitution in consumption, and γ is the inverse of the Frisch elasticity of labour supply. A demand shock (ϑ_t) is added to capture exogenous changes in the household's intertemporal preferences and can therefore also be referred to as a taste shock. This demand disturbance follows an $AR(1)$ process,

$$\vartheta_t = (\vartheta)^{1-\rho_\vartheta} (\vartheta_{t-1})^{\rho_\vartheta} \exp\left(s.d(\alpha^\vartheta) \cdot \alpha_t^\vartheta \right), \tag{2}$$

where ϑ is the steady state value of the discount factor shock, ρ_ϑ is the degree of persistence, and α_t^ϑ is a random shock distributed as standard normal with a constant standard deviation given by $s.d(\alpha^\vartheta)$.[7]

Households enter period t with real cash holdings of M_t. They receive their wage bill $W_t H_t$ paid as cash at the start of the period, with W_t denoting real wages. This cash is then used to make deposits D_t at the bank. The households remaining cash balances of $M_t + W_t H_t - D_t$ become available to purchase consumption goods (C_t), subject to a cash-in-advance constraint, $C_t \leq M_t + W_t H_t - D_t$. This constraint represents the implicit cost of holding intra-period deposits that yield interest but that cannot be used for transaction services. At the end of the period, households receive all real profit income from financial intermediation (J_t^{FI}), the IG firms ($\int_0^1 J_{j,t}^{IG} dj$) and a lump-sum transfer from the central bank (T_t).[8] Furthermore, households earn the after-tax

[7] Steady state values are denoted by dropping the time subscript.

[8] Households also receive profits from the final good firm, but these profits are equal to zero in equilibrium.

interest payments on their private assets, $\left(1+\left(R_t^D-1\right)\left(1-\tau_t^D\right)\right)D_t$, with R_t^D representing the gross deposit (policy) rate, and τ_t^D the tax rate on net capital returns.[9] The real value of cash carried over to period $t+1$ is,

$$M_{t+1}\frac{P_{t+1}}{P_t}=M_t+W_tH_t-D_t-C_t+\left(1+\left(R_t^D-1\right)\left(1-\tau_t^D\right)\right)D_t+J_t^{FI}+\int_0^1 J_{j,t}^{IG}dj+T_t. \quad (3)$$

With a positive deposit rate, $R_t^D>1$, and taking real wages (W_t), prices (P_t) and taxes (τ_t^D) as given, the first-order conditions of the household's problem with respect to C_t, D_t and H_t can be summarized as,

$$C_t^{-\varsigma}=\beta\left(1+\left(R_t^D-1\right)\left(1-\tau_t^D\right)\right)\mathbb{E}_t\left(C_{t+1}^{-\varsigma}\frac{\vartheta_{t+1}}{\vartheta_t}\frac{P_t}{P_{t+1}}\right), \quad (4)$$

$$H_t^{\gamma}C_t^{\varsigma}=W_t. \quad (5)$$

Equation (4) represents the Euler equation with respect to deposits. Notice that capital income taxation directly impacts the *effective* rate of return on financial assets and therefore distorts the households' intertemporal consumption-savings decision. Moreover, with households deposits used to facilitate working-capital loans provided by the bank, a tax on net deposit returns can also be treated as a tax / subsidy on bank liquidity. Equation (5) defines the optimal labour supply.

Each IG firm $j\in(0,1)$ faces the following linear production function,

$$Y_{j,t}=\varepsilon_{j,t}H_{j,t}, \quad (6)$$

where $H_{j,t}$ is employment by firm j in period t, and $\varepsilon_{j,t}$ represents an idiosyncratic shock with a constant variance distributed uniformly over the interval $(\underline{\varepsilon},\bar{\varepsilon})$.[10] The IG firm must borrow from the bank in order to pay households wages in advance. Let $L_{j,t}$ be the amount borrowed by firm j, so the demand for loans is,

$$L_{j,t}=W_tH_t. \quad (7)$$

In each period, a fraction χ_t of the IG firm's expected output value ($Y_{j,t}$) must be pledged as collateral in order to secure working-capital loans from the commercial bank. Moreover, we assume that the IG firm (borrower) has the option to 'run away' and default on its debt. In the good states of nature, each IG firm pays back the bank principal plus interest on credit, with the gross lending rate denoted by R_t^L. The firm decides to default if its value after non-payment is greater than its expected value after repaying back the loan in full. Specifically,

$$\left(1-\chi_t\right)Y_{j,t}>Y_{j,t}-R_t^LL_{j,t}, \quad (8)$$

with $\left(1-\chi_t\right)Y_{j,t}$ denoting the expected value of the IG firm after 'running away', and $\chi_tY_{j,t}$ representing the share of expected collateralized output the bank is able to retain in case of firm default.

[9] The qualitative results and policy implications derived in this paper follow through also when the tax rate is applied to the *gross* capital returns, i.e., $\left(1-\tau_t^D\right)R_t^DD_t$.

[10] We use the uniform distribution in order to generate plausible data-consistent steady state credit spreads as explained in the parameterization section. This simple distribution also enables a closed-form expression for credit risk. See also Faia and Monacelli (2007) who adopt a similar approach.

It is further assumed that χ_t follows the $AR(1)$ shock process,

$$\chi_t = (\chi)^{1-\rho_\chi} \left(\chi_{t-1}\right)^{\rho_\chi} \exp\left(s.d\left(\alpha^\chi\right) \cdot \alpha_t^\chi\right), \tag{9}$$

where $\chi \in (0,1)$ is the steady state value of this fraction, ρ_χ is the degree of persistence, and α_t^χ is a random shock with a normal distribution and a constant standard deviation denoted by $s.d\left(\alpha^\chi\right)$. A shock to the probability of collateral recovery (χ_t) represents a financial (credit) shock in this model, as it directly impacts credit risk at the firm level as well as bank credit spreads, as shown below.[11] Using (6) and (7), and re-arranging (8) results in the threshold value $(\varepsilon_{j,t}^M)$ below which the IG firm defaults,

$$\varepsilon_{j,t}^M = \frac{R_t^L W_t}{\chi_t}. \tag{10}$$

Therefore, the cut-off point is related to aggregate credit shocks, the funding costs and real wages, and is *identical* across all IG firms (as in Agénor and Aizenman (1998)).[12] Given the uniform properties of ε_t, the closed-form expression for the probability of default is,

$$\Phi_t = \int_{\underline{\varepsilon}}^{\varepsilon_t^M} f(\varepsilon_t) d\varepsilon_t = \frac{\varepsilon_t^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}. \tag{11}$$

Finally, the pricing decision during period t takes place in two stages. In the first stage, each IG producer minimizes the cost of employing labour, taking its effective costs $(R_t^L W_t)$ as given. This minimization problem yields the real marginal cost,[13]

$$mc_t = \frac{R_t^L W_t}{\varepsilon_{j,t}}. \tag{12}$$

In the second stage, each IG producer chooses the optimal price for its good. Here Calvo (1983)-type contracts are employed, where a portion of ω firms keep their prices fixed while a portion of $1-\omega$ firms adjust prices optimally given the going marginal cost and the loan rate. Solving the standard IG firm's problem yields the familiar form of the log-linear New Keynesian Phillips Curve (NKPC): $\widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + k_p \widehat{mc}_t$, with $k_p \equiv (1-\omega)(1-\omega\beta)/\omega$.[14]

2.2 Financial Intermediation

The bank raises D_t funds via the households at the gross deposit rate $\left(R_t^D\right)$ and also receives a liquidity injection (X_t) from the central bank, which is also remunerated at the same risk-free policy rate.[15] All funds are used to finance the working-capital needs of IG firms and thus act as liabilities

[11] Tayler and Zilberman (2016) also motivate a similar type of financial / credit / collateral / risk shock that directly hits borrowing costs.

[12] As we solve explicitly for the risk of default using a threshold condition, the collateral constraint in this model, from which we derive the cut-off point, is always binding.

[13] Below we show that the bank sets the loan rate based on the IG firm's default decision and threshold default value. Therefore, the risk of default has also a direct effect on the IG firms marginal cost through its endogenous impact on the cost of borrowing. In other words, firms internalize the possibility of default in their optimal pricing behaviour once they borrow at the going lending rate.

[14] Log-linear variables are denoted by '^'.

[15] Introducing a liquidity injection (X_t) is simply to allow the markets to clear (as in Ravenna and Walsh (2006)). In the model, the bank is indifferent between borrowing deposits from households and receiving a central bank loan

to the households and to the central bank. The bank's balance sheet in real terms reads as,

$$L_t = D_t + X_t. \tag{13}$$

The loan rate is set at the beginning of the period, before firms engage in their production activity and prior to their labour demand and pricing decisions. The bank breaks-even from its intermediation activity, such that the expected income from lending to a continuum of IG firms is equal to the total costs of borrowing these funds. The lending bank's expected intra-period zero profit condition from lending to firm j is,

$$\int_{\varepsilon_{j,t}^M}^{\bar{\varepsilon}} R_t^L L_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} \chi_t Y_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} = (D_t + X_t) R_t^D, \tag{14}$$

where $f(\varepsilon_{j,t})$ is the probability density function of $\varepsilon_{j,t}$. The first element on the left hand side is the expected repayment to the bank in the non-default states, while the second element is the expected return in the default states, measured in terms of collateralized output ($\chi_t Y_{j,t}$). The terms $D_t R_t^D$ and $X_t R_t^D$ are the total costs of deposits and central bank liquidity, respectively. To derive the lending rate, we use the balance sheet equation (13), constraint (10) for $\chi_t \varepsilon_{j,t}^M H_{j,t} = R_t^L L_{j,t}$, divide by $L_{j,t}$ and substitute the production function (6), such that (14) boils down to,

$$R_t^L - \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} (\varepsilon_{j,t}^M - \varepsilon_{j,t}) \chi_t \frac{H_{j,t}}{L_{j,t}} f(\varepsilon_{j,t}) d\varepsilon_{j,t} = R_t^D. \tag{15}$$

To find an explicit expression for the risk premium and the loan rate, we use the uniform distribution properties of the idiosyncratic shock. Specifically, the probability density of $\varepsilon_{j,t}$ is $1/(\bar{\varepsilon} - \underline{\varepsilon})$ and its mean is $\mu_\varepsilon = (\bar{\varepsilon} + \underline{\varepsilon})/2$. Using this information and re-substituting (10) in (15) yields the loan rate equation,[16]

$$R_t^L = \nu_t R_t^D, \tag{16}$$

with $\nu_t \equiv \left[1 - \left(\frac{\bar{\varepsilon} - \underline{\varepsilon}}{2\varepsilon_t^M}\right) \Phi_t^2\right]^{-1} > 1$ defined as the risk premium, and $\Phi_t = \int_{\underline{\varepsilon}}^{\varepsilon_t^M} f(\varepsilon_t) d\varepsilon_t = \frac{\varepsilon_t^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$ representing the probability of default. The loan rate is therefore set as an endogenous premium over the policy rate due to the possibility of firm default.

2.3 Monetary Policy and Asset Income Taxation

The central bank sets the nominal interest rate on deposits according to the following Taylor (1993) rule,

$$R_t^D = \max(R_t^{D,NOT}, 1), \tag{17}$$

with $R_t^{D,NOT}$ denoting the desired (or notional) gross policy rate,

$$R_t^{D,NOT} = \left(R^{D,NOT}\right)^{(1-\phi)} \left(R_{t-1}^{D,NOT}\right)^{\phi} \left(\frac{\pi_t}{\pi}\right)^{(1-\phi)\phi_\pi}. \tag{18}$$

such that the loan rate pricing decision is unaffected.

[16] The cut-off value $\varepsilon_{j,t}^M$ depends on the state of the economy and hence it is identical across all IG firms. Similarly, real wages and labour employed by each IG firm are identical such that the volume of loans supplied by the lending bank is also the same. Thus, the subscript j is dropped in what follows.

The term $\phi \in (0,1)$ is the degree of interest rate smoothing, $\phi_\pi > 0$ is the policy coefficient measuring the relative weight on inflation from its steady state, and $R^{D,NOT} = \beta^{-1}$ is the long-run value of the nominal gross policy rate.[17] The central bank sets $R_t^D = R_t^{D,NOT}$ if and only if the policy rate reaction implies a non-zero nominal net interest rate.

Delegating financial stability considerations to the policy maker may require additional policy tools beyond conventional Taylor rules that target price stability, especially when the nominal policy rate strikes the ZLB and becomes largely ineffective. In this model, we consider capital income taxation (τ_t^D) as an extra policy instrument available to the benevolent central bank, and which is chosen optimally to minimize households welfare losses following various shocks. In particular, we explore the stabilization roles and welfare implications of this policy instrument within our framework, which also allows for various financial frictions such as the credit cost channel, endogenous credit spreads and the lower bound for the nominal policy rate and credit spreads.

2.4 Equilibrium

We assume that the size of the liquidity injection from the central bank is $X_t = M_{t+1}\frac{P_{t+1}}{P_t} - M_t$. Following the financial intermediation process, the central bank receives $R_t^D X_t = J_t^{FI}$, while tax revenues are given by $T_t = \tau_t^D \left(R_t^D - 1\right) D_t$. Both the financial intermediation profits and tax revenues are paid back to households as a lump-sum.[18] In a symmetric equilibrium, we substitute the IG firms profits, total profits from the financial intermediation process, the equilibrium condition in the market for loans ($W_t H_t = D_t + X_t$), lump-sum taxes and the size of the liquidity injection in identity (3) to obtain the goods market clearing condition, $Y_t = C_t$.

To solve the model, we log-linearize the behavioral equations and the resource constraint around the non-stochastic, zero inflation ($\pi = 1$) steady state. Using the log-linear versions of (5), (12) and $\hat{Y}_t = \hat{C}_t$ allows us to write the NKPC as,

$$\widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + k_p (\varsigma + \gamma) \hat{Y}_t + k_p \hat{R}_t^L, \tag{19}$$

with $k_p \equiv (1-\omega)(1-\omega\beta)/\omega$.

The Euler equation (4) in log-linear form is,

$$\hat{Y}_t = \mathbb{E}_t \hat{Y}_{t+1} - \varsigma^{-1} \left[\frac{\left(1-\tau^D\right) R^D}{1 + (R^D - 1)(1-\tau^D)} \hat{R}_t^D - \frac{\tau^D \left(R^D - 1\right)}{1 + (R^D - 1)(1-\tau^D)} \hat{\tau}_t^D - \mathbb{E}_t \widehat{\pi}_{t+1} - \hat{r}_t^e \right], \tag{20}$$

where $\hat{r}_t^e \equiv \mathbb{E}_t \left(\hat{\vartheta}_t - \hat{\vartheta}_{t+1}\right)$ is a function of the taste shock. The aggregate level of lending is procured from the log-linear versions of (5), (6), (7) and $\hat{Y}_t = \hat{C}_t$, and is given by,

$$\hat{L}_t = (1 + \varsigma + \gamma) \hat{Y}_t. \tag{21}$$

Turning now to derive the loan rate, we first log-linearize equations (6), (10), (11) and (12) to

[17] We ignore a response to output in the Taylor rule as we find that reacting to this variable adds only negligible welfare gains. This allows us also to clearly establish the relationship between inflation targeting monetary policy rules and optimal state-contingent asset income taxation policies.

[18] The bank is perfectly competitive and therefore earns zero profits.

obtain the log-linearized risk of default,

$$\hat{\Phi}_t = \frac{\varepsilon^M}{\varepsilon^M - \underline{\varepsilon}} \left[\hat{R}_t^L + (\varsigma + \gamma) \hat{Y}_t - \hat{\chi}_t \right]. \tag{22}$$

By log-linearizing (16) and using (22), the equation determining credit spreads can be written as,

$$\hat{R}_t^L - \hat{R}_t^D = \left(\frac{\Psi}{1 - \Psi} \right) \left[\hat{R}_t^D + (\varsigma + \gamma) \hat{Y}_t - \hat{\chi}_t \right], \tag{23}$$

with $\Psi \equiv \frac{(\varepsilon^M + \underline{\varepsilon})(\varepsilon^M - \underline{\varepsilon})}{\left[2\varepsilon^M (\bar{\varepsilon} - \underline{\varepsilon}) - (\varepsilon^M - \underline{\varepsilon})^2\right]} \in (0, 1)$. The term $\varepsilon^M = (pm)^{-1} (\chi)^{-1} \mu_\varepsilon$ is the steady state *reduced-form* threshold value below which the IG firm defaults, where $pm \equiv \lambda / (\lambda - 1)$ denotes the price mark-up resulting from monopolistic competition in the goods market, and λ captures the constant elasticity of substitution between intermediate goods. The steady state risk of default is therefore $\Phi = \frac{\left[(pm)^{-1} \mu_\varepsilon / \chi\right] - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$ while the long-run loan rate is $R^L = \nu R^D$, with $\nu \equiv \left[1 - \left(\frac{\bar{\varepsilon} - \underline{\varepsilon}}{2\varepsilon^M}\right) \Phi^2\right]^{-1}$ and $R^D = \frac{\beta^{-1} - 1}{(1 - \tau^D)} + 1$. Therefore, in the long-run, a positive $\tau^D > 0$ acts as a banking sector tax incurring a higher lending rate (R^L) passed on to IG firms. The steady state level of output is given by $Y = \left(\frac{(pm)^{-1} \mu_\varepsilon}{R^L}\right)^{\frac{1}{\gamma + \varsigma}}$ so a higher τ^D in the long-run lowers the steady state level of aggregate demand. This outcome is a manifestation of the Chamley (1986) and Judd (1985) results, who show that assets should *not* be taxed in the *long-run*.

Notice that from (23) and (21), credit spreads can be directly and positively related to the level of outstanding working-capital debt. Contributing to Cúrdia and Woodford (2010, 2016) and Woodford (2011), who employ a reduced-form credit spread function, the positive relationship between loans and credit spreads in our setup is micro-founded, and does not hamper upon the analytical tractability of the model. In fact, substituting (23) in (19), and re-writing the policy rate rule (18) in a log-linearized form, the model can be expressed in terms of three equations involving inflation (the *AS* curve), output (the *AD* curve), and the policy rate. Specifically,

$$\widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + k_p \left(\frac{1}{1 - \Psi} \right) \left[\hat{R}_t^D + (\varsigma + \gamma) \hat{Y}_t - \Psi \hat{\chi}_t \right], \tag{24}$$

$$\hat{Y}_t = \mathbb{E}_t \hat{Y}_{t+1} - \varsigma^{-1} \left[\frac{(1 - \tau^D) R^D}{1 + (R^D - 1)(1 - \tau^D)} \hat{R}_t^D - \frac{\tau^D (R^D - 1)}{1 + (R^D - 1)(1 - \tau^D)} \hat{\tau}_t^D - \mathbb{E}_t \widehat{\pi}_{t+1} - \hat{r}_t^e \right], \tag{25}$$

$$\hat{R}_t^D = \max(\phi \hat{R}_{t-1}^D + (1 - \phi) \phi_\pi \widehat{\pi}_t, 0). \tag{26}$$

The stochastic process for $\hat{\tau}_t^D$ and the $AR(1)$ processes for the financial and demand shocks close the model.

The competitive approximate equilibrium can now be defined as a collection of real allocations $\left\{\hat{Y}_t\right\}_{t=0}^{\infty}$, prices $\{\widehat{\pi}_t\}_{t=0}^{\infty}$, interest rates $\left\{\hat{R}_t^D\right\}_{t=0}^{\infty}$ and private asset income tax policies $\left\{\hat{\tau}_t^D\right\}_{t=0}^{\infty}$ such that for a given sequence of exogenous shock processes $\left\{\hat{\vartheta}_t, \hat{\chi}_t\right\}_{t=0}^{\infty}$, equations (24), (25) and (26) are satisfied.

A novel aspect of our model is that the finance premium and consequently the loan rate are driven primarily by the elements of the marginal cost (see equations (22) and (23)). Therefore, output or debt, both of which are proxies for the marginal cost, largely determine credit spreads, and provide an additional channel through which monetary policy as well as state-contingent capital income tax policies alter borrowing costs and the economic activity. To fix ideas, we will refer to this mechanism as the *risk premium channel* that operates through the wider *credit cost channel* linking the loan rate to inflation and output. The term that measures the degree of financial market imperfections and that quantifies the risk-adjusted credit cost channel is given by Ψ or $(1-\Psi)^{-1}$, which are *negatively* correlated to the fraction of collateralized output received in case of default (χ). Indeed, note that our model nests the standard cost channel framework of Ravenna and Walsh (2006) by setting $\Psi = 0$ and $\tau^D = 0$, as well as the standard textbook New Keynesian setup by ignoring the term $k_p (1-\Psi)^{-1} \hat{R}_t^D$ in equation (24) and setting again $\Psi = 0$ and $\tau^D = 0$.

3 The Transmission Channels of Asset Income Taxation

Before turning to optimal state-contingent taxation policy, in this section we provide an analytical solution to the model, and examine the role of financial frictions in explaining the effectiveness of taxation on asset returns in normal times and in a liquidity trap. We closely follow the solution strategy of Eggertsson (2011), albeit with a focus on the impact of the credit cost channel and financial risk on the capital taxation transmission mechanism.

To facilitate the derivation of the analytical results with a *positive* nominal policy rate, the model is simplified by setting $\phi = 0$, $\varsigma = 1$ and by ignoring the taste shock ($\hat{r}_t^e = 0$). Hence, we first focus on the inflationary adverse financial shock to credit spreads ($\hat{\chi}_t$) such that the ZLB initially does not bind (see equations (23), (24) and (26)).

In particular, consider a temporary negative credit shock, $\hat{\chi}_t < \chi$, that persists with probability p and returns back to its long-run level (χ) with a probability of $1-p$ every period. We set $p = \rho_\chi$ so the persistence of the shock is comparable with the probability of the disturbance being away from its steady state value. It is also assumed that the central bank activates the tax policy instrument ($\hat{\tau}_t^D \neq 0$) when the economy is hit by a financial shock, and sets $\hat{\tau}_t^D = 0$ otherwise. In the steady state, deviations in inflation and output are zero ($\widehat{\pi}_t = \hat{Y}_t = 0$) and $\tau^D = 0.32$. The term p therefore also denotes the probability of the tax rate differing from its long-run rate, associated with output and inflation deviating from their steady state values. Under this shock specification, and with $R^D = \frac{\beta^{-1}-1}{(1-\tau^D)} + 1$ and $\hat{R}_t^D > 0$, the *AS* and *AD* curves (equations (24) and (25)) can be respectively written as,

$$\widehat{\pi}_t = \frac{k_p \left(\frac{1}{1-\Psi}\right)}{\left[(1-\beta p) - k_p \left(\frac{1}{1-\Psi}\right) \phi_\pi\right]} \left[(1+\gamma)\hat{Y}_t - \Psi \hat{\chi}_t\right], \tag{27}$$

and,

$$\hat{Y}_t = \frac{(1-\beta)}{(1-p)} \left\{ -\frac{\left[\left(1-\beta\tau^D\right)\phi_\pi - p\right]}{(1-\beta)} \widehat{\pi}_t + \frac{\tau^D}{(1-\tau^D)} \hat{\tau}_t^D \right\}. \tag{28}$$

Solving (27) and (28) results in the following solutions for inflation and output as functions of the model parameters, the tax rate on net private asset returns and the financial shock,

$$\widehat{\pi}_t = \frac{k_p \left(\frac{1}{1-\Psi}\right)(1-p)}{\Upsilon} \left[-\Psi \hat{\chi}_t + \frac{(1+\gamma)(1-\beta)}{(1-p)} \frac{\tau^D}{(1-\tau^D)} \hat{\tau}_t^D \right], \tag{29}$$

and,

$$\hat{Y}_t = \frac{k_p\left(\frac{1}{1-\Psi}\right)\left[\left(1-\beta\tau^D\right)\phi_\pi - p\right]}{\Upsilon}\left[\Psi\hat{\chi}_t + \frac{(1-\beta)\left((1-\beta p) - k_p\left(\frac{1}{1-\Psi}\right)\phi_\pi\right)}{k_p\left(\frac{1}{1-\Psi}\right)\left[\left(1-\beta\tau^D\right)\phi_\pi - p\right]}\frac{\tau^D}{\left(1-\tau^D\right)}\hat{\tau}_t^D\right], \tag{30}$$

where the denominator Υ is defined as,

$$\Upsilon \equiv (1-p)(1-\beta p) + (1+\gamma)\, k_p\left(\frac{1}{1-\Psi}\right)\left[\left(\frac{\gamma+p}{1+\gamma} - \beta\tau^D\right)\phi_\pi - p\right] > 0. \tag{31}$$

Following a negative financial shock, credit spreads increase, which through the credit cost channel, induce a rise in the marginal cost and price inflation. With $\phi_\pi > 1$, the central bank raises the policy rate, thereby generating a fall in output and hence giving rise to a trade-off between inflation and output stabilization following financial shocks (see also Gilchrist, Schoenle, Sim and Zakrajsek (2017)). The presence of financial frictions, as captured by Ψ and $(1-\Psi)^{-1} > 1$, amplifies the surge in inflation and the drop in output. Intuitively, a higher degree of financial market imperfections (as also measured by a lower steady state collateral recovery rate, χ) intensifies the hike in the risk premium and borrowing costs. This upshot leads to a more pronounced increase in inflation, and therefore to a stronger policy rate reaction that accelerates the contraction in GDP. At the same time, the decline in output dampens the rise in credit spreads and inflation via both the standard demand channel of monetary policy as well as the credit cost channel. This mitigation effect is captured by the second term on the right hand side of (31), but does not reverse the direct inflationary impact of a spike in borrowing costs that stems from the exogenous deterioration in collateral recovery.

Suppose the policy maker decides to cut capital income taxation in reaction to the negative collateral shock with the aim to facilitate liquidity to the banking sector. Cutting this tax rate, all else equal, raises the *effective* savings rate faced by households, thereby increasing the incentives to save with the bank, and lowering current demand through an intertemporal substitution effect (see also Eggertsson (2011)).[19] A reduction in capital taxation may therefore further depress the economic activity in the short-run, but also translate to dis-inflationary pressures through the NKPC. The degree of financial market imperfections, $(1-\Psi)^{-1} > 1$, contributes to an additional fall in inflation following the tax cut. Intuitively, the lower output and dis-inflation stemming from the liquidity tax stimulus place downward pressure on credit risk and borrowing costs, both of which intensify the drop in prices and prompt the central bank to lower the policy rate more aggressively. The more substantial policy rate cut that follows the capital tax reduction may, on the one hand, magnify the descend in inflation through the *credit cost channel*, but on the other, may cushion the drop in output and inflation expectations via an *intertemporal substitution effect*. Hence, away from the ZLB and with an active credit cost channel, cutting the tax on capital income may conflict with standard monetary policy. It is therefore imperative to study how such policy tools interact with one another. This analysis requires simulation methods that are performed in

[19] The effective deposit rate that accounts for changes in deposit income taxation is equal to,

$$\frac{\left(1-\tau^D\right)R^D}{1+\left(R^D-1\right)\left(1-\tau^D\right)}\hat{R}_t^D - \frac{\tau^D\left(R^D-1\right)}{1+\left(R^D-1\right)\left(1-\tau^D\right)}\hat{\tau}_t^D.$$

the next sections.

How do imperfect financial markets affect the transmission of capital income taxation when the economy enters a liquidity trap? Consider a large temporary demand shock, $\hat{r}_t^e < r^e$, that persists with probability p and returns back to its steady state (r^e) with a probability of $1-p$ every period. We assume that the central bank sets $\hat{\tau}_t^{D,Z} \neq 0$ in the lower bound, and $\hat{\tau}_t^D = 0$ in the absence of the negative demand shock and/or in the long-run.

Denoting $\widehat{\pi}_t^Z$ and $\hat{Y}_t^Z$ as the values of inflation and output in a liquidity trap, and using the above characteristics of the preference shock while setting $\hat{\chi}_t = 0$, then solving equations (24) and (25) for inflation and output in the ZLB yields,

$$\widehat{\pi}_t^Z = \frac{k_p\left(\frac{1}{1-\Psi}\right)(1+\gamma)(1-\beta)}{\left[(1-\beta p)(1-p)-(1+\gamma)k_p\left(\frac{1}{1-\Psi}\right)p\right]}\left[\frac{1}{(1-\beta)}\hat{r}_t^e + \frac{\tau^D}{(1-\tau^D)}\hat{\tau}_t^{D,Z}\right], \tag{32}$$

and,

$$\hat{Y}_t^Z = \frac{(1-\beta p)(1-\beta)}{\left[(1-\beta p)(1-p)-(1+\gamma)k_p\left(\frac{1}{1-\Psi}\right)p\right]}\left[\frac{1}{(1-\beta)}\hat{r}_t^e + \frac{\tau^D}{(1-\tau^D)}\hat{\tau}_t^{D,Z}\right]. \tag{33}$$

Compared to the existing New Keynesian literature that largely abstracts from the role of credit frictions in explaining the effectiveness of various taxation policies in a liquidity trap, the term $(1-\Psi)^{-1} > 1$ exacerbates the decline in inflation and output following a negative shock to $\hat{r}_t^e$, and also has a meaningful impact on the efficacy of asset income taxation in stimulating the economy.

To start with, following a large adverse demand shock, output, the marginal cost and prices plummet. Beyond this direct effect, the dip in the marginal cost places downward pressure on credit risk, which, in turn, lowers the lending rate via the risk premium channel. Through the credit cost channel, the fall in credit spreads magnifies the deflationary impact of the shock and deepens the economic recession. Similar to De Fiore and Tristani (2013), without credit or supply side shocks that lead to a rise in borrowing costs, demand shocks generate a procyclical lending rate. In our model, this relationship can be detrimental to the economic activity when the policy rate is constrained to the ZLB. In other words, by amplifying the slump in prices, falling credit spreads keep the real policy rate elevated such that the economic activity remains depressed.

Given the positive relationship between output, inflation and the tax on private asset returns, then a rise in the latter can be an effective tool in stimulating the economy and in restoring the target levels of output and inflation. This banking tax policy is particularly useful following adverse disturbances moving output and inflation in the same negative direction (such as preference shocks). Furthermore, we find that a *lower* fraction of output received in case of default (χ) increases Ψ, raises $(1-\Psi)^{-1} > 1$, inflates the steady state values of default risk and credit spreads, and hence accentuates the relative improvement in inflation and output following an increase in $\hat{\tau}_t^D$. Intuitively, with a lower collateral recovery rate (χ), and given the procyclical relationship between credit spreads and output (in the absence of credit shocks), the immediate output rise that follows the tax hike places upward pressure on credit spreads, which, in turn, generate an additional relative advancement in prices. The credit cost channel mechanism reinforces the standard demand channel, implying that financial market imperfections and credit risk considerably magnify the expansionary outcome of a capital income tax increase in a liquidity trap.[20] As we also demonstrate in the simulations section below, if the economy faces a liquidity trap, and/or when the loan rate is an

[20] Similarly, Eggertsson (2011) finds that a capital tax increase in a liquidity trap can produce expansionary out-

endogenous mark-up over the risk-free policy rate, varying capital taxation provides the policy maker an extra degree of freedom to stabilize the economy following various shocks.

4 Parameterization and Solution Strategy

The baseline parameterization used to simulate the model is summarized in Table 1. Most parameters are standard in the literature and are chosen to match observed ratios and interest rate spreads in the U.S.

Table 1: Benchmark Parameterization

Parameter	Value	Description
β	0.99	Discount factor
ς	1.00	Inverse of elasticity of intertemporal substitution
γ	0.50	Inverse of the Frisch elasticity of labour supply
ϑ	1.00	Average taste shock value
λ	6.00	Elasticity of demand for intermediate goods
ω_p	0.82	Degree of price stickiness
$\bar{\varepsilon}$	1.20	Idiosyncratic productivity shock upper range
$\underline{\varepsilon}$	0.80	Idiosyncratic productivity shock lower range
χ	0.97	Fraction of collateral seized in default states
τ^D	0.32	Asset income tax rate in steady state
ϕ	0.00	Degree of persistence in monetary policy rule
ϕ_π	2.00	Response of policy rate to inflation deviations
$\rho_\vartheta = p$	0.88	Degree of persistence - Demand shock
$\rho_\chi = p$	0.88	Degree of persistence - Financial shock
$s.d(\alpha^\vartheta)$	0.009	Standard deviation - Demand shock
$s.d(\alpha^\chi)$	0.06	Standard deviation - Financial shock

Elaborating on some of the unique parameters to this model. The subjective discount factor is set to $\beta = 0.99$, while the deposit income tax rate in steady state is $\tau^D = 0.32$ (as in Fernández-Villaverde (2010)). These values imply a risk-free interest rate of 4 percent. The idiosyncratic productivity range is set between $(0.8, 1.2)$, which together with the fraction of output received in case of default pinned to $\chi = 0.97$, and a price mark-up of 20 percent, yields an annual credit spread of 2.04 percent and a loan to GDP ratio of 81.7 percent. All these estimates roughly correspond with the long-run U.S. data.

The Taylor (1993) rule parameters are given by $\phi_\pi = 2$ and $\phi = 0$. We set the smoothing parameter in the interest rate rule to zero in favour of a tractable model solution and a more transparent illustration of our analytical results, as presented above and also below.[21] As for the main shocks examined in our paper, we fix the persistence parameters governing the evolution of financial and demand shocks, $\rho_\chi = p$ and $\rho_\vartheta = p$, both to 0.88, while the standard deviations

comes. However, in this model we account for meaningful financial frictions, and examine explicitly the transmission channels and welfare implications of capital income taxation following financial and demand shocks, including the corresponding state-contingent optimal policy and interactions with standard monetary policy.

[21] Nevertheless, adding persistence to the Taylor rule does not qualitatively alter our optimal policy implications presented below.

associated with these shocks are $s.d(\alpha^{\chi}) = 0.06$ and $s.d(\alpha^{\vartheta}) = 0.009$, respectively. These numbers are fairly consistent and within range of the calibrated values obtained in Benes and Kumhof (2015), Christiano, Motto and Rostagno (2014) - for financial shocks; and Eggertsson (2011), Denes, Eggertsson and Gilbukh (2013) - for demand shocks.

Finally, to quantitatively solve the model with occasionally binding constraints, we implement the methodology developed in Guerrieri and Iacoviello (2015), who propose a piecewise-linear approach that: i) combines multiple regimes of the same model; and ii) solves for the model-implied expected future prices. Specifically, in our setup we define two regimes: the first when the lower bound binds and the second when it does not. The combination of the two different regimes generates strong non-linearities in the model variables, and constructs a piecewise-linear approximation to the original non-linear model. This approximation is then applied to determine the duration and probability of procuring the ZLB, both of which endogenously impact the dynamics and moments of key variables.[22]

5 Optimal State-Contingent Taxation Policy and Welfare

In this section, we calculate optimal tax policies in response to inflationary financial shocks and deflationary demand shocks that can push the economy towards a liquidity trap. The central bank's objective function is given by a second-order approximation of the household's *ex-ante* expected utility,[23]

$$\sum_{t=0}^{\infty} \beta^t U_t \approx U - \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{\lambda}{\kappa_p} \right) \hat{\pi}_t^2 + (1+\gamma)\, \hat{Y}_t^2 \right]. \tag{34}$$

We measure the welfare gain of policy j as a fraction of the consumption path under the benchmark case (defined below for each shock and denoted by I) that must be given up in order to obtain the benefits of welfare associated with the various optimal policies: $\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^j, H_t^j \right) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left((1-\Lambda)\, C_t^I, H_t^I \right)$, where Λ is a measure of welfare gain in units of steady state consumption. Given the utility function adopted and with $\varsigma = 1$, the expression for the consumption equivalent (Λ) in percentage terms is,

$$\Lambda = \left\{ 1 - \exp\left[(1-\beta) \left(\mathbb{W}_t^j - \mathbb{W}_t^I \right) \right] \right\} \times 100,$$

with $\mathbb{W}_t^j = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^j, H_t^j \right)$ representing the unconditional expectation of lifetime utility under Policy j, and $\mathbb{W}_t^I = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^I, H_t^I \right)$ the welfare associated with the benchmark policy. A higher

[22] In the context of a perfect foresight setup, as in this model, using either the Guerrieri and Iacoviello (2015) solution method or Holden's (2016) algorithm, produces the exact same results. In general, Holden's (2016) algorithm can be applied to higher order pruned perturbations (thereby providing higher accuracy results) and account for future uncertainty, all of which are absent from the Guerrieri and Iacoviello (2015) method.

[23] The richer borrowing cost channel, featuring default risk and capital taxation, therefore does not change the *structure* of the loss function compared to standard New Keynesian models with just a monetary policy cost channel (see also Ravenna and Walsh (2006)). The detailed derivation of the loss function is provided in the Appendix.

positive Λ implies a larger welfare gain and hence indicates that the policy is more desirable from a welfare perspective.

5.1 Financial Shocks

For financial shocks, we compare between the following policies: Policy I (benchmark case) - a standard Taylor rule policy where the central bank sets $\phi_\pi = 2$, and a constant capital income rate ($\tau^D = 0.32$ and $\hat{\tau}_t^D = 0$). Policy II - central bank optimally varying the capital income tax rate given a standard Taylor rule (adjusting optimally $\hat{\tau}_t^D$ for a given $\phi_\pi = 2$). Policy III - central bank optimally reacting to inflation in the Taylor rule, and optimally adjusting capital taxation (ϕ_π and $\hat{\tau}_t^D$ optimized). For the purpose of Policy III, the optimal ϕ_π is grid-searched within the range $\phi_\pi = [1 : 100]$ with step of 0.01.[24]

Table 2 reports the simulated standard deviations (measured in annual percentage terms) of key variables following a 1 standard deviation shock to χ_t, and the welfare gain (Λ) of the various optimal policy combinations relative to benchmark Policy I,

Table 2: Optimal Policy and Welfare - Financial Shock

Policy I	Policy II	Policy III
$\phi_\pi = 2.0$	$\phi_\pi = 2.0$	$\phi_\pi = 86.1$
$\hat{\tau}^D = 0.0$	$\hat{\tau}^D$ optimized	$\hat{\tau}^D$ optimized
$\Lambda = -$	$\Lambda = 0.0490$	$\Lambda = 0.0596$
$s.d\,(\widehat{\pi}_t) = 1.066$	$s.d\,(\widehat{\pi}_t) = 0.022$	$s.d\,(\widehat{\pi}_t) = 0.083$
$s.d(\widehat{Y}_t) = 1.080$	$s.d(\widehat{Y}_t) = 1.196$	$s.d(\widehat{Y}_t) = 0.036$

The optimal capital income tax rate in Policies II and III which maximizes welfare is obtained by maximizing (34) subject to (29) and (30) with respect to $\hat{\tau}_t^D$. For a given ϕ_π, this problem yields the following optimal tax rate that should be set following credit shocks,

$$\hat{\tau}_t^D = \frac{k_p\left(\frac{\Psi}{1-\Psi}\right)\left\{\lambda\left(\frac{1}{1-\Psi}\right)(1-p) - \left[(1-\beta p) - k_p\left(\frac{1}{1-\Psi}\right)\phi_\pi\right]\left[\left(1-\beta\tau^D\right)\phi_\pi - p\right]\right\}}{(1-\beta)\left\{(1+\gamma)\,\lambda k_p\left(\frac{1}{1-\Psi}\right)^2 + \left[(1-\beta p) - k_p\left(\frac{1}{1-\Psi}\right)\phi_\pi\right]^2\right\}}\frac{\left(1-\tau^D\right)}{\tau^D}\hat{\chi}_t. \tag{35}$$

For our benchmark calibration with $\phi_\pi = 2$, the coefficient multiplying $\hat{\chi}_t$ in (35) is equal to 2.38. That is, an *adverse* financial shock ($\hat{\chi}_t < 0$) requires the policy maker to significantly *cut* the capital income tax rate to minimize welfare losses.[25] Figure 1 depicts the impulse response

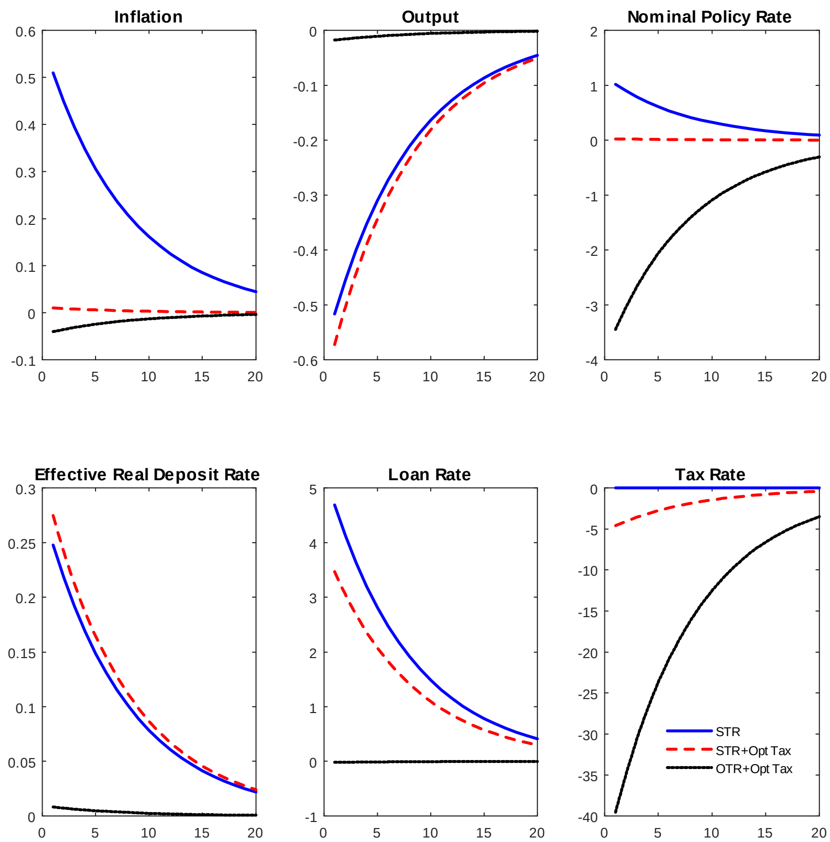
[24] We find that the optimal ϕ_π always lies between [1 : 100] for various parameter configurations Hence, our *unbounded* results for ϕ_π and corresponding simulations are in line with Ramsey monetary optimal policy. In this paper, nonetheless, we opt to focus on the interactions between optimal taxation and *optimal* monetary policy *rules*. See also Christiano, Eichenbaum and Rebelo (2011), who calculate optimal government spending for a *given* Taylor (1993) rule.

[25] It is worth noting that from (29) and (34), the policy maker can achieve full price stability ($\widehat{\pi}_t^2 = 0$) by setting the tax rate to,

$$\hat{\tau}_t^D = \frac{(1-p)}{(1-\beta)(1+\gamma)}\frac{\left(1-\tau^D\right)}{\tau^D}\Psi\hat{\chi}_t.$$

functions associated with the different policies examined in Table 2, with the optimal tax rate in Policies II and III determined by (35).

Figure 1 - Adverse Financial Shock



i) Figure 1 compares between Policy I (Standard Taylor Rule, ‘STR’), Policy II (Standard Taylor Rule + Optimal Tax Policy, ‘STR+Opt Tax’), and Policy III (Optimal Taylor Rule + Optimal Tax Policy, ‘OTR+Opt Tax’). (ii) Interest rates, inflation and the capital income tax rate are measured in annualized percentage point deviations from steady state. Output is measured in annualized percentage deviations.

In benchmark Policy I, an adverse collateral shock raises the risk premium and the lending rate, and results in an immediate rise in price inflation. With $\phi_\pi > 1$, the policy rate increases, which, in turn, raises the effective real savings rate and therefore discourages aggregate demand. As explained

Using our benchmark parameter values, the coefficient multiplying $\hat{\chi}_t$ in the above tax equation is equal to 2.44, a larger value than the one obtained in (35) for the tax rate that maximizes overall welfare. Put differently, while a larger cut in capital taxation can promote full price stability, the more substantial decline in output associated with such a policy would result in a more accute economic recession and higher welfare losses compared to Policy II.

earlier, and similar to Gilchrist, Schoenle, Sim and Zakrajsek (2017), a negative financial shock in this model is inflationary, and gives rise to a trade-off between inflation and output stabilization.[26]

Examining Policy II, optimal taxation policy calls for a decrease in the capital income tax rate in the face of adverse financial shocks, intrinsic in higher borrowing costs and deteriorating lending conditions. Upon impact, cutting the capital income tax rate by around 5 percentage points (from the benchmark case of 32 percent) increases the effective savings rate faced by households, and consequently generates an exaggerated decline in output. At the same time, with a reduced tax rate, the bank can afford to charge a lower loan rate due to the positive impact that capital taxation inflicts on output and the risk premium. This outcome propagates the dis-inflationary effects emanating from the initial fall in GDP. More specifically, the demand-pull dis-inflation, linked with the higher effective savings rate, results in a lower nominal policy rate and in an easing of borrowing cost pressures. The latter, in turn, reinforces the fall in prices and contributes further to price stability. While this effect may potentially promote an improvement in output, the higher effective savings rate required to curb inflationary pressures results in a more severe slump in output. Hence, despite the higher volatility and welfare costs associated with a larger fall in GDP, the far less pronounced rise in inflation translates into an overall and relevant welfare enhancement of 0.0490 percent with a micro-founded welfare loss function.

In Policy III, we ask how does optimal taxation alter the optimal transmission mechanism of monetary policy and vice versa? We find that a policy mix of subsidizing capital income and a stronger feedback from inflation to the monetary policy rule attains the highest welfare gain. Due to the demand-pull dis-inflation inherent in the fall in taxes, the central bank can react more aggressively to prices in the Taylor rule, resulting in a lower nominal policy rate and an additional fall in borrowing costs. In turn, this cost-push dis-inflation subdues the descend in aggregate demand, and prompts a *shorter and less persistent* recession. While Policy III implies larger volatility in prices (compared to Policy II), the significant relative improvement in GDP yields an overall welfare gain of 0.0596 percent compared to Policy I. Moreover, increasing ϕ_π from its standard level to its optimal value, raises the coefficient multiplying $\hat{\chi}_t$ in (35) from 2.38 to 20.59. Hence, a more hawkish type Taylor rule must be coordinated with a much larger subsidy to holding private assets with the bank. Put differently, if the large subsidy to the banking system is not coordinated with a significant policy rate cut, then the risks of greater dis-inflation outweigh the gains from moderating the fall in output.

However, note that the optimal tax rate drives the policy rate close to the ZLB territory under our benchmark parameterization (see the behaviour of the nominal policy rate falling just shy of 3.5 percentage points annually). Suppose the size of the financial shock is now of scale $1.5 \times s.d(\alpha^\chi)$, with $s.d(\alpha^\chi) = 0.06$ remaining constant. In this case, the optimal policy combination described in Policy III would send the economy to a liquidity trap, impeding upon the ability of the central bank to further lower the nominal policy rate. In turn, at the ZLB and for $\hat{R}_t^D = 0$ or $\phi_\pi = 0$, the

[26] For a discussion on risk premium shocks that initially produce a co-movement between output and inflation, thereby pushing the nominal refinance rate towards the ZLB, see Amano and Shukayev (2012), and Carrillo and Poilly (2013). These models do *not* feature a credit cost channel as in our model, but do include investment and physical capital.

welfare-maximizing tax rate becomes,

$$\hat{\tau}_t^{D,Z} = \frac{k_p\left(\frac{\Psi}{1-\Psi}\right)\left[\lambda\left(\frac{1}{1-\Psi}\right)(1-p)+p\,(1-\beta p)\right]}{(1-\beta)\left[(1+\gamma)\,\lambda k_p\left(\frac{1}{1-\Psi}\right)^2+(1-\beta p)^2\right]}\frac{\left(1-\tau^D\right)}{\tau^D}\hat{\chi}_t, \tag{36}$$

with the coefficient now multiplying $\hat{\chi}_t$ equal to 2.67. That is, the required drop in the tax rate following an adverse credit shock, and in the face of a liquidity trap must be of a smaller magnitude in order to offset the deflationary impact resulting from the rise in the effective real deposit rate and the plummet in borrowing costs. The exacerbated decline in GDP is driven by the inability of the nominal policy to optimally adjust. Figure 2 plots the impulse response functions following a large adverse financial shock of magnitude $1.5 \times s.d(\alpha^{\chi})$, and with the tax rate set to its optimal level away from and at the ZLB. Specifically, we compare between the following three regimes: i) Regime I ('Old Opt Tax - Linear') - the tax rate following (35) and $\phi_\pi = 86.1$, disregarding the ZLB; ii) Regime II ('Old Opt Tax - Piecewise-Linear (PL)') - the tax rate still set according to (35) and $\phi_\pi = 86.1$, but with the nominal policy rate occasionally hitting the ZLB; iii) Regime III - ('New Opt Tax - Piecewise-Linear (PL)') - the tax rate following (36) and the policy rate allowed to reach the ZLB.

Figure 2 - Adverse Financial Shock - Optimal Policy and the ZLB

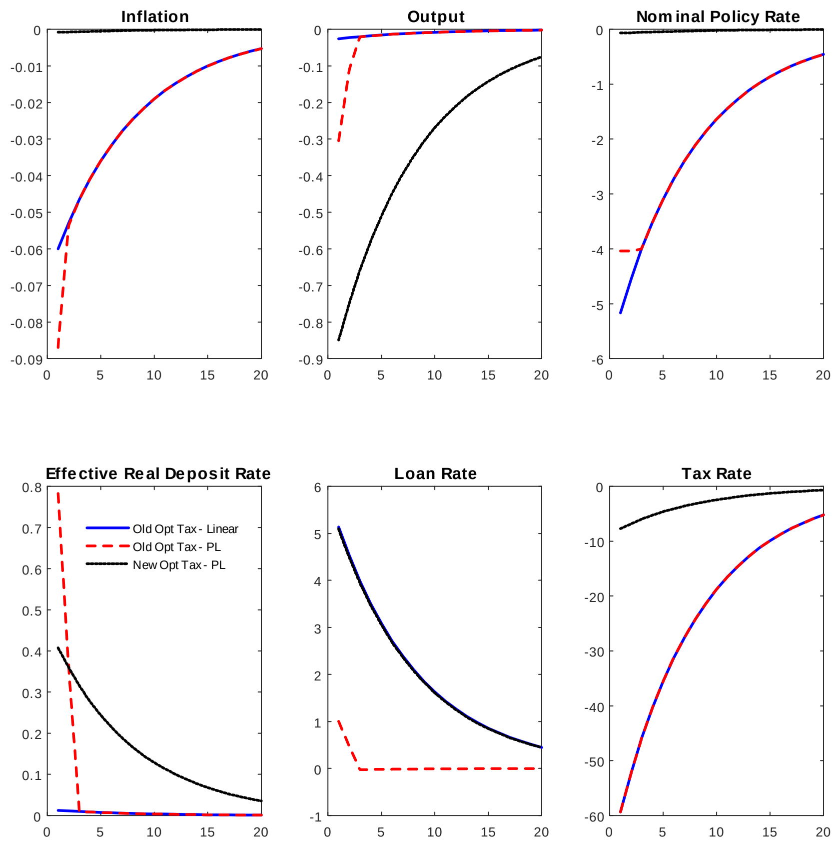


Figure 2 compares between Regime I ('Old Opt Tax - Linear'), Regime II ('Old Opt Tax - Piecewise-Linear (PL)'), and Regime III ('New Opt Tax - Piecewise-Linear (PL)'). ii) Interest rates, inflation and the capital income tax rate are measured in annualized percentage point deviations from steady state. Output is measured in annualized percentage deviations.

Once the policy rate hits the ZLB and becomes impotent in mitigating the fall in output induced by the subsidy to private asset income, the decline in output and inflation is considerably amplified due to the rise in the effective real deposit rate and the fall in credit spreads. From our discussion above and away from the lower bound, a subsidy to private asset income together with an expansionary monetary policy is crucial to achieve combined inflation and output stability. However, in a liquidity trap, the collapse in inflation and output is accentuated, thereby hindering upon the neutralizing effects of the otherwise optimal fiscal-monetary policy combination. Interestingly, to shield the economy from the ZLB, optimal policy calls for a *muted* decline in the asset income tax rate, associated with a more significant short-run slump in output, but with deflationary pressures essentially being subdued. While the short-run decline in output becomes more severe, such an optimal policy as described in Regime III minimizes the asymptotic standard deviations in inflation and GDP, as well reduces the long-run risk of entering a liquidity trap to zero. Table 3 summarizes the above discussion, and quantifies the welfare losses from Regimes II and III relative to the unconstrained Regime I. This table also shows the asymptotic standard deviations in key variables and the probability of reaching the ZLB under the different policy regimes.

Table 3: Optimal policy, standard deviations and welfare costs at the ZLB

	Regime I	Regime II	Regime III
	$s.d\,(\widehat{\pi}_t) = 0.083$ $s.d(\widehat{Y}_t) = 0.036$	$s.d\,(\widehat{\pi}_t) = 1.862$ $s.d(\widehat{Y}_t) = 2.488$	$s.d\,(\widehat{\pi}_t) = 1.1 \times 10^{-3}$ $s.d(\widehat{Y}_t) = 1.183$
Probability of hitting ZLB (percent)	–	23.90	0
Welfare Cost/Gain	–	−0.2061	−0.0101

Notes: i) The standard deviations of key variables are represented in annualized rates.
ii) The welfare cost is the percentage consumption equivalent, measured relative to Regime I - 'Linear' model.

To conclude, the policy implications of our model following a negative financial shock are fairly consistent with what may have been in the minds of policy makers during the peak of the financial crisis in 2008, when liquidity injections to the banking system and unconventional expansionary fiscal policies became operative, and the federal funds rate was lowered substantially. While this model does not explicitly account for liquidity injections, central bank's balance sheet policies or the interest payment on reserves, all of which facilitate bank liquidity, a tax cut on capital income in this framework is in line with such operations. The policy rate pushed closer towards the ZLB is quantitatively also supported by this model. Our counterfactual analysis suggests that the interaction between unconventional banking taxation (or subsidy) policies and standard monetary policy is crucial and can be utilized to counteract the welfare detrimental effects of an inflationary financial shock, at least when the policy rate is away from its lower bound. However, and most importantly, following larger financial shocks, the optimal fiscal-monetary policy mix can send the economy to a liquidity trap, which then requires a more moderated fall in the savings income tax rate in order to prevent the occurrence of the ZLB. Our model therefore suggests that the uncertainty regarding the size of the shock can hamper the effectiveness of an otherwise optimal fiscal-monetary expansionary policy combination. In fact, such policy can significantly increase the risk of entering an unintended liquidity trap if the policy maker miscalculates the magnitude of the credit shock.

5.2 Demand Shocks

We now turn to examine the implications of the lower bound on the dynamics and standard deviations of key variables, as well as the stabilization properties of optimal capital taxation in a liquidity trap environment generated by a large negative preference shock. In studying the dynamics of key variables following a sizeable adverse demand shock, we also restrict credit spreads from falling below zero so $\hat{R}_t^L \geq \hat{R}_t^D$ or $\hat{R}_t^L - \hat{R}_t^D = \max(\hat{R}_t^L - \hat{R}_t^D, 0)$. Hence, we solve the model with *two* occasionally binding constraints, one for the main policy rate and the other for credit spreads.

The optimal ZLB capital income tax rate is obtained from the optimization of (34) subject to (32) and (33) with respect to $\hat{\tau}_t^{D,Z}$. This problem yields,

$$\hat{\tau}_t^{D,Z} = -\frac{\left(1-\tau^D\right)}{\tau^D}\left(1-\beta\right)^{-1}\hat{r}_t^e. \tag{37}$$

Therefore, the optimal tax rate is independent of ϕ_π, and can *completely* minimize welfare losses following preference shocks. Optimal policy calls for *increasing* the tax rate in the face of an *adverse* demand disturbance ($\hat{r}_t^e < 0$) that drives the economy into a liquidity trap.

To illustrate the intuition behind this result, we study the effects of a sizeable adverse preference shock that creates a negative co-movement between inflation and output, such that the central bank lowers the policy rate until the ZLB constraint becomes binding. For this purpose, we examine a preference shock with a magnitude of $5 \times s.d(\alpha^\vartheta)$, with $s.d(\alpha^\vartheta) = 0.009$ remaining constant. The multiplicative scale of the shock is set to cause the economy to reach a liquidity trap upon impact, and stay there for 6 periods (under a benchmark monetary policy rule: $\phi_\pi = 2$ and $\phi = 0$). Admittedly, the duration of the ZLB on the policy rate in our model is shorter than the one observed for the United States and the Eurozone for nearly a decade (see also Carrillo and Poilly (2013)). Nevertheless, given our parameterization choices and with $s.d(\alpha^\vartheta) = 0.009$, our model attains the ZLB with a frequency of 9 percent, consistent with long-run empirical evidence.

Figure 3 shows the response of key variables to a negative unexpected large preference shock that drags the economy into a liquidity trap. The figure compares between three different scenarios: i) Scenario I ('Linear') - a standard Taylor rule ($\phi_\pi = 2$) that disregards the lower bound; ii) Scenario II ('Piecewise (PL)') - a piecewise-linear solution where the policy rate and credit spreads are occasionally struck by their lower bound; iii) Scenario III ('Piecewise (PL)+Optimal Tax Policy (Opt Tax)') - a piecewise-linear solution where the lower bound may be occasionally binding, the refinance rate follows a standard rule, and the tax rate set according to (37).

Figure 3 - Adverse Preference Shock - Liquidity Trap

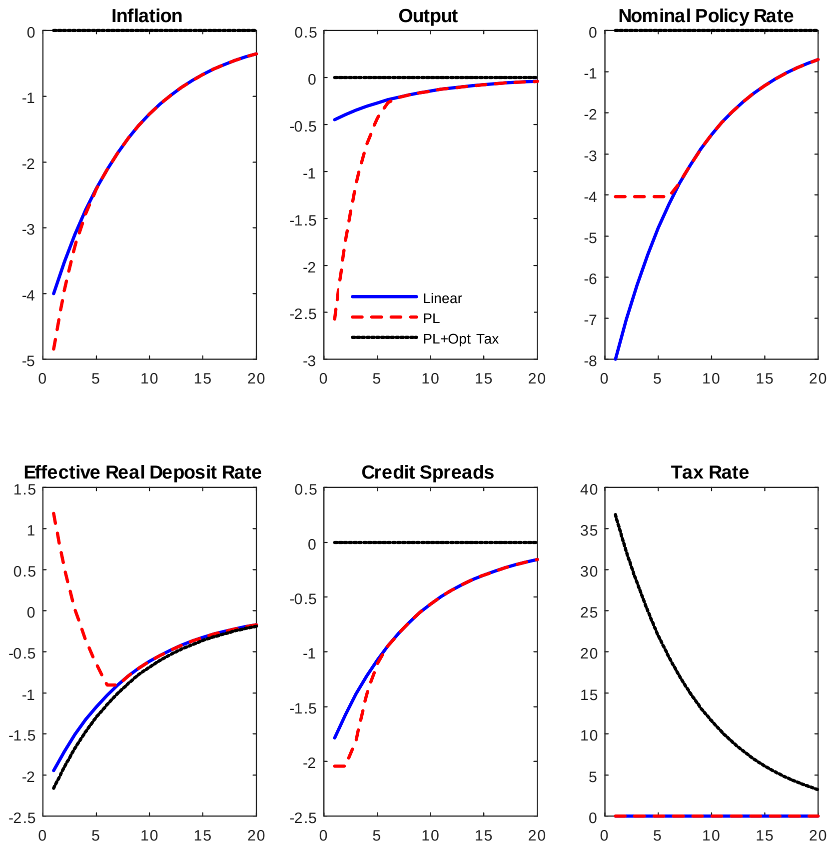


Figure 3 compares between Scenario I (linear model - 'Linear'), Scenario II (piecewise-linear model - 'PL'), and Scenario III (piecewise-linear model with optimal capital taxation - 'PL+Opt Tax'). ii) Interest rates, credit spreads, inflation and the capital income tax rate are measured in annualized percentage point deviations from steady state. Output is measured in annualized percentage deviations.

In Scenario I, an adverse demand shock delivers a direct dip in GDP, which leads to a plummet in prices through a standard demand channel affecting the NKPC, and also to a decline in the risk premium. The fall in the latter acts to lower credit spreads and therefore exacerbate price deflation via the credit cost channel. In response to the falling price level, and without the lower bound being imposed on the policy rate and credit spreads, the central bank lowers the (shadow) policy rate, giving rise to two conflicting effects on the economic activity. On the one hand, a more drastic cut in the nominal policy rate amplifies deflation due to the direct monetary policy cost channel mechanism. On the other, such an expansionary monetary policy cushions the drop in output and hence in inflation. Given our standard parameterization, the direct demand effect of GDP on inflation dominates such that output declines, prices decrease and the main interest rates are lowered. Therefore, following a preference shock, credit spreads are *procyclical* with respect to output, as opposed to the consequences arising from a financial shock.[27]

As is evident from Figure 3, there is a striking difference in the behaviour of macro and financial variables implied by the piecewise-linear Scenario II compared to the linear Scenario I. The fall in inflation and output is more pronounced when the policy rate and credit spreads are constrained by their lower bound and are thus unable to further adjust in order to mitigate the slump in output. Because the demand channel of monetary policy dominates its cost channel mechanism, the central bank would find it welfare-enhancing to cut the refinance rate more substantially despite the deflationary effects linked with the monetary policy cost channel. However, as the central bank cannot accommodate for the decline in output and inflation using the policy rate alone, the effective real policy rate increases, thereby generating a prolonged and aggravated economic recession.

Under the piecewise-linear Scenario III, we find that a liquidity trap can be prevented with the implementation of an optimal contractionary tax policy. Intuitively, taxation on households wealth renders the policy maker an extra degree of freedom in seeking its primary objectives via the risk premium component of the credit cost channel (see also (23), (24), (25)). Commitment to a higher tax rate over the course of the shock encourages an expansion in output and a rise in the loan rate, two mechanisms that contribute to restoring the target level of inflation. As a result, the rise in prices also averts the policy rate from tumbling into the ZLB, consequently inducing downward pressure on the effective real policy rate faced by households and reinforcing the improvement in the economic activity. In this way, an *automatic* increase in capital taxation that counteracts a negative demand shock (as implied by (37)) releases the policy rate and credit spreads from the lower bound territory, and insulates the economy from the repercussions of a liquidity trap both in the short and long-run. Hence, the optimal tax policy can be considered also as a banking sector tax instrument that is aimed at bringing down the effective real policy rate and increasing borrowing costs against the backdrop of deflationary pressures.

The differences between the three various scenarios are further reflected in Table 4. This table shows the simulated standard deviations in key variables, the probability of attaining the ZLB, and the relative welfare cost/gain of the piecewise-linear model without and with optimal tax policy compared to the unconstrained linear case. Interestingly, a significant rise in the tax on net deposit returns of around 37 percentage points following a $5 \times s.d(\alpha^{\vartheta})$ shock completely insures against a liquidity trap, minimizes the standard deviations in key macro variables, and achieves a meaningful welfare benefit of 0.1234 percent.

Table 4: Standard deviations and welfare gains from optimal policy at the lower bound

	Linear	Piecewise-Linear	Piecewise-Linear - Optimal Tax
	$s.d\,(\widehat{\pi}_t) = 1.672$ $s.d(\widehat{Y}_t) = 0.186$	$s.d\,(\widehat{\pi}_t) = 1.712$ $s.d(\widehat{Y}_t) = 0.387$	$s.d\,(\widehat{\pi}_t) = 0$ $s.d(\widehat{Y}_t) = 0$
Probability of hitting ZLB (percent)	–	9.00	0
Welfare Cost/Gain	–	−0.0083	0.1234

Notes: i) The standard deviations of key variables are represented in annualized rates.
ii) The welfare cost / gain is the percentage consumption equivalent, measured relative to the 'Linear' scenario.

[27] De Fiore and Tristani (2013) also find that following some shocks, the risk premium and the cost of borrowing are procyclical with respect to GDP.

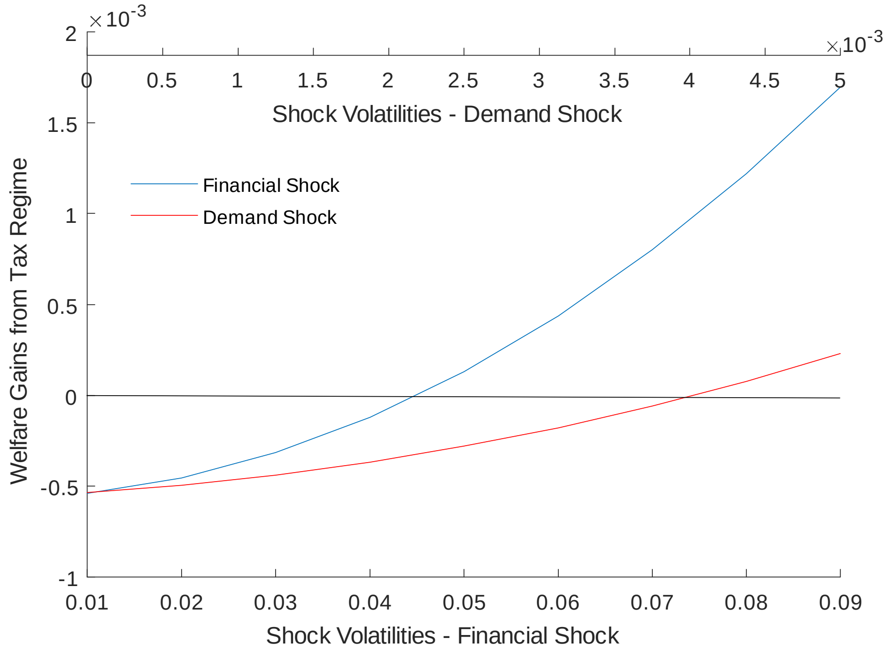
While this model does not explicitly account for the bank reserves market, the recent implementation of the negative interest on reserves policy by the ECB is equivalent to taxing the banking sector or lowering the effective savings rate faced by households. Therefore, increasing the tax rate on private asset returns in a liquidity trap, as we advocate for in this model, is not inconsistent with the recent attempts taken by the ECB to lower deposit rates and to increase credit spreads in light of the persistent low inflation experienced in the Eurozone. We show that such a policy can be achieved by varying the tax rate on net asset returns.

5.3 Welfare Gains from Dynamic Tax Regimes

The analysis so far has explored the welfare *improving* properties of optimal state-contingent private asset taxation in a stochastic environment. However, as mentioned earlier in the equilibrium section, a positive tax in steady state, $\tau^D > 0$, induces a lending rate above the level that would prevail with $\tau^D = 0$. Such tax is then passed on to IG firms and generates an inefficient long-run level of output and thus lower welfare. Therefore, the optimal policy in steady state would be to set a zero capital tax rate.

This section analyses more broadly the welfare costs and benefits from the optimal *dynamic* behaviour of asset taxation, as discussed in the previous sections, against a regime where $\tau_t^D = 0$ for all t. In a deterministic environment, the regime absent of savings taxation is always preferred due the mitigated distortions transmitted from the lending rate to output. In a stochastic environment, nonetheless, the relative welfare gains of state-contingent asset taxation, used as an additional instrument to smooth the business cycle and to mitigate welfare losses, depend on the type and volatility of the shock hitting the economy. As shock volatilities increase, so do the relative welfare gains from the additional degree of freedom arising from the implementation of unconventional dynamic taxation. In other words, the welfare gains from applying state-contingent taxation overcomes the steady state welfare losses induced by setting $\tau_t^D \neq 0$.

Figure 4 - Shock Volatilities, Dynamic Taxation and Welfare



Note: The top horizontal axis measures the shock volatilities for demand shock, whereas the bottom horizontal axis represents the financial shock volatilities.

Figure 4 indeed confirms that for financial shocks with volatilities above 0.045, and for preference shock with volatilities higher than 0.004, the welfare gains from varying private asset taxation are strictly positive. These standard deviations thresholds are *significantly lower* than empirical estimates found in Christiano, Motto and Rostagno (2014), and Denes, Eggertsson and Gilbukh (2013), among others. Therefore, we advocate for tax flexibility, associated with higher tax volatility, to shield the economy from the repercussions of volatile shocks that can result in liquidity trap episodes and significant welfare losses. Contributing to Eggertsson (2011), who mainly focuses on fiscal policy multipliers, we quantify the welfare gains of unconventional optimal taxation policies and find the optimal capital tax rate that can be used as an automatic stabilizer in the presence of both financial and demand shocks.

6 Concluding Remarks

By employing a standard three-equation New Keynesian model modified for a credit cost channel, endogenous financial risk, a banking sector and the lower bound, we shed new insights on the welfare implications, stabilization properties and transmission mechanisms of optimal private asset taxation. We show that varying capital taxation according to the state of the business and financial cycles has meaningful effects on the behaviour of macroeconomic and financial variables, and alters the transmission of standard monetary policy. The distinctive risk premium channel highlighted in our model presents an additional motivation for applying state-contingent capital taxation by affording the policy maker an extra degree of freedom to pursue its primary mandates. Our model supports the use of state-contingent capital taxation policies and specifically the implementation of subsidies to the banking sector following inflationary financial shocks, and taxing households liquid private assets against the backdrop of deflationary demand shocks.

This paper also advances an alternative answer to the highly topical question: how can a liquidity trap be avoided, and which optimal policies can help to achieve this goal? Blanchard, Dell'Ariccia and Mauro (2010) propose increasing the inflation target, as well as argue for a better integration between monetary and fiscal policy. Nakov (2008) suggests to change the monetary policy strategy such that in times of low inflation the central bank promises to raise inflation to its target level. Finally, Adam and Billi (2007) put forward the idea to increase the aggressiveness of monetary policy in order to reduce the probability of hitting the ZLB. Contributing to this literature, we advocate for cyclically adjusting the tax on asset returns based on the nature and scale of the shock hitting the economy, and show how financial frictions, liquidity traps and monetary policy modify the behaviour of such unconventional taxation policy.

Like Correia, Farhi, Nicolini, and Teles (2013), our state-contingent policy recommendations require taxes to be flexible and rather volatile. It is well known that fiscal policy tools are not as flexible as monetary policy instruments, and require a long legislative process until they can actually be executed. The recent Great Recession, however, has led to somewhat more flexibility in terms of implementing fiscal and financial policies, despite the main focus still placed on government spending since the American Recovery and Reinvestment Act (ARRA) of 2009, and countercyclical regulation associated with the gradual imposition of Basel III. Either way, we make a normative point that taxes (or financial policies) should be at least as flexible as monetary policy, so long as the policy maker can correctly identify the source and the size of the shock distorting the economy.

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7 Appendix - Welfare Function Derivation (Not for publication)

The derivation of the loss function as presented in the paper strictly follows Woodford (2003) and Ravenna and Walsh (2006, online appendix).

To derive a second-order approximation of the representative utility function, it is first necessary to clarify some additional notation. For any variable $\mathbb{X}_t$, let $\mathbb{X}$ be its steady state value, $\mathbb{X}_t^e$ be its efficient level, $\widetilde{\mathbb{X}}_t = \mathbb{X}_t - \mathbb{X}$ be the deviation of $\mathbb{X}_t$ around its steady state, and finally $\hat{\mathbb{X}}_t = \log(\mathbb{X}_t/\mathbb{X})$ be the log-deviation of $\mathbb{X}_t$ around its correspondent steady state. Using a second order Taylor approximation, the variables $\tilde{\mathbb{X}}_t$ and $\hat{\mathbb{X}}$ can be related using the following equation,

$$\frac{\mathbb{X}_t}{\mathbb{X}} = 1 + \log\left(\frac{\mathbb{X}_t}{\mathbb{X}}\right) + \frac{1}{2}\left[\log\left(\frac{\mathbb{X}_t}{\mathbb{X}}\right)\right]^2 = 1 + \hat{\mathbb{X}}_t + \frac{1}{2}\hat{\mathbb{X}}_t^2. \tag{38}$$

As we can write $\tilde{\mathbb{X}}_t = \mathbb{X}\left(\frac{\mathbb{X}_t}{\mathbb{X}} - 1\right)$, it follows that $\tilde{\mathbb{X}}_t \approx \mathbb{X}\left(\hat{\mathbb{X}}_t + \frac{1}{2}\hat{\mathbb{X}}_t^2\right)$.

Utility is assumed to be separable in consumption and leisure,

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\vartheta_t C_t^{1-\varsigma}}{1-\varsigma} - \frac{\vartheta_t H_t^{1+\gamma}}{1+\gamma} \right\}. \tag{39}$$

We start by approximating the utility from consumption. With the steady state value of the discount factor shock (ϑ) equal to 1, the second order expansion for $U(C_t, \vartheta_t)$ yields,

$$\begin{aligned} U(C_t, \vartheta_t) \quad &\approx \quad U(C,1) + U_C(C,1)\tilde{C}_t + \frac{1}{2}U_{CC}(C,1)\tilde{C}_t^2 + \\ &\quad\quad U_\vartheta(C,1)\widetilde{\vartheta}_t + \frac{1}{2}U_{\vartheta,\vartheta}\widetilde{\vartheta}_t^2 + U_{C,\vartheta}\widetilde{\vartheta}_t\tilde{C}_t, \end{aligned} \tag{40}$$

which according to our utility function (39) results in,

$$\begin{aligned} U(C_t, \vartheta_t) \quad &\approx \quad U(C,1) + U_C(C,1)\tilde{C}_t + \frac{1}{2}U_{CC}(C,1)\tilde{C}_t^2 + \\ &\quad\quad U_\vartheta(C,1)\widetilde{\vartheta}_t + \frac{1}{2}U_{\vartheta,\vartheta}\widetilde{\vartheta}_t^2 + U_{C,\vartheta}\widetilde{\vartheta}_t\tilde{C}_t, \end{aligned}$$

using $\widetilde{\vartheta}_t \approx \widehat{\vartheta}_t$ and $U_{C,\vartheta} = U_C(C,1)$, the above becomes,

$$\begin{aligned} U(C_t, \vartheta_t) \quad &\approx \quad U(C,1) + U_C(C,1)C\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right) - \frac{1}{2}\varsigma U_C(C,1)C\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right)^2 + \\ &\quad\quad + U_\vartheta(C,1)\widehat{\vartheta}_t + \frac{1}{2}U_{\vartheta,\vartheta}\widehat{\vartheta}_t^2 + U_C(C,1)C\widehat{\vartheta}_t\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right), \end{aligned}$$

ignoring the terms $\mathbb{X}^i$ for $i > 2$ yields,

$$\begin{aligned} U(C_t, \vartheta_t) \quad &\approx \quad U(C,1) + U_C(C,1)C\left[\left(1 + \widehat{\vartheta}_t\right)\hat{C}_t + \frac{1}{2}\left(1-\varsigma\right)\hat{C}_t^2\right] + \\ &\quad\quad + U_\vartheta(C,1)\widehat{\vartheta}_t + \frac{1}{2}U_{\vartheta,\vartheta}\widehat{\vartheta}_t^2, \end{aligned} \tag{41}$$

We next derive an expression for the disutility from labour. The Taylor expansion for $V(H_t, \vartheta_t)$ gives,

$$V(H_t, \vartheta_t) \approx V(H,1) + V_H(H,1)\tilde{H}_t + \frac{1}{2}V_{HH}(H,1)\tilde{H}_t^2 + \\ +V_\vartheta(C,1)\widetilde{\vartheta}_t + \frac{1}{2}V_{\vartheta,\vartheta}\widetilde{\vartheta}_t^2 + V_{C,\vartheta}\widetilde{\vartheta}_t\tilde{H}_t, \tag{42}$$

where aggregate employment is,

$$\tilde{H}_t = \int_0^1 \tilde{H}_{j,t} dj,$$

and employment at firm j,

$$\tilde{H}_{j,t} \approx H\left[\hat{H}_{j,t} + \frac{1}{2}\hat{H}_{j,t}^2\right].$$

For the purpose of calculating the *ex-ante* loss function, we ignore the effects of the idiosyncratic shock that takes place at the end of the period such that $\hat{\varepsilon}_{j,t} = 0$. We therefore only examine the ex-ante uniform properties of this shock to calculate welfare. Using this assumption, each firm faces the following technology function,

$$\hat{H}_{j,t} = \hat{Y}_{j,t}.$$

Thus, we can define employment as,

$$\hat{H}_t = H\left[\int_0^1 \hat{Y}_{j,t} dj + \frac{1}{2}\int_0^1 \hat{Y}_{j,t}^2 dj\right]. \tag{43}$$

Substituting (43) into (42) and using $H = Y$ results in,

$$V(H_t, \vartheta_t) \approx V(Y,1) + V_H(Y,1)Y\left[\int_0^1 \hat{Y}_{j,t} dj + \frac{1}{2}\int_0^1 \hat{Y}_{j,t}^2 dj\right] + \\ +\frac{1}{2}V_{HH}(Y,1)Y^2\left[\int_0^1 \hat{Y}_{j,t} dj\right]^2 + V_\vartheta(C,1)\left(\widehat{\vartheta}_t + \frac{1}{2}\widehat{\vartheta}_t^2\right) + \\ +\frac{1}{2}V_{\vartheta,\vartheta}\left(\widehat{\vartheta}_t + \frac{1}{2}\widehat{\vartheta}_t^2\right)^2 + \\ +V_{H,\vartheta}Y\left(\widehat{\vartheta}_t + \frac{1}{2}\widehat{\vartheta}_t^2\right)\left[\int_0^1 \hat{Y}_{j,t} dj + \frac{1}{2}\int_0^1 \hat{Y}_{j,t}^2 dj\right].$$

ignoring terms of $\mathbb{X}^i$ for $i > 2$ yields,

$$\begin{aligned} V(H_t, \vartheta_t) &\approx V(Y,1) + V_H(Y,1)Y\left[\int_0^1 \hat{Y}_{j,t}dj + \frac{1}{2}\int_0^1 \hat{Y}_{j,t}^2 dj\right] + \\ &+\frac{1}{2}V_{HH}(Y,1)Y^2\left(\int_0^1 \hat{Y}_{j,t}dj\right)^2 + \\ &+V_\vartheta(Y,1)\widehat{\vartheta}_t + \frac{1}{2}V_{\vartheta,\vartheta}\widehat{\vartheta}_t^2 + V_{H,\vartheta}Y\widehat{\vartheta}_t\int_0^1 \hat{Y}_{j,t}dj. \end{aligned} \tag{44}$$

Given the demand function of each firm j, aggregate output is approximated by,

$$\hat{Y}_t = \int_0^1 \hat{Y}_{j,t}dj + \frac{1}{2}\left(\frac{\lambda-1}{\lambda}\right) var_j \hat{Y}_{j,t},$$

hence,

$$\left(\int_0^1 \hat{Y}_{j,t}dj\right)^2 = \left[\hat{Y}_t - \frac{1}{2}\left(\frac{\lambda-1}{\lambda}\right) var_j \hat{Y}_{j,t}\right]^2 \approx \hat{Y}_t^2,$$

and,

$$\int_0^1 \hat{Y}_{j,t}^2 dj = \left(\int_0^1 \hat{Y}_{j,t}dj\right)^2 + var_j \hat{Y}_{j,t}.$$

Therefore,

$$\int_0^1 \hat{Y}_{j,t}^2 dj \approx \widehat{Y}_t^2 + var_j \hat{Y}_{j,t},$$

and,

$$\int_0^1 \hat{Y}_{j,t}dj \approx \hat{Y}_t.$$

Using $V_{H,\vartheta} = V_H(Y,1)$ and the above results, (44) becomes,

$$\begin{aligned} V(H_t, \vartheta_t) &\approx V(Y,1) + V_H(Y,1)Y\left\{\left(1+\widehat{\vartheta}_t\right)\hat{Y}_t + \frac{1}{2}\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} + \frac{1}{2}(1+\gamma)\hat{Y}_t^2\right\} + \\ &+V_\vartheta(Y,1)\widehat{\vartheta}_t + \frac{1}{2}V_{\vartheta,\vartheta}\widehat{\vartheta}_t^2 \end{aligned} \tag{45}$$

To determine total utility we subtract (45) from (41) to obtain,

$$
\begin{aligned}
U(C_t,\vartheta_t)-V(H_t,\vartheta_t) \quad = \quad & U(C,1)-V(Y,1)+U_C(C,1)C\left[\left(1+\widehat{\vartheta}_t\right)\hat{C}_t+\frac{1}{2}\left(1-\varsigma\right)\hat{C}_t^2\right]- \\
& -V_H(Y,1)Y\left\{\left(1+\widehat{\vartheta}_t\right)\hat{Y}_t+\frac{1}{2}\left(\frac{1}{\lambda}\right)var_j\hat{Y}_{j,t}+\frac{1}{2}\left(1+\gamma\right)\hat{Y}_t^2\right\} \\
& +\left(U_\vartheta(C,1)-V_\vartheta(Y,1)\right)\widehat{\vartheta}_t+\frac{1}{2}\left(U_{\vartheta,\vartheta}-V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2. \qquad (46)
\end{aligned}
$$

Note that the steady state labour market equilibrium condition is $\frac{V_H}{U_C}=W=\frac{\mu_\varepsilon}{(pm)R^L}$, where $\mu_\varepsilon=1$ is the average of the idiosyncratic shock, and $pm\equiv\frac{\lambda}{(\lambda-1)}$ is the price mark-up. We define Ξ such that,

$$1-\Xi\equiv\frac{1}{(pm)\,R^L}.$$

Then $V_H(H,1)Y$ can be written as $U_C\,(C,1)\,Y(1-\Xi)$. As in Ravenna and Walsh (2006), given that the Ξ is small, terms such as $(1-\Xi)\widehat{Y}^2$ simply boil down to $\widehat{Y}_t^2$.[28] With these assumption we can now rewrite equation (46) as,

$$
\begin{aligned}
U(C_t,\vartheta_t)-V(H_t,\vartheta_t) \quad = \quad & U(C,1)-V(Y,1)+U_C(C,1)C\left[\left(1+\widehat{\vartheta}_t\right)\hat{C}_t+\frac{1}{2}\left(1-\varsigma\right)\hat{C}_t^2\right]- \\
& -U_C\,(C,1)\,Y(1-\Xi)\left[\begin{array}{c}\left(1+\widehat{\vartheta}_t\right)\hat{Y}_t+ \\ +\frac{1}{2}\left(\frac{1}{\lambda}\right)var_j\hat{Y}_{j,t}+\frac{1}{2}\left(1+\gamma\right)\hat{Y}_t^2\end{array}\right]+ \\
& +\left(U_\vartheta(C,1)-V_\vartheta(Y,1)\right)\widehat{\vartheta}_t+\frac{1}{2}\left(U_{\vartheta,\vartheta}-V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2. \qquad (47)
\end{aligned}
$$

Using $C=Y$ and collecting terms,

$$
\begin{aligned}
U(C_t,\vartheta_t)-V(H_t,\vartheta_t) \quad = \quad & U(C,1)-V(Y,1) \\
& +U_C(C,1)Y\left\{\begin{array}{c}\left(1+\widehat{\vartheta}_t\right)\left[\hat{C}_t-(1-\Xi)\hat{Y}_t\right]+ \\ +\frac{1}{2}\left(1-\varsigma\right)\hat{C}_t^2-\frac{1}{2}\left(1+\gamma\right)\hat{Y}_t^2\end{array}\right\} \\
& -\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right)var_j\hat{Y}_{j,t}+ \\
& +\left(U_\vartheta(C,1)-V_\vartheta(Y,1)\right)\widehat{\vartheta}_t+\frac{1}{2}\left(U_{\vartheta,\vartheta}-V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2
\end{aligned}
$$

[28] Note that like Ravenna and Walsh (2006), the value of Ξ is increasing with the price markup and the loan rate, which in our model is larger due to the presence of the various financial frictions.

Substituting the log-linear representation of consumption, $\hat{C}_t = \hat{Y}_t$, gives,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ &+U_C(C,1)Y\left\{ \begin{array}{c} \left(1+\widehat{\vartheta}_t\right)\Xi\hat{Y}_t + \\ \frac{1}{2}(1-\varsigma)\hat{Y}_t^2 - \frac{1}{2}(1+\gamma)\hat{Y}_t^2 \end{array} \right\} \\ &-\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} + \\ &+ \left(U_\vartheta(C,1) - V_\vartheta(Y,1)\right)\widehat{\vartheta}_t + \frac{1}{2}\left(U_{\vartheta,\vartheta} - V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2, \end{aligned}$$

or,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ &+\frac{1}{2}U_C(C,1)Y\left\{[(1-\varsigma)-(1+\gamma)]\hat{Y}_t^2 + 2\left(1+\widehat{\vartheta}_t\right)\Xi\hat{Y}_t\right\} \\ &-\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} + \\ &+ \left(U_\vartheta(C,1) - V_\vartheta(Y,1)\right)\widehat{\vartheta}_t + \frac{1}{2}\left(U_{\vartheta,\vartheta} - V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2, \end{aligned}$$

collecting terms,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ &+\frac{1}{2}U_C(C,1)Y\left\{-(\varsigma+\gamma)\left[\hat{Y}_t^2 - 2\frac{\Xi}{(\varsigma+\gamma)}\left(1+\widehat{\vartheta}_t\right)\hat{Y}_t\right]\right\} \\ &-\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} + \\ &+ \left(U_\vartheta(C,1) - V_\vartheta(Y,1)\right)\widehat{\vartheta}_t + \frac{1}{2}\left(U_{\vartheta,\vartheta} - V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2, \end{aligned}$$

or,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ &+\frac{1}{2}U_C(C,1)Y\left\{-(\varsigma+\gamma)\left[\left(\hat{Y}_t - \frac{\Xi}{(\varsigma+\gamma)}\left(1+\widehat{\vartheta}_t\right)\right)^2 - \left(\frac{\Xi}{-(\varsigma+\gamma)}\right)^2\left(1+\widehat{\vartheta}_t\right)^2\right]\right\} \\ &-\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} + \left(U_\vartheta(C,1) - V_\vartheta(Y,1)\right)\widehat{\vartheta}_t + \frac{1}{2}\left(U_{\vartheta,\vartheta} - V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2. \end{aligned}$$

Collecting all terms that are independent of policy stabilization and denoting them as *tip* results in,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ &+\frac{1}{2}U_C(C,1)Y\left\{ \begin{array}{c} -(\varsigma+\gamma)\left[\left(\hat{Y}_t - \frac{\Xi}{(\varsigma+\gamma)}\right)^2 + \left(\frac{\Xi}{(\varsigma+\gamma)}\right)^2\right] \\ -\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} \end{array} \right\} + tip. \end{aligned}$$

Assuming that the term $\frac{\Xi}{(\varsigma+\gamma)}$ is a small constant, the above boils down to,

$$\begin{aligned} U(C_t,\vartheta_t)-V(H_t,\vartheta_t) &= U(C,1)-V(Y,1) \\ &+\frac{1}{2}U_C(C,1)Y\left\{-(\varsigma+\gamma)\hat{Y}_t^2-\left(\frac{1}{\lambda}\right)var_j\hat{Y}_{j,t}\right\}+tip. \end{aligned} \tag{48}$$

Given the demand function for each intermediate good, $Y_{j,t}=Y_t\left(\frac{P_{j,t}}{P_t}\right)^{-\lambda}$, we have,

$$\log Y_{j,t}=\log Y_t-\lambda\left(\log P_{j,t}-\log P_t\right),$$

so,

$$var_j\log Y_{j,t}=\lambda^2 var_j\log P_{j,t}.$$

Note the price adjustment mechanism involves a randomly chosen fraction $(1-\omega)$ of all firms acting optimally by adjusting prices in each period. Defining $\Delta_t\equiv var_j\log P_{j,t}$ then Woodford (2003, pp. 694-696) shows that,

$$\Delta_t\approx\omega\Delta_{t-1}+\left(\frac{\omega}{1-\omega}\right)\hat{\pi}_t^2.$$

Assuming Δ_{t-1} is the initial degree of price dispersion, then,

$$\sum_{t=0}^{\infty}\beta_t^t\Delta_t=\left[\frac{\omega}{(1-\omega)(1-\omega\beta)}\right]\sum_{t=0}^{\infty}\beta^t\hat{\pi}_t^2+tip. \tag{49}$$

Combining (49) with (48), the present discounted value of the representative household welfare is,

$$\mathbb{W}_t\equiv\sum_{t=0}^{\infty}\beta^tU_t\approx U-\sum_{t=0}^{\infty}\beta^t\mathcal{L}_t,$$

where the associated losses from welfare are given by,

$$\sum_{t=0}^{\infty}\beta^t\mathcal{L}_t=\frac{1}{2}U_C(C,1)Y\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{\lambda}{\kappa_p}\right)\hat{\pi}_t^2+(\varsigma+\gamma)\hat{Y}_t^2\right], \tag{50}$$

with $\kappa_p=\frac{(1-\omega)(1-\omega\beta)}{\omega}$. With $\varsigma=1$ and $Y=C$, (50) boils down to,

$$\sum_{t=0}^{\infty}\beta^t\mathcal{L}_t=\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{\lambda}{\kappa_p}\right)\hat{\pi}_t^2+(1+\gamma)\hat{Y}_t^2\right]. \tag{51}$$

Welfare Measure

In considering optimal taxation policies, we measure the welfare benefit of a particular optimal policy j as a fraction of the consumption path under the benchmark case (Policy I) that must be given up in order to obtain the benefits of welfare associated with the various optimal policies; $\mathbb{E}_t\sum_{t=0}^{\infty}\beta^tU_t\left(C_t^j,H_t^j\right)=\mathbb{E}_t\sum_{t=0}^{\infty}\beta^tU_t\left((1-\Lambda)C_t^I,H_t^I\right)$, where superscript j refers to Policies II and III and superscript I refers to Policy I. Given the utility function adopted and with $\varsigma=1$, the expression for Λ in percentage terms is,

$$\Lambda = \left\{1 - \exp\left[(1-\beta)\left(\mathbb{W}_t^j - \mathbb{W}_t^I\right)\right]\right\} \times 100,$$

where $\mathbb{W}_t^j = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^j, H_t^j\right)$ represents the unconditional expectation of lifetime utility under policy $j = II, III$, and $\mathbb{W}_t^I = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^I, H_t^I\right)$ is the welfare associated with the benchmark Policy I. Converting the loss function to the welfare measure gives,

$$\mathbb{W}_t \equiv U - \frac{1}{2}\frac{U_C C}{(1-\beta)}\left[\left(\frac{\lambda}{\kappa_p}\right) var(\hat{\pi}_t) + (1+\gamma)\, var\left(\hat{Y}_t\right)\right].$$