Changing the Basics

Toward More Use of Quantile Regressions in Hospitality and Tourism Research

Abstract

The aim of this paper is to encourage more use of Quantile Regressions (QRs) in Hospitality and Tourism Research. More importantly, we focus on the Bayesian estimation of QRs and discuss its advantages over the traditional estimation techniques. We also discuss a Bayesian QR model that accounts for heteroscedasticity. We illustrate the performance of the two models using an interesting application on corporate social responsibility and firm value.

Introduction

Testing hypotheses using conventional regression methods (CRM) has always been the norm in hospitality and tourism research. These methods usually "fit models using the conditional mean approach; that is, scholars report how the mean of the dependent variable changes for each unit change in the value of predictor variables" (Li, 2015, p. 72). CRM are easy to fit, straightforward to interpret, and have many other desirable advantages (Koenker, 2005), but since they focus on the estimation of conditional mean functions, we believe it is "fruitful to move beyond these models" to obtain more generalizable findings (Yu and Moyeed, 2001).

Quantile regression models have recently emerged as an attractive alternative to CRM. Unlike the mean or traditional regression models, QRs belong "to a robust model family, which can give an overall assessment of the covariate effects at different quantiles of the dependent variable" (Yuan and Yin, 2010, p.105). They provide a nice extension to the results obtained by CRM, particularly in contexts where the interest is not only in central locations (e.g. assessing the effectiveness of corporate social responsibility beyond its impact on average firm value). There has been recently some applications of QRs in the hospitality literature (Masiero et al. 2015), and other related fields (Ramdani and Witteloostuijn, 2010), but its use remains limited. Recent papers have also emerged calling for more use of QRs in social science applications (Li, 2015).

The goal of this paper is to take the issue further and describe for the first time the Bayesian quantile regression (BQR), and illustrate its advantages using a hospitality application. The paper also describes a BQR model that accounts for heteroscedasticity. Traditional QRs ignore the heteroscedasticity that is quite common in cross-sectional and panel data. Ignoring heteroscedasticity leads to a misspecification of the distribution whose quantiles are estimated. This is because the quantile regression problem has a likelihood interpretation that corresponds to a Laplace distribution (Tsionas, 2003). To our knowledge, heteroscedasticity in QRs has not been clearly addressed in the literature, as it is difficult to modify the objective function. In this paper, we discuss a formulation of the quantile regression model with an explicit form for heteroscedasticity that is allowed to be different across quantiles. Therefore, not only regression coefficients but also the coefficients of the variance function are allowed to vary by quantile. This provides valuable information on how not only the mean but also the variance effects change by quantile.

The use of the Bayesian approach adds important advantages to the estimation of QRs. Unlike the frequentist approach, the Bayesian approach does not rely on asymptotic theory. In addition, the "frequentist approach cannot tell us the distribution of the parameters whereas the Bayesian approach provides us with his information" (Li, 2015, p. 76). We provide below some details about the difference between the Bayesian and frequentist approach.

Quantile Regressions

As mentioned, a single mean regression function is not always enough, as it does not describe the data accurately enough for practical purposes. Specifically, we would like to see the *distribution* of an outcome given the covariates. For example the slope coefficients at the median of the distribution may be different from the slope coefficients at the lower end (10% and 20% quantiles) and the upper end (80% and 90% quantiles) of the distribution. This is, of course, a particular instance of heterogeneity in the data: There is no single slope coefficient or effect of a regressor on the dependent variable (β) but a whole array of slopes $\beta(\tau)$ where τ is the particular quantile.

Suppose we have a regression model of the form:

$$y_i = x_i'\beta + u_i,\ u_i \sim (0, \sigma^2). \tag{1}$$

where x_i is a $k \times 1$ vector of regressors, β is a $k \times 1$ vector of parameters and u_i is the usual statistical error. In this problem, the effect of the vector of regressors on the dependent variable is β so these effects are assumed constant. In the quantile regression problem we seek estimates in the following model:

$$y_i = x_i'\beta^{(\tau)} + u_i^{(\tau)},\ u_i \sim (0, \sigma^{2,(\tau)}),$$

where the quantile is denoted by τ. Therefore, we seek estimates that are different across quantiles. In different terms, we parameterize the distribution of the response given the regressors in the form:

$$Q(y_i, \tau \mid x_i) = x_i'\beta^{(\tau)},$$

where $Q(y_i, \tau \mid x_i)$ denotes the τ th quantile of the distribution of y_i given x_i.

The quantile regression estimator $\hat{\beta}\ \tau$ solves the problem:

$$\min : \mathcal{S}\ \beta\ = \sum_{i=1}^{n} \rho_\tau\ \ y_i - x_i'\beta\ \ , \tag{2}$$

for a given $\tau \in (0,1)$ where:

$$\rho_\tau(u) = \begin{cases} \tau u, & u \geq 0, \\ -(1-\tau)u, & u < 0. \end{cases} \tag{3}$$

When τ =0.5 the problem is equivalent to minimizing $\sum_{i=1}^{n} \left| y_i - x_i'\beta \right|$. This problem is known as Least Absolute Deviations (LAD) regression and the solution corresponds to finding the slopes at the median of the distribution of y given the x's. This approach is known to deliver estimates that are more robust compared to CRM.

The Bayesian Estimation of Quantile Regressions

The Bayesian approach introduces many advantages to the estimation of regression models. There is only limited information that we can generate with the traditional (i.e. frequentist or sampling theory approach) estimation of these models. Unlike the frequentist estimation, the Bayesian estimation "provides one with the entire distribution of the parameter of interest" (Yu and Moyeed, 2001, p.438). For example, with traditional QR we can only say whether "x" has a significant impact on "y" at a certain quantile. Although confidence intervals can be provided, these only have an asymptotic justification. As mentioned, the Bayesian approach provides the whole distribution of the parameters of interest, and allows one to make direct probability statements about the parameters. Therefore, one can avoid the above limitations. The Bayesian approach has also the additional advantage of allowing for "parameter uncertainty to be taken into account when making predictions", and has better small sample properties (Yu and Moyeed, 2001, p.438)[1].

For Bayesian analysis of the QR model, one requires first a prior over the coefficients, which we denote by $p(\theta)$ where $\theta = \left[\beta', \gamma'\right]'$ is the parameter vector. The parameter space is $\Theta \subseteq \mathbb{R}^D$ where D is the dimensionality of the parameter. Using Bayes' theorem, the posterior distribution is:

$$p(\theta; \tau \mid Y) \propto L(\theta; \tau, Y)p(\theta). \tag{4}$$

where Y denotes the available data, $p(\theta)$ is the prior distribution and $L(\theta; \tau, Y)$ is the likelihood of the model for a particular quantile. There are different choices for the prior distribution. One of the most common choices is to use a flat prior, indicating that we do not have detailed prior information about the parameters:[2]

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[1] We refer the reader to Assaf et al. (2017) for a more detailed discussion of the Bayesian approach. The "Journal of Management" has also recently dedicated a special issue on the topic (Zyphur and Oswald, 2015).
[2] Our techniques accommodate general priors. For example it is possible to use a multivariate normal prior, $N_D(\theta_o,\Omega_o)$, for a given prior mean θ_o and a prior covariance matrix Ω_o.

$$p(\theta) \propto \text{const.} \tag{5}$$

One of the challenges with the Bayesian estimation of QRs is that it can be difficult to find the likelihood function of the model, as QRs do not assume any distribution on the error term (Li, 2015). This is of course not the case with simple models such as linear regression, where it is easy to find the likelihood. Yu and Moyeed's (2001) and Tsionas (2003) have addressed the challenge in the QR context by showing "that the set of parameters that minimizes the absolute value of residuals in (2) also maximizes the likelihood function formed by combining the independently distributed asymmetric Laplace density function" (Li, 2015, p.75).

To illustrate, if we introduce a scale factor σ, it is known that we have the Laplace distribution (Tsionas, 2003), yielding the likelihood:

$$L\ \beta,\sigma;\tau,Y\ = \left(\frac{\tau(1-\tau)}{\sigma}\right)^{n} \exp\left\{-\sum_{i=1}^{n} \rho_{\tau}\left(\frac{y_i - x_i'\beta}{\sigma}\right)\right\}, \tag{6}$$

We can write the likelihood in different form as follows:

$$L\ \beta,\sigma;\tau,Y\ = \sigma^{-n} \exp\ -\sigma^{-1}\sum_{i=1}^{n} \left|u_i\right|\ \tau I_{[0,\infty)}(u_i) + (1-\tau) I_{(-\infty,0]}(u_i)\ \ . \tag{7}$$

Since we have the prior and likelihood we can proceed with Bayesian analysis. To derive posterior moments, we use Markov Chain Monte Carlo (MCMC) methods. Specifically, we use the Gibbs sampler to implement Bayesian inference in the model whose likelihood is (7) under the prior in (5). If the parameter vector is $\theta = \left[\theta_1,\ldots,\theta_d\right]$ we draw random samples from the posterior conditional distributions $p(\theta_d \mid \theta_{(-d)}, Y), d = 1,\ldots,D$ where $\theta_{(-d)}$ denotes all elements of the parameter vector with the exception of element θ_d. We use an adaptive Metropolis-Hastings algorithm (Tierney, 1994) to draw from this posterior conditional distribution. Specifically, given the previous draw $\theta_d^{(s-1)}$ we draw a candidate θ_d^{*} from a uniform distribution in the interval $[a_d, b_d]$. The draw is accepted with probability:

$$\min\left\{1,\ \frac{p(\theta_d^{*} \mid \theta_{(-d)}^{(s-1)}, Y)}{p(\theta_d^{(s)} \mid \theta_{(-d)}^{(s-1)}, Y)}\right\},$$

otherwise we repeat the previous draw, viz. $\theta_d^{(s)} = \theta_d^{(s-1)}$. The interval $[a_d, b_d]$ is adjusted during the burn-in phase so that, approximately, 25% of the candidates are eventually accepted during the main phase of the MCMC algorithm. This is repeated for each $d = 1,\ldots,D$, i.e. for each element of the parameter vector. In all our MCMC experiments we use 60,000 draws the first 10,000 of which are discarded to mitigate possible start-up effects and tune the interval $[a_d, b_d]$ for each $d = 1,\ldots,D$.

These methods yield a long sample $\{\theta_{\tau}^{(s)}, s=1,\ldots,S\}$ whose stationary distribution is the one whose kernel density appears in (4).

Using MCMC we can obtain, for each quantile, the posterior means

$$\overline{\beta}_{\tau} = S^{-1}\sum_{s=1}^{S}\beta_{\tau}^{(s)},$$

and the posterior covariance matrix:

$$V_{\beta,\tau} = S^{-1}\sum_{s=1}^{S}(\beta_{\tau}^{(s)} - \overline{\beta}_{\tau})(\beta_{\tau}^{(s)} - \overline{\beta}_{\tau})',$$

whose diagonal elements represent posterior variances of the parameters. Using these expressions we can provide plots of the posterior mean estimates of β along with plus/minus two standard error bands.

Bayesian Heteroskedastic Quantile Regressions

In this section, we present a QR model that accounts for heteroscedasticity. To our knowledge, we are not aware of any previous discussion of a Bayesian Heteroskedastic QR (BHQR) model in the literature. The usefulness of the heteroskedastic BHQR model is that it addresses better the problems that we often encounter in cross-sectional data, where heteroscedasticity is common. We all know that ignoring heteroscedasticity can lead to incorrect statistical tests of significance when we wrongly assume that the modelling errors are uncorrelated and uniform. In QRs, it can lead to biased and inconsistent estimates (if ignored), because the model is nonlinear and the likelihood functions of HQRs and QRs are quite different.

The BHQR model we want to estimate has the following form:

$$y_i = x_i'\beta^{(\tau)} + u_i^{(\tau)},\ \ u_i \sim (0, \sigma_i^2),$$

$$\log \sigma_i = x_i'\gamma^{(\tau)}.$$

The proper interpretation is that in (7) the scale parameter is a linear function of the regressors:

$$\log \sigma_i = x_i'\gamma, \tag{8}$$

where γ is a $k \times 1$ vector of parameters. The likelihood function is given by:

$$L\ \ \beta,\gamma;\tau,Y\ \ \propto \exp\ -nx_i'\gamma\ \ \exp\ \ -\exp\ \ -x_i'\gamma\ \sum_{i=1}^{n}\left|u_i\right|\ \ \tau I_{[0,\infty)}(u_i) + (1-\tau)I_{(-\infty,0]}(u_i)\ \ \ .$$

(9)

The log-likelihood function is:

$$\log L\ \ \beta,\gamma;\tau,Y\ \ = -n\sum_{i=1}^{n} x_i'\gamma - \exp\ \ -x_i'\gamma\ \ \sum_{i=1}^{n}\left|u_i\right|\ \ \tau I_{[0,\infty)}(u_i) + (1-\tau) I_{(-\infty,0]}(u_i)\ \ , \tag{10}$$

which can be maximized using standard numerical techniques to obtain $\hat{\beta}\ \ \tau$ and $\hat{\gamma}\ \ \tau$. The first order conditions of the problem are (for given τ):

$$\frac{\partial \log L\ \ \hat{\beta},\hat{\gamma}}{\partial \beta} = \sum_{i=1}^{n} \frac{\delta \rho_\tau(\hat{u}_i)}{\delta u_i} x_i' \exp\ \ -\tfrac{1}{2} x_i'\hat{\gamma}\ \ = 0, \tag{11}$$

$$\frac{\partial \log L\ \ \hat{\beta},\hat{\gamma}}{\partial \gamma} = -\tfrac{n}{2} x_i + \tfrac{1}{2}\sum_{i=1}^{n} \frac{\delta \rho_\tau(\hat{u}_i)}{\delta u_i}\ \ y_i - x_i'\hat{\beta}\ \ x_i \exp\ \ -\tfrac{1}{2} x_i'\hat{\gamma}\ \ = 0, \tag{12}$$

where $\hat{u}_i = y_i - x_i'\hat{\beta}$ and $\dfrac{\delta \rho_\tau(u)}{\delta u_i} = \lim\limits_{h\to 0} \dfrac{\rho_\tau(u+h) - \rho_\tau(u-h)}{2h}$.

Bayesian analysis proceeds using the MCMC procedure that we described in the previous section. The parameter vector now consists of both β and γ.

Evaluation of constancy coefficients in quantile regression

Although graphical techniques can be used to plot the coefficients of quantile regression along with standard error bands, less attention has been paid to formal tests of constancy of coefficients in quantile regression. Suppose we want to test the constancy of coefficients across quantiles for β (the analysis for γ is similar). We consider the function of interest $\delta^{(s)} = \beta_\tau^{(s)} - \beta_{0.5}^{(s)}$ which compares quantile regression coefficients relative to the median, for each MCMC draw. Our "test" is to present the density of $\{\delta^{(s)}, s = 1,\ldots,S\}$ (for each coefficient or all coefficients jointly) and see whether there is concentration of posterior probability around zero, in which case constancy of coefficients in quantile regression can be accepted.

Application

To illustrate both the BQR and BHQR models, we aim in this application to analyze the impact corporate social responsibility on firm value. While this hypothesis has been tested extensively in the literature (Kang et al. 2010; Lee et al. 2013), it would be interesting to see how the impact of CSR on firm value varies across different quantiles using both the BQR and BHQR models. We use data on

US restaurants covering an unbalanced sample of 22 publicly traded companies[3] from 2001 to 2012 (232 observations). In our estimation we also control for advertising spending, firm size and financial leverage (Assaf et al. 2017). We measured advertising spending as the reported firm advertising expenditure in the COMPUSTAT database. For CSR data, we used the KLD Research and Analytics' KLD STAT, one of the most frequently used databases in the strategy and management literature (Kang et al. 2010)[4].

We measured firm value using the Market Value Added (MVA), calculated as: MVA= market value-capital, where market value reflects the equity market valuation of the firm and capital reflects the debt and equity invested in the firm (Hillman and Keim, 2001).

Results

Table 1 provides some descriptive statistics for the variables used in our application. We estimate both the BQR and BHQR models using the Gauss software. For comparison, we also estimate a simple OLS model. The codes are available in the online Appendix. As the Bayesian estimation of BQRs and BHQRs is usually challenging, we believe that providing such codes may encourage more use of the Bayesian approach for the estimation of QRs in hospitality and tourism research. We provide in Tables 2 and 3, the estimates of the BQR and BHQR models, respectively.

Looking at the CSR posterior estimates in Table 2, we observe some very interesting findings. For example, the impact of CSR on firm value only becomes significant at the median and higher level quantiles, while it is insignificant at the lower quantiles. In other words, CSR investments seem to be more effective for firms that already enjoy a higher level of firm value. This contradicts the results of the OLS model, which indicates a significant impact of CSR on firm value. What is even more alarming is that when we account for heteroscedasticity, the impact of CSR becomes insignificant for all quantiles.

It is evident from the variance parameters (see equation 8) that heteroscedasticity is present in the data. For instance, we can see that the variance parameter of some variables (including CSR) is significant at most or all quantiles. We can also observe from the constancy test in Figure 1 that the results for CSR do not seem to differ much across quantiles (i.e. centered around zero), while the variance parameter does. Hence, the BHQR model is more reliable in this context. The fact that CSR does not have a significant impact on firm value, but significant impact on the dispersion (i.e. risk) of firm value (as indicated by the variance parameter) might not be surprising after all. For example, Rost and Ehrmann (2017) recently conducted a meta-analysis on 162 CSR studies and found weak evidence that CSR really leads to greater financial success. According to them, most studies do not always discuss the pros and cons of CSR and only focus on the positive aspects. In

[3] We focus here on publicly traded firms as we have firm value as one of our performance measures.

[4] Note that the same dataset was previously used by Assaf at el. (2017)

fact, CSR does also bring cost to the firm (Science News, 2015). The authors further highlighted that most CSR studies are methodologically weak and not driven by theoretical foundations.

Conclusions

As hospitality and tourism researchers, it is time to start thinking outside the box. We need to change the basics and move toward more comprehensive methods like the BQR and BHQR. In general, the use of quantile regression leads to more robust and comprehensive hypothesis testing, even when the results do not significantly vary across quantiles.

Importantly, this paper discussed and highlighted the advantages of using the Bayesian approach for the estimation of QRs. We also discussed a Bayesian quantile regression that accounts for heteroscedasticity. Such model can have strong practical implications, as many applications in hospitality and tourism suffer from this heteroscedasticity problem. We illustrate the application of the models using an interesting application on CSR and firm value. The findings provided further support that the heavy reliance on the linear regression model might be masking some important information we do not really observe about some interesting and influential hypotheses in our field.

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TABLE 1. DESCRIPTIVE STATISTICS

Variable	Mean	S.D.	Median
MVA	2980.09	8864.25	329.15
CSR	1.36	6.21	0.00
Advertising spending	106.65	186.79	28.85
Size	3.00	0.56	2.90
Financial Leverage	0.64	0.61	0.51

TABLE 2. HOMOSKEDASTIC QUANTILE REGRESSION

	OLS	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
CSR	**445.695** (69.126)	18.093 (17.723)	10.733 (22.347)	20.900 (28.272)	52.136 (36.259)	**101.741** (41.921)	**188.846** (54.411)	**257.634** (50.375)	**364.175** (60.114)	**388.619** (58.876)
Adv	**34.123** (3.934)	**4.755** (1.813)	**9.061** (1.971)	**14.113** (2.111)	**18.124** (1.817)	**20.050** (1.914)	**23.1085** (2.801)	**26.984** (2.486)	**46.487** (3.650)	**58.158** (3.141)
size	-1770.699 (1387.770)	**-916.882** (273.513)	**-1061.493** (279.613)	**-1095.362** (301.870)	**-1091.681** (317.675)	**-992.738** (467.153)	-652.445 (638.342)	-185.745 (678.466)	590.888 (611.117)	897.226 (729.033)
leverage	-737.090 (626.117)	-104.419 (159.910)	-110.696 (169.222)	-130.557 (172.510)	-173.272 (191.962)	-214.868 (216.728)	-306.307 (238.196)	-417.575 (231.313)	**-688.365** (229.995)	**-737.124** (248.737)
constant	4523.713 (3864.202)	**2075.781** (656.892)	**2469.400** (695.445)	**2565.079** (768.182)	**2617.242** (834.441)	**2530.891** (1244.565)	1996.843 (1695.616)	1225.307 (1813.192)	-194.569 (1682.575)	-373.414 (1995.842)

Note: Numbers in parentheses represent the posterior standard deviations

TABLE 3. HETEROSKEDASTIC QUANTILE REGRESSION

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
CSR	3.967 (4.506)	5.227 (4.618)	3.105 (6.893)	-0.703 (9.308)	-0.385 (9.854)	-2.225 (11.220)	-5.4891 (11.435)	-7.265 (14.625)	-13.111 (18.652)
Adv	**3.619** (1.099)	**5.056** (1.575)	**7.324** (1.991)	**7.952** (2.094)	**6.736** (2.576)	**8.247** (2.487)	**9.835** (2.933)	**14.028** (1.894)	**14.632** (2.071)
size	**672.914** (91.326)	**-611.046** (84.219)	**-504.231** (140.158)	**-371.040** (163.223)	-15.684 (172.179)	193.295 (158.512)	**387.155** (150.750)	**691.843** (161.001)	**1464.892** (196.958)
leverage	-61.919 (63.146)	-44.349 (46.668)	-76.797 (58.716)	-116.928 (77.168)	-178.005 (103.350)	-147.039 (110.361)	-97.312 (98.434)	-86.273 (104.972)	-54.527 (89.908)
constant	**1504.061** (207.197)	**1387.646** (191.493)	**1177.925** (306.990)	**936.364** (351.947)	214.175 (379.292)	-211.845 (353.617)	-609.150 (329.031)	**-1220.530** (360.380)	**-2807.119** (424.629)
Variance Parameters									
CSR	0.015 (0.011)	**0.022** (0.011)	**0.025** (0.011)	**0.031** (0.011)	**0.036** (0.011)	**0.041** (0.012)	**0.047** (0.012)	**0.066** (0.014)	**0.086** (0.013)
Adv	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
size	**2.399** (0.211)	**2.266** (0.263)	**2.291** (0.250)	**2.195** (0.252)	**2.161** (0.238)	**2.181** (0.237)	**2.238** (0.240)	**2.118** (0.261)	**2.108** (0.245)
leverage	0.096 (0.107)	0.061 (0.098)	0.070 (0.099)	0.092 (0.106)	0.153 (0.119)	0.132 (0.120)	0.094 (0.112)	0.047 (0.107)	-0.019 (0.114)
constant	**-2.293** (0.583)	-1.336 (0.732)	-1.090 (0.696)	-0.649 (0.704)	-0.496 (0.669)	-0.498 (0.656)	-0.653 (0.667)	-0.394 (0.734)	-0.703 (0.684)

Note: Numbers in parentheses represent the posterior standard deviations

FIGURE 1. CONSTANCY OF COEFFICIENTS IN THE BHQR