The Price of Correlation Risk: New Evidence from Commodity Options

Yuan Xu*

August 31, 2017

Abstract

This paper investigates the price that the market pays in order to alleviate or dissociate from the impacts of variations in intra-commodity correlations; in other words, the correlation risk in commodity markets. Using a novel source from Deutsche Bank Commodity Index (DBCI), a liquid, representative, straightforward and low-tracking-error index, and option data of all the index's components, this paper shows that i) Contrary to recent empirical findings of significantly priced correlation risk premium embedded in equity index options, our findings are not supportive of the hypothesis that intra-commodity correlation risk (as implied from options using model-free methodology) is a pricing factor; ii) The above finding is robust against changes in assumptions; iii) The normality of correlations between extreme returns among energy, agricultural and metal sectors can not be rejected, which is consistent with the finding that systemic correlation risk is not priced. This paper's finding is favorable towards the view that commodities are still to a large extent priced as a distinctive asset class rather than an asset that is fully financial.

JEL classification: G12, G13, C61

Keywords: Commodities, Variance risk premia, Correlation risk, Index Options

*Department of Accounting and Finance, Lancaster University Management School, LA1 4YX, United Kingdom, phone: +44 7745 940126, email: y.xu8@lancs.ac.uk.

1 Introduction

The variance-covariance matrix of a basket of assets evolves over time, which represents a major source of risk. For example, Bollerslev, Engle & Wooldridge (1988) sets up a multivariate GARCH process for returns to bills, bonds, and stocks, finding that the conditional covariances, which present large variations over time, are significant pricing factors for the time-varying risk premium. In particular, such risks are most pronounced in scenarios where volatilities and correlations of equity markets increase simultaneously, jointly enlarging the systematic risk exposure of an equity portfolio, such as the case of financial crises.[1]Hence, it is natural to hypothesize that the exposure to such correlation risk exposure should be priced.

On the other hand, during the last decade, the demand for commodity derivatives by either traditional commercial traders of commodities or by financial institutions holding diversified portfolios has grown dramatically. It is therefore important to investigate how correlation risk embedded in commodity portfolios is priced, and how it may differ from the case of equity portfolios. The understanding of this research question shapes the role of commodities in the context of a well-diversified portfolio: If the cross-market-linkages among commodities are similarly priced as equities, then such findings would provide additional support to the assertion that commodities are priced more like financial assets than a seperate, segmented asset class. On the other hand, if empirical evidence is not conclusive about whether intra-commodity correlations attract a premium, such a conclusion has important implications on how commodities and related derivatives fit into the asset allocation decision from an investor's viewpoint as a diversifier.

Instead of explicitly modelling the process of commodity returns and estimating the price of correlation risk, this paper approaches the question through the lens of index options, as pioneered by Driessen, Maenhout & Vilkov (2009). As a stylized fact, stock index options tend to be more expensive than their corresponding index components would imply. The premium of index options relative to individual options is well-documented in equity markets, and the statistical and economic significance of such pricing gaps attracts much research interest. This paper empirically revisits

[1]Specifically, Longin & Solnik (2001) model the distribution of stock return tails using extreme value theory. They derive the empirical distribution of extreme correlation, finding that correlation increases only in bear-tail but not in bull-tail states. Their conclusion is important in that extreme correlation impacts the distribution of portfolio returns asymmetrically with an unfavorable fatter left-tail but no boost to upside.

the well-known index option premium puzzle using a novel option data of Deutsch Bank Commodity Index (DBCI); a liquid, representative, straight-forward and low-tracking-error commodity index, and the corresponding option data of all the index's components. There are two important characteristics of the commodity sector that make this study a meaningful extension to the literature. First and foremost, the crash fearing nature in stock market, or crashophobia in the sense of Rubinstein (1994), features both a surge in volatility and correlations among stocks in bearish states, but not in bullish states. In contrast, the idea of crashophobia can not be straightfowardly transplanted without modification to the commodity sector: both a surge or a crash in oil price, for instance, may cause crashophobia, hence potentially priced correlation risk. Second, it is important to note that the commodity sector in general and the energy markets in particular are characterized by more prevalent and dramatic jumps, as well as higher capital and margin constraints of market intermediaries in response to the jumps. Consequently, according to the demand pressure theory as Garleanu, Pedersen & Poteshman (2009) suggest, the surge in cost of delta-hedging may result in a pricing premium. One may therefore reasonably hypothesize that part of unhedgeable risk may increase and hence a larger portion of risk to be priced according to demand for insurance protection.

This paper contributes to the literature in two specific ways. Firstly, our finding suggests that correlation exposure is not necessarily regarded as a risk factor in every market. In fact, we show that despite the large magnitude of variance risk premium embedded in 13 out of 14 commodity options that we have studied, it is statistically very hard to extract any convincing risk premium that is conclusively attributable to correlation risk. In order to understand why correlation risk is priced in equity markets but likely not priced at all in commodity markets, we apply extreme value theory and model the bivariate distribution of commodities from different sectors, namely the energy, agricultural and metal sectors. We find that commodities from different sectors demonstrate limited co-movement in either extremely positive or extremely negative return scenarios, demonstrating excellent diversification during unfavorable states. Hence, correlation risk is almost absent in commodity sectors.

The finding of this paper also provides an interesting aspect to evaluate the recent emerging argument of commodity financialisation. In theory, the commodity financialisation arguments would suggest favorable conditions for the existence of correlation risk premium. Tang & Xiong (2012) show that since the early 2000s, prices of non-energy commodity futures in the United States have become increasingly correlated with oil:

a trend that is signicantly more pronounced for commodities in S&P Goldman Sachs Commodity Index and Dow Jones-UBS Commodity Index. Cheng & Xiong (2013) argued that investment inflows distort commodity prices by affecting risk sharing and information discovery in commodity markets. Silvennoinen & Thorp (2013) find that increases in VIX and nancial traders short open interest not only raise future-market returns volatility for many commodities but also increases commodity returns correlation with equity returns, hence closer return integration. Büyükşahin & Robe (2014) utilize data on trader positions and show that the return correlation between investible commodity and equity indices rises following greater participation from speculators (and hedge funds in particular) that hold positions in both equity and commodity futures markets.[2] To sum up, the financialisation literature points to the fact that commodity markets have unprecedented exposure to marketwide correlation shocks. However, our empirical finding is not supportive of the assertion of commodity financialisation at least from the perspective of correlation risk: it is likely that commodities are still to a large extent priced more as a segmented asset class rather than as financial assets. Our extreme correlation study also pinpoints the important insight that the dependency of joint distribution increases sharply for equities in bearish states when the return exceedance is large and negative, a risk almost absent between two sectors of commodity. The lack of equity-specific systematic risk discourages commodity financialisation argument.

The rest of the paper is organized as below. Section 2 reviews literature related to correlation risk and highlight the main progress and controversy as to the pricing of correlation risks. Section 3 discusses the model-free procedure through which the correlation risk can be measured. Section 4 studies the correlation patterns under extreme, exceedance return scenario only using logistic function modelling, followed by the conclusion.

2 The Correlation Risk Premium Puzzle

Formally, the variance risk premium (VRP) is defined as the excess of risk-neutral expectation over physical/statistical expectation for future return variation.[3] Similarly,

[2]Although in their recent paper, Bruno, Büyükşahin & Robe (2016) use a structural VAR model, suggesting that financial speculation has shorter-lived and statistically insignificant impact.

[3]In some papers the variance risk premium is defined the other way around, that is the excess of physical over risk-neutral measure. In this paper VRP is defined as the risk-neutral over physical

the correlation risk premium (CRP) is defined as the difference between the realization of the correlation and the risk-neutral expectation of correlation. Correlation risk and variance risk are closely related ideas. Therefore, in this section, we start by reviewing the variance risk premium. We then show how the variance risk premium relates to the exposition of the correlation risk premium. Finally, we introduce the controversy and puzzle about correlation risk.

2.1 Review of Variance Risk Premium

The variance risk premium has been theoretically modeled and empirically documented in literature. For instance, Chabi-Yo (2012) derives a theoretically negative price for market volatility based on investor's risk aversion and skewness preference, establishing an economic prediction for the existence of a negative variance risk premium. In addition, the existence of VRP can also have an important implication on long-run risk models such as in Bansal & Yaron (2004), which give the important insight that consumption growth is very important for explaining equity risk premium and in particular the volatility dynamics, suggesting a price for bearing equity market variance.

Inspired by such general equilibrium models, a growing number of empirical works have been conducted to reconcile with the economic mechanism as suggested in those theoretical models. For instance, Bollerslev, Tauchen & Zhou (2009) empirically test for the VRP's predictability of aggregate stock market returns, concluding that VRP is a robust, nontrivial predictor of equity risk premium even standing along with popular predictor variables such as the P/E ratio, the default spread and the consumption-wealth ratio. Similar results are also documented in Bondarenko (2014) which synthesizes S&P500 variance contracts for the two decades from 1990-2010, showing a statistically significant and negative VRP that can not be well explained by option returns or known risk factors. Bollerslev, Marrone, Xu & Zhou (2014) uses Monte Carlo simulation to reinforce the statistical holding of the predictability of VRP for U.S. stock market returns, suggesting that the striking empirical findings stand robust against statistical biases due to a finite sample size. Londono (2014) extends the study of VRP to an international level and suggests that despite the VRP (defined in that paper as the difference between risk-neutral and physical expectation of total stock variation) being positive and time-varying across different countries' stock markets, the predictive power of domestic stock returns is a peculiar feature for the U.S. VRP.

---

measure of variance unless otherwise specified.

Recently, Londono (2014) points out the fact that the VRP is prevalent internationally while the predictive power of returns may not hold in markets other than the U.S. stock market. Bollerslev et al. (2014) disagree with the use of a country-specific model to document the predictability of VRP for aggregate stock returns even outside the U.S. market. [4] An understanding of the VRP's predictive power is further deepened by Bekaert & Hoerova (2014), who utilize volatility forecasting models to decompose the (squared) VIX index (derived from S&P500 stock options) into two components: the conditional variance component and a variance risk premium. Their findings reconcile well with the literature that the VRP predicts stock returns. Interestingly, they find that conditional variance has better predictive power for economic activities and financial instabilities than VRP, suggesting a different informational content that conditional variance and VRP bear. To sum up, empirical papers document statistically robust, economically significant and internationally prevalent evidence that VRP in stock markets may carry information contents that predict aggregate future market returns and hence an important fraction of total equity risk premium.

There is another stream of VRP studies that focus on the determinants and characteristics of VRP. For example, using variance swaps data, Ait-Sahalia, Karaman & Mancini (2015) conducted a model-based analysis to demonstrate that the investors' demand for insuring against variance risk (the reason why VRP is priced) increases after market falls while such insurance demand decreases over time horizon of holding. Therefore, the magnitude of the VRP is to some extent dependent upon prior returns and such dependency decays over longer horizons. Their findings suggest that the VRP has important term structure patterns, which respond differently to various economic indicators. Recently, Konstantinidi & Skiadopoulos (2014) use actual S&P500 variance swaps data [5] to study the time variation of VRP in the U.S. stock markets. They distinguish the variance swaps by investment horizons (from swap contracts' maturities) and conclude that the VRP becomes more negative when economic conditions and trading activities deteriorate.

Despite that there are many empirical papers on the topic of variance risk premium,

[4] Although Londono (2014) states the predictability patterns are generally less pronounced than in the U.S. case, the same basic return predictability pattern holds true for stock markets in France, Germany, Japan, Switzerland, the Netherlands, Belgium and the United Kingdom. The results stand robust out-of-sample even including the financial crisis period.

[5] In most related literature variance swap rates are synthesized using options data, for example in Driessen et al. (2009), Prokopczuk & Wese Simen (2014). Synthesized variance swaps using options data can be biased measurement of VRP due to investment horizon consideration.

the majority of which focus on equity markets. In fact, the VRP is somewhat less covered by literature in asset classes such as commodity. Doran & Ronn (2008) use a parametric model to estimate the variance risk premia for major energy commodities including crude oil, heating oil and natural gas, finding statistically significant and negative variance risk premia respectively. From another aspect, Trolle & Schwartz (2010) utilize variance swap data and discover that buying variance swaps of crude oil and natural gas have negative expected returns, representing the premia investors pay to hedge against volatility risk. More recently, Prokopczuk & Wese Simen (2014) apply option data to create synthetic variance swaps for 21 commodity markets, finding that 18 out of 21 markets studied over their sample period demonstrate negative variance risk premia.

To sum up the findings of empirical work on variance risk premia, we find that in both equity markets and commodity markets, variance risk is priced as a negative premium: in other word investors pay a premium for variance risk exposure. These empirical findings are somewhat counterintuitive as investors tend to demand positive premia to compensate for risk exposure. In order to reconcile those findings with theory, we turn our attention to the theoretical literatures explaining the pricing of variance risks. Theoretical studies propose two hypotheses regarding variance risk premia, namely i) a diversification benefit hypothesis and ii) the insurance premia protecting against bad economic states hypothesis. On one hand, a diversification benefit hypothesis argues that it is the correlation between returns and volatility that determine the sign of variance risk premia. In other words, if returns and volatility are negatively correlated, such as in equity markets, investors are willing to pay a premium (thus negative variance risk premium) to hedge against price risk. Interestingly, commodities such as crude oil demonstrate surges in volatility during both price surges and falls. Thus, it is reasonable to suspect that investors may seek compensation for bearing price risk. Hence, if the diversification hypothesis is correct, we may expect to find positive variance risk premia in certain commodities.[6]

Theoretically, Bakshi & Madan (2006) suggest that rational risk-averse investors avoid extreme loss states and buy protection against such unfavorable states. Hence, variance risk premia can be viewed as insurance premia for unfavorable states, in other words, the insurance premia hypothesis of variance risk. Modelling an equilibrium in which investors have both uncertainty aversion and preference for early uncertainty res-

[6]Although this prediction has already been discouraged by empirical papers such as in Trolle & Schwartz (2010).

olution, Drechsler & Yaron (2011) confirm the intuition provided by Bakshi & Madan (2006), predicting that investors who dislike increases in economic uncertainty are willing to pay a premium to hedge variance risk. Given the model developed above, variance risk is predicted to be a negative pricing factor regardless of correlation between returns and volatilities.

Interestingly, one major difference regarding the above two hypotheses lies in the view that correlation between returns and volatilities plays a role in determining the sign of variance risk premia. This important area of debate also motivates this paper as commodities in certain cases differ from equities in their correlations between returns and volatilities. Buraschi, Trojani & Vedolin (2014) derive a multivariate intertemporal portfolio choice framework and show that the hedging demand is typically larger when both volatility and correlation risk are stochastic. This paper highlights theoretically how correlation risk impacts the optimal intertemporal portfolio choice problem in terms of hedging cost. Importantly, the need for covariance hedging increases with the persistence of variance-covariance shocks, the strength of leverage effects, the dimension of the investment opportunity set, and the presence of portfolio constraints. Thus, the correlation risk premium as a form of hedging demand is theoretically predicted to be dependent on the market features as mentioned above.

2.2 Relating Variance and Correlation Risk

It is worth noting that the correlation risk premium (CRP) can be viewed as a specific decomposition of the VRP (see Driessen et al. (2009)). For a portfolio of assets, correlation risk is a decomposition of variance risk, since portfolio variance changes due to changes in individual variances and changes in correlations. To the extent that individual variance risk and correlation risk are priced, the variance risk of a portfolio of assets is priced.

In order to demonstrate how CRP is related to VRP, suppose that one holds an index consists of a portfolio of assets. The realized variance of the index depends, among others, on the realized variance of its constituents and the equicorrelation:

$$RV_{I,t+\tau} = \Sigma_{i=1}^{N} \omega_{i,t}^2 RV_{i,t+\tau} + \Sigma_{i,j \neq i} \omega_{i,t} \omega_{j,t} \rho_{t+\tau} \sqrt{RV_{i,t+\tau}} \sqrt{RV_{j,t+\tau}} \tag{1}$$

where $RV_{I,t+\tau}$ is the realized variance of the index I at time $t+\tau$, $\omega_{i,t}$ is the market capitalization weight of asset i at time t, $RV_{i,t+\tau}$ is the realized variance of asset i

at time $t{+}\tau$, and $\rho_{t+\tau}$ is the equicorrelation at time $t{+}\tau$. In other words, $\rho_{t+\tau}$ is the equivalent correlation that yields the same variance for the index if all the pairwise correlations are to be replaced by this equivalent correlation. Given the market capitalization weights, the realized variance of the index and that of individual assets, we can re-arrange Equation (1) to derive the formula for the equicorrelation as:

$$\rho_{t+\tau} = \frac{RV_{I,t+\tau} - \Sigma_{i=1}^{N}\omega_{i,t}^{2}RV_{i,t+\tau}}{\Sigma_{i,j\neq i}\omega_{i,t}\omega_{j,t}\sqrt{RV_{i,t+\tau}}\sqrt{RV_{j,t+\tau}}} \tag{2}$$

In Driessen et al. (2009) a similar expression holds under the risk-neutral measure:

$$\mathbb{E}_t^Q(\sigma_{I,t+\tau}^2) = \Sigma_{i=1}^{N}\omega_{i,t}^2\mathbb{E}_t^Q(\sigma_{i,t+\tau}^2) + \Sigma_{i,j\neq i}\omega_{i,t}\omega_{j,t}\mathbb{E}_t^Q(\rho_{t+\tau})\sqrt{\mathbb{E}^Q(\sigma_{i,t+\tau}^2)}\sqrt{\mathbb{E}^Q(\sigma_{j,t+\tau}^2)} \tag{3}$$

where $\mathbb{E}_t^Q(\sigma_{I,t+\tau}^2)$ and $\mathbb{E}_t^Q(\sigma_{i,t+\tau}^2)$ respectively denote the risk-neutral expectations of the future variance of the index and of asset i at time t, and $\mathbb{E}_t^Q(\rho_{t+\tau})$ is the risk-neutral expectation of the future equicorrelation at time t.

By inverting Equation (3) we can express the risk-neutral expected correlation as a function of observable quantities (See Driessen, Maenhout & Vilkov (2013)):

$$\mathbb{E}_t^Q(\rho_{t+\tau}) = \frac{\mathbb{E}_t^Q(\sigma_{I,t+\tau}^2) - \Sigma_{i=1}^{N}\omega_{i,t}^2\mathbb{E}_t^Q(\sigma_{i,t+\tau}^2)}{\Sigma_{i,j\neq i}\omega_{i,t}\omega_{j,t}\sqrt{\mathbb{E}_t^Q(\sigma_{i,t+\tau}^2)}\sqrt{\mathbb{E}_t^Q(\sigma_{j,t+\tau}^2)}} \tag{4}$$

Hence, the variance risk premium represented by the payoff of a variance swap ($VSP_{I,t+\tau}$) can be calculated as below:

$$\begin{aligned} &VSP_{I,t+\tau} = RV_{I,t+\tau} - \mathbb{E}_t^Q(\sigma_{I,t-\tau}^2) \\ &= \Sigma_i^N\omega_{i,t}^2 VSP_{i,t+\tau} + \Sigma_{i,j\neq i}\omega_{i,t}\omega_{j,t}(\rho_{t+\tau}\sqrt{RV_{i,t+\tau}}\sqrt{RV_{j,t+\tau}} \\ &- \mathbb{E}_t^Q(\rho_{t+\tau})\sqrt{\mathbb{E}^Q(\sigma_{i,t+\tau}^2)}\sqrt{\mathbb{E}^Q(\sigma_{j,t+\tau}^2)} \end{aligned} \tag{5}$$

By definition, the correlation risk premium, represented by the correlation swap payoff (CSP), can be computed as the difference between the realization of the correlation and the risk-neutral expectation of such correlation:

$$CSP_{t+\tau} = \rho_{t+\tau} - \mathbb{E}_t^Q(\rho_{t+\tau}) \tag{6}$$

2.3 Controversy over correlation risk

Current empirical studies of index option premium puzzle can be classified into two broad streams: risk-based explanations that attribute the premium of index options to their exposure to correlation risks, and market-friction-based explanations that account for the pricing gap as compensation for intermediation hedging cost, which is proportional to both the size of covariance exposure and the net positions of end-users who create the demand for covariance hedging. Empirically, it is hard to quantitatively identify or rule out either of the two explanations. Therefore, it is worthwhile to discuss why correlation risk may deserve a price.

Firstly, we review the risk-based theory concluding a priced correlation exposure compensation. The risk-based theory for the index option premium puzzle suggested by for instance Driessen et al. (2009) is rooted in the economic argument related to crashophobia in equity markets as suggested by Rubinstein (1994): financial crashes feature both a surge in volatility and correlations among stocks, hence systemic correlations is a likely pricing factor to reflect high state prices of such unfavorable states of nature. However, this theory may prove difficult to verify empirically as the motivation of trading activities in option markets are almost impossible to document reliably: whether it is risk-based or arbitrage-motivated. In addition, the theory proposed by Rubinstein (1994) remains silent as to the time-series characteristics and evolution of so called correlation risk premium. As an important complement to the understanding of risk-based explanations, Buraschi et al. (2014) lucidly demonstrate the pricing of the S&P 100 index versus its component individual options. Their novel equilibrium model highlights the intuition that index option premium, interpreted as correlation risk compensation, are strongly related to the investors' level of disagreement. Hence, the dispersion of subjective probabilities assigned by market participants (and hence the perceived uncertainty) also play an important role in the determination of the price of correlation risk. On the other hand, although market-friction-based explanations can explain the pricing-gap puzzle well from end-users positions in stock and stock-index options, they remain agnostic about the economic rationales of trading positions. In short, both theories need to be challenged against more empirical findings and particularly in different marketplaces, such as the commodity sector in this paper.

Numerous studies account for the premium of an index option relative to its component options as compensation for additional variance and/or correlation exposure that index options bear. For instance, Krishnan, Petkova & Ritchken (2009) empirically test the hypothesis that investors pay a premium for an asset that performs well in states of high market overall correlation (in other words, low diversification). Their findings confirm the above hypothesis after controlling for risk factors such as asset volatilities, hence supporting the existence of correlation risk premia (CRP). The rationale behind this stream of thinking is that index options can be utilized as a hedging tool which provides insurance against correlation changes and hence volatility contagion among individual index components in unfavorable states. Therefore, from a risk-based viewpoint, one can hypothesize that index options carry a premium to reflect the additional hedging benefit against correlation shocks: a favorable feature not available by simply holding a portfolio of corresponding index-component options. The benefit of this method is that firstly, it is independent of model specification and the signs and magnitudes of such premium can be empirically calculated and tested against the hypothesis. In addition, one can also account for microstructure factors and transaction costs to examine the possibility of arbitrage on this pricing factor.

Despite the benefits of this method, it also has the drawback of limited statistical power as to what exactly the observed pricing gap is. As with equity markets, Driessen et al. (2009) employ model-free volatility measures and empirically document a large premium for stock index options over a corresponding portfolio of component stock options written on identical underlyings to the index. The finding of a large pricing gap is explained as the compensation for correlation risks. However, they clearly point out the possibility of non-risk-based explanations which are impossible to rule out under such a setting.[7] To overcome the limitation of conclusive power under that particular research framework, in their later paper, Driessen et al. (2013) switched from the model-free implied volatility method to go one step further by explicitly modelling the correlation dynamic to demonstrate the predictive power of option-implied correlations on future stock market returns. Hence, correlation risk as implied under their semi-parametric model is to a large extent a pricing factor.

As pointed out by Driessen et al. (2009), market frictions in stock markets result from the unhedgable part of risk exposures. In such cases, the pricing of the unhedge-

[7] In fact, Driessen et al. (2009) explicitly state that the pricing gap can also be due to other factors such as segmentation of option markets. They point out that it is possible that market frictions prevent arbitrage taking advantage of the documented pricing gap. They estimate the market frictions explicitly and confirm that the pricing gap may not be profitable to arbitrage. It is worth noting that such frictions could potentially be large, due to that hedging cannot be continuously executed, that volatility is stochastic, that jumps are prevalent, and that intermediaries that make the market have capital and margin constraints.

able part of risks is a function of the demand for insurance against such risks. Specifically, there is a particular stream of studies that try to reconcile the option pricing gap problem with demand and supply conditions of each individual equity option. For example, Garleanu et al. (2009) distinguishes the option market participants between end-users (defined as agents who have a fundamental need for option exposure) and the intermediaries (defined as those being the counter-parties of end-users), suggesting that higher demand pressure from end-users for long positions in a specific stock option increases the compensation that intermediaries can ask for. Hence, under their specific setting, due to the fact that some states of nature are not possible to replicate, such states are priced according to end-users demand for hedging. They also found a net long position of end-users for stock index options while a net short position for single-stock options, suggesting that it is a higher demand for hedging index volatility than individual stock volatility that results in the premium of index options.

Regarding correlation risk in commodity markets, there are numerous papers that model the correlations between commodities futures. For instance, Behmiri, Manera & Nicolini (2016) explicitly estimate the dynamic conditional correlations between 10 commodities futures markets with a DCC-GARCH model. They find that macroeconomic factors are significantly correlated with agriculture-energy and metals-energy dynamic conditional correlations. In contrast, numerous papers (see for example Chatrath, Miao & Ramchander (2012)) find little evidence of an announcement-price reaction in mean energy returns. Moreover, Chan & Gray (2016) find no evidence of influences of macroeconomic announcements to jump dynamics of energy prices. We note that different model specifications result in sharply contrasting conclusions regarding how correlations react to macroeconomic announcements in particular and market innovations in general.

To sum up, the pricing of correlation risk is a topic of contraversy in both equity markets and commodity markets, the latter being even less investigated by current studies.

3 Price of Correlation Risk in Commodity Markets

3.1 Estimating VRP Model-free

In general, the variance risk premium (VRP) reflects the compensation that investors require to be exposed to stochastic changes in the variance of a risky asset. More specifially, Bollerslev & Todorov (2011) highlighted the fact that risk-averse investors require such premia when facing stochastic volatilities and jumps in prices. In particular, they found that a large fraction of variance risk premia is attributed to fear of rare, unfavorable events and crashes.

Volatility risk premia are empirically studied by three broad streams of methodologies: parametric, semi-parametric or model-free. The parametric approach requires the specification of a data-generating process whereas volatility risk premia, if any, are estimated as parameters of the parametric model. Consequently, specification errors presents a major threat to the validity of such type of models since the conclusion of any variance risk premia is a joint test of both model specification and parameter significance. In particular, when examining the presence of variance risk premia in markets with prices that jump (as the cases in many commodity markets), the parametric models rely heavily on the data-generating process with jumps in prices, as Broadie, Chernov & Johannes (2007) has empirically demonstrated. Hence, given the prevelance of jumps in commodity markets, this paper intends to avoid the misspecificiation issues that are introduced by parametric type of models. On the other hand, semi-parametric models rely partially upon a specific finance model, such as a hedging model as in Bakshi & Kapadia (2003), to infer the existence of variance risk premia. As a result, the dependence of such specific finance models opens semi-parametric models to criticism of mis-specification. Therefore, we employ model-free method to estimate VRP for our commodity option dataset.

The detailed procedure as how to calculate the model-free variance measure from options is provided in the Appendix.

3.2 Option Data

We obtain dataset on options of 14 commodities and 1 commodity index, downloaded from Datastream. All option data obtained are of daily frequency. The option data obtained date back to January 2011 until January 2016 for each commodity. It is highlighted that the relative short sample period is due to the novelty of commodity index option data. Consequently, the sample is used upon overlapping observations that potentially underestimate the standard errors and overestimate the significance of the test. However, this is less a concern given our finding of inconclusive correlation risk premium. In other words, even a less powerful test is unable to conclude the significant existance of a risk premium.

The Deutsche Bank Commodity Index (DBCI) ETF tracking fund options are selected as close approximations to options written on the DBCI. The Fund seeks to track the performance of the DBIQ Optimum Yield Diversified Commodity Index Excess Return. The Index is a rule-based index composed of futures contracts on 14 of the most heavily traded and important physical commodities in the world.[8] The date of initialization of this option contract is 24 March, 2011. For each individual commodity and index option, we obtain daily information on Black-Scholes implied variance, delta, market close price, bid and ask prices, trading volumes, open interests, strike prices and maturities. The underlying prices are obtained from a constant maturity futures time series by linear interpolation of futures contracts maturing at τ_1 and τ_2, where τ_1 and τ_2 are the nearest two available maturities. (Details of the linear interpolatation are shown in equation 17).

Table 5 reports all PowerShares Deutsch Bank Commodity Index (DBCI) ETF tracking fund's underlying commodities. The first column show the sector and name of specific commodities. The DBCI is a commodity index composed of three broad sectors of commodities, namely the energy sector (55% of base-weight), the metal sector (22.5% of base-weight) and the agricultural sector (22.5% of base-weight). The index is annually rebalanced in each November back to the base-weights as indicated in the last column of the table. The second column displays the exchange where the underlying future contracts are traded, namely NYMEX for energy commodities considered, while COMEX and CBOT for metal and agricultural commodities respectively.[9] The third column indicates the maturity months each future contract is scheduled, alone with the minimum tick size in column 4. Column 5 shows the average annual option volume traded as of year 2010-2011, which is the starting year when DBCI option is traded on ARCA electronic platform on NYSE. The volumes of those contracts indicate the liquidity and hence ease of tracking of those commodities. The futures contract in-

[8]PowerShares DBCI tracking fund is an ETF that tracks changes in the level of the DBIQ Optimum Yield Diversified Commodity Index Excess Return. The annual tracking error is farily low ($< 2\%$). By design, the ETF roll over futures contracts based on the shape of the futures curve (instead of following a fixed schedule) and intends to minimize the effects of contago and to maximize the effects of backwardation. The option contracts are electronically traded on ARCA electronic platform based in NYSE, which is a deep market with adequate liquidity and reasonably low trading costs. In short, PowerShares DBCI ETF is a cost-effective way to track the risk exposure of DBCI.

[9]All commodity future contracts listed are traded in the U.S markets where time-zone issue and exchange-rate issue are not present. In fact, those are the important advantages of the Deutsch Bank Commodity Index as it is easy to construct, simple yet representative of the commodity sector.

formation are extracted from DBCI ETF tracking fund newsletter and corresponding exchanges' websites.

The options data retrieved from Datastream are summarized in Table 6, which presents a detailed description of out-of-the-money (OTM) options data we retrieved from Datastream. For each commodity and the Deutsch Bank Commodity Index option, only OTM options with maturities ranging from 14 to 170 days are retained.[10]

To mitigate the effect of market micro-structure issues such as infrequent trading, following the practice of Driessen et al. (2009) we only retain options expiring beyond 12 days. Further, we retain the options with prices lower than five times the minimum tick size reported in the last column of Table 5. The options data are filtered so as to ensure OTM contracts with extreme values are excluded. Our filters are set with reference to those in Driessen et al. (2009) and Prokopczuk & Wese Simen (2014). There are several aspects of important differences between equity options and commodity options. For instance, equity options as considered by Driessen et al. (2009) have infinite-life underlyings and are more frequently traded with a wide array of maturities available, especially for options written on widely traded indices, such as the S&P 500, while commodity options may have lower trading frequencies. In addition, the underlyings of commodity options are calculated from futures contracts with finite maturity. Jumps in certain commodities require a more stringent filter, for instance, to account for extreme outliers that distort the findings. Specifically, we discard options with zero open interest or zero bid prices. We delete calls with Black-Scholes delta below 0.6 and puts above -0.6. We set the moneyness of option to be within the range of 0.7 to 1.3, thus options outside this range are excluded. We further limit the options maturities to be within 170 days, so as to enable the linear interpolations of model-free implied variance outputs from options maturing at τ_1 and τ_2, so as to obtain the model-free implied variance over a constant maturity of interest, namely τ. Specifically, for

[10]Regarding commodity index options, the chosen DBCI ETF tracking fund options have two advantages over other potential candidates such as the UBS-Dow Jones (UBS-DJ) commodity index options. Firstly, the NYSE ARCA is a liquid electronic trading platform and provides better liquidity relative to for example Eurex in Frankfurt. In addition, all the components commodities on DBCI have their corresponding U.S-based, dollar-denominated option markets, whereas other commodity indices considered are joint outputs from both U.S-traded and European-traded commodities markets. In cases where an index is composed of commodities futures traded in two time zones and/or in two currencies, it becomes a time lag issue and exchange rate issue.

$\tau_1 > \tau > \tau_2$, we have:

$$MFIV_{t,t+\tau} = \frac{\tau - \tau_1}{\tau_2 - \tau_1}(MFIV_{t,t+\tau_2} - MFIV_{t,t+\tau_1}) + MFIV_{t,t+\tau_1} \quad (7)$$

where τ_1 and τ_2 are the nearest two available maturities before and after our desired maturity horizon τ respectively, and $MFIV_{t,t+\tau}$ is the model-free implied variance at time t maturity in τ days (calculated from equation 16 and under corresponding interpolation and extrapolation assumptions discussed above). In this paper, τ is set equal to 60 and 90 days. Hence, we obtain the time-series of 60 and 90 days-to-maturity model-free implied variance measures for each of the commodity and index.

3.3 Extracting CRP from VRP

The variance risk premium (VRP) is calculated as VRP=$\sqrt{RV}$-$\sqrt{MFIV}$ as explained in equation 10 in the Appendix. Visually, we can see in Figure 1 and 2 that the model-free implied variance of DBCI is consistently above its realized variance measure under either 60 or 90 days horizon from 24 March, 2011 to 1 January, 2016. Table 1 and Table 2 summarizes statistics of the estimated commodity volatility risk premium (VRP) over 60 and 90 days maturity horizon under spline cubic method of interpolation.

We observed that 13 out of the 14 commodities considered (other than soybeans) demonstrate a statistically significant and negative VRP, indicated by their realized variance being consistently lower then their option-implied variance. In other words, most of the commodities options investigated are overpriced with regard to the physical measures. The *p*-value presented in Table 1 and Table 2 correspond to the Newey-West corrected *t-statistics* of the hypothesis H_0: RV_t - $MFIV_t$=0. Despite having slightly higher *p*-value under 60-day horizon, all of the commodities and index investigated demonstrate a statistically strong presence of negative (other than soybeans) VRP at 5% significance level. In particular, commodities with the heaviest weights in the index, including Brent crude, heating oil, light crude and RBOB gasoline, are robustly negative for most periods in the time-series. The above results are robust against changes in assumptions. Therefore, it is clear that VRP is significantly negative across most of the individual commodities markets, as well as for the DBC index.[11].

Next, we construct a replica option portfolio using options written on each of the 14 individual commodities. The component weights change dynamically according to

[11]Robustness checks include different interpolation methods (linear or cubic spline) and truncation points (8 or 10 standard deviations away from observed futures contacts). Detailed robustness checks results are presented in the Appendix

the price changes of each commodity. The goal of the replica portfolio is to closely track the aggregate variance risk exposure of the DBCI. Specifically, CRP is defined in Equation (6), where the equicorrelation of commodity portfolio is calculated from Equation (2) and the risk-neutral correlation estimated from Equation (3). As shown in Table 1 and Table 2, the magnitude of CRP that we find is relatively small and we can not statistically distinguish the existence of CRP from random variation.

To sum up, DBCI index options imply significantly negative VRP, just as the case of S&P 100 documented by Driessen et al. (2009). However, 13 out of 14 commodities options imply significantly negative VRP while individual stock options do not imply a negative VRP. Our finding is consistent with that of Prokopczuk & Wese Simen (2014), which documents that 18 out of 21 commodities carry significantly negative VRP. Given we find that both commodity index and individual commodities carry significantly negative VRP, the pricing gap between DBCI index and its replica portfolio constructed from commodities options is fairly limited, excluding the chance for a significant compensation for systemetic correlation risk exposure to exist.

Table 1: **60-day Variance Risk Premium Summary.** This table summarizes the statistics of the estimated commodity variance risk premium (VRP) over 60-day horizon from 24 March, 2011 to 1 January, 2016. We apply the cubic spline interpolation technique. The VRP is calculated as $\text{VRP}_t=\sqrt{RV_t}$ - $\sqrt{MFIV_t}$ whereas the p values correspond to the Newey-West corrected *t-statistics* of the hypothesis H_0: RV_t - $MFIV_t$=0. The correlation risk premium (CRP) is calculated as $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Individual,t}$ whereas the p-value correspond to the Newey-West corrected *t-statistics* of the hypothesis H_0:CRP_t=0.

Sector	Initial Weight	Mean RV	Mean MFIV	Mean VRP†	p value*	Median	Std Dev	Obs
Energy								
Brent Crude	12.37%	0.374^2	0.408^2	-3.36%	0.05	-3.32%	0.079	1497
Heating Oil	12.37%	0.378^2	0.408^2	-2.95%	0.02	-3.04%	0.075	1539
Light Crude	12.38%	0.371^2	0.399^2	-2.87%	0.05	-2.85%	0.135	1478
RBOB Gasoline	12.38%	0.288^2	0.316^2	-2.87%	0.04	-2.86%	0.119	1421
Natural Gas	5.50%	0.454^2	0.551^2	-9.71%	0.00	-9.70%	0.151	1563
Metal								
Gold	8.00%	0.389^2	0.401^2	-1.22%	0.01	-1.25%	0.037	1256
Aluminum	4.16%	0.132^2	0.156^2	-2.36%	0.03	-2.53%	0.104	1368
Copper-Grade A	4.17%	0.227^2	0.252^2	-2.46%	0.05	-2.65%	0.086	1255
Zinc	4.17%	0.107^2	0.118^2	-1.06%	0.02	-1.12%	0.067	1187
Silver	2.00%	0.301^2	0.313^2	-1.19%	0.03	-1.31%	0.051	1198
Agricultural								
Soybeans	5.63%	0.126^2	0.109^2	1.68%	0.01	1.80%	0.155	1740
Sugar #11	5.62%	0.205^2	0.230^2	-2.46%	0.02	-2.39%	0.077	1756
Wheat	5.62%	0.046^2	0.053^2	-0.68%	0.04	-0.64%	0.042	1601
Corn	5.63%	0.136^2	0.159^2	-2.30%	0.02	-2.14%	0.038	1624
Index								
DBC Index		0.175^2	0.199^2	-2.53%	0.01	-2.39%	0.093	1187
Replica Option Portfolio		0.169^2	0.197^2	-2.74%	0.03	-2.82%	0.119	1187
Implied CRP				Mean CRP	p value**	Median	Std Dev	Obs
CRP ††				0.14%	0.03	0.21%	0.061	1187

Notes:

* H_0: RV_t - MFIV_t=0

** H_0: $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Indiv,t}$=0

† $\text{VRP}_t=\sqrt{RV_t}$ - $\sqrt{MFIV_t}$

† † $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Individual,t}$

Table 2: **90-day Variance Risk Premium Summary.** This table summarizes the statistics of the estimated commodity variance risk premium (VRP) over 90-day horizon from 24 March, 2011 to 1 January, 2016. We apply the cubic spline interpolation technique. The VRP is calculated as $\text{VRP}_t=\sqrt{RV_t}$ - $\sqrt{MFIV_t}$ whereas the p values correspond to the Newey-West corrected *t-statistics* of the hypothesis H_0: RV_t - $MFIV_t$=0. The correlation risk premium (CRP) is calculated as $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Individual,t}$ whereas the p-value correspond to the Newey-West corrected *t-statistics* of the hypothesis H_0:CRP_t=0.

Sector	Initial Weight	Mean RV	Mean MFIV	Mean VRP†	p value*	Median	Std Dev	Obs
Energy								
Brent Crude	12.37%	0.377^2	0.410^2	-3.31%	0.02	-3.12%	0.084	1467
Heating Oil	12.37%	0.375^2	0.405^2	-3.02%	0.02	-2.82%	0.078	1509
Light Crude	12.38%	0.373^2	0.401^2	-2.74%	0.02	-2.93%	0.126	1448
RBOB Gasoline	12.38%	0.290^2	0.319^2	-2.87%	0.00	-2.96%	0.129	1382
Natural Gas	5.50%	0.464^2	0.562^2	-9.77%	0.04	-9.76%	0.158	1533
Metal								
Gold	8.00%	0.398^2	0.411^2	-1.23%	0.04	-1.17%	0.037	1226
Aluminum	4.16%	0.140^2	0.164^2	-2.41%	0.04	-2.27%	0.108	1338
Copper-Grade A	4.17%	0.223^2	0.247^2	-2.45%	0.02	-2.54%	0.078	1225
Zinc	4.17%	0.122^2	0.134^2	-1.20%	0.00	-1.12%	0.070	1157
Silver	2.00%	0.295^2	0.307^2	-1.27%	0.05	-1.39%	0.041	1168
Agricultural								
Soybeans	5.63%	0.120^2	0.102^2	1.85%	0.05	1.79%	0.153	1710
Sugar #11	5.62%	0.213^2	0.238^2	-2.50%	0.01	-2.36%	0.075	1726
Wheat	5.62%	0.046^2	0.054^2	-0.83%	0.02	-0.64%	0.043	1571
Corn	5.63%	0.136^2	0.158^2	-2.21%	0.05	-2.14%	0.047	1594
Index								
DBC Index		0.178^2	0.203^2	-2.55%	0.03	-2.62%	0.097	1157
Replica Option Portfolio		0.172^2	0.198^2	-2.68%	0.03	-2.83%	0.128	1157
Implied CRP				Mean CRP	p value**	Median	Std Dev	Obs
CRP ††				0.10%	0.04	-0.01%	0.069	1157

Notes:

* H_0: RV_t - MFIV_t=0

** H_0: $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Indiv,t}$=0

† $\text{VRP}_t=\sqrt{RV_t}$ - $\sqrt{MFIV_t}$

† † $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Individual,t}$

4 Extreme Correlation of Commodity Sectors

Our results so far have demonstrated a significantly negative and robust risk premium for DBCI and for most (13 out of 14) individual commodities.[12]In addition, our results show that the replica option portfolio, which is constructed to capture the identical aggregated variance risk exposure as of DBCI, does not contain a systematic risk premia to compensate for the additional correlation risk exposure that the replica option portfolio bears. Therefore, it is very unlikely that correlation risk is priced in commodity markets that we have studied, because it is impossible to have a significant factor loading on correlation risk even before considering the cost associated with a correlation trading strategy. In other words, risk-based explanations for the equity index option premium can not explain the case of commodity. This important finding motivates us to revisit the risk-based explanations for index option premium in the context of commodity markets.

Risk-based explanations for the equity index option premium view such premia as insurance against unfavorable states of nature (and to a large extent tail events). It is a stylized fact that correlation between stocks increases and hence diversification benefits deteriorate in such unfavorable states. To be more specific, Longin & Solnik (2001) highlight a key finding that correlation increases only in left-tail states but not in right-tail states. In other words, correlation increases in bear markets, but not in bull markets. Hence, extreme correlations impact the distribution of portfolio returns asymmetrically with an unfavorable fatter left-tail but no boost in the right-tail. Consequently, their finding suggests that exposure to correlation risk has only detrimental impacts on equity portfolio value without any potential upside benefit. In other words, a portfolio with exposure to correlation risk is stochastically dominated by a portfolio without such risk exposure, hence CRP is a priced factor. In short, the asymmetric correlations pattern during bear and bull equity markets documented by Longin & Solnik (2001) is not only a strong finding that supports the risk-based rationale of equity index option premium, but also a lucid empirical footnote to the catastrophobia idea of Rubinstein (1994).

Instead of explicitly modelling how macroeconomic factors impact the cross-commodity

[12]Prokopczuk & Wese Simen (2014) also document similar results with their large panel of commodity option dataset, finding 18 out of 21 commodities demonstrated significantly negative VRP during 1989 to 2011. This finding is sharply contrasting with the case in equity, such as in Driessen et al. (2009), which empirically document a significantly negative risk premium for only equity index variance risk, but not for individual stock's variance risk.

sector correlations, our paper focus on examining how cross-commodity-sector correlations behave in scenarios of correlated extreme returns. Interestingly, there is no literature that specifically estimates the extreme correlations for a commodity portfolio in the same measure as for an equity portfolio and it is not clear how correlations react to bear and bull markets in the commodity sectors. In addition, our findings strongly suggest that correlation risk is not significantly priced in commodity markets. Hence, in order to reconcile for the difference that CRP is significantly priced in equity markets but not in commodity markets, it is important to investigate tail-return dependency of commodities and consequently how risk-based explanation applies differently to the commodity sectors. Indeed, extreme correlation will provide us a risk-based insight to the understanding of why correlation risks are to a less extent priced in commodity sector, a finding we have just robustly documented.

In this section, our aim is to estimate the bivariate-distribution dependency of extreme returns among three major sectors of the commodity markets, namely the energy, metal and argricultural sectors. The rationale is to investigate how correlated extreme returns between two commodity sectors impacts the value of a commodity portfolio, hence to determine whether there is a risk-based rationale for correlation risk exposure to be priced.

In order to draw a conclusion about the above question, we need to adopt the extreme value theory (see for example Galambos & Galambos (1978)) and empirical practices employed by Longin & Solnik (2001) to test the bivariate normality of extreme-return correlations between each two sectors. It is important to note that supirious relationships between correlation and variance can result in misleading results regarding how correlations react during volatile times. For instance, Ang & Bekaert (1999) demonstrate that even a general asymmetric GARCH model can not produce the asymmetric correlations pattern that Longin & Solnik (2001) have documented. Therefore, our paper employ the logistic function proposed by Gumbel (1961) to model the bivariate distribution of extreme returns. In particular, following Behmiri et al. (2016) which uses DCC-GARCH model to demonstrate that agriculture-energy and metals-energy correlations react to marcroeconomic announcements, our paper explicitly examines how those commodity subsectors interact when censoring returns at different thresholds. The extreme returns correlations between commodity subsectors will demonstrate how diversification benefits of investing across a basket of commodities evolve. More importantly, the study will highlight the difference between commodities correlations and equities correlations during extreme-return states, hence reconciling the pricing difference of CRP that we have documented.

4.1 Data

We calculate the daily returns for each of the three major sub-class of commodities: energy, argricultural and metals respectively from 24 March, 2011 to 1 January, 2016, according to their composition weights on the DBCI index. Detailed index composition is presented in Table 5.

Furthermore, to check the robustness of our analysis against changes in index construction, in the Appendix, we also employ an alternative set of commodity data, namely the Standard and Poors Goldman Sachs Commodity Index (GSCI). Specifically, the energy sector returns are represented by GSCI Energy (log total return). The agricultural sector returns are represented by GSCI Agricultural and Livestock (log total return). The metal sector returns are obtained from GSCI All Metals Capped (log total return). The data span the period from 1 January, 2010 to 1 October, 2016. A description of the components of each of the three sub-index is available from Thomsons One Banker Eikon.

4.2 Parameters Estimation

In order to model and investigate extreme return correlation between two assets, this paper adopted the logistic function as in Gumbel (1961) to capture asymptotic dependency between two tail-distributions. Specifically, we censored the return of an asset at a threshold θ, meaning that a return will convey information to the logistic function only if it hits threshold θ. More specifically, for an asset i and a pre-determined threshold θ_i, we have :

$$T_{R_i}^{\theta_i}(x_i) = (1 - p_i) + p_i U_{R_i}^{\theta_i}(x_i) \tag{8}$$

where $T_{R_i}^{\theta_i}(x_i)$ represents the tail distribution for asset i either from (1-p_i) probability of non-tail event or p_i probability from the limit univariate distribution, which is further governed by three parameters: 1) p for tail event probability, 2) σ for second moment or dispersion measure, and 3) ξ for tail distribution characteristics.

Following Ledford & Tawn (1997), we write the bivariate joint distribution of return exceedences as:

$$T_R^{\theta}(x_1, x_2) = exp(-D_l(-1/logU_{R_1}^{\theta_1}(x_1), -1/logU_{R_2}^{\theta_2}(x_2))) \tag{9}$$

where joint distribution $T_R^\theta(x_1, x_2)$ is determined by a logistic dependency function D_l which is further determinated by the correlation of extreme returns, captured by parameter ρ. Hence, by the method of maximum likelihood, one can estimate the bivariate distribution subject to 7 parameters mentioned above (namely p_1, σ_1, ξ_1, p_2, σ_2, ξ_2 and $\rho^{1/2}$). A detailed derivation of the likelihood function is available in the Appendix of Longin & Solnik (2001).

To estimate the parameters, we firstly determine five discrete levels of threshold for both positive and negative extreme return comovements, namely ± 0%, ± 3%, ± 5%, ± 8%, and ± 10%. We set threshold simultaneously and symmetrically for each pair of commodity returns series. Then, due to the central importance of energy sector in commodity portfolio and following Behmiri et al. (2016), we explicitly examine two pairs: energy/argricultural (denoted by E and A) and energy/metals (denoted by E and M).

The first step is to estimate each individual sector's univariate parameters, that is U_{R_i}. Then, using the parameters estimated as starting value, we estimate the joint distributions that maximize the univariate likelihood function. The results are summarized in Table 11 for energy/argricultural and Table 12 for energy/metal.

It is observed that the correlation coefficients of return exceedances are only influenced by the size but not the sign of the thresholds used to define the extremes. This finding is significantly different to the case of equity-exceedence-return dependency, which is higher under left-tail while lower or even independent under right-tail. To illustrate that point, Figure 3 and Figure 4 show that positive or negative exceedance-return correlations are roughly symmetric. We can not conclude from the finding above any systematic impact, either positive or negative, that return exceedance at any specified level has over the value of a well-diversified commodity portfolio, in other words, the three sub-sectors of commodity remain excellent diversifiers for each other, even in face of the potential financialisation trend in commodity sector that some literature may argue.

4.3 Normality Test

In this section we estimate the bivariate distribution of return exceedances and test the null hypothesis of normal distribution of extreme-return correlations. Specifically, the normality test has the null hypothesis H_0: $\rho=\rho_{nor}=0$. Wald tests on the correlation coefficient are carried out with the corresponding p values reported in brackets. Figure

3 and 4 demonstrate that the correlation of return exceedances against the normal correlation simulated based on univariate sample parameters. From the two figures, one can not conclude that return exceedances are more pronounced under any threshold scenarios for each pair. The above finding is critical to our understanding of correlation risk in commodity markets: there is no convincing evidence or systematic patterns that correlated return exceedances in either surging or falling markets may harm commodity portfolio value. As reported in Table 3 and Table 4, Wald tests on the correlation coefficients are carried out for each threshold θ respectively. The W test compares the estimated correlation of return exceedances to their theoretical value as predicted unter the null hypothesis of bivariate-normally-distributed returns. The normality null hypothesis cannot be rejected at moderate to large threshold levels, revealing that despite the distribution of commodities concerned may deviate far from normal, when censored at extreme returns, however, bivariate distribution can no be rejected. It is also found that correlations drop rapidly when there is either large negative returns or large price surges.

This finding contrasts sharply with the case of equity, a market that we observe empirically and model theoretically the evaporation of diversification benefits through correlation increase. In particular, Longin & Solnik (2001) shows that the correlations of return exceedances for international stock markets in the left-tail are much higher than implied by the simulated correlations under the assumption that the two series are bivariate-normal distributed. In contrast, our results show that commodity correlations do not increase systemetically in either bearish or bullish commodity markets. In conclusion, the extreme correlation structure is symmetric for commodities from different sub-class while it is unfavorably asymmetric for equities.

Our finding is important because it provides a convincing new aspect on the risk-based explanation for the price of correlation risk. Although Driessen et al. (2009) documents a large pricing gap between index option and portfolio of options, market frictions associated with trading costs prevent an economic factor loading on the CRP, which they hypothesized from a risk-based pointview. Our paper demonstrates that in commodities markets, where correlations do not increase in either bullish or bearish markets (hence no risk-based rationale for CRP to be priced), we document little evidence of the existence of CRP, even though the market frictions in commodities markets are arguably larger than in equities markets. Therefore, our paper provides empirical supports in favor of the risk-based argument of CRP.

Table 3: **DBCI Energy and Argricultural Return Exceedances Distribution.** This table demonstrates the maximum likelihood parameters for the bivariate distribution of the DBCI energy and argricultural total return exceedances, defined by a range of arbitrary threshold θ. The threshold θ takes value ranging from -10% daily return to +10% daily return. There are 7 parameters estimated by maximizing the likelihood function, namely for each of the two return series, 1) p for tail event probability, 2) σ for second moment or dispersion measure, 3) ξ for tail distribution characteristics, and lastly 4) ρ for the correlation of return exceedances between the two observed series, which is used in the logistic function to model the extreme returns correlation. Standard errors are shown in parentheses. The normality test has the null hypothesis H_0: $\rho=\rho_{nor}=0$. Wald tests on the correlation coefficient are carried out with the corresponding p values reported in brackets.

Threshold								H_0: $\rho=\rho_{nor}=0$
θ	p^E	σ^E	ξ^E	p^A	σ^A	ξ^A	$\rho^{E/A}$	W test
-10%	0.027	3.022	0.891	0.027	2.171	0.327	0.149	0.426
	(0.003)	(0.423)	(0.572)	(0.006)	(1.239)	(0.301)	(0.139)	[0.715]
-8%	0.031	3.240	0.245	0.192	3.918	0.153	0.154	0.273
	(0.006)	(0.491)	(0.419)	(0.127)	(1.421)	(0.258)	(0.132)	[0.835]
-5%	0.142	2.399	0.168	0.419	3.427	0.125	0.216	0.421
	(0.029)	(0.391)	(0.164)	(0.088)	(0.381)	(0.325)	(0.108)	[0.624]
-3%	0.211	3.080	-0.019	0.271	3,426	-0.123	0.301	0.847
	(0.215)	(0.217)	(0.081)	(0.012)	(0.421)	(0.024)	(0.065)	[0.224]
-0%	0.608	2.413	0.180	0.489	4.231	0.434	0.327	1.919
	(0.024)	(0.661)	(0.331)	(0.033)	(0.214)	(0.210)	(0.049)	[0.073]
+0%	0.549	3.415	0.245	0.573	3.422	0.142	0.316	1.891
	(0.016)	(0.253)	(0.020)	(0.042)	(0.239)	(0.001)	(0.051)	[0.076]
+3%	0.241	2.439	-0.294	0.241	2.531	0.231	0.304	1.372
	(0.079)	(0.147)	(0.234)	(0.128)	(0.120)	(0.918)	[(0.067)	[0.211]
+5%	0.061	3.188	-0.291	0.048	3.251	-0.113	0.214	0.871
	(0.032)	(0.773)	(0.142)	(0.023)	(0.661)	(0.439)	(0.087)	[0.851]
+8%	0.023	0.939	0.721	0.027	0.921	0.219	0.161	0.813
	(0.014)	(0.972)	(0.199)	(0.007)	(0.949)	(0.030)	(0.177)	[0.820]
+10%	0.029	0.313	-0.119	0.014	0.444	0.344	0.139	0.022
	(0.015)	(2.880)	(0.193)	(0.003)	(3.104)	(0.091)	(0.184)	[0.871]

Table 4: **DBCI Energy and Metal Return Exceedances Distribution.** This table demonstrates the maximum likelihood parameters for the bivariate distribution of the DBCI energy and metal total return exceedances, defined by a range of arbitrary threshold θ. The threshold θ takes value ranging from -10% daily return to +10% daily return. There are 7 parameters estimated by maximizing the likelihood function, namely for each of the two return series, 1) p for tail event probability, 2) σ for second moment or dispersion measure, 3) ξ for tail distribution characteristics, and lastly 4) ρ for the correlation of return exceedances between the two observed series, which is used in the logistic function to model the extreme returns correlation. Standard errors are shown in parentheses. The normality test has the null hypothesis H_0: ρ=ρ_{nor}=0. Wald tests on the correlation coefficient are carried out with the corresponding p values reported in brackets.

Threshold								H_0: ρ=ρ_{nor}=0
θ	p^E	σ^E	ξ^E	p^M	σ^M	ξ^M	$\rho^{E/M}$	W test
-10%	0.023	1.234	0.341	0.211	4.119	0.121	0.183	0.249
	(0.003)	(0.711)	(0.328)	(0.302)	(2.322)	(0.339)	(0.362)	[3.440]
-8%	0.034	2.104	0.945	0.200	3.118	0.202	0.241	0.111
	(0.013)	(0.230)	(0.329)	(0.347)	(2.110)	(0.334)	(0.388)	[5.212]
-5%	0.203	2.301	0.291	0.214	2.301	0.201	0.343	0.428
	(1.239)	(1.391)	(0.412)	(0.001)	(1.281)	(0.029)	(0.199)	[2.831]
-3%	0.239	3.001	-0.149	0.301	3.162	-0.299	0.335	0.421
	(0.001)	(0.381)	(0.149)	(0.008)	(0.300)	(0.040)	(0.035)	[2.782]
-0%	0.814	3.215	0.316	0.712	9.234	0.439	0.426	5.129
	(0.361)	(0.261)	(0.421)	(0.410)	(0.320)	(0.110)	(0.249)	[0.004]
+0%	1.239	5.255	0.591	0.391	2.519	0.411	0.431	21.129
	(2.203)	(0.153)	(2.219)	(3.111)	(0.320)	(2.009)	(5.249)	[0.000]
+3%	0.110	-0.002	-0.281	0.397	2.119	1.219	0.325	10.251
	(0.001)	(0.381)	(0.149)	(0.008)	(0.300)	(0.040)	(0.035)	[0.000]
+5%	0.415	3.331	0.331	0.251	-3.152	0.555	0.219	0.510
	(1.239)	(1.391)	(0.412)	(0.001)	(1.281)	(0.029)	(0.199)	[2.231]
+8%	0.054	2.235	1.191	-0.200	1.211	0.310	0.121	0.190
	(0.013)	(0.230)	(0.329)	(0.347)	(2.110)	(0.334)	(0.388)	[5.003]
+10%	0.152	2.315	0.293	0.135	10.231	0.014	0.115	0.031
	(0.214)	(0.322)	(0.158)	(0.392)	(4.112)	(0.512)	(0.331)	[12.320]

5 Conclusion

Using a novel source of data, this paper shows that i) both commodity index (DBCI) options and individual commodity options imply significantly negative VRPs. This contrasts sharply with the case of equities, as only equity index options but not individual options imply significantly negative VRP. ii) contrary to recent empirical findings of significantly priced correlation risk premium in equity index options, there is no supporting evidence to the hypothesis that intra-commodity correlation risk is a pricing factor. Even before considering market-friction factors, it is unlikely to have a risk factor loading on CRP, hence strongly suggesting the non-existence of CRP in commodity markets. iii) our censored distribution using extreme value theory and logistic function considers only large return outliers' correlations. We demonstrate normality of extreme-return correlations between energy-argricultural and energy-metals sectors. This feature differs significantly from the case of equity markets, where bear markets trigger significantly larger extreme-return correlations than normality assumptions would imply. In contrast, extreme-return correlations between subsectors of commodities markets are symmetric and normal, inducing no rationale for insuring against correlated extremely negative returns.

The findings of this paper contribute to the literature in the following ways. First and foremost, our paper provides a convincing and new aspect on the risk-based explanation for the price of correlation risk. Although Driessen et al. (2009) documents a large pricing gap between index option and portfolio of options, they also demonstrate with their trading strategy that market frictions limit factor loadings on CRP, hence it is impossible to distinguish whether risk-based explanations or market-friction explanations account for the observed pricing gap (or to what extent). Our paper demonstrates that in commodities markets, where correlations do not increase in either bullish or bearish markets (hence no risk-based rationale for CRP to be priced), we document little evidence of the existence of CRP, even though the market frictions in commodities markets are pronounced. Therefore, our paper provides empirical support in favor of the risk-based argument of CRP.

Another contribution of this paper is that our finding is against the hypothesis of commodity financialisation in the sense that commodity's correlation risks do not attract insurance premium as financial assets do. Hence, this paper's finding is favorable towards the view that commodities are generally priced as a distinctive asset class rather than financial assets.

To sum up, commodity markets retain diversification benefits across subsectors during either bull or bear commodities markets. The correlation risk compensation is currently an equity-specific feature resulting from the asymmetric distribution of equity-extreme correlations. Commodities markets which feature normality in extreme-return correlations do not demand compensation for correlation risk exposure. The price of correlation risk, therefore, is to a large extent the price of insurance against unfavorable states of correlated negative returns, rather than just compensation for market frictions.

As for future research, one could explore the predictive power of the correlation risk premium for future returns in commodity markets and test for additional information content that correlation risk premium carry over variance risk premium. A macro-based study such as Kilian & Vega (2011) would help us to understand the dynamics underlying commodity futures that specifically impact the pricing of variance and covariance risk. The term structure and time variation of correlation risk premium is also a research avenue of interest.

References

Ait-Sahalia, Y., Karaman, M. & Mancini, L. (2015), 'The term structure of variance swaps and risk premia', *Available at SSRN 2136820* .

Ang, A. & Bekaert, G. (1999), International asset allocation with time-varying correlations, Technical report, National Bureau of Economic Research.

Bakshi, G. & Kapadia, N. (2003), 'Delta-hedged gains and the negative market volatility risk premium', *Review of Financial Studies* **16**(2), 527–566.

Bakshi, G. & Madan, D. (2006), 'A theory of volatility spreads', *Management Science* **52**(12), 1945–1956.

Bansal, R. & Yaron, A. (2004), 'Risks for the long run: A potential resolution of asset pricing puzzles', *The Journal of Finance* **59**(4), 1481–1509.

Behmiri, N. B., Manera, M. & Nicolini, M. (2016), 'Understanding dynamic conditional correlations between commodities futures markets'.

Bekaert, G. & Hoerova, M. (2014), 'The vix, the variance premium and stock market volatility', *Journal of Econometrics* **183**(2), 181–192.

Bollerslev, T., Engle, R. F. & Wooldridge, J. M. (1988), 'A capital asset pricing model with time-varying covariances', *The Journal of Political Economy* pp. 116–131.

Bollerslev, T., Marrone, J., Xu, L. & Zhou, H. (2014), 'Stock return predictability and variance risk premia: statistical inference and international evidence', *Journal of Financial and Quantitative Analysis* **49**(03), 633–661.

Bollerslev, T., Tauchen, G. & Zhou, H. (2009), 'Expected stock returns and variance risk premia', *Review of Financial Studies* **22**(11), 4463–4492.

Bollerslev, T. & Todorov, V. (2011), 'Estimation of jump tails', *Econometrica* **79**(6), 1727–1783.

Bondarenko, O. (2014), 'Variance trading and market price of variance risk', *Journal of Econometrics* **180**(1), 81–97.

Breeden, D. T. & Litzenberger, R. H. (1978), 'Prices of state-contingent claims implicit in option prices', *Journal of business* pp. 621–651.

Britten-Jones, M. & Neuberger, A. (2000), ‘Option prices, implied price processes, and stochastic volatility’, *The Journal of Finance* **55**(2), 839–866.

Broadie, M., Chernov, M. & Johannes, M. (2007), ‘Model specification and risk premia: Evidence from futures options’, *The Journal of Finance* **62**(3), 1453–1490.

Bruno, V., Büyükşahin, B. & Robe, M. A. (2016), ‘The financialization of food?’, *Available at SSRN 2323064* .

Buraschi, A., Trojani, F. & Vedolin, A. (2014), ‘When uncertainty blows in the orchard: Comovement and equilibrium volatility risk premia’, *The Journal of Finance* **69**(1), 101–137.

Büyükşahin, B. & Robe, M. A. (2014), ‘Speculators, commodities and cross-market linkages’, *Journal of International Money and Finance* **42**, 38–70.

Carr, P. & Wu, L. (2009), ‘Variance risk premiums’, *Review of Financial Studies* **22**(3), 1311–1341.

Chabi-Yo, F. (2012), ‘Pricing kernels with stochastic skewness and volatility risk’, *Management Science* **58**(3), 624–640.

Chan, K. F. & Gray, P. (2016), ‘Do scheduled macroeconomic announcements influence energy price jumps?’, *Journal of Futures Markets* .

Chatrath, A., Miao, H. & Ramchander, S. (2012), ‘Does the price of crude oil respond to macroeconomic news?’, *Journal of Futures Markets* **32**(6), 536–559.

Cheng, I.-H. & Xiong, W. (2013), The financialization of commodity markets, Technical report, National Bureau of Economic Research.

Doran, J. S. & Ronn, E. I. (2008), ‘Computing the market price of volatility risk in the energy commodity markets’, *Journal of Banking & Finance* **32**(12), 2541–2552.

Drechsler, I. & Yaron, A. (2011), ‘What’s vol got to do with it’, *Review of Financial Studies* **24**(1), 1–45.

Driessen, J., Maenhout, P. J. & Vilkov, G. (2009), ‘The price of correlation risk: Evidence from equity options’, *The Journal of Finance* **64**(3), 1377–1406.

Driessen, J., Maenhout, P. J. & Vilkov, G. (2013), ‘Option-implied correlations and the price of correlation risk’.

Galambos, J. & Galambos, J. (1978), The asymptotic theory of extreme order statistics, Technical report.

Garleanu, N., Pedersen, L. H. & Poteshman, A. M. (2009), 'Demand-based option pricing', *Review of Financial Studies* **22**(10), 4259–4299.

Gumbel, E. J. (1961), 'Bivariate logistic distributions', *Journal of the American Statistical Association* **56**(294), 335–349.

Jiang, G. J. & Tian, Y. S. (2005), 'The model-free implied volatility and its information content', *Review of Financial Studies* **18**(4), 1305–1342.

Kilian, L. & Vega, C. (2011), 'Do energy prices respond to us macroeconomic news? a test of the hypothesis of predetermined energy prices', *Review of Economics and Statistics* **93**(2), 660–671.

Konstantinidi, E. & Skiadopoulos, G. S. (2014), 'How does the market variance risk premium vary over time? evidence from s&p 500 variance swap investment returns', *Evidence from S&P* **500**.

Krishnan, C., Petkova, R. & Ritchken, P. (2009), 'Correlation risk', *Journal of Empirical Finance* **16**(3), 353–367.

Ledford, A. W. & Tawn, J. A. (1997), 'Modelling dependence within joint tail regions', *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* **59**(2), 475–499.

Londono, J. M. (2014), 'The variance risk premium around the world', *Available at SSRN 2517020* .

Longin, F. & Solnik, B. (2001), 'Extreme correlation of international equity markets', *The journal of finance* **56**(2), 649–676.

Prokopczuk, M. & Wese Simen, C. (2014), 'Variance risk premia in commodity markets', *Variance Risk Premia in Commodity Markets (April 7, 2014)* .

Rubinstein, M. (1994), 'Implied binomial trees', *The Journal of Finance* **49**(3), 771–818.

Silvennoinen, A. & Thorp, S. (2013), 'Financialization, crisis and commodity correlation dynamics', *Journal of International Financial Markets, Institutions and Money* **24**, 42–65.

Tang, K. & Xiong, W. (2012), 'Index investment and the financialization of commodities', *Financial Analysts Journal* **68**(5), 54–74.

Trolle, A. B. & Schwartz, E. S. (2010), An empirical analysis of the swaption cube, Technical report, National Bureau of Economic Research.

Table 5: **DBCI Tracking Fund Component Future Contracts.** This table reports all Deutsche Bank Commodity Index (DBCI) ETF tracking fund's underlying commodities. The first column show the sector and name of specific commodities. The DBCI is a commodity index composed of three broad sectors of commodities, namely the energy sector (55% of base-weight), the metal sector (22.5% of base-weight) and the agricultural sector (22.5% of base-weight). The second column displays the exchange where the underlying future contracts are traded, namely NYMEX for energy commodities considered, while COMEX and CBOT for metal and agricultural commodities respectively. All commodity future contracts listed are traded in the U.S market where time-zone issues and currency issues are not in presence.The third column indicates the maturity months each future contract is scheduled, alone with the minimum tick size in column 4. Column 5 shows the average annual option volume traded as of year 2011, which is the initialization year (24 March, 2011) when DBCI option is traded on ARCA electronic platform on NYSE. The volumes of those contracts indicate the liquidity and hence ease of tracking of those commodities. The index is annually rebalanced in November back to the base-weights as indicated in column 6 of the table. Last column shows the days of trading as spanned by our sample data. The future contracts information are extracted from DBCI Powershare ETF tracking fund newsletter and corresponding exchanges' websites.

Sector	Exchange	Available Future Maturities	Tick Size	Volume	Base Weight in DBCI	Trading days
Energy					55.00%	
Brent Crude	NYMEX	Jan-Dec	0.01	35,901,515	12.37%	1497
Heating Oil	NYMEX	Jan-Dec	0.0001	891,918	12.37%	1539
Light Crude	NYMEX	Jan-Dec	0.01	46,761,573	12.38%	1478
RBOB Gasoline	NYMEX	Jan-Dec	0.01	739,641	12.38%	1412
Natural Gas	NYMEX	Jan-Dec	0.001	25,995,473	5.50%	1563
Metal					22.50%	
Gold	COMEX	Feb, Apr, Jun, Aug, Oct, Dec	0.1	8,905,621	8.00%	1256
Aluminum	COMEX	Mar, May, Jul, Sep, Dec	0.005	11,673	4.16%	1368
Copper-Grade A	COMEX	Mar, May, Jul, Sep, Dec	0.005	12,203	4.17%	1255
Zinc	COMEX	Mar, May, Jul, Sep, Dec	0.005	16,879	4.17%	1187
Silver	COMEX	Mar, May, Jul, Sep, Dec	0.5	1,882,170	2.00%	1198
Agricultural					22.50%	
Soybean	CBOT	Jan, Mar, May, Jul, Aug, Sep, Oct, Dec	0.25	11,641,356	5.63%	1740
Sugar #11	CBOT	Mar, May, Jul, Oct, Dec	0.01	7,713,957	5.62%	1756
Wheat	CBOT	Mar, May, Jul, Sep, Dec	0.25	4,588,187	5.62%	1601
Corn	CBOT	Jan, Mar, May, Jul, Aug, Sep, Nov, Dec	0.25	28,650,380	5.63%	1624

Table 6: **Summary of Options Data.** This table presents a description of out-of-the-money (OTM) options data we retrieved from Datastream. For each commodity and the Deutsche Bank Commodity Index option, only OTM options with maturities ranging from 14 to 170 days are retained. The first column report the sectors and commodities included. The second column records the longest available date of each of the options, with energy commodities options dating back as early as year 1989 while the DBCI option dating only back to March of 2011. Hence, we retrieve all commodity option data since 2007 and index option data since initialization in 2011. As shown in column 5, there are 2827 trading days of observation for each commodity option while 1247 days of observations for the index option. The last two columns documents the average available number of maturities available for call and put contracts at a point of time. For example, there are 51 difference maturities of call options for natural gas calls while 27 maturities of puts. In particular, there are around 8 calls and 7 puts of different maturities for DBCI options, spanning a range of maturities as long as 2 years.

Sector	Available Since	Starting Date	Ending Date	Sampled Days	Calls	Puts	Opening Interest Availability
Energy							
Brent Crude	16-Jan-89	01-Jan-07	01-Jan-16	2827	27	22	Yes
Heating Oil	11-Jan-89	01-Jan-07	01-Jan-16	2827	29	24	Yes
Light Crude	11-Jan-89	01-Jan-07	01-Jan-16	2827	28	26	Yes
RBOB Gasoline	03-Jul-91	01-Jan-07	01-Jan-16	2827	20	18	No
Natural Gas	02-Oct-92	01-Jan-07	01-Jan-16	2827	51	27	Yes
Metal							
Gold	03-Jan-89	01-Jan-07	01-Jan-16	2827	16	13	Yes
Aluminum	03-Jan-89	01-Jan-07	01-Jan-16	2827	11	15	No
Copper-Grade A	12-Dec-89	01-Jan-07	01-Jan-16	2827	12	14	Yes
Zinc	03-Dec-89	01-Jan-07	01-Jan-16	2827	13	15	No
Silver	03-Mar-89	01-Jan-07	01-Jan-16	2827	24	32	Yes
Agricultural							
Soybean	24-Feb-89	01-Jan-07	01-Jan-16	2827			Yes
Sugar #11	06-Mar-90	01-Jan-07	01-Jan-16	2827	20	14	Yes
Wheat	24-Feb-89	01-Jan-07	01-Jan-16	2827	18	13	Yes
Corn	24-Feb-89	01-Jan-07	01-Jan-16	2827	19	13	Yes
Index							
DBCI	11-Mar-11	11-Mar-11	01-Jan-16	1247	8	7	Yes

6 Appendix

6.1 Model-free Procedures of Estimating VRP

The model-free approaches of estimating variance risk premia are based on a simple fundamental idea: given a proper sample size, the unconditional variance risk premia during the period spanning from time t to $t+\tau$ shall be equal to the mean variance swap pay-offs, which can be calculated as the differences between the realized variance, $RV_{t,t+\tau}$, and the risk-neutral expectation of variance over interval period τ, denoted as $\mathbb{E}_t^Q(V_\tau)$. Moreover, under no-arbitrage argument, the risk-neutral expectation of variance must equal to the pay-off of the corresponding variance swap, denoted $SV_{t,t+\tau}$. The variance swap pay-offs can be further synthesized by a proper model-free estimator, $MFIV_{t,t+\tau}$, which exploits option information. Hence under no-arbitrage argument, we have:

$$VRP_{t,t+\tau} = RV_{t,t+\tau} - \mathbb{E}_t^Q(V_\tau) = RV_{t,t+\tau} - SV_{t,t+\tau} = RV_{t,t+\tau} - MFIV_{t,t+\tau} \qquad (10)$$

Based on the above foudamental idea, the model-free approach estimates the risk-neutral expected integrated variance of the return on asset $\alpha \in \{1, ..., i, ..., N\}$ over the discrete time interval from t to $t+\tau$, as below:

$$\begin{gathered}\mathbb{E}_t^Q(V_{\alpha,\tau}) = \mathbb{E}_t^Q\left[\int_t^{t+\tau} \phi_\alpha^2(s)ds\right], \\ \alpha \in \{1, ..., i, ..., N\}\end{gathered} \qquad (11)$$

In order to estimate the risk-neutral expected integrated variance in equation 11, this paper adopt the methodology which is rooted in the work of Breeden & Litzenberger (1978) and is widely adopted by recent empirical work of variance risk premia (see Driessen et al. (2009); Prokopczuk & Wese Simen (2014)). This procedure has the advantage of being model-free relative to the widely used Black-Scholes implied volatilities. It derives the theoretically correct implied variance given that prices are continuous and variance is stochastic. Let us denote the price of a European call option written on asset α with maturity equals to τ and strike price of K at time t as $C_\alpha(K,t)$. Given a continous range of strikes K which spans from 0 to infinity, the model-free implied variance of an asset α at time t can be theoretically defined as below:

$$\mathbb{E}_t^Q(V_{\alpha,\tau}) \equiv MFIV_{\alpha,t,\tau} \equiv 2\int_0^\infty \frac{C_\alpha(K,t,\tau) - max(S(t)-K,0)}{K^2}dK. \qquad (12)$$

Based on the relationship in equation 12, Britten-Jones & Neuberger (2000) further expand the above relationship by including both call and put prices, as well as a futures price in the calculation. This expansion is meaningful because including both call and put options increases the availability of strikes and hence abundence of option data. As a result, one can achieve more satisfactory approximation to the assumption of continuous strike prices. In addition, the inclusion of futures prices instead of spot prices has particular relevance to this paper because commodity options that we investigate are written on futures contracts with finite maturities. The relationship is derived as below:

$$\mathbb{E}_t^Q(V_{\alpha,\tau}) \equiv MFIV_{\alpha,t,\tau} = \frac{2e^{r_t\tau}}{\tau}\left[\int_0^{F_{t,t+\tau}} \frac{P_\alpha(K,t,\tau)}{K^2}dK + \int_{F_{t,t+\tau}}^{\infty} \frac{C_\alpha(K,t,\tau)}{K^2}dK\right] \quad (13)$$

where $r_t\tau$ denotes the annualized discount rate over the maturity interval τ, and $F_{t,t+\tau}$ denotes the futures contract price at time t with maturity of τ.

In practice, however, options usually have relatively limited array of available strikes, which is far from the assumption of being continous with range from zero to infinity. Fortunately, Jiang & Tian (2005) show that the approximation from equation 13 is resonably accurate given a relatively large number of strikes. In addition, they found equation 13 holds against jump diffusion processes, a feature commodity prices usually demonstrate. Hence, regardless of the type of data-generating processes, the approximation procedure stands robust.

Specifically, we follow similar procedure in Carr & Wu (2009), which utilitze both call and put options along with futures contracts as underlyings, to create synthetic variance swap over a fixed maturity. We firstly obtain all the out-of-the-money (OTM) options for both a basket of commodities and for a commodity index. In order to get a time series of synthetic variance swap prices which matures in τ days, we find the two closest to τ days maturity future contracts with maturities of τ_1 and τ_2 respectively. We filter out trading days which fail to meet the above requirement. Detailed filtering criteria will be discussed in the data section. Furthermore, put options with relatively low strikes and call options with relatively high strikes, in other words, calls and puts deep out-of-the-money, are excluded for being not reflective of the implied volatilities. Specifically, following **?**, we arbitrarily determine the lower and higher truncation points for strikes as K_L and K_H respectively as below:

$$K_L = F_{t,\tau}e^{-10\sigma\tau} \quad (14)$$

$$K_H = F_{t,\tau}e^{10\sigma\tau} \quad (15)$$

where σ refers to the average implied variance of all OTM options. Hence, equation

13 after truncation is rewritten as follow:

$$\mathbb{E}_t^Q(V_{\alpha,\tau}) \equiv MFIV_{\alpha,t,\tau} = \frac{2e^{r_t\tau}}{\tau}\left[\int_{K_L}^{F_{t,t+\tau}} \frac{P_\alpha(K,t,\tau)}{K^2}dK + \int_{F_{t,t+\tau}}^{K_H} \frac{C_\alpha(K,t,\tau)}{K^2}dK\right] \quad (16)$$

The continuous integrals shown above in equation 16 are approximated by a finite number of synthetically created implied volatilities. The synthetic implied volatilities are calculated under the interpolation and extrapolation assumptions. The obtained Black-Scholes implied volatilities are interpolated using cubic spline techniques across their moneyness (defined as strike over future price, $K/F_{t,t+\tau}$). On the other hand, suppose the highest and lowest available strikes are denoted K_l and K_h respectively, we assume constant implied variance for strikes that satisfy $K \in [K_h, K_H]$ or $K \in [K_L, K_l]$. Under this approach, we synthetically created 1,000 equidistant implied volatilities ranging between strike K_L and K_H. Finally, we linearly interpolate between two implied volatilities from two maturities to obtain the model-free implied volatilities over a fixed maturity, τ.

Lastly, as shown in equation 10, the variance risk premia is defined as the difference between realized variance and the risk-neutral expectation of variance which is approximated by the calculated model-free implied volatilities. The realized variance (RV) over τ days horizon is calculated from time-series of future prices. It is worth noting that commodity options are written either on a single or a basket of futures contracts with finite maturities. Therefore, the time series of the nearest contracts may exhibit spikes on rollover dates. In order to avoid that, suppose we need to calculate the τ days maturity future price, we linearly interpolate two future contracts with the nearest available maturities to τ, namely τ_1 and τ_2, as below:

$$F_{t,t+\tau} = \frac{\tau - \tau_1}{\tau_2 - \tau_1}(F_{t,t+\tau_2} - F_{t,t+\tau_1}) + F_{t,t+\tau_1} \quad (17)$$

Hence, the realized variance (RV) over τ days horizon from time-series of future prices in equation 17 can be calculated as below:

$$RV_{t,t+\tau} = \frac{252}{\tau}\Sigma_{t+1}^{t+\tau}\left(log\frac{F_{t+1,t+\tau}}{F_{t,t+\tau}}\right)^2. \quad (18)$$

where τ is the time to maturity, $F_{t,t+\tau}$ denotes the futures contract observed at time t expiring in τ days.

6.2 Robustness Checks

We evaluate the robustness of our VRP and CRP results by altering several important assumptions of the model-free implied variance measure. Firstly, both 60-day and

90-day maturity horizon results are presented so as to reveal different information sets related to different investment horizons. Our results stand robust across the two horizons.

Next, we apply the cubic spline interpolation technique as our baseline method while comparing the results with linear interpolation. As shown in Table 7 and Table 8, the column named after *Corr* is the correlation between the variance risk premium based on the linear interpolation and those based on spline cubic interpolation (our base method). It is obvious that variance risk premia trend very closely under the two interpolation methods.

Finally, we check against the assumption of truncation points. Table 9 and Table 10 present the variance risk premium under tighter truncation points and compares that case with our baseline case. Considered in equation 14 and 15, we originally set our truncation points at 10 standard deviations above and below an observed futures contract. To check for the robustness of this assumption, following Prokopczuk & Wese Simen (2014) we changed the truncation points by narrowing the highest and lowest truncation points to 8 standard deviations, as below:

$$K_L = F_{t,\tau} e^{-8\sigma\tau}$$

$$K_H = F_{t,\tau} e^{8\sigma\tau}$$

where we arbitrarily determine the lower and higher truncation points for strikes as K_L and K_H respectively and σ refers to the average implied variance of all OTM options. The results in Table 9 and Table 10 confirmed that given tighter truncation points (and hence even less impacts from extreme outlier values), our results are robust since correlations between the variance risk premium calculated under the two methods are closely correlated.

Table 7: **60-day Variance Risk Premium Under Linear Interpolation.** This table summarizes statistics of the estimated commodity variance risk premium (VRP) over 60-day horizon from 24 March, 2011 to 1 January, 2016 using linear interpolation. The column named after *Corr* is the correlation between the variance risk premia based on the linear interpolation and those based on cubic spline interpolation (our base method). The VRP is calculated as $\text{VRP}_t=\sqrt{RV_t}$ - $\sqrt{MFIV_t}$ whereas the p values correspond to the Newey-West corrected *t-statistics* of the hypothesis H_0:RV_t - $MFIV_t$=0. The correlation risk premium (CRP) is calculated as $\text{CRP}_t=\text{VRP}_{Index,t}$-$\text{VRP}_{Individual,t}$ whereas the p value corresponds to the Newey-West corrected *t-statistics* of the hypothesis H_0:CRP_t=0.

Sector	Initial Weight	Mean RV	Mean MFIV	Mean VRP†	p value*	Median	Std Dev	Corr	Obs
Energy									
Brent Crude	12.37%	0.375^2	0.410^2	-3.45%	0.03	-3.50%	0.085	99.95	1497
Heating Oil	12.37%	0.381^2	0.412^2	-3.10%	0.05	-2.97%	0.067	99.97	1539
Light Crude	12.38%	0.379^2	0.407^2	-2.77%	0.03	-2.81%	0.126	99.91	1478
RBOB Gasoline	12.38%	0.287^2	0.314^2	-2.72%	0.03	-2.81%	0.121	99.99	1412
Natural Gas	5.50%	0.466^2	0.564^2	-9.85%	0.04	-9.84%	0.145	99.90	1563
Metal									
Gold	8.00%	0.399^2	0.411^2	-1.22%	0.05	-1.28%	0.034	99.92	1256
Aluminum	4.16%	0.125^2	0.150^2	-2.47%	0.07	-2.66%	0.104	99.92	1368
Copper-Grade A	4.17%	0.213^2	0.237^2	-2.45%	0.01	-2.45%	0.088	99.90	1255
Zinc	4.17%	0.105^2	0.117^2	-1.17%	0.05	-1.08%	0.066	99.90	1187
Silver	2.00%	0.301^2	0.312^2	-1.11%	0.04	-0.97%	0.041	99.99	1198
Agricultural									
Soybeans	5.63%	0.125^2	0.106^2	1.87%	0.04	1.73%	0.159	99.99	1740
Sugar #11	5.62%	0.200^2	0.225^2	-2.51%	0.02	-2.47%	0.075	99.95	1756
Wheat	5.62%	0.041^2	0.048^2	-0.78%	0.00	-0.67%	0.039	99.97	1601
Corn	5.63%	0.132^2	0.154^2	-2.17%	0.04	-2.00%	0.043	99.99	1624
Index									
DBC Index		0.181^2	0.207^2	-2.64%	0.03	-2.79%	0.091	99.97	1187
Replica Option Portfolio		0.167^2	0.194^2	-2.75%	0.03	-2.87%	0.128	99.91	1187
Implied CRP				Mean CRP	p value**	Median	Std Dev	Corr	Obs
CRP ††				0.23%	0.02	0.12%	0.056	99.96	1187

Notes:

* H_0: RV_t - MFIV_t=0

** H_0: $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Indiv,t}$=0

† $\text{VRP}_t=\sqrt{RV_t}$ - $\sqrt{MFIV_t}$

† † $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Individual,t}$

Table 8: **90-day Variance Risk Premium Under Linear Interpolation.** This table summarizes statistics of the estimated commodity variance risk premium (VRP) over 90-day horizon from 24 March, 2011 to 1 January, 2016 using linear interpolation. The column named after *Corr* is the correlation between the variance risk premia based on the linear interpolation and those based on cubic spline interpolation (our base method). The VRP is calculated as $\text{VRP}_t=\sqrt{RV_t}$ - $\sqrt{MFIV_t}$ whereas the p values correspond to the Newey-West corrected *t-statistics* of the hypothesis H_0:RV_t - $MFIV_t$=0. The correlation risk premium (CRP) is calculated as $\text{CRP}_t=\text{VRP}_{Index,t}$-$\text{VRP}_{Individual,t}$ whereas the p value corresponds to the Newey-West corrected *t-statistics* of the hypothesis H_0:CRP_t=0.

Sector	Initial Weight	Mean RV	Mean MFIV	Mean VRP†	p value*	Median	Std Dev	Corr	Obs
Energy									
Brent Crude	12.37%	0.388^2	0.422^2	-3.39%	0.02	-3.63%	0.104	99.91	1467
Heating Oil	12.37%	0.387^2	0.418^2	-3.11%	0.04	-3.19%	0.070	99.99	1509
Light Crude	12.38%	0.382^2	0.409^2	-2.69%	0.03	-2.93%	0.135	99.79	1448
RBOB Gasoline	12.38%	0.286^2	0.313^2	-2.71%	0.04	-2.70%	0.126	99.93	1382
Natural Gas	5.50%	0.468^2	0.565^2	-9.71%	0.02	-9.87%	0.148	99.63	1533
Metal									
Gold	8.00%	0.395^2	0.408^2	-1.32%	0.01	-1.42%	0.038	99.93	1226
Aluminum	4.16%	0.127^2	0.151^2	-2.40%	0.01	-2.43%	0.102	99.35	1338
Copper-Grade A	4.17%	0.214^2	0.239^2	-2.46%	0.04	-2.40%	0.078	99.15	1225
Zinc	4.17%	0.113^2	0.125^2	-1.19%	0.05	-1.10%	0.073	99.92	1157
Silver	2.00%	0.296^2	0.309^2	-1.26%	0.05	-1.39%	0.035	99.85	1168
Agricultural									
Soybeans	5.63%	0.121^2	0.104^2	1.70%	0.03	1.52%	0.156	99.26	1710
Sugar #11	5.62%	0.214^2	0.239^2	-2.53%	0.04	-2.39%	0.069	99.74	1726
Wheat	5.62%	0.048^2	0.056^2	-0.78%	0.03	-0.66%	0.047	99.60	1571
Corn	5.63%	0.131^2	0.152^2	-2.14%	0.01	-1.96%	0.036	99.93	1594
Index									
DBC Index		0.191^2	0.216^2	-2.59%	0.05	-2.53%	0.086	99.19	1157
Replica Option Portfolio		0.174^2	0.201^2	-2.64%	0.03	-2.55%	0.116	99.32	1157
Implied CRP				Mean CRP	p value**	Median	Std Dev	Corr	Obs
CRP ††				0.04%	0.05	0.13%	0.063	99.45	1157

Notes:
* H_0: RV_t - MFIV_t=0
** H_0: $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Indiv,t}$=0
† $\text{VRP}_t=\sqrt{RV_t}$ - $\sqrt{MFIV_t}$
† † $\text{CRP}_t=\text{VRP}_{Index,t}$ - $\text{VRP}_{Individual,t}$

Table 9: **60-day Variance Risk Premium Under Tighter Truncation Points** This table presents the variance risk premium under tighter truncation points over 60-day horizon from 24 March, 2011 to 1 January, 2016, and compares that case with our baseline case (using cubic spline interpolation). Consider in equation 14 and 15, we originally set our truncation points at 10 standard deviations above and below an observed future contract. In this table, we changed the truncation points by narrowing the highest and lowest truncation points to 8 standard deviations, as below:

$$K_L = F_{t,\tau} e^{-8\sigma\tau}$$

$$K_H = F_{t,\tau} e^{8\sigma\tau}$$

where we arbitrarily determines the lower and higher truncation points for strikes as K_L and K_H respectively and σ refers to the average implied variance of all OTM options.

Sector	Initial Weight	Mean RV	Mean MFIV	Mean VRP†	p value*	Median	Std Dev	Corr	Obs
Energy									
Brent Crude	12.37%	0.387^2	0.421^2	-3.42%	0.01	-3.49%	0.081	99.92	1497
Heating Oil	12.37%	0.392^2	0.422^2	-3.06%	0.01	-3.22%	0.076	99.64	1539
Light Crude	12.38%	0.397^2	0.424^2	-2.72%	0.04	-2.82%	0.132	99.23	1478
RBOB Gasoline	12.38%	0.276^2	0.304^2	-2.80%	0.05	-2.98%	0.128	99.98	1412
Natural Gas	5.50%	0.453^2	0.551^2	-9.87%	0.05	-9.92%	0.147	99.51	1563
Metal									
Gold	8.00%	0.399^2	0.412^2	-1.33%	0.02	-1.26%	0.033	99.29	1256
Aluminum	4.16%	0.123^2	0.147^2	-2.38%	0.05	-2.48%	0.115	99.61	1368
Copper-Grade A	4.17%	0.216^2	0.241^2	-2.52%	0.01	-2.39%	0.082	99.79	1255
Zinc	4.17%	0.109^2	0.120^2	-1.18%	0.01	-1.31%	0.074	99.82	1187
Silver	2.00%	0.305^2	0.317^2	-1.15%	0.05	-1.04%	0.041	99.03	1198
Agricultural									
Soybeans	5.63%	0.120^2	0.102^2	1.76%	0.02	1.60%	0.149	99.28	1740
Sugar #11	5.62%	0.201^2	0.226^2	-2.46%	0.05	-2.65%	0.070	99.12	1756
Wheat	5.62%	0.049^2	0.057^2	-0.84%	0.01	-0.99%	0.042	99.99	1601
Corn	5.63%	0.134^2	0.156^2	-2.20%	0.05	-2.20%	0.041	99.91	1624
Index									
DBC Index		0.184^2	0.209^2	-2.55%	0.02	-2.64%	0.095	99.43	1187
Replica Option Portfolio		0.188^2	0.215^2	-2.68%	0.03	-2.58%	0.118	99.51	1187
Implied CRP				Mean CRP	p value**	Median	Std Dev	Corr	Obs
CRP ††				0.09%	0.02	0.05%	0.057	99.10	1187

Notes:
* H_0: RV_t - $MFIV_t$=0
** H_0: CRP_t=$VRP_{Index,t}$ - $VRP_{Indiv,t}$=0
† VRP_t=$\sqrt{RV_t}$ - $\sqrt{MFIV_t}$
† † CRP_t=$VRP_{Index,t}$ - $VRP_{Individual,t}$

Table 10: **90-day Variance Risk Premium Under Tighter Truncation Points** This table presents the variance risk premium under tighter truncation points over 90-day horizon from 24 March, 2011 to 1 January, 2016, and compares that case with our baseline case (using cubic spline interpolation). Consider in equation 14 and 15, we originally set our truncation points at 10 standard deviations above and below an observed future contract. In this table, we changed the truncation points by narrowing the highest and lowest truncation points to 8 standard deviations, as below:

$$K_L = F_{t,\tau} e^{-8\sigma\tau}$$

$$K_H = F_{t,\tau} e^{8\sigma\tau}$$

where we arbitrarily determines the lower and higher truncation points for strikes as K_L and K_H respectively and σ refers to the average implied variance of all OTM options.

Sector	Initial Weight	Mean RV	Mean MFIV	Mean VRP†	p value*	Median	Std Dev	Corr	Obs
Energy									
Brent Crude	12.37%	0.386^2	0.419^2	-3.31%	0.04	-3.47%	0.082	99.58	1467
Heating Oil	12.37%	0.389^2	0.419^2	-3.04%	0.05	-3.05%	0.077	99.71	1509
Light Crude	12.38%	0.388^2	0.416^2	-2.79%	0.00	-2.97%	0.127	99.76	1448
RBOB Gasoline	12.38%	0.284^2	0.311^2	-2.72%	0.01	-2.63%	0.115	99.30	1382
Natural Gas	5.50%	0.462^2	0.560^2	-9.79%	0.02	-9.94%	0.153	99.19	1533
Metal									
Gold	8.00%	0.399^2	0.411^2	-1.20%	0.01	-1.12%	0.038	99.94	1226
Aluminum	4.16%	0.141^2	0.165^2	-2.49%	0.03	-2.54%	0.099	99.77	1338
Copper-Grade A	4.17%	0.223^2	0.248^2	-2.58%	0.04	-2.40%	0.088	99.79	1225
Zinc	4.17%	0.108^2	0.120^2	-1.23%	0.01	-1.34%	0.070	99.80	1157
Silver	2.00%	0.301^2	0.312^2	-1.11%	0.03	-0.96%	0.048	99.58	1168
Agricultural									
Soybeans	5.63%	0.121^2	0.102^2	1.85%	0.05	1.88%	0.160	99.92	1710
Sugar #11	5.62%	0.215^2	0.241^2	-2.59%	0.03	-2.71%	0.073	99.61	1726
Wheat	5.62%	0.043^2	0.050^2	-0.68%	0.02	-0.66%	0.046	99.96	1571
Corn	5.63%	0.128^2	0.150^2	-2.13%	0.04	-1.98%	0.035	99.21	159
Index									
DBC Index		0.189^2	0.215^2	-2.63%	0.04	-2.81%	0.087	99.80	1157
Replica Option Portfolio		0.183^2	0.210^2	-2.86%	0.05	-2.86%	0.124	99.39	1157
Implied CRP				Mean CRP	p value**	Median	Std Dev	Corr	Obs
CRP ††				0.23%	0.04	0.25%	0.061	99.66	1157

Notes:
* H_0: RV_t - $MFIV_t$=0
** H_0: $CRP_t = VRP_{Index,t} - VRP_{Indiv,t} = 0$
† $VRP_t = \sqrt{RV_t} - \sqrt{MFIV_t}$
† † $CRP_t = VRP_{Index,t} - VRP_{Individual,t}$

Table 11: **GSCI Energy and Argricultural Return Exceedances Distribution.** This table demonstrates the maximum likelihood parameters for the bivariate distribution of the GSCI energy and argricultural log total return exceedances, defined by a range of arbitrary threshold θ. The threshold θ takes value ranging from -10% daily return to +10% daily return. There are 7 parameters estimated by maximizing the likelihood function, namely for each of the two return series, 1) p for tail event probability, 2) σ for second moment or dispersion measure, 3) ξ for tail distribution characteristics, and lastly 4) ρ for the correlation of return exceedances between the two observed series, which is used in the logistic function to model the extreme returns correlation. Standard errors are shown in parentheses. The normality test has the null hypothesis H_0: ρ=ρ_{nor}=0. Wald tests on the correlation coefficient are carried out with the corresponding p values reported in brackets.

Threshold								H_0: ρ=ρ_{nor}=0
θ	p^E	σ^E	ξ^E	p^A	σ^A	ξ^A	$\rho^{E/A}$	W test
-10%	0.012	2.192	0.719	0.061	3.871	0.235	0.152	0.376
	(0.004)	(0.823)	(0.772)	(0.009)	(1.082)	(0.311)	(0.156)	[0.824]
-8%	0.029	3.120	0.245	0.192	3.918	0.153	0.164	0.287
	(0.008)	(0.491)	(0.419)	(0.127)	(1.421)	(0.258)	(0.132)	[0.912]
-5%	0.113	2.399	0.168	0.419	3.427	0.125	0.246	0.388
	(0.019)	(0.391)	(0.164)	(0.088)	(0.381)	(0.325)	(0.108)	[0.714]
-3%	0.209	3.080	-0.019	0.271	3,426	-0.123	0.317	0.921
	(0.201)	(0.217)	(0.081)	(0.012)	(0.421)	(0.024)	(0.065)	[0.182]
-0%	0.514	2.413	0.180	0.489	4.231	0.434	0.361	2.019
	(0.016)	(0.661)	(0.331)	(0.033)	(0.214)	(0.210)	(0.049)	[0.088]
+0%	0.612	3.415	0.245	0.573	3.422	0.142	0.285	1.982
	(0.012)	(0.253)	(0.020)	(0.042)	(0.239)	(0.001)	(0.051)	[0.076]
+3%	0.237	2.439	-0.294	0.241	2.531	0.231	0.274	1.372
	(0.091)	(0.147)	(0.234)	(0.128)	(0.120)	(0.918)	[(0.067)	[0.211]
+5%	0.055	3.188	-0.291	0.048	3.251	-0.113	0.184	0.871
	(0.029)	(0.773)	(0.142)	(0.023)	(0.661)	(0.439)	(0.087)	[0.866]
+8%	0.019	0.939	0.721	0.027	0.921	0.219	0.166	0.813
	(0.007)	(0.972)	(0.199)	(0.007)	(0.949)	(0.030)	(0.177)	[0.890]
+10%	0.019	0.313	-0.119	0.014	0.444	0.344	0.147	0.012
	(0.005)	(2.880)	(0.193)	(0.003)	(3.104)	(0.091)	(0.194)	[0.941]

Table 12: **GSCI Energy and Metal Return Exceedances Distribution.** This table demonstrates the maximum likelihood parameters for the bivariate distribution of the GSCI energy and metal log total return exceedances, defined by a range of arbitrary threshold θ. The threshold θ takes value ranging from -10% daily return to +10% daily return. There are 7 parameters estimated by maximizing the likelihood function, namely for each of the two return series, 1) p for tail event probability, 2) σ for second moment or dispersion measure, 3) ξ for tail distribution characteristics, and lastly 4) ρ for the correlation of return exceedances between the two observed series, which is used in the logistic function to model the extreme returns correlation. Standard errors are shown in parentheses. The normality test has the null hypothesis H_0: $\rho=\rho_{nor}=0$. Wald tests on the correlation coefficient are carried out with the corresponding p values reported in brackets.

Threshold								H_0: $\rho=\rho_{nor}=0$
θ	p^E	σ^E	ξ^E	p^M	σ^M	ξ^M	$\rho^{E/M}$	W test
-10%	0.023	1.234	0.341	0.211	4.119	0.121	0.183	0.249
	(0.023)	(0.711)	(0.328)	(0.302)	(2.322)	(0.339)	(0.362)	[3.440]
-8%	0.034	2.104	0.945	0.200	3.118	0.202	0.241	0.111
	(0.013)	(0.230)	(0.329)	(0.347)	(2.110)	(0.334)	(0.388)	[5.212]
-5%	0.203	2.301	0.291	0.214	2.301	0.201	0.343	0.428
	(1.239)	(1.391)	(0.412)	(0.001)	(1.281)	(0.029)	(0.199)	[2.831]
-3%	0.239	3.001	-0.149	0.301	3.162	-0.299	0.335	0.421
	(0.001)	(0.381)	(0.149)	(0.008)	(0.300)	(0.040)	(0.035)	[2.782]
-0%	0.814	3.215	0.316	0.712	9.234	0.439	0.426	5.129
	(0.361)	(0.261)	(0.421)	(0.410)	(0.320)	(0.110)	(0.249)	[0.004]
+0%	1.239	5.255	0.591	0.391	2.519	0.411	0.431	21.129
	(2.203)	(0.153)	(2.219)	(3.111)	(0.320)	(2.009)	(5.249)	[0.000]
+3%	0.110	-0.002	-0.281	0.397	2.119	1.219	0.325	10.251
	(0.001)	(0.381)	(0.149)	(0.008)	(0.300)	(0.040)	(0.035)	[0.000]
+5%	0.415	3.331	0.331	0.251	-3.152	0.555	0.219	0.510
	(1.239)	(1.391)	(0.412)	(0.001)	(1.281)	(0.029)	(0.199)	[2.231]
+8%	0.054	2.235	1.191	-0.200	1.211	0.310	0.121	0.190
	(0.013)	(0.230)	(0.329)	(0.347)	(2.110)	(0.334)	(0.388)	[5.003]
+10%	0.152	2.315	0.293	0.135	10.231	0.014	0.115	0.031
	(0.214)	(0.322)	(0.158)	(0.392)	(4.112)	(0.512)	(0.331)	[12.320]

Figure 1: Deutsche Bank Commodity Index 60-day Implied versus Realized Variance

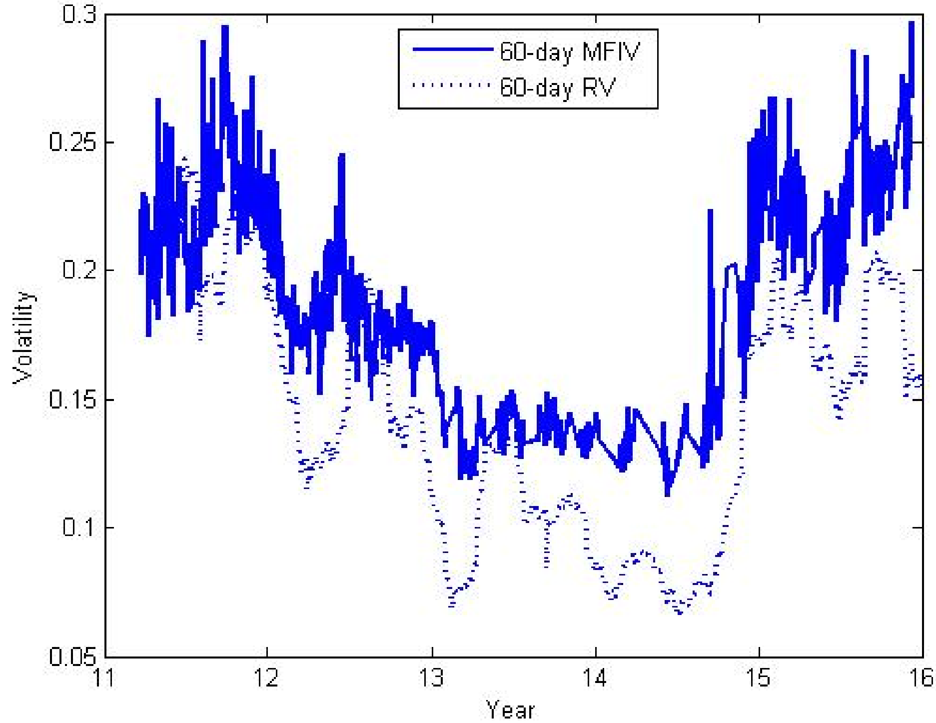

Figure 2: Deutsche Bank Commodity Index 90-day Implied versus Realized Variance

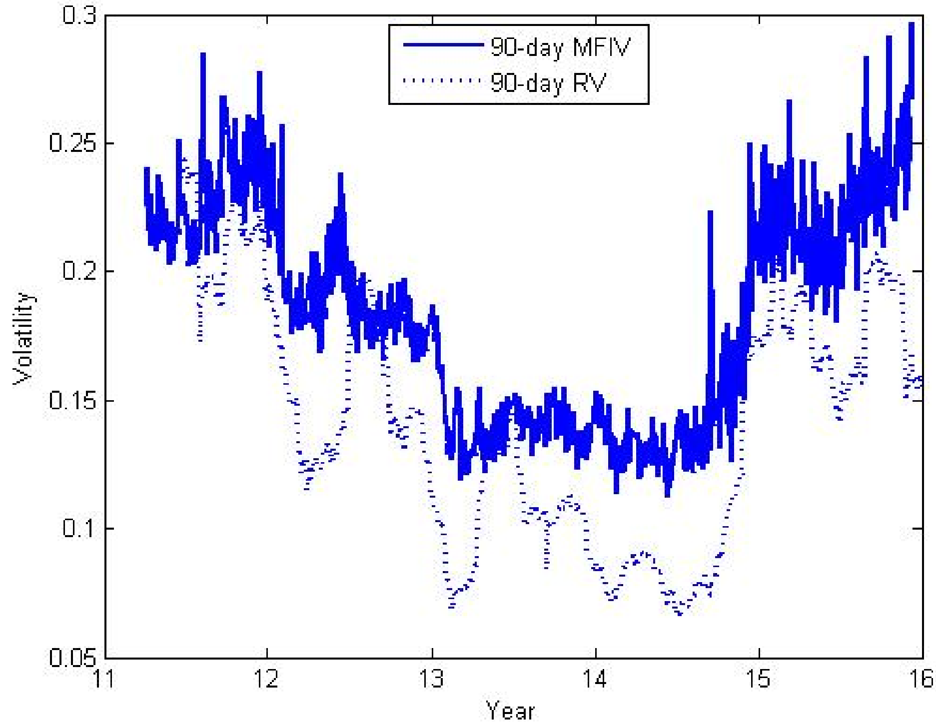

Figure 3: **Correlation between GSCI Energy and Argricultral return exceedances.** This figure depicts the correlation structure of return exceedances between the GSCI Energy log total return and the GSCI Agricultural and Livestock log total return. The solid line shows the correlation between realized return exceedances obtained from the bivariate distribution modeled with the logistic function, as in Table 11. The dotted line dipicts the simulated correlation by assuming a multivariate-normally-distributed return with parameters set to be the sample point estimates. The horizontal axis represents the threshold θ above which a return is defined as exceedance.

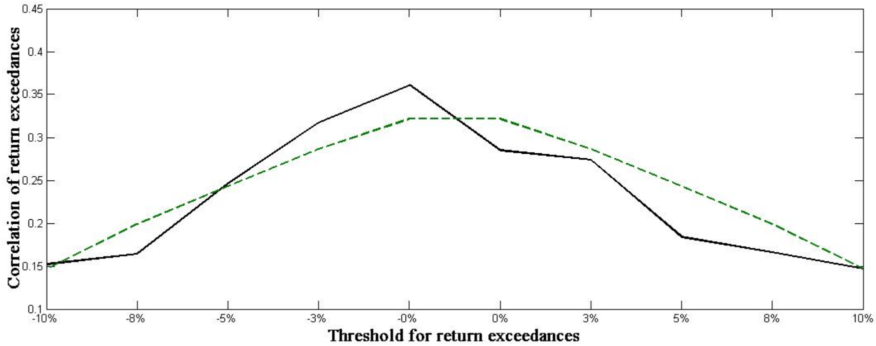

Figure 4: **Correlation between GSCI Energy and Metal return exceedances.** This figure depicts the correlation structure of return exceedances between the GSCI Energy log total return and the GSCI All Metals Capped log total return. The solid line shows the correlation between realized return exceedances obtained from the bivariate distribution modeled with the logistic function, as in Table 12. The dotted line dipicts the simulated correlation by assuming a multivariate-normally-distributed return with parameters set to be the sample point estimates. The horizontal axis represents the threshold θ above which a return is defined as exceedance.