On the Beat of the Drum:
Improving the Flow Shop Performance of the Drum-Buffer-Rope Scheduling Mechanism

Matthias Thürer (corresponding author) and Mark Stevenson

Name: Prof. Matthias Thürer
Institution: Jinan University
Address: Institute of Physical Internet
School of Electrical and Information Engineering
Jinan University (Zhuhai Campus)
519070, Zhuhai, PR China
E-mail: matthiasthurer@workloadcontrol.com

Name: Prof. Mark Stevenson
Institution: Lancaster University
Address: Department of Management Science
Lancaster University Management School
Lancaster University
LA1 4YX - U.K.
E-mail: m.stevenson@lancaster.ac.uk

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Abstract

One of the main elements of the Theory of Constraints is its Drum-Buffer-Rope (DBR) scheduling (or release) mechanism, which controls the release of jobs to the system. Jobs are not released directly to the shop floor – they are withheld in a backlog and released in accordance with the output rate of the bottleneck (i.e. the drum). The sequence in which jobs are considered for release from the backlog is determined by the schedule of the drum, which also determines the order that jobs are processed or dispatched on the shop floor. In the DBR literature, the focus is on the urgency of jobs and the same procedure is used both for backlog sequencing and dispatching. In this study, we explore the potential of using different combinations of rules for sequencing and dispatching to improve DBR performance. Based on controlled simulation experiments in a pure and general flow shop we demonstrate that, although the original procedure works well in a pure flow shop, it becomes dysfunctional in a general flow shop where job routings vary. Performance can be significantly enhanced by switching between a focus on urgency and a focus on the shortest bottleneck processing time during periods of high load.

Keywords: *Drum-Buffer-Rope; Theory of Constraints; Order Release; Dispatching; Flow Shop.*

1. Introduction

The Theory of Constraints – originating in the seminal work of Goldratt (e.g. Goldratt & Cox, 1984; Goldratt, 1990) – is a powerful production planning and control concept for shops with bottlenecks. Many successful implementations have been reported in the literature, with 80% of companies reporting improvements in lead time and due date performance (Mabin & Balderstone, 2003). One of the main elements of the Theory of Constraints is Optimized Production Technology (OPT), its scheduling (or release) mechanism, that is now more commonly known as Drum-Buffer-Rope (DBR) – a descriptor of the way order release is realized (Simons & Simpson, 1997).

Since its inception, DBR has received much research attention. A first stream of early academic literature sought to clarify the meaning of the original concept (e.g. Schragenheim & Ronen, 1990; Luebbe & Finch, 1992). A second stream has compared DBR with other production planning and control concepts, including Material Requirements Planning (MRP; e.g. Duclos & Spencer, 1995; Steele *et al*., 2005), infinite loading (e.g. Chakravorty, 2001), ConWIP (Gilland, 2002), and *kanban* systems (e.g. Lambrecht & Segaert, 1990; Chakravorty & Atwater, 1996; Watson & Patti, 2008) – for a review, the reader is referred to Rahman (1998) and Gupta & Snyder (2009). A third stream of literature has focussed on the performance impact of environmental factors such as set-up times and the percentage of non-bottleneck jobs (e.g. Chakravorty & Atwater, 2005; Golmohammadi, 2015). But despite this broad research attention, few studies have examined the performance contribution made by each component of the DBR system (Chakravorty & Atwater, 2005). As a consequence, the potential to improve performance by refining these components may have been overlooked.

DBR controls (or subordinates) the release of jobs to the system in accordance with the bottleneck (i.e. the constraint or drum). In other words, jobs are not released directly to the shop floor – they are withheld in a backlog (similar to ConWIP; see, e.g. Spearman *et al*., 1990) from where they are released in accordance with the output rate of the bottleneck (drum). While feedback on the output rate of the drum determines *when* a job is released, it does not determine *which* particular job is released. This latter decision is determined by the drum schedule, which also determines the sequence in which released jobs are processed on the shop floor (the dispatching decision).

Both decisions, i.e. the backlog sequencing and dispatching decision, realize the drum schedule. They are thus important components of the DBR system. Both also have a significant impact on performance. For example, backlog sequencing has been recognized as an important means of improving performance in the ConWIP literature (e.g. Leu, 2000;

Framinan *et al*., 2001; Thürer *et al*., 2017a). Meanwhile, it is well established that shop floor dispatching has a significant impact on performance (e.g. Panwalker & Iskander, 1977; Blackstone *et al.*, 1982). Yet the procedure for determining the drum schedule embedded in DBR has remained largely unchanged since its introduction. Moreover, DBR uses the same procedure for backlog sequencing and dispatching. In contrast, in this study, we will explore the potential of different combinations of backlog sequencing and shop floor dispatching rules to improve DBR performance.

The remainder of this paper is organized as follows. In Section 2, we introduce DBR and develop our research question. The simulation model used is then described in Section 3 before the results are presented, discussed, and analyzed in Section 4. Finally, conclusions are drawn in Section 5, where limitations and future research directions are also outlined.

2. Literature Review

A DBR system is depicted in Figure 1 for a single bottleneck station. Its essential parts can be described as follows:

- *Drum*: This is the constraint (e.g. the bottleneck station, the market, etc.) and its schedule.
- *Buffer*: This is both the constraint buffer (i.e. the buffer before the bottleneck) and the shipping buffer (i.e. finished goods inventory; see, e.g. Watson *et al*., 2007). Buffers are time (e.g. Radovilsky, 1998; Rahman, 1998; Schragenheim & Ronen, 1990; Simons & Simpson, 1997; Chakravorty & Atwater, 2005) or a time-equivalent amount of work-in-process. Note that the shipping buffer does not exist in this study since we consider jobs to be delivered immediately upon completion.
- *Rope*: This is the communication channel for providing feedback from the drum to the beginning of the system, i.e. order release. Based on this feedback, order release aligns the input of work with the output rate of the bottleneck. In other words, a maximum limit on the number of jobs released to the bottleneck but not yet completed is established and a job is released whenever the number of jobs is below the limit (e.g. Ashcroft, 1989; Lambrecht & Segaert, 1990; Duclos & Spencer, 1995; Chakravorty & Atwater, 1996; Chakravorty, 2001; Watson & Patti, 2008). There are two ropes: Rope 1 determines the schedule at the bottleneck to exploit the constraint according to the organization's goal (Schragenheim & Ronen, 1990); Rope 2 then subordinates the system to the constraint (the bottleneck station).

[Take in Figure 1]

The drum schedule determines the sequence in which jobs are released to the shop floor (the backlog sequence) and the sequence in which jobs are selected for processing on the shop floor (the dispatching sequence). There are two scenarios. If production is fairly repetitive then the drum schedule is driven by the product mix (see e.g. Luebbe & Finch, 1992; Fredendall & Lea, 1997). If production is high-variety then the drum is driven by urgency considerations. In this study, we focus on a high-variety make-to-order context. A feasible drum schedule in this context is typically derived via a two-step process: backward infinite loading from the customer due date followed by levelling to resolve any overlaps. In other words, a time allowance is subtracted from the due date. Any overlap is then resolved by a simple rule; for example, Chakravorty & Atwater (2005) pushed the schedule back on a first-come-first-served basis. The sequence in which jobs are released from the backlog is then determined by backward infinite loading from the drum schedule, i.e. a second time allowance is subtracted from the bottleneck schedule.

2.1 Problem Statement: The Drum Schedule

An essential element of DBR is the constraint buffer, which is measured in time or a time-equivalent amount of work-in-process. The constraint buffer determines when a job can be released, i.e. as soon as the number of jobs released and on their way to the bottleneck is below a certain limit. While the constraint buffer determines when a job can be released, drum scheduling determines which job can be released, typically by using lead time offsets or allowances to determine the urgency of jobs. Despite its importance, drum scheduling has received relatively little research attention. A major contribution was presented in Radovilsky (1998), which focussed on determining the size of the time allowance used for calculating planned bottleneck start times and release dates. Meanwhile, Simons & Simpson (1997) and Wu & Yeh (2006) explored the use of 'rods' (i.e. a specific time allowance in-between bottlenecks) in shops with more than one bottleneck and shops with re-entrant flows, respectively. Finally, Sirikrai & Yenradee (2006) explored the use of a finite scheduling procedure in jobs with non-identical parallel machines, i.e. in which processing times vary according to which machine is used. However, these procedures assume jobs are known in advance. To the best of our knowledge, the backward infinite loading procedure itself for determining the drum schedule in a stochastic context has never been questioned. The DBR drum schedule calculates a planned bottleneck start time and a planned release date by backward scheduling and bases both the backlog sequencing and the dispatching decision on these urgency measures.

Recent wider literature however has demonstrated the potential of load-based sequencing rules to improve order release performance. For example, Thürer *et al.* (2017a) demonstrated that load-based sequencing can enhance ConWIP's workload balancing capabilities. While workload balancing becomes functionless when there is a strong bottleneck (Thürer *et al.*, 2017b), the positive effect of avoiding starvation through shortest processing time effects still remains. Similar load-based dispatching rules, such as the shortest processing time rule, have long since been shown to reduce flow times (Conway *et al.*, 1967). We therefore ask:

How is the performance of DBR affected by using alternative backlog sequencing and dispatching rules?

An exploratory study based on controlled simulation experiments will be used to provide an answer to this question.

3. Simulation Model

To improve the generalizability of the findings and to avoid interactions that might inhibit full understanding of the effects of the experimental factors, we use a stylized model of a pure flow shop and a general flow shop. The shop and job characteristics modeled in the simulations are first summarized in Section 3.1. How we model DBR and its drum schedule is described in Section 3.2. Finally, the experimental design is summarized and the measures used to evaluate performance are presented in Section 3.3.

3.1 Overview of Modeled Shop and Job Characteristics

Two different flow shops are modeled to assess the impact of shop characteristics on the performance of our backlog sequencing and dispatching rules. The two shops have different degrees of routing variability, but both are characterized by a direct flow – as is typical for DBR simulations (e.g. Lambrecht & Segaert, 1990; Duclos & Spencer, 1995; Chakravorty & Atwater, 1996). This avoids any interaction with potential re-entrant flows. A simulation model of a general flow shop and a pure flow shop has been implemented in the Python$^{©}$ programming language using the SimPy$^{©}$ simulation module. Both shops contain seven stations, where each station is a single resource with constant capacity. This means we do not consider non-identical parallel resources, which allows us to keep our study focused. There is one bottleneck station – Station 4.

As in previous research on bottleneck shops (e.g. Enns & Prongue-Costa, 2002; Fernandes *et al.*, 2014), non-bottlenecks are created by reducing the corresponding processing times.

The reduction in our study is 15%. Note that we experimented with different levels of bottleneck severity but this did not affect our conclusions. Therefore, only one level will be considered in this study. An equal adjustment was applied to all non-bottlenecks since the position of protective capacity is argued to have no effect on flow times (see Craighead *et al*., 2001). Operation processing times – before adjustment – follow a truncated 2-Erlang distribution with a mean of 1 time unit after truncation and a maximum of 4 time units.

For the *general flow shop,* the routing length of jobs varies uniformly from one to seven operations. The routing length is first determined before the routing sequence is generated randomly without replacement. The resulting routing vector (i.e. the sequence in which stations are visited) is then sorted such that the routing becomes directed and there are typical upstream and downstream stations. For the *pure flow shop*, all jobs visit all stations in increasing station number. The inter-arrival time of jobs follows an exponential distribution with a mean of 0.635 time units for the general flow shop and 1.111 time units for the pure flow shop. Both settings deliberately result in a utilization level of 90% at the bottleneck.

Finally, due dates are set exogenously by adding a random allowance factor, uniformly distributed between 28 and 36 time units, to the job entry time. The minimum value will be sufficient to cover a minimum shop floor throughput time corresponding to the maximum processing time (3.4 time units for non-bottleneck operations and 4 time units for the bottleneck operation, which totals 24.4 time units across the seven stations) plus an allowance for the waiting or queuing time. The simulated shop and job characteristics are summarized in Table 1. While in practice any individual high-variety shop will certainly differ from our stylized model, our model captures the high routing variability, processing time variability, and arrival variability that defines this context.

[Take in Table 1]

3.2 Drum-Buffer-Rope

As in previous simulation studies on DBR (e.g. Lambrecht & Segaert, 1990; Duclos & Spencer, 1995; Chakravorty & Atwater, 2005), it is assumed that all jobs are accepted, materials are available, and all necessary information regarding shop floor routings, processing times, etc. is known. Once an order arrives, it flows into the backlog and awaits release. While all jobs visit the bottleneck station in the pure flow shop; in the general flow shop, a job may or may not visit the bottleneck. As in Chakravorty & Atwater (2005), jobs that do not include the bottleneck in their routing are released immediately upon arrival. Twelve buffer limits are applied from 9 to 20 jobs. These limits are based on preliminary simulation experiments. As a baseline, we also include experiments where jobs are released immediately to the shop floor.

3.2.1 The Drum Schedule

Jobs that visit the bottleneck always receive priority over jobs that do not visit the bottleneck, since this was argued to be the best policy for one-of-a-kind production and negligible set-up times (Golmohammadi, 2015). Non-bottleneck jobs are simply processed according to the earliest due date rule. The drum schedule for bottleneck jobs is determined by the different backlog sequencing and dispatching rules, as summarized in Table 2.

[Take in Table 2]

In addition to First Come First Served (FSFS), which is used as a baseline, three alternative types of backlog sequencing/dispatching rules will be considered: (i) urgency based rules, in the form of the Planned Release Date (PRD) sequencing and the Planned Bottleneck Start Time (PBST) dispatching rules; (ii) load-based rules, in the form of the Shortest Bottleneck Processing Time (SBPT) rule; and, (iii) combined urgency and load-based rules, in the form of the Modified Planned Release Date (MODPRD) and the Modified Planned Bottleneck Start Time (MODPBST) dispatching rules.

The calculation of the PBST$_{ij}$ for the i^{th} operation of a job j follows Equation (1) below. A constant allowance c for the operation throughput time is successively subtracted from the planned start time of the preceding operation beginning at the due date δ_j. As in Chakravorty & Atwater (2005), this constant allowance is based on the realized operation throughput times (i.e. the waiting time plus the processing time). It is given by the cumulative moving average, i.e. the average of all operation throughput times realized until the current simulation time. The PBST of the first operation in the routing of a job – PBST$_{1j}$ – is equal to the PRD.

$$PBST_{ij} = \delta_j - (n_j - i + 1) * c \qquad i; 1 \dots n_j \tag{1}$$

n_j – Routing length, i.e. the number of operations in the routing of job j

For MODPRD and MODPBST, orders are divided into two classes: urgent orders for which the PRD has already passed; and non-urgent orders. Urgent orders always receive priority over non-urgent orders and are released according to the SBPT rule. Non-urgent orders are released based on the PRD/PBST rule. Both rules shift between a focus on PRD/PBSTs to complete jobs on time and a focus on speeding up jobs – through SPT effects – during periods of high load, i.e. when multiple jobs exceed their ODD (Land *et al.*, 2015).

3.3 Experimental Design and Performance Measures

The experimental factors are: (i) the two different shop types (General Flow Shop and Pure Flow Shop); (ii) the four different backlog sequencing rules (FCFS, PRD, SBPT, MODPRD); (iii) the four different dispatching rules (FCFS, PBST, SBPT, MODPBST); and (iv) the twelve different buffer limit levels for our release methods (from 9 to 20 jobs). A full factorial design was used with 384 cells (2*4*4*12), where each cell was replicated 100 times. Results were collected over 10,000 time units following a warm-up period of 3,000 time units. These parameters allowed us to obtain stable results while keeping the simulation run time to a reasonable level.

The principal performance measures considered in this study are as follows: the *lead time* – the mean of the completion date minus the pool entry date across jobs; the *percentage tardy* – the percentage of jobs completed after the due date; and, the *mean tardiness* $T_j = \max(0, L_j)$, with L_j being the lateness of job *j* (i.e. the actual delivery date minus the due date of job *j*). In addition to these three main performance measures, we also measure the *shop floor throughput time* as an instrumental performance variable. While the lead time includes the time that an order waits before release, the shop floor throughput time only measures the time after release to the shop floor.

4. Results

Statistical analysis has been conducted by applying ANOVA. ANOVA is here based on a block design with the buffer limit level as the blocking factor, i.e. the different levels of the DBR limit (from 9 to 20 jobs) are treated as different systems. A block design allowed the main effect of the buffer limit and both the main and interaction effects of our four backlog sequencing and dispatching rules to be captured. As can be observed from Table 3, all main effects and two-way interactions were shown to be statistically significant. Meanwhile the dispatching rule has a stronger main effect than the backlog sequencing rule.

[Take in Table 3]

The Scheffé multiple-comparison procedure was used to further prove the significance of the performance differences. Test results, as given in Table 4 for backlog sequencing rules and in Table 5 for the dispatching rules, show significant differences for most rules for at least one performance measure. The only exceptions are the PRD and FCFS sequencing rules, which perform statistically equivalent in the pure flow shop. Detailed performance results for the general flow shop are presented next in Section 4.1 before Section 4.2 presents the results for the pure flow shop.

[Take in Table 4 & Table 5]

4.1. Performance Assessment – General Flow Shop

Figure 2a to Figure 2d show the lead time, percentage tardy, and mean tardiness results over the shop floor throughput time results for FCFS, PBST, SBPT, and MODPBST dispatching, respectively. Only results for the general flow shop are shown as the results for the pure flow shop are assessed in Section 4.2. To aid interpretation, the simulation results are presented in the form of performance curves. The left-hand starting point of the curves represents the lowest DBR limit level (9 jobs). The limit level increases step-wise by moving from left to right on each curve, with each data point representing one limit level. Increasing the limit increases the level of work-in-process and, as a result, increases shop floor throughput times. Meanwhile, under immediate release, jobs are not withheld in the pool; therefore, the backlog sequencing rule is inactive, which results in all backlog sequencing rules converging on the same point. This single point is located to the far right since it leads to the highest level of work-in-process and, consequently, the longest shop floor throughput times.

[Take in Figure 2]

The following can be observed from the results on the performance of the various backlog sequencing and dispatching rules and on the performance of different combinations of rules:

- *Backlog Sequencing Rule*: The performance of the backlog sequencing rules can be evaluated by comparing the curves within each figure. As expected, SBPT reduces lead times and the percentage tardy; it is the best-performing backlog sequencing rule in terms of these two measures, but this is at the expense of mean tardiness performance. The best mean tardiness performance is achieved by MODPRD. Meanwhile, PRD – which is the original backlog sequencing rule embedded in DBR systems – leads to the worst mean tardiness performance. This effect is similar to the one observed in Thürer *et al.* (2017a) in the context of ConWIP systems. The procedure for calculating the PRD considers the routing length, i.e. the number of stations in the routing of jobs. As a result, the more stations there are in the routing of a job, the higher the priority of the job amongst jobs with similar due dates.

- *Shop Floor Dispatching Rule*: The performance of the shop floor dispatching rules in isolation can be evaluated by comparing the results for immediate release (IMM, the single right-hand point) across Figure 2a to Figure 2d, i.e. where no backlog sequencing rule is applied. Surprisingly, in the light of findings from balanced shops (e.g. Land *et al*., 2015), PBST dispatching (Figure 2b) results in a higher percentage tardy and mean tardiness than FCFS dispatching (Figure 2a). This may be explained by the fact that, for PBST, the more stations there are after the bottleneck station, the higher the priority of the job amongst jobs with similar due dates. As expected, SBPT (Figure 2c) leads to the shortest lead times while MODPBST (Figure 2d) outperforms all other dispatching rules in terms of percentage tardy and mean tardiness.
- *Backlog Sequencing x Dispatching Rule*: The interaction effect between backlog sequencing and dispatching rules can be evaluated by comparing the performance curves across Figure 2a to Figure 2d. Performance differences between backlog sequencing rules are similar for FCFS and PBST dispatching (Figure 2a and Figure 2b, respectively). Meanwhile, and as expected, performance differences between backlog sequencing rules diminish under SBPT dispatching (Figure 2c). Finally, we see a shift in terms of percentage tardy under MODPBST dispatching (Figure 2d), where PRD backlog sequencing becomes the worst-performing rule. Arguably the best performance in the general flow shop can be achieved by combining MODPRD backlog sequencing and MODPBST dispatching. It is therefore this combination that should be applied in general flow shops in practice.

4.2. Performance Assessment – Pure Flow Shop

Figure 3a to Figure 3d show the lead time, percentage tardy and mean tardiness results over the shop floor throughput time results for the pure flow shop for FCFS, PBST, SBPT, and MODPBST dispatching, respectively.

[Take in Figure 3]

The following can be observed from the results:

- *Backlog Sequencing Rule*: As suggested by our statistical analysis, PRD and FCFS result in similar performance outcomes. A key factor determining the performance of PRD in the general flow shop was that PRD considers the routing length and thus prioritizes jobs with long routings. This factor disappears in the pure flow shop since all jobs have to visit all stations in the same sequence. Again, SBPT reduces the percentage tardy at the expense of mean tardiness while MODPRD arguably leads to the best trade-off in terms of percentage tardy and mean tardiness performance.

- *Shop Floor Dispatching Rule*: As for the general flow shop, SBPT (Figure 3c) leads to the lowest lead time while MODPBST (Figure 3d) outperforms all other dispatching rules in terms of the percentage tardy. For BPST, the negative effect of prioritizing jobs with more stations downstream of the bottleneck disappears since all jobs have to visit all stations in the same sequence. As a consequence, PBST (Figure 3b) outperforms FCFS (Figure 3a) in terms of the percentage tardy and, overall, it is this rule that now leads to the best performance in terms of mean tardiness.
- *Backlog Sequencing x Dispatching Rule*: As for the general flow shop, performance differences between backlog sequencing rules are similar for FCFS and PBST dispatching (Figure 3a and Figure 3b respectively) and diminish under SBPT dispatching (Figure 3c). However, which combination of backlog sequencing and dispatching rule to choose to adopt in practice is less clear in the pure flow shop. While we argue that MODPRD and MODPBST is still the best choice (Figure 3d), the MODPBST backlog sequencing rule could also be substituted for the simpler FCFS rule (or even PBST).

5. Conclusions

One of the main elements of the Theory of Constraints is its Drum-Buffer-Rope (DBR) scheduling (or release) mechanism. DBR controls (or subordinates) the release of jobs to the system; jobs are not released directly to the shop floor – they are withheld in a backlog from where they are released in accordance with the output rate of the bottleneck (the drum). While feedback on the output rate from the drum determines *when* a job is released, it does not determine *which* job is released. The latter decision is determined by the drum schedule, which also creates the sequence in which jobs are to be processed on the shop floor (the dispatching decision). Since the inception of the DBR approach, the same backward infinite loading procedure has been used for both backlog sequencing and dispatching. First, a planned bottleneck start time is calculated by subtracting a time allowance from the due date. Second, the planned release date is calculated by subtracting a time allowance from the planned bottleneck start time. In contrast, we have asked: How is the performance of DBR affected by using alternative backlog sequencing and dispatching rules?

Based on controlled simulation experiments in a pure and general flow shop, we have demonstrated that performance can be significantly enhanced through the use of our modified backlog sequencing/dispatching rules that switch from a focus on urgency – as in the original procedure – to a focus on load in the form of the shortest bottleneck processing time. This switch in focus takes place during high load periods when many jobs become urgent. Meanwhile, although the original procedure works well in a pure flow shop (i.e. where all jobs visit all stations in the same sequence) it has been shown to become dysfunctional in a general flow shop where job routings vary. Before implementing DBR, managers in practice should therefore carefully check the shop's prevailing routing characteristics.

5.1 Limitations and Future Research

A first limitation of our study is that we have only considered one bottleneck position. The bottleneck position determines where in the routing of an order the bottleneck is located and consequently may have an impact on performance in the general flow shop. A second limitation is our focus on simple backlog sequencing and dispatching rules. While this focus is justified by our stochastic make-to-order environment, future research could consider more repetitive production contexts that allow for more advanced drum scheduling procedures, possibly including product mix considerations. A third limitation is the complexity of the environmental setting. While we considered two shop types with different degrees of routing variability, both have directed routings thus avoiding issues such as re-entrant flows. Future research could therefore examine the impact of the drum schedule in more complex contexts such as shops with re-entrant flows, non-identical parallel resources, or convergent/divergent assembly operations.

Our study has re-emphasized the importance of switching between different backlog sequencing/dispatching rules in response to a changing shop situation. The measure that determined when to switch between rules was the urgency of jobs and the set of rules that we switched between were urgency and load-based rules. However, there may be other ways to implement switching behaviour in practice. Thus, future research could explore different measures for determining when to switch and different sets of rules to switch between. Finally, while it is arguably the best known, DBR is only one type of bottleneck-oriented release method. Future research could therefore extend our study to consider other bottleneck-oriented release methods, e.g. in the context of Workload Control.

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Table 1: Summary of Simulated Shop and Job Characteristics

		General Flow Shop	Pure Flow Shop
Shop Characteristics	Routing Variability	Random routing; no-re-entrant flows	Fixed sequence; no-re-entrant flows
	Routing Direction	directed routing	directed routing (PFS)
	No. of Stations	7	7
	Interchangeability of Stations	No interchange-ability	No interchange-ability
	Station Capacities	All equal	All equal
Job Characteristics	No. of Operations per Job	Discrete Uniform[1, 7]	7
	Operation Processing Times (bottleneck)	Truncated 2–Erlang; (mean = 1; max = 4)	Truncated 2–Erlang (mean = 1; max = 4)
	Operation Processing Times (non-bottleneck)	Truncated 2–Erlang (mean = 1; max = 4) times 0.85	Truncated 2–Erlang (mean = 1; max = 4) times 0.85
	Due Date Determination Procedure	Due Date = Entry Time + *d*; *d* U ~ [28, 36]	Due Date = Entry Time + *d*; *d* U ~ [28, 36]
	Inter-Arrival Times	Exp. Distribution; mean = 0.635	Exp. Distribution; mean = 1.111

Table 2: Drum Schedule – Backlog Sequencing and Dispatching Rules

	Bottleneck Jobs		Non-bottleneck Jobs	
Rule Type	Backlog Sequencing	Dispatching	Backlog Sequencing	Dispatching
Baseline - Arguably the simplest rule	*First-Come-First-Served (FCFS):* Orders are released based on their time of arrival.	*First-Come-First-Served (FCFS):* Orders are selected for processing based on their time of arrival.	None	*Earliest Due Date (EDD):* Orders are selected for processing based on their due date
Urgency based - This is the rule originally used in DBR systems in the literature	*Planned Release Date (PRD):* Orders are released based on their PRD. A PRD is calculated by backward scheduling from the planned bottleneck start time.	*Planned Bottleneck Start Time (PBST):* Orders are selected for processing based on their PBST, which is calculated by backward scheduling from the due date.		
Load-based	*Shortest Bottleneck Processing Time (SBPT):* Orders are released based on the processing time at the bottleneck station.	*Shortest Bottleneck Processing Time (SBPT):* Orders are selected for processing based on the processing time at the bottleneck station.		
Urgency and load-based - This is a variant of the Modified Operation Due Date rule (MODD; e.g. Baker & Kanet, 1983)	*Modified Planned Release Date (MODPRD):* Orders are subdivided into two classes: urgent orders for which the PRD has already passed and non-urgent orders. Urgent orders always receive priority over non-urgent orders and are released based on SBPT. Non-urgent orders are released based on PRD.	*Modified Planned Bottleneck Start Time (MODPBST):* Orders are subdivided into two classes: urgent orders for which the PBST has already passed and non-urgent orders. Urgent orders always receive priority over non-urgent orders and are released based on SBPT. Non-urgent orders are released based on PBST.		

Table 3: ANOVA Results

Shop Type	Performance Measure	Source of Variance	Sum of Squares	df[1]	Mean Squares	F-Ratio	p-Value
General Flow Shop	Lead Time	Limit	198.017	11	18.002	22.720	0.000
		Sequencing (S)	361.869	3	120.623	152.250	0.000
		Dispatching (D)	11674.846	3	3891.615	4912.060	0.000
		S x D	93.462	9	10.385	13.110	0.000
		Error	15189.956	19173	0.792		
	Percentage Tardy	Limit	0.028	11	0.003	15.650	0.000
		Sequencing (S)	0.251	3	0.084	523.890	0.000
		Dispatching (D)	1.231	3	0.410	2567.220	0.000
		S x D	0.273	9	0.030	189.640	0.000
		Error	3.064	19173	0.000		
	Mean Tardiness	Limit	183.578	11	16.689	107.490	0.000
		Sequencing (S)	419.068	3	139.689	899.690	0.000
		Dispatching (D)	559.573	3	186.524	1201.340	0.000
		S x D	144.138	9	16.015	103.150	0.000
		Error	2976.859	19173	0.155		
Pure Flow Shop	Lead Time	Limit	1229500.800	11	111772.800	158.190	0.000
		Sequencing (S)	76286.161	3	25428.720	35.990	0.000
		Dispatching (D)	263703.500	3	87901.167	124.410	0.000
		S x D	133612.150	9	14845.795	21.010	0.000
		Error	13546844.000	19173	706.558		
	Percentage Tardy	Limit	24.684	11	2.244	1064.000	0.000
		Sequencing (S)	6.857	3	2.286	1083.680	0.000
		Dispatching (D)	11.932	3	3.977	1885.920	0.000
		S x D	3.584	9	0.398	188.820	0.000
		Error	40.437	19173	0.002		
	Mean Tardiness	Limit	957635.880	11	87057.807	124.450	0.000
		Sequencing (S)	88682.924	3	29560.975	42.260	0.000
		Dispatching (D)	116048.190	3	38682.731	55.300	0.000
		S x D	124542.440	9	13838.049	19.780	0.000
		Error	13412734.000	19173	699.564		

[1]) degrees of freedom

Table 4: Results for Scheffé Multiple Comparison Procedure: Backlog Sequencing Rules

Shop Type	Sequencing Rule (x)	Sequencing Rule (y)	Lead Time		Percentage Tardy		Mean Tardiness	
			lower[1)]	upper	lower	upper	lower	upper
General Flow Shop	PRD	FCFS	0.016	0.118	-0.006	-0.005	0.319	0.364
	SBPT	FCFS	-0.343	-0.242	-0.010	-0.009	0.041	0.086
	MODPRD	FCFS	-0.070*	0.032	-0.003	-0.002	-0.057	-0.012
	SBPT	PRD	-0.410	-0.309	-0.005	-0.003	-0.300	-0.255
	MODPRD	PRD	-0.137	-0.036	0.002	0.004	-0.398	-0.353
	MODPRD	SBPT	0.222	0.324	0.006	0.008	-0.120	-0.075
Pure Flow Shop	PRD	FCFS	-1.527*	1.507	-0.004*	0.001	-1.531*	1.488
	SBPT	FCFS	2.405	5.439	-0.049	-0.044	3.156	6.175
	MODPRD	FCFS	2.522	5.556	-0.021	-0.016	2.318	5.337
	SBPT	PRD	2.415	5.449	-0.048	-0.043	3.178	6.196
	MODPRD	PRD	2.533	5.566	-0.020	-0.015	2.339	5.358
	MODPRD	SBPT	-1.400*	1.634	0.026	0.031	-2.348*	0.671

[1)] 95% confidence interval; * not significant at α=0.05

Table 5: Results for Scheffé Multiple Comparison Procedure: Shop Floor Dispatching Rules

Shop Type	Dispatching Rule (x)	Dispatching Rule (y)	Lead Time		Percentage Tardy		Mean Tardiness	
			lower[1)]	upper	lower	upper	Lower	upper
General Flow Shop	PBST	FCFS	-0.196	-0.095	0.005	0.006	-0.099	-0.054
	SBPT	FCFS	-1.953	-1.851	0.008	0.009	0.256	0.301
	MODPBST	FCFS	-0.228	-0.127	-0.013	-0.012	-0.204	-0.159
	SBPT	PBST	-1.807	-1.705	0.002	0.004	0.333	0.378
	MODPBST	PBST	-0.083*	0.019	-0.019	-0.017	-0.128	-0.083
	MODPBST	SBPT	1.674	1.775	-0.022	-0.020	-0.483	-0.438
Pure Flow Shop	PBST	FCFS	-6.971	-3.937	-0.011	-0.006	-6.924	-3.905
	SBPT	FCFS	-11.588	-8.555	-0.028	-0.023	-6.549	-3.530
	MODPBST	FCFS	-9.031	-5.998	-0.067	-0.062	-7.797	-4.778
	SBPT	PBST	-6.134	-3.101	-0.020	-0.015	-1.134*	1.885
	MODPBST	PBST	-3.577	-0.544	-0.059	-0.054	-2.382*	0.637
	MODPBST	SBPT	1.040	4.074	-0.042	-0.036	-2.757*	0.262

[1)] 95% confidence interval; * not significant at α=0.05

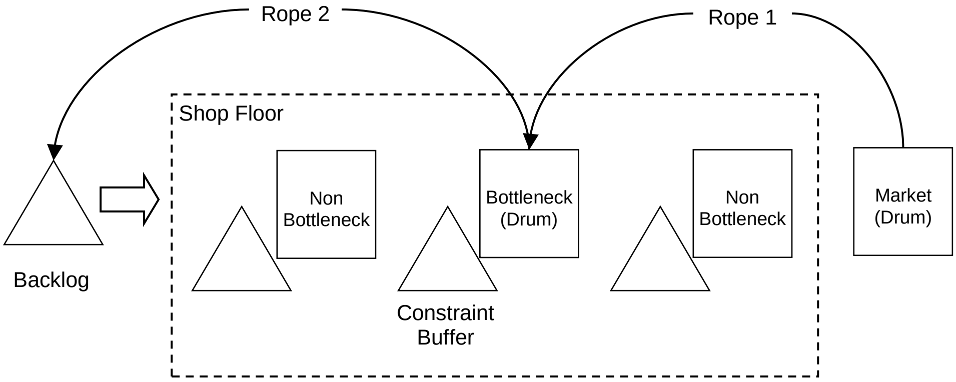
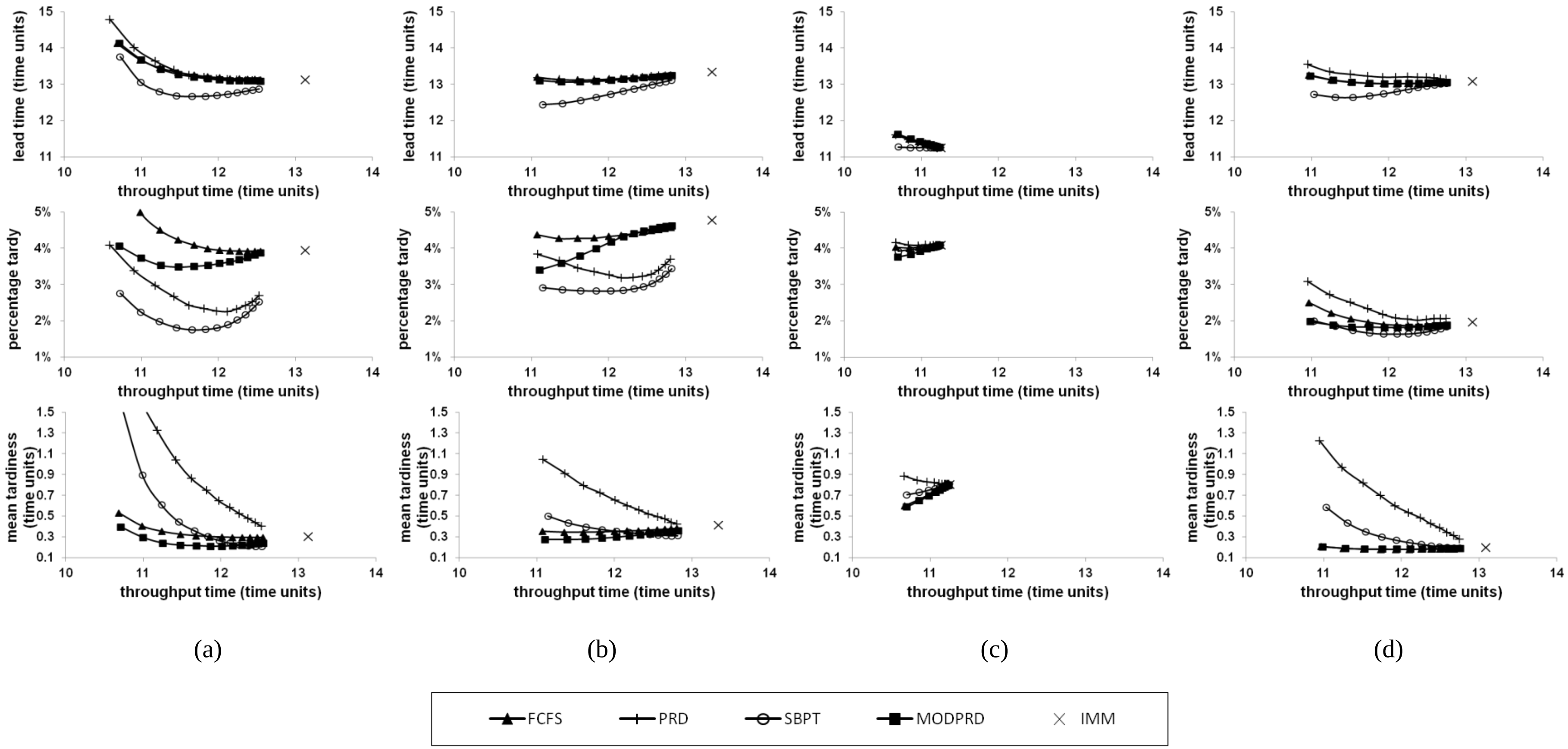
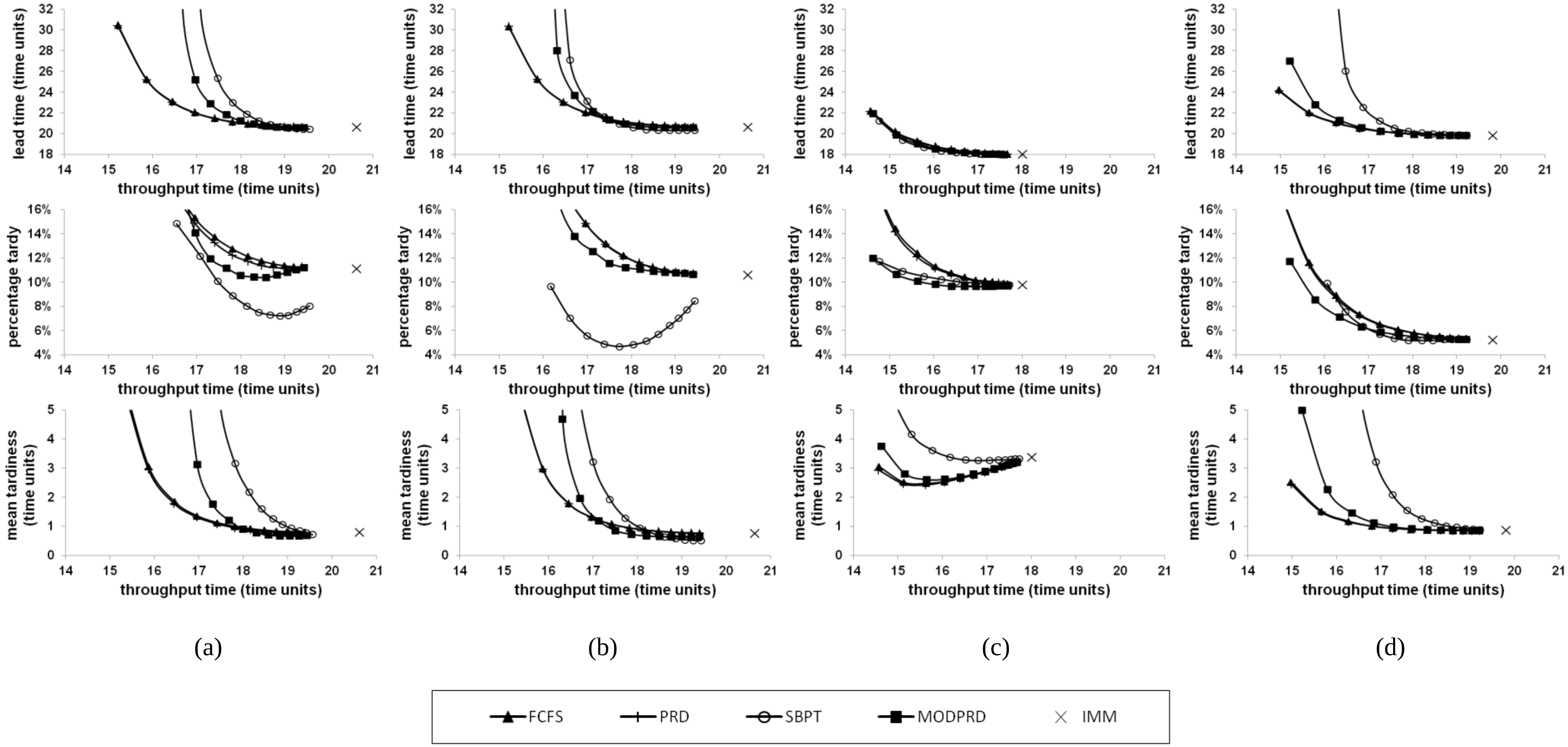


Figure 1: Drum-Buffer-Rope



(a) (b) (c) (d)

Figure 2: Results for the General Flow Shop and (a) FCFS, (b) PBST, (c) SBPT, and (d) MODPBST Dispatching



(a) (b) (c) (d)

Figure 3: Results for the Pure Flow Shop and (a) FCFS, (b) PBST, (c) SBPT, and (d) MODPBST Dispatching

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