NORDITA 2017-101

The spectrum of symmetric teleparallel gravity

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(Dated: October 17, 2017)

General Relativity and its higher derivative extensions have *symmetric teleparallel* reformulations in terms of the non-metricity tensor within a torsion-free and flat geometry. These notes present a derivation of the exact propagator for the most general infinite-derivative, even-parity and generally covariant theory in the symmetric teleparallel spacetime. The action made up of the non-metricity tensor and its contractions is decomposed into terms involving the metric and a gauge vector field and is found to complement the previously known non-local ghost- and singularity-free theories.

The recent detection of gravitational waves [1], or fluctuations in the gravitational field, fully agree with the predictions of General Relativity (GR). As a theory of the metric gravitational field, however, GR remains incomplete in the ultra-violet. Simple but infinite-derivative-order actions that alleviate the singular structure of GR without introducing new degrees of freedom [2–5], have lead to promising results in recent investigations into e.g. quantum loops [6, 7], scattering amplitudes [8, 9], inflation [10, 11], bouncing cosmology [12–14] and black holes [15, 16].

The purpose of this note is to generalise the classification of metric theories from Riemannian (see Ref. [5]) to a more general geometry. Recently it has been suggested that a reconciliation between gravitation as a gauge theory of translations [17, 18] and as a gauge theory of the general linear transformation $GL(4)$ [19, 20] could be achieved by stipulating that the former group of transformations should be the unbroken remainder of the latter, in the frame where inertial effects are absent [21] (where by 'unbroken' we mean that the gauge is fixed in such a way that the affine connection always remains a translation). This logic leads us to the symmetric teleparallel spacetime [22], see also [21, 23–25].

An affine connection $\Gamma^a{}_{bc}$ is invariantly characterised by its curvature,

$$R^a{}_{bcd} = 2\partial_{[c}\Gamma^a{}_{d]b} + 2\Gamma^a{}_{[c|e|}\Gamma^e{}_{d]b}\,, \tag{1}$$

and its torsion, $T^a{}_{bc} = 2\Gamma^a{}_{[bc]}$. In a *teleparallel* spacetime, where $R^a{}_{bcd} = 0$, the inertial connection is given by a general linear transformation $J^a{}_b$ of the trivial vanishing connection solution or "coincident gauge", as $\Gamma^a{}_{bc} = (J^{-1})^a{}_d\partial_b J^d{}_c$, where $(J^{-1})^a{}_d$ are the components of the inverse matrix that parameterises the $GL(4)$ transformation. In a *symmetric teleparallel* spacetime, the torsion also vanishes $T^a{}_{bc} = 0$. It follows that $(J^{-1})^a{}_d\partial_{[b}J^d{}_{c]} = 0$, which indeed leaves us with the coordinate-changing diffeomorphism $J^a{}_b = \partial_b\xi^a$ which was identified with translations (by a vector ξ^a in the tangent space) in the gauging of the Poincaré group already in Ref. [26]. We refer the reader to Fig. 1 for the relations between the eight types of affinely connected spacetimes.

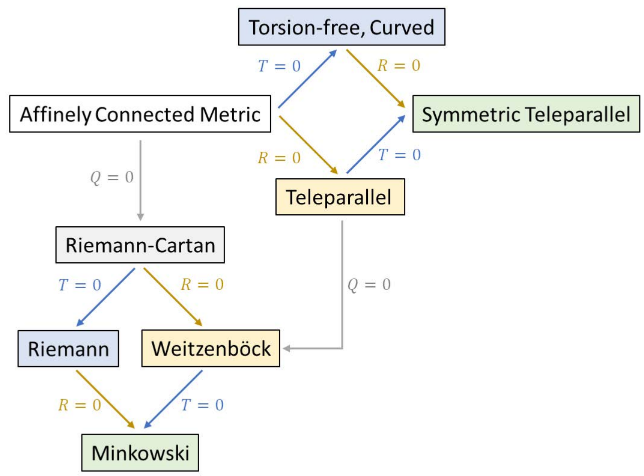

Figure 1. Diagram of affinely connected metric spacetimes illustrating all permutations of the non-metricity tensor Q_{abc}, the Riemann curvature $R^a{}_{bcd}$ and the torsion $T^a{}_{bc}$, where indices have been suppressed for presentation purposes. For more details on these spacetimes see Refs. [18–27].

In the presence of a metric g_{ab}, we can not only define the curvature and torsion but also the non-metricity $Q_{abc} = \nabla_a g_{bc}$, where ∇_a is the covariant derivative with respect to the affine connection $\Gamma^a{}_{bc}$. The non-metricity has two independent traces, which we denote as $Q_a = Q_a{}^b{}_b$ and $\tilde{Q}_a = Q_b{}^b{}_a$. The quadratic form

$$Q^2 = \frac{1}{4}Q_{abc}Q^{abc} - \frac{1}{2}Q_{abc}Q^{bca} - \frac{1}{4}Q_aQ^a + \frac{1}{2}Q_a\tilde{Q}^a\,, \tag{2}$$

is equivalent to the metric Ricci scalar $\mathcal{R}$, up to a boundary term [21]. To be more precise, $\mathcal{R}$ is contraction of the Riemann tensor (1) via $\mathcal{R} = g^{ac}g^{bd}\mathcal{R}_{abcd}$, where the affine connection $\Gamma^a{}_{bc}$ is none other than the Christoffel symbol, while the boundary term is given by [21] $\mathcal{D}_a(Q^a - \tilde{Q}^a)$, where $\mathcal{D}_a$ is the covariant derivative of the Christoffel symbol.

Beltrán *et al* [21] introduced the Palatini formalism for teleparallel and symmetric teleparallel gravity theories. In these notes, however, we adopt the inertial variation [27] as the recipe to obtain the field equations in a covariant form within the desired geometry. Our degrees of freedom are the fluctuations h_{ab} in the metric g_{ab}, and the vector fluctuations u^a that determine the connection $\Gamma^c{}_{ab}$,

$$g_{ab} = \eta_{ab} + h_{ab}\,, \quad \Gamma_{cab} = \partial_a \partial_b u_c\,. \tag{3}$$

From the above, it is straightforward to verify that the components of the non-metricity tensor are given by

$$Q_{abc} = \partial_c (h_{ab} - 2\partial_{(a} u_{b)}), \tag{4}$$

which is by construction a tensor and invariant under the infinitesimal diffeomorphism $\delta_\xi h_{ab} = 2\xi_{(a,b)}$, $\delta_\xi u_a = \xi_a$.

We consider the most general covariant, parity-even Lagrangian that is quadratic in non-metricity. There are 10 possible terms:

$$\begin{aligned}
-2\mathcal{L}_G = &-\frac{1}{2}\left[Q^{abc} a_1(\Box) + 2Q^{bac} b_1(\Box)\right] Q_{abc} \\
&+ \left[\nabla_e Q^{aec} b_2(\Box) + \frac{1}{2}\nabla_e Q^{cea} f_2(\Box)\right] \Box^{-1} \nabla^b Q_{abc} \\
&- \left[Q^a c_2(\Box) + \frac{1}{2}\tilde{Q}^a f_3(\Box)\right] \Box^{-1} \nabla^b \nabla^c Q_{abc} \\
&- \frac{1}{2}\left[Q^a d_1(\Box) Q_a + 2\tilde{Q}^a c_1(\Box) Q_a + 2\tilde{Q}^a b_3(\Box) \tilde{Q}_a\right] \\
&- \frac{1}{2} Q^{aef} f_1(\Box) \Box^{-2} \nabla_e \nabla_f \nabla^b \nabla^c Q_{abc}\,.
\end{aligned} \tag{5}$$

The functions $a_i(\Box)$, $b_i(\Box)$, $c_i(\Box)$, $d_i(\Box)$, $f_i(\Box)$ are analytic functions of the D'Alembertian operator $\Box \equiv g^{ab}\nabla_a\nabla_b$, modulated by a mass scale m, so as to remain dimensionless. The special case with the five non-vanishing constants a_1, b_1, c_1, d_1 and b_3 has been considered previously [21, 23, 24].

We include matter sources by considering the total Lagrangian density to be given by $\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M$, while taking into account the currents at the linear order in the perturbative expansion (3), like so

$$\tau_{ab} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g^{ab}}\,, \quad \tau_a = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta u^a}\,. \tag{6}$$

These are the (linearised) stress energy tensor and the hyperstress vector, respectively. By substituting the expansion (4) into the Lagrangian (5) and varying w.r.t. h_{ab} and u_a, we obtain the equations of motion for the two fields. Thus, the field equation for the metric field can be written as

$$\begin{aligned}
-\tau^{ab} = &\, a\Box \left(h^{ab} - 2\partial^{(a} u^{b)}\right) \\
&+ 2b\left(\partial^c \partial^{(a} h_c{}^{b)} - \partial^{(a} \Box u^{b)} - \partial^c \partial^a \partial^b u_c\right) \\
&+ c\left[\partial^a \partial^b h - 2\partial^a \partial^b \partial^d u_d + \eta^{ab}\left(\partial^e \partial^c h_{ec} - 2\partial^c \Box u_c\right)\right] \\
&+ \eta^{ab} d\Box\left(h - 2\partial^c u_c\right) \\
&+ f\Box^{-1}\left(\partial^a \partial^c \partial^e \partial^b h_{ec} - 2\partial^a \partial^b \partial^c \Box u_c\right)\,,
\end{aligned} \tag{7}$$

while the field equation for the connection field is given by

$$\begin{aligned}
-\frac{1}{2}\tau^a = &\left[a+b\right]\Box\left(\partial^b h_b{}^a - \Box u^a\right) \\
&- \left[(a+b) + 2(c+d) + 2(b+c+f)\right]\Box \partial^c \partial^a u_c \\
&+ \left[b+c+f\right]\partial^b \partial^c \partial^a h_{bc} + \left[c+d\right]\Box \partial^a h\,.
\end{aligned} \tag{8}$$

The reader should note that we have eased the burden of notation by dropping the arguments from the 10 functions appearing in (5) and by grouping them into the 5 functions defined by

$$\begin{aligned}
a &\equiv a_1\,, \\
b &\equiv b_1 + b_2 + b_3\,, \\
c &\equiv c_1 + c_2\,, \\
d &\equiv d_1\,, \\
f &\equiv f_1 + f_2 + f_3\,.
\end{aligned} \tag{9}$$

The first port of call is to establish consistency with known results in the familiar Riemannian geometry. To this end, let us turn our attention to what happens if we eliminate the vector field u^a from the theory. Substituting $u^a = 0$ into (7) and we find, as expected, that the metric field equation does indeed reduce to the known field equation in Riemannian geometry, see [5]. Furthermore, it is straightforward to check that the Bianchi identity i.e. $\partial_a \tau^{ab} = 0$ dictates that the constraints

$$u^a = 0 \quad \Rightarrow \quad a+b = c+d = b+c+f = 0\,, \tag{10}$$

which were established in [5] hold also in this case. Moreover, substitution of these constraints into (8) results in the field equations vanishing identically in the absence of a source, i.e. $\tau^a = 0$.

Having obtained commonality with known results for the most general, parity-even action that is quadratic in curvature within Riemannian geometry, we now turn our attention to GR. From (2), we know that the equivalent of the Einstein-Hilbert Lagrangian is given by $\mathcal{L}_G = Q^2$. Upon reflection, we observe that the Lagrangian (5) reduces to that of GR for the non-vanishing parameters $a_1 = -b_1 = c_1 = -d_1 = 1$, which obey (10), as required. At the linear order, there are indeed many other equivalent theories that remain invariant consistently within the $\Gamma^a{}_{bc} = 0$ gauge, i.e. (10) is satisfied. To be explicit, the action with five free functions,

$$\begin{aligned}
\mathcal{L}_G = &\, Q^2 + b_2(\Box)\left(\tilde{Q}^a Q_a - 2Q^{bac} Q_{abc}\right) \\
&+ b_3(\Box)\left(\tilde{Q}^a Q_a - \nabla_e Q^{aec} \Box^{-1} \nabla^b Q_{abc}\right) \\
&+ c_1(\Box) Q^a \left(\tilde{Q}_a - \Box^{-1} \nabla^b \nabla^c Q_{abc}\right) \\
&+ f_2(\Box)\left(\nabla_e Q^{cea} + \tilde{Q}^a \nabla^b\right) \Box^{-1} \nabla^c Q_{abc} \\
&+ f_3(\Box)\left(\tilde{Q}^a - Q^{aef}\frac{\nabla_e \nabla_f}{\Box}\right) \frac{\nabla^b \nabla^c}{\Box} Q_{abc}\,,
\end{aligned} \tag{11}$$

represents the class of theories that reduce to (symmetric teleparallel) equivalents of GR at the quadratic order. The first line consists of the terms without higher derivatives.

By taking into account the inertial connection, we retain consistency with the Bianchi identities for any choice of parameters to all orders [21]. We demonstrate this important fact, at the linear order, by taking the divergence of (7),

$$
\begin{aligned}
-\partial_a\tau^{ab} &= (a+b)\Box\left(\partial_a h^{ab} - \Box u^b - \partial^c\partial^b u_c\right)\\
&+ (c+d)\left(\Box\partial^b h - 2\Box\partial^c\partial^b u_c\right)\\
&+ (b+c+f)\left(\partial^c\partial^e\partial^b h_{ec} - 2\Box\partial^c\partial^b u_c\right). \quad (12)
\end{aligned}
$$

Comparison with (8) confirms the relation

$$
\partial_a\tau^{ab} = \frac{1}{2}\tau^b\,, \quad (13)
$$

which is our desired conservation law. At this point, we make a further simplification by assuming that matter is *minimally coupled* in that the field u^a does not enter into the matter lagrangian $\mathcal{L}_M$. This is identically true in a vacuum and for canonical scalar and vector fields. For spinor matter the minimal coupling assumes the usual metric Levi-Civita connection recast in non-metric geometry [24].

Our intention now is to express the field equation (7) purely in terms of the metric h_{ab}. In the minimally coupled system, we can neglect the hyperstress vector (6) (i.e. $\tau^a = 0$) and integrate out the connection. By the divergence of the equation of motion (8), we can relate the divergence of the vector u^a to the derivatives of the metric fluctuation like so,

$$
\Box\partial^c u_c = \frac{\alpha-(c+d)}{2\alpha}\partial_e\partial^d h_d{}^e + \frac{(c+d)}{2\alpha}\Box h\,. \quad (14)
$$

Further still, by returning this result into the equation of motion (8), it is possible to express the vector u^a purely in terms of the derivatives of the metric fluctuation,

$$
\begin{aligned}
&-(a+b)\,\Box^2 u^a = -(a+b)\,\Box\partial^b h_b{}^a - \Big\{(b+c+f)\\
&-\frac{[2\alpha-(a+b)]\,[\alpha-(c+d)]}{2\alpha}\Big\}\partial^b\partial^c\partial^a h_{bc}\\
&-\Big\{(c+d) - \frac{[2\alpha-(a+b)]\,[c+d]}{2\alpha}\Big\}\partial^a\Box h\,. \quad (15)
\end{aligned}
$$

Here, we have defined a short-hand notation introducing a parameter α that vanishes in the pure-metric case (10),

$$
\alpha \equiv (a+b) + (c+d) + (b+c+f)\,. \quad (16)
$$

As a cross-check of these relations, we note that (14) and (15) do indeed result in a vanishing divergence of the field equation (7), in accordance with the condition (13) in our minimally coupled prescription $\tau^a = 0$. To eliminate the inertial connection from (7) entirely, we require the combination $\partial^{(a}\Box u^{b)}$ which can be easily deduced from (15).

Substitution then reveals the field equations with the source τ^{ab} purely in terms of the field h^{ab}:

$$
\begin{aligned}
-\tau^{ab} &= a\left(\Box h^{ab} - 2\partial^{(a}\partial^c h_c{}^{b)} + \Box^{-1}\partial^a\partial^c\partial^e\partial^b h_{ec}\right)\\
&+\left[d - \frac{(c+d)^2}{\alpha}\right]\Big[\eta^{ab}(\Box h - \partial^e\partial^c h_{ec})\\
&-\partial^a\partial^b h + \Box^{-1}\partial^a\partial^c\partial^e\partial^b h_{ec}\Big]. \quad (17)
\end{aligned}
$$

As a final piece of book-keeping, let us define

$$
\begin{aligned}
A &\equiv a\,,\\
C &\equiv \frac{(c+d)^2}{(a+b)+(c+d)+(b+c+f)} - d\,. \quad (18)
\end{aligned}
$$

so that the field equation (17) can be expressed in the following useful form

$$
\begin{aligned}
-\tau^{ab} &= A(\Box)\left(\Box h^{ab} - 2\partial^{(a}\partial^c h_c{}^{b)}\right)\\
&- C(\Box)\left(\eta^{ab}\Box h - \eta^{ab}\partial^e\partial^c h_{ec} - \partial^a\partial^b h\right)\\
&+ [A(\Box) - C(\Box)]\,\Box^{-1}\partial^a\partial^c\partial^e\partial^b h_{ec}\,. \quad (19)
\end{aligned}
$$

We now move on to our main task, the propagator for the theory (5). The metric propagator Π_{abcd} is defined via $\Pi^{-1}_{abcd}h^{cd} = \tau_{ab}$ and is determined by the above form of the field equation [5] in terms of the Barnes-Rivers spin projectors [28]. If we first define the transversal projectors for the wavevectors k^a in the Fourier space $\Box \to -k^2$ as

$$
\theta_{ab} = \eta_{ab} - \frac{k_a k_b}{k^2}\,, \quad (20)
$$

we can present the two projectors relevant to us as

$$
P^{(2)}_{abcd} = \theta_{c(a}\theta_{b)d} - \frac{1}{2}\theta_{ab}\theta_{cd}\,, \quad P^{(0)}_{abcd} = \frac{1}{3}\theta_{ab}\theta_{cd}\,, \quad (21)
$$

for spin-2 and spin-0, respectively. The propagator, which is well-known to allow no vector excitations in flat space, is given by

$$
\Pi_{abcd} = \frac{P^{(2)}_{abcd}}{Ak^2} + \frac{3\,(A-C)}{2A\,(A-3C)}\frac{P^{(0)}_{abcd}}{k^2}\,. \quad (22)
$$

Depending on the functions $A(\Box)$ and $C(\Box)$ in (18), the spectrum can contain an arbitrary number of spin-2 and spin-0 modes. The GR form (2) is given by $A(-k^2) = C(-k^2) = 1$, for which the scalar sector decouples. It is then the $k^{-2}P^{(2)}_{abcd}$ sector that propagates the graviton as in the massless representation with two polarisations.

Let us first consider some specific examples of theories that give rise to the spin-0 and the spin-2 components that feature in our main result (22). In terms of the irreducible representations of the non-metricity tensor [20] we can separate a "conom" term that excites the longitudinal mode of the Weyl vector Q^a (and only that when the projective mode carried by $\tilde{Q}^a$ is removed) like so

$$
\mathcal{L}_G = \frac{1}{6}\left(Q^a - \frac{1}{2}\tilde{Q}^a\right)Q_a \quad \Rightarrow \quad \Pi_{abcd} = \frac{P^{(0)}_{abcd}}{k^2}\,. \quad (23)
$$

Furthermore, the pure massless spin-2 mode can be excited, for example, by a "trinom" term

$$
\mathcal{L}_G = \frac{1}{4}Q_{abc}Q^{abc} - \frac{1}{8}Q_a\tilde{Q}^a \quad \Rightarrow \quad \Pi_{abcd} = \frac{P^{(2)}_{abcd}}{k^2}\,. \quad (24)
$$

As is clear from (18) and (9), there are a vast number of theories that reduce to GR at the quadratic order - such as the example (24). Because we take into account the inertial connection, these theories are covariant to all orders [21]. A classification of the theories that remain viable at a non-perturbative order is an interesting task to undertake in future study.

It is easy to see that equivalents of all Riemannian metric theories are contained within the symmetric teleparallel geometry. To show that there also exists theories that do not have a metric equivalent, it is sufficient to consider the Lagrangian (5), where the only non-zero parameter is $a_1 = 1$:

$$\mathcal{L}_G = \frac{1}{4}Q_{abc}Q^{abc} \quad \Rightarrow \quad \Pi_{abcd} = \frac{P^{(2)}_{abcd}}{k^2} + \frac{3P^{(0)}_{abcd}}{2k^2} . \tag{25}$$

The minimal action has the massless graviton, and a massless scalar mode with positive residue[1]. However, this combination appears to be ruled out experimentally as the Newtonian limit would generate the usual time but not the space distortion around spherical sources [30]. The introduction of a mass scale m may lead to a viable theory, where $1/m$ is much smaller than the radius of the Solar system. A simple theory that achieves this is

$$\mathcal{L}_G = \frac{1}{4}Q_{abc}Q^{abc} + \frac{1}{4}\left(\tilde{Q}^a - 2Q^a\right)\left(1 - \frac{\Box}{6m^2}\right)Q_a . \tag{26}$$

For the scalaron to propagate, we must include higher derivatives into the action, since the non-metricity tensor is only first order in terms of derivatives.

Finally, we consider the case of infinite-derivative actions ála string field theory. By modulating the massless GR propagator by a suitable[2] function, such as $A(\Box) = C(\Box) = e^{-\frac{\Box}{M^2}}$, we can improve the scaling $\sim k^{-2}$ of the GR propagator that leads to divergences in the ultra-violet. This is realised by the following infinite-derivative generalisation of the symmetric teleparallel equivalent:

$$\mathcal{L}_G = \frac{1}{4}Q_{abc}e^{-\frac{\Box}{M^2}}P^{abc} , \tag{27}$$

where

$$P_{abc} = \frac{1}{2}Q_{abc} - Q_{(cb)a} - \frac{1}{2}Q_a g_{bc} + \tilde{Q}_{(b}g_{c)a} . \tag{28}$$

If we allow for the introduction of the gauge vector field, we can write an equivalent realisation of (27) like so

$$\mathcal{L}_G = \frac{1}{4}Q_{abc}e^{-\frac{\Box}{M^2}}\left(Q^{abc} - \frac{1}{2}g^{a(b}Q^{c)}\right) , \tag{29}$$

to obtain an asymptotically-free vector-tensor theory of gravitation with one spin-2 field in its spectrum. The model in both gauges reproduces the metric dynamics of the Riemannian theory

$$\mathcal{L}_G \sim \mathcal{R} + \mathcal{R}_{ab}\left(\frac{\exp\left(-\frac{\mathcal{D}^2}{M^2}\right) - 1}{\mathcal{D}^2}\right)\left(\mathcal{R}^{ab} - \frac{1}{2}\mathcal{R}\right) . \tag{30}$$

Thus, the Lagrangians (27), (29) and (30) all have the same propagator in flat space: $\Pi_{abcd} = e^{\frac{-k^2}{M^2}}k^{-2}P^{(2)}_{abcd}$.

Formulation in the coincident frame (27), and its translated equivalent (29), suggest novel realisations for the preferred class of ghost-free and non-singular propagators that were recognised in the exhaustive analysis of metric theories [5]. The elegance of these formulations gives rise to optimism for technical progress in the investigations into infinite-derivative gravity [4–16], but also hint at a possible shortcut towards a finite quantum theory. Non-locality has been recognised as a key to reconcile unitarity with renormalisability [2–5]. In the newly *purified gravity* [21], we further avoid the conceptual difficulty of reconciling the local character of the equivalence principle and the the non-local character of the quantum uncertainty principle [31, 32]. In teleparallel gravity [18], in contrast to GR, it is possible to separate the inertial effects from gravitation and to consider its quantisation. This separation is in-built to our geometry.

[1] Because of the higher derivatives of the metric contained in the quadratic Riemannian invariants, it is impossible to retain the canonical graviton part $a(\Box) = 1$ while simultaneously eliminating the piece that modifies the scalar part, $c(\Box) = 0$ to produce (25), without resorting to non-analytic functions with problematical integral operators such as $1/\Box$ [29, 30].

[2] For a suitable function: 1) $A(-k^2 \to 0) \to 1$ so that GR is recovered in the infra-red, 2) $A(\Box) = e^{\gamma(\Box)}$, where $\gamma(\Box)$ is an entire function, so there are no additional poles, and 3) falls off sufficiently fast as $A(-k^2 \to -\infty) \to \infty$ so as to tame ultra-violet singularities. In the following examples, $\gamma(\Box) = -\Box/M^2$.

[1] B. P. Abbott *et al.* (Virgo, LIGO Scientific), Phys. Rev. Lett. **116**, 061102 (2016), arXiv:1602.03837 [gr-qc].

[2] N. V. Krasnikov, Theor. Math. Phys. **73**, 1184 (1987), [Teor. Mat. Fiz.73,235(1987)].

[3] E. T. Tomboulis, (1997), arXiv:hep-th/9702146 [hep-th].

[4] L. Modesto, Phys. Rev. **D86**, 044005 (2012), arXiv:1107.2403 [hep-th].

[5] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, Phys. Rev. Lett. **108**, 031101 (2012), arXiv:1110.5249 [gr-qc].

[6] S. Talaganis, T. Biswas, and A. Mazumdar, Class. Quant. Grav. **32**, 215017 (2015), arXiv:1412.3467 [hep-th].

[7] L. Modesto and L. RachwaE●, Nucl. Phys. **B900**, 147 (2015), arXiv:1503.00261 [hep-th].

[8] P. Doná, S. Giaccari, L. Modesto, L. Rachwal, and

Y. Zhu, JHEP **08**, 038 (2015), arXiv:1506.04589 [hep-th].
[9] S. Talaganis and A. Mazumdar, Class. Quant. Grav. **33**, 145005 (2016), arXiv:1603.03440 [hep-th].
[10] F. Briscese, L. Modesto, and S. Tsujikawa, Phys. Rev. **D89**, 024029 (2014), arXiv:1308.1413 [hep-th].
[11] A. S. Koshelev, L. Modesto, L. Rachwal, and A. A. Starobinsky, JHEP **11**, 067 (2016), arXiv:1604.03127 [hep-th].
[12] T. Biswas, A. Mazumdar, and W. Siegel, JCAP **0603**, 009 (2006), arXiv:hep-th/0508194 [hep-th].
[13] T. Biswas, T. Koivisto, and A. Mazumdar, JCAP **1011**, 008 (2010), arXiv:1005.0590 [hep-th].
[14] A. Conroy, A. S. Koshelev, and A. Mazumdar, JCAP **1701**, 017 (2017), arXiv:1605.02080 [gr-qc].
[15] V. P. Frolov, Phys. Rev. Lett. **115**, 051102 (2015), arXiv:1505.00492 [hep-th].
[16] A. Conroy, A. Mazumdar, and A. Teimouri, Phys. Rev. Lett. **114**, 201101 (2015), arXiv:1503.05568 [hep-th].
[17] R. P. Feynman, *Feynman lectures on gravitation*, edited by F. B. Morinigo, W. G. Wagner, and B. Hatfield (1996).
[18] V. C. de Andrade, L. C. T. Guillen, and J. G. Pereira, Phys. Rev. Lett. **84**, 4533 (2000), arXiv:gr-qc/0003100 [gr-qc].
[19] C. J. Isham, A. Salam, and J. Strathdee, Annals of Physics **62**, 98 (1971).
[20] F. W. Hehl, J. D. McCrea, E. W. Mielke, and Y. Ne'eman, Phys. Rept. **258**, 1 (1995), arXiv:gr-qc/9402012 [gr-qc].
[21] J. Beltran, L. Heisenberg, and T. Koivisto, "Coincident General Relativity" (2017).
[22] J. M. Nester and H.-J. Yo, Chin. J. Phys. **37**, 113 (1999), arXiv:gr-qc/9809049 [gr-qc].
[23] M. Adak, M. Kalay, and O. Sert, Int. J. Mod. Phys. **D15**, 619 (2006), arXiv:gr-qc/0505025 [gr-qc].
[24] M. Adak, z. Sert, M. Kalay, and M. Sari, Int. J. Mod. Phys. **A28**, 1350167 (2013), arXiv:0810.2388 [gr-qc].
[25] I. Mol, Adv. Appl. Clifford Algebras **27**, 2607 (2017), arXiv:1406.0737 [physics.gen-ph].
[26] T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961).
[27] A. Golovnev, T. Koivisto, and M. Sandstad, Class. Quant. Grav. **34**, 145013 (2017), arXiv:1701.06271 [gr-qc].
[28] P. Van Nieuwenhuizen, Nucl. Phys. **B60**, 478 (1973).
[29] S. Deser and R. P. Woodard, Phys. Rev. Lett. **99**, 111301 (2007), arXiv:0706.2151 [astro-ph].
[30] A. Conroy, T. Koivisto, A. Mazumdar, and A. Teimouri, Class. Quant. Grav. **32**, 015024 (2015), arXiv:1406.4998 [hep-th].
[31] C. Lammerzahl, Gen. Rel. Grav. **28**, 1043 (1996), arXiv:gr-qc/9605065 [gr-qc].
[32] R. Y. Chiao, (2003), arXiv:gr-qc/0303100 [gr-qc].