Innovation across cities

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Abstract

This paper examines the distribution of patenting activity across cities in the OECD, using a sample of 218 cities from 2000 to 2008. We obtain three main results. First, patenting activity is more concentrated than population and GDP. Second, patenting activity is less persistent than population and GDP, especially in the middle of the distribution. Third, in a parametric model, patenting does not exhibit mean-reversion, and is positively associated with GDP and population density. Our results suggest that policymakers can influence the amount of innovative activity through the use of appropriate policies.

JEL Classification: R1, O3.

Keywords: Patents; Zipf's Law; transition probability; dynamic panel data; local linear estimator.

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"The mysteries of the trade become no mysteries; but are as it were in the air…"

Alfred Marshall (1920), *Principles of Economics*, 8th Edition, p.225.

1. INTRODUCTION

Since at least Marshall (1920) it has been argued that forces of agglomeration may lead to the formation of industrial clusters, and by extension, cities. As has been discussed in greater detail elsewhere (Krugman, 1991, Fujita et al, 1999), Marshall identified three reasons for the spatial concentration of economic activity: knowledge spillovers, thick markets for specialised skills, and the backward and forward linkages associated with large local markets. Because of the presence of knowledge spillovers, cities are not only the centre of economic activity, but also the focal point of innovative activity. Indeed, if it is argued that innovative activity makes use of all three of Marshall's external economies, then innovative activity should be even more concentrated than economic activity in general. Anecdotal evidence supports this idea; for instance, in 2008 Tokyo had 27 percent of Japan's population, but 32.3 percent of GDP, and 34.3 percent of the number of patents.

This paper explores the distribution of patenting activities across cities, the persistence and growth of patenting in cities, and the determinants of patenting activity. In so doing, we make use of methods developed for the analysis of city populations, and city population thus acts as a useful benchmark to compare with patents. We use a sample of 218 cities from OECD countries, from 2000 to 2008, and obtain three main results. First, patenting is more unevenly distributed across cities than population or GDP. Second, patenting is less persistent than both population and GDP, especially in the middle of the distribution. Third, even after controlling for the endogeneity of some explanatory variables, the number of patents is positively associated with GDP and population density. Taken together our results suggest that it may be possible for policymakers to implement policies that encourage innovation in cities.

Usually in this literature the analysis is performed using a sample of cities within a country. One reason for this is that different countries may have different institutional settings, which may influence the distribution of innovative activity in the country. Our use of data for cities across OECD countries may be defended along the following lines. First, we focus on innovations, and innovators are often highly skilled, footloose people, who may be more likely to move across international borders. In this context, the cities in the sample are the largest cities in each country with a minimum population of 500,000, so may be viewed as substitutes (even if imperfect) by innovators. Second, some of the theoretical literature on city systems (for instance, Gabaix, 1999) shows that, if each region or country follows Zipf's Law (which in turn arises from Gibrat's Law), then the overall distribution will follow Zipf's Law as well. Hence, using a larger geographic region as the sample should not materially influence the analysis. Indeed, we are unable to reject the null hypothesis that Gibrat's Law of proportional growth holds for patents, thus suggesting that the mechanism identified by Gabaix (1999) may apply in our sample. Third, in estimating the determinants of innovative activity, we make use of methods which control for unobserved city-specific effects, so institutional frameworks which are different across countries should not influence the results.

This paper is related to three strands of literature. First, the literature on the production of knowledge in cities is discussed in Audretsch and Feldman (1996, 1999) and has been surveyed in Audretsch and Feldman (2004). This line of research is mainly focussed on the impact of industrial concentration and diversity on the productivity of R&D ("spillovers"). A closely related line of work in Glaeser et al (1992, 1995) investigates the effects of different industrial composition on economic growth in cities. Unlike this literature, our focus is not on R&D spillovers, but rather on the distribution of innovation across cities, and the factors that may explain the distribution.

There is an associated branch of the literature which examines innovative and creative activities in cities. This includes OhUallachain (1999), Berry and Glaeser (2005), Bettencourt et al (2007, 2010), and Strumsky and Thill (2013). However, much of this literature focuses on US cities, and is primarily interested in describing the distribution of innovative activity across cities. In the present paper, we use an international dataset comprising the largest cities in the OECD, thus allowing us to see whether any trends that we observe operate across national boundaries. In addition, whilst we are also interested in how innovative activity is distributed across cities, we extend the analysis to consider the persistence and evolution of innovative activity over time.

Methodologically, since the paper presents evidence on the distribution and growth of innovation in cities, it is related to the literature on the size distribution and growth of cities, as discussed in Gabaix and Ioannides (2004), Eaton and Eckstein (1997), Black and Henderson (2003), Dobkins and Ioannides (1999, 2001), Ioannides and Overman (2001, 2003, 2004), Soo (2005, 2007), and Bosker et al (2008). On the distribution of innovation in cities, we make use of the concept of Zipf's Law (Zipf, 1949), that the size of cities follows a Pareto distribution. Gabaix and Ibragimov (2011) develop a simple way of improving the performance of OLS estimates of Zipf's Law. On the persistence of innovation in cities, we make use of the concept of transition probability matrices. Finally, on the growth of innovation over time, we make use of both parametric and non-parametric approaches to describe the growth patterns of innovation, and the determinants of innovative growth.

The next section discusses the data used in this paper. This is followed in Section 3 by the analysis of the distribution of innovative activity, in Section 4 by the persistence of innovative activity, in Section 5 by the growth of innovative activity, and in Section 6 by the determinants of innovative activity. Because of the wide range of methods used, they will be discussed within each section to maximise clarity. The final section concludes.

2. DATA

The data is obtained from the OECD Metropolitan Database, which contains data for metro areas with a population of 500,000 or more across OECD countries. Metro areas are defined following a harmonised functional definition developed by the OECD in OECD (2012). This is important, since studies using data across countries can be affected by the fact that the data may not be defined consistently across countries. We avoid this by using data from the OECD Metropolitan Database. There are a total of 275 cities from 28 OECD countries. Patent data is available for 218 metro areas from 16 countries from 2000 to 2008, and represents a count of the number of patent applications by the city of the inventor[1]. The dataset also includes other variables, such as population, geographical and administrative information, labour markets, and GDP (measured in US$ in constant prices and constant PPPs with a base year of 2005).

< Place Table 1 here >

< Place Table 2 here >

Table 1 shows the distribution of cities across countries in the data. Most major OECD countries are represented, with the notable exceptions being Canada, Korea, Spain and the

United Kingdom, for which patent data are not available. Table 2 reports the correlation between patenting activity, economic activity as measured by GDP, and population in our sample, for 2008. There is high correlation between all three variables; large cities are also cities with lots of economic activity, and lots of innovative activity. Figure 1 graphically represents the same information as in Table 2[2].

< Place Figure 1 here >

< Place Table 3 here >

Table 3 presents the ten cities with the largest number of patents in 2008, along with their population and GDP, with their 2000 ranks in parentheses. Although in general the cities with the most patents also have the most population and the highest GDP, there are some anomalies. For instance, San Francisco is associated with Silicon Valley, and has a larger number of patents than would be predicted by its population or GDP. Similarly, Boston is associated with biotechnology and the IT cluster of Route 128, while San Diego is a centre for biotechnology and communications technology. Two other features of Table 3 are noteworthy. First, comparing rankings between 2008 and 2000 shows that populations are persistent over time, whereas GDP and patents are less so; we shall return to this in Section 4 below. Second, cities in the United States dominate the table, occupying seven of the top ten patenting cities in 2008; the equivalent number in 2000 was five of the top ten from the United States. This emphasises the United States' dominance in innovation, although it may be partially driven by cities in countries which have been omitted from our sample due to lack of data, for instance London and Seoul.

3. THE DISTRIBUTION OF INNOVATIVE ACTIVITY

In this section we compare the distribution of patents across cities with the distribution of population and economic activity. If the idea behind Marshall's external economies is correct, then we would expect that patents are going to be more highly concentrated than economic activity in general, and that economic activity is in turn going to be more highly concentrated than population.

< Place Table 4 here >

A simple way to compare the distribution of the three variables is to compare the standard deviations of the natural logs of the variables. This is reported in Table 4, where it is clear that population has the smallest standard deviation, followed by GDP and patents. That is, as predicted by Marshall's theory, patents are more concentrated in a small number of cities than economic activity and population. Figure 2 plots the coefficient of variation (standard deviation divided by the mean) over time for the three variables. Not only does Figure 2 show the same patterns as in Table 4, in addition it shows that the coefficient of variation of patenting is decreasing over time, unlike for population and GDP, which have remained fairly constant over the time period of Figure 2. This suggests that patents are becoming less concentrated over time. Figure 3 plots the kernel density functions for the three variables in 2008 (in natural logs), using an Epanechnikov kernel and the Silverman (1986) rule of thumb bandwidth. This figure again shows the greater dispersion of patents compared to the other two variables, which indicates greater concentration of patents in the cities which undertake the most patenting.

< Place Figure 2 here >

< Place Figure 3 here >

An alternative way of comparing the distribution of these variables, which has been popular in the city size literature, is to use Zipf's Law, which states that the size distribution follows a simple Pareto distribution with shape parameter equal to 1. To operationalise this idea, let:

$$R = AS^{-\alpha}, \tag{1}$$

where R is the rank of a city in terms of its size (with the largest city being ranked 1), S is the size of the city used in constructing R, and A and α are parameters. Taking natural logs of equation (1) and adding a random error term ϵ gives:

$$\ln R = \ln A - \alpha \ln S + \epsilon. \tag{2}$$

Thus the Zipf's Law prediction is that there is a linear relationship between the natural log of the rank and the natural log of the size. The parameter α is a measure of the inequality of the distribution; the larger is α, the more equal is the distribution across cities.

< Place Figure 4 here >

Figure 4 plots the scatter diagram of the rank of a city versus its size as measured by population, GDP and number of patents, for 2008, on a log scale with the largest value normalised to 1. The figure shows that, whilst there appears to be a roughly linear relationship between log of rank and log of population, there is pronounced curvature for GDP and especially for patents. Another observation that can be made from Figure 4 is that, overall, population is more equally distributed than GDP, which in turn is more equally distributed than patents. If Marshall's external economies argument is correct, then this is what we would expect; that larger cities are more productive than smaller cities, and this is especially true for innovative activity where proximity to other innovating agents will yield greater external economies than other types of economic activity.

Gabaix and Ioannides (2004) show that OLS estimation of equation (2) leads to biased results, while Gabaix and Ibragimov (2011) show that a simple way to improve OLS estimation of equation (2) is instead to estimate the following equation:

$$\ln\left(R-\tfrac{1}{2}\right)=\ln A-\alpha\ln S+\epsilon, \qquad (3)$$

with the standard error of α being given by $(2/n)^{1/2}\alpha$, where n is the number of cities. The results of estimating equation (3) for each year for population, GDP and patents are presented in Table 5, which reports the values of α. Comparing across the three variables, the coefficients for population are always larger than for GDP, which in turn are always larger than for patents. This confirms the visual inspection of Figure 4 discussed above; population is the most equally distributed across cities, followed by GDP, with patents being the most unequally distributed.

< Place Table 5 here >

Comparing the coefficients across time, the coefficient for population is almost constant over time. The coefficient for GDP shows greater variation over time (although part of the variation is driven by data availability), while the coefficient for patents shows the greatest variation over time. Especially for patents, there appears to be a trend of rising coefficients, which indicates that patenting activity is becoming more dispersed over time. This may indicate that the Marshallian external economies in innovative activity are becoming weaker over time, perhaps in response to developments in communication technology, and supports the analysis using the coefficient of variation in Figure 2. In terms of Zipf's Law (the hypothesis that $\alpha=1$), for this sample of cities, Zipf's Law holds for GDP, but not for population and patents. City populations are more equal in size than would be predicted by Zipf's Law, whereas patents are less equally distributed than the Zipf's Law prediction[3].

4. THE PERSISTENCE OF INNOVATIVE ACTIVITY

In this section we examine how persistent is innovative activity, relative to population and GDP. We make use of transition probability matrices first introduced into the economic growth literature by Quah (1993), and used in the city population literature by Eaton and Eckstein (1997), Dobkins and Ioannides (2000), and Black and Henderson (2003). We group the sample of cities into ten cells in each year. Let F_t be a 10×1 vector which denotes the distribution of sizes across cities at time t. Assume that F_t evolves according to:

$$F_{t+1} = MF_t, \qquad (4)$$

where M is a 10×10 transition probability matrix, mapping the assignment from period t into an assignment in period $t + 1$. Following Dobkins and Ioannides (2000), we define the vector F_t based on the deciles of the distribution[4]. Since we have data from 2000 to 2008, and since population changes only slowly, we present results for the 8-year transition matrix between 2000 and 2008[5].

< Place Table 6 here >

Table 6 presents the results, arranged so as to make the comparison between the three variables (population, GDP and patents) as clear as possible. Overall, patents exhibit less persistence than population and GDP; the diagonal elements of the matrix (in **bold** type) are, on average, smaller for patents than for population and GDP. This is especially true in the middle of the distribution. On the other hand, population and GDP appear to be quite similar in terms of how persistent they are over time[6]. Indeed, the mobility of a city both up and down the distribution of patents is quite large; a city which in the year 2000 was between the 60th and 70th percentiles of the distribution of patents, could by the year 2008 lie anywhere between the 30th and 90th percentiles.

Nevertheless, where patenting activity does exhibit considerable persistence, is at both ends of the distribution. Cities in the bottom 10th percentile of the distribution of patents in the year 2000 only had a 13.6 percent chance of moving up to the 20th percentile by 2008, which is a lower likelihood of transition than for both population and GDP. A similar though less pronounced pattern can be observed at the top of the distribution. What this suggests is that cities that start off with low levels of patenting activity, struggle to develop any innovation capacity (or perhaps choose to specialise in non-innovation-intensive activities); cities with lots of patenting activity benefit from Marshallian external economies, while cities in between may end up in either a virtuous or a vicious cycle of innovation.

5. THE GROWTH OF INNOVATIVE ACTIVITY

In this section we make use of both parametric and nonparametric approaches to examine the growth of innovative activity. Perhaps a natural starting point is to assume that city growth and city size are independently distributed; that is, that city growth obeys Gibrat's Law. We follow Black and Henderson (2003) in estimating the following equation:

$$\ln(S_{it}) - \ln(S_{it-1}) = \beta_i + \delta_t + \gamma \ln(S_{it-1}) + \epsilon_{it}, \quad (5)$$

where β_i are city fixed effects and δ_t are time fixed effects. Hence, both here and in Section 6, the coefficients are identified from within-city, across-time variation in the explanatory variables (and therefore any city- or country-specific effects such as different institutional arrangements, are partialled-out). Equation (5)[7] may be rewritten as follows:

$$\ln(S_{it}) = \beta_i + \delta_t + (1+\gamma)\ln(S_{it-1}) + \epsilon_{it}. \quad (6)$$

The null hypothesis implied by Gibrat's Law is that $\gamma = 0$ or $1+\gamma = 1$. Given the null hypothesis of Gibrat's Law, the error term cannot be serially correlated, so we use a conventional fixed-effects model to estimate equation (6). Here, unlike in the previous section, we make use of data on an annual basis.

< Place Table 7 here >

The estimated values of $1+\gamma$ for population, GDP and patents using the conventional fixed-effects model are reported in columns (1) to (3) of Table 7. Standard errors are clustered by city to allow for heteroskedasticity and within-city correlation in the residuals, and all results reported include both year and city fixed effects. For all three variables of interest, the Gibrat's Law null hypothesis that $1+\gamma = 1$ is rejected in favour of the alternative that $1+\gamma < 1$. That is, rather than random growth, we find evidence of mean-reversion; large cities grow more slowly than small ones. The coefficient is smallest (hence mean reversion is the quickest) for patents, followed by GDP and population. Similarly to the results of the previous section, patents exhibit less persistence than GDP and especially population.

However, parametric models such as equation (6) do not give a complete picture of the relationship between size and growth of cities. Therefore, we supplement equation (6) with a non-parametric estimator. Consider the following general model of the relationship between the size and growth of a city[8]:

$$\Delta S_{it} = m(S_{it}) + \epsilon_{it}, \qquad \epsilon_{it} \sim iid(0, \sigma_\epsilon^2), \quad (7)$$

where $\Delta S_{it} = S_{it+1} - S_{it}$. However, the functional form $m(\cdot)$ is not specified. $m(\cdot)$ may be estimated using a local weighted average estimator:

$$\hat{m}(S_0) = \sum_{i=1}^{N} w_{i0,h}\, \Delta S_{it}, \tag{8}$$

where the weights $w_{i0,h} = w(S_{it}, S_0, h)$ sum to 1. The weights increase as S_{it} becomes closer to S_0. The Nadaraya-Watson or kernel regression estimator (used for instance in the cities literature by Ioannides and Overman, 2003 and Eeckhout, 2004) uses a kernel weighting function $K(\cdot)$, so that:

$$\hat{m}(S_0) = \frac{\frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{S_{it}-S_0}{h}\right)\Delta S_{it}}{\frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{S_{it}-S_0}{h}\right)}. \tag{9}$$

The constant h is the bandwidth of the kernel function. The kernel regression estimator can be obtained by minimising $\sum_{i=1}^{N} K\left(\frac{S_{it}-S_0}{h}\right)(\Delta S_{it} - m_0)^2$ with respect to m_0. That is, the kernel regression estimator is a local constant estimator, because it assumes that $m(S)$ is a constant in the local neighbourhood of S_0. Instead, one can let $m(S)$ be linear in the neighbourhood of S_0, so that $m(S) = a_0 + b_0(S - S_0)$ in the neighbourhood of x_0. The local linear estimator minimises:

$$\sum_{i=1}^{N} K\left(\frac{S_{it}-S_0}{h}\right)\left(\Delta S_{it} - a_0 - b_0(S_{it} - S_0)\right)^2 \tag{10}$$

with respect to a_0 and b_0, where $K(\cdot)$ is a kernel weighting function. Then $\hat{m}(S) = \hat{a}_0 + \hat{b}_0(S - S_0)$ in the neighbourhood of S_0. Fan (1992) and Fan and Gijbels (1996) argue that the local linear estimator has many attractive properties. The local linear estimator is the best among all linear smoothers, and has a smaller bias than the Nadaraya-Watson estimator, especially at the boundaries of the support of S_{it}. Compared to other local regression estimators such as LOWESS (Cleveland, 1979), the local linear estimator is much less computationally intensive. On the other hand, Hansen (2017) argues that the Nadaraya-Watson estimator outperforms the local linear estimator when $m(S)$ is close to a flat line, but the opposite is true when $m(S)$ is non-constant.

In implementing a nonparametric estimator of the type just described, there are three considerations[9]. The first consideration is the degree of polynomial used. Although the local constant and local linear estimators have been described above, these can be extended to local polynomial estimators. However, high-order local polynomial estimators may face the curse of dimensionality; that is, data sparsity becomes more of a problem for higher order polynomials. This increases the variance of the estimate. We therefore use the local linear estimator for its superior performance relative to the Nadaraya-Watson estimator, but also to keep the order of the polynomial low to reduce the curse of dimensionality. A second consideration is the kernel weighting function used; possible choices include the Gaussian, Epanechnikov, uniform, biweight and triweight kernels. The Epanechnikov kernel has the smallest integrated mean squared error (IMSE; see Wand and Jones, 1995), so we use this kernel. However, the difference between the Epanechnikov kernel and other kernels is often small.

A third consideration is the bandwidth h used. Larger values of h will reduce the variance, since more points will be included in the estimate. However, as h increases, the average distance between the local points ans S_0 will also increase, which can result in a larger bias and oversmoothing. We use the rule-of-thumb plugin estimator of the asymptotically optimal constant bandwidth (note this is not the same as Silverman's (1986) rule of thumb bandwidth estimator). A confidence interval is also reported for the local linear estimator. The residual variance at each smoothing point is estimated by locally fitting a polynomial of order 3, and the bandwidth used for the confidence interval is $1.5 \times h$. To implement this estimator, we standardise the size and growth of cities by subtracting the annual mean from the raw data and dividing by the standard deviation. This allows us to pool observations across years.

< Place Figure 5 here >

Figure 5 reports the results of the nonparametric estimates, for the three variables population, GDP and patents, together with a 95 percent confidence interval; the scatterplot of data points has been omitted for clarity. From this figure it can be seen that GDP most closely follows the Gibrat's Law null hypothesis of no relationship between GDP and GDP growth. Even here there is some evidence that cities with larger GDP exhibit slower growth than cities with smaller GDP. For population, cities in the middle of the population distribution grow faster than those at both ends of the distribution. For patents, the confidence bands are much narrower than for the other two variables, and, consistently with the parametric results in Table 7, it is cities with the fewest patents that experience the fastest patent growth rates. However, cities between 1 and 2 standard deviations below the mean experience slower patent growth rates on average. Without additional analysis it is difficult to interpret this finding. However, the result bears some similarity with those obtained by Davis and Weinstein (2008), hence may indicate the presence of mean-reversion or multiple equilibria. Whilst it may be relatively easy for cities with few patents to rapidly increase their patenting rate, it may be more difficult to step up to the next level and join the ranks of the major innovating centres.

6. THE DETERMINANTS OF INNOVATIVE ACTIVITY

In the previous section, one general conclusion that emerged was that cities with relatively fewer patents, experience more rapid growth in patenting activity. In this section we explore this further, and investigate the possible determinants of innovative activity in a city. Similarly to Black and Henderson (2003), we extend equation (6) in the previous section to include additional explanatory variables:

$$\ln(S_{it}) = \beta_i + \delta_t + (1+\gamma)\ln(S_{it-1}) + \boldsymbol{\psi}\mathbf{X_{it}} + \epsilon_{it}, \tag{11}$$

where the vector $\mathbf{X_{it}}$ may include both time-varying and time-invariant variables. Including the lagged dependent variable in equation (11) means that conventional OLS, fixed- and random-effects estimates are all biased. We therefore use the Blundell and Bond (1998) system GMM method in its asymptotically efficient, two-step form. The method estimates a system of two equations; the equation in levels, and in orthogonal deviations (each observation is subtracted from the average of all future available observations). Because of the inclusion of the levels equation, it is possible to recover the coefficients on time-invariant explanatory variables. The reported standard errors are clustered by city so are robust to heteroskedasticity and arbitrary serial correlation within panels, and are corrected for downward bias using the Windmeijer (2005) correction. Time dummies are included in all regressions to reduce the contemporaneous correlation across cities.

The lagged dependent variable is assumed to be endogenous and needs to be instrumented. Under standard system GMM, the variables in the levels equation are instrumented with lags of their own first differences, while the variables in the orthogonal transformed equation are instrumented with lags of the variables in levels. However, this results in the number of instruments being quadratic in the time dimension. To avoid the problem of too many instruments in system GMM (see Roodman (2009b)), we follow the recent literature (Mehrhoff (2009), Kapetanios and Marcellino (2010), Bai and Ng (2010)) and replace the GMM instruments with their principal components. Principal components analysis is run on the correlation matrix of the GMM instruments, and the principal components with the largest eigenvalues are selected as instruments. Additional statistics reported in Table 8 show that in each specification the principal components explain most of the variation in the instruments, and that they perform well based on the Kaiser-Meyer-Olkin measure of sampling adequacy.

In the Appendix, to check the robustness of our results to the instrument selection process just described, we report the results of estimating the same specifications as in Table 8, for (1) the full set of GMM instruments as in Blundell and Bond (1998), and (2) results using bootstrap resampling of the principal-components-based estimates.

As a first step, we re-estimate equation (6) for the three variables population, GDP and patents using the GMM method outlined above. The results are reported in columns (4) to (6) of Table 7. Notably, the GMM results are quite different from the fixed-effects results. The standard errors in all three cases are much larger than those obtained using fixed-effects. This means that we cannot reject the Gibrat's Law null hypothesis that $1+\gamma=1$ for all three variables. That is, we find little evidence of mean reversion in all three variables. That the results are quite different when the GMM method is used, suggests that the endogeneity of the lagged dependent variable is an important issue.

< Place Table 8 here >

Table 8 presents the results of estimating equation (11), which add additional controls to the bivariate regression of equation (6). In columns (1) and (2), population and GDP are included. In column (1), these two variables are assumed to be exogenous, while in column (2) they are assumed to be endogenous and are instrumented in the same way as the lagged dependent variable. Including population and GDP results in a slight increase in the size and significance of the coefficient on lagged patents, when compared with the results in column (6) of Table 7. Controlling for the endogeneity of the other two variables, in column (2), both population and GDP have a positive and significant effect on patents.

Columns (3) and (4) include additional controls. This includes the number of local governments per 100,000 inhabitants of the metropolitan area (capturing the fragmentation of local government), the number of non-contiguous core areas in the metro area (the polycentricity of the city), the share of the total metropolitan population living in the core areas of the city, the population density, and an indicator for whether there is a top-100 university in the city. By including core population, population density and polycentricity, we seek to explore whether the concentration of people (Marshall's knowledge spillovers) affects the degree of innovative activity. The fragmentation of local government may affect the coordination of government policies across local governments, which again may influence innovation. In column (3), all these additional variables are assumed to be exogenous, whereas in column (4), population density and the share of the total metropolitan population living in the core areas of the city are treated as endogenous and are instrumented in the usual way.

Including the presence of a top-100 university as an explanatory variable comes from the idea that knowledge spillovers from university research and research collaborations with local universities may spur private sector research. Early research on such relationships includes Jaffe (1989), and more recently Abramovsky et al (2007). There are three major global university rankings: the Academic Ranking of World Universities (ARWU or the Shanghai Ranking), the Times Higher Education World University Rankings, and the QS World University Rankings. The QS World University Rankings were not available for our sample period, and the other two rankings are available only since 2003 (ARWU) and 2004 (Times). The results reported below make use of the ARWU rankings in 2008, and we code all cities with a top-100 university according to this ranking equal to 1, and all other cities equal to zero. A total of 32 cities in our sample includes at least one top-100 university according to this measure[10].

Once the endogeneity of population density and population share of the core are controlled for in column (4), population density is positively and significantly associated with patenting activity. This suggests that if Marshall's knowledge spillovers are active, one channel via which they operate is through increased interaction because of greater population density. The other four additional variables – the degree of polycentricity, the share of metropolitan population living in the core, the degree of local government fragmentation, and the presence of a top-100 university – do not have statistically significant effects on patenting. Inclusion of these additional variables leaves the coefficients on lagged patents and GDP unchanged in terms of both size and significance. However, population is no longer statistically significant in columns (3) and (4)[11]. This suggests that, controlling for these additional variables, the level of economic activity is more strongly associated with innovative activity than the mere presence of a larger population.

In Table 8, we are unable to reject the null hypothesis of Gibrat's Law, that $1 + \gamma = 1$, for all specifications. That is, there is no evidence of mean reversion; alternatively, we find strong evidence of persistence in patenting. The results of Table 8 sit somewhat uncomfortably with the results in previous sections. For instance, in Table 6 in Section 4, the transition probability matrix showed that patents are persistent at both tails of the distribution, but not in the middle of the distribution. In Figure 5 in Section 5, the local linear estimates show evidence of nonlinearity in the relationship between patenting and the growth of patenting. We speculate that this apparent disparity is due to the fact that the (parametric) approach adopted in this section restricts the relationship between patents and growth of patents to be linear, whereas the nonparametric approaches in previous sections are better-able to capture the true relationship, which differs in different parts of the distribution[12].

We also include a set of diagnostic statistics in Table 8 (similar statistics are reported for the GMM estimates in columns (4) to (6) of Table 7). First, we report the number of instruments used, which ranges from 19 to 33 instruments. These are fairly low, which should mitigate the problem of having too many instruments (see Roodman (2009b))[13], and as discussed above, is because we have used the principal components of the GMM instruments; if we had not done so, column (4) of Table 8 would have had over 150 instruments. Second, we report the Hansen test of over-identification. In the baseline column (1), this has a p-value of 0.13, and takes on similar values as we include additional controls. This suggests reasonable confidence in the validity of our instruments. A third set of test statistics reported is the Arellano and Bond (1991) tests for first- and second-order serial correlation in the first-differenced residuals. We find evidence of first-order serial correlation, but not second-order serial correlation, across all specifications in Table 8. First-order serial correlation is expected in a dynamic panel; that we do not find second-order serial correlation provides evidence that our use of lags as instruments is valid.

7. CONCLUSIONS

Competition among firms drives innovation in a capitalist economy, as firms seek to gain a competitive edge over their rivals. Hence as urbanisation proceeds and economic activity becomes increasingly concentrated in cities, so too does innovative activity. What this paper has set out to do, is to describe and explain the distribution of innovative activity across OECD cities. Although there has been much research on innovation in cities, to our knowledge this is the first paper to compare the distribution of innovation to the distribution of population and economic activity across cities.

Our first main result is that innovation is more highly concentrated than both population and general economic activity. This is suggestive of the role of Marshall's knowledge spillovers as a key driver of innovation. Our second main result is that innovation is less persistent than population or economic activity, especially in the middle of the distribution. Even in the relatively short time period in our sample, cities can become much more (or less) innovative. This gives policymakers hope, that government policy can influence how innovative a city is. Our third main result is that, even after controlling for the endogeneity of some explanatory variables, innovation is positively related to general economic activity and population density. Again this gives policymakers a handle on what types of policies may be more effective at promoting innovation.

The present paper's focus on cities as centres of innovative activity yields both advantages and disadvantages. On the one hand, cities are undoubtedly important; in the OECD, the vast majority of the population lives and works in cities. So thinking about government policies in terms of cities may be the more natural unit of analysis. On the other hand, precisely because cities have not historically been the default unit of analysis, our analysis suffers from data limitations that not only restrict our sample, but also prevent us from digging deeper into the determinants of innovative activity as in Audretsch and Feldman (1996, 1999). Such data is available for different geographical units, and analysis using this data should serve as an important next step in this line of research. In addition, the use of firm-level data on productivity and innovation would enable us to present more direct evidence on knowledge spillovers, rather than the indirect evidence which we obtain here.

Finally, although we find that Gibrat's Law holds in our sample, more could also be done with regard to the assumption made, that it makes sense to combine data across major OECD cities. For instance, the Gibrat's Law equation (6) could be augmented with a set of country indicators interacted with lagged city population; this would yield a set of country-specific coefficients, and a test could be performed for the equality of the country-specific coefficients across countries. Failure to reject the null of equality would indicate that the relationship between size and growth is common across countries, and would be supportive of the aggregation of cities across countries. We have not performed such a test in this paper, since, as shown in Table 1, each country has a different number of cities in the sample, which represents a different fraction of each country's urban population. In this situation, the proposed test may over-reject the null, as we are not performing a like-for-like comparison between countries. As suggested above, the use of data for different geographical units may be one way of addressing this issue.

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NOTES

[1] Are patents an input or an output in the knowledge creation process? Griliches (1990) provides an insightful discussion on the use of patent statistics in Economics, and concludes that, in the absence of detailed R&D data, patent data can be used as an indicator of both inventive input and output.

[2] A simple regression of the natural log of patents against the natural log of population for any one year yields a coefficient which is always larger than 1; this implies that a 1 percent increase in population has a greater than 1 percent effect on patents. A similar result is obtained for a regression of the natural log of patents against the natural log of GDP.

[3] Some papers in the literature (for instance, Eeckhout, 2004) have suggested that a lognormal distribution may be more appropriate for city sizes. Along similar lines, Clauset et al (2009) present a set of techniques to validate and quantify the existence of power laws. We do not pursue these lines of inquiry in this paper.

[4] Ioannides and Overman (2001) discuss further the implications of this way of defining F_t as compared to that used by Eaton and Eckstein (1997) and Black and Henderson (2003), which is based on fractions of the contemporaneous mean. In this paper, since we are comparing the distributions of different variables, a decile-based definition seems more appropriate.

[5] Many of the papers which make use of transition probability matrices on city populations go on to obtain the long run, implied ergodic distribution of city sizes. We do not do so, because the relatively short time period of our sample means there are relatively few off-diagonal elements of the transition matrices, making the calculations sensitive to the choice of cell boundaries. In addition, it would require F_t to be defined based on fractions of the contemporaneous mean (see the previous note) as opposed to our decile-based definition.

[6] Because of the many zero entries in the table, it is not possible to perform a chi-squared test of the similarity between the distributions of the three variables.

[7] Equation (5) is of course just the equation that is estimated in a panel unit root test. Conventional panel unit root tests cannot be used for our data because of the limited time dimension and the fact that we have an unbalanced panel for GDP. See for instance Bosker et al (2008) for an application of tests of this type to German city sizes.

[8] The following exposition follows that in Cameron and Trivedi (2005) and Hansen (2017).

[9] We performed a series of sensitivity analyses based on each of the three considerations below. Whilst there are some differences in the results depending on the choices made, the justification for the results reported is discussed in the text.

[10] Results using the Times ranking are qualitatively similar.

[11] This change in results is not driven by the change in sample size between column (2) and column (4).

[12] It is of course possible to include nonlinear (quadratic, cubic) terms in the parametric regression analysis. Exploratory analysis suggested that it is difficult to obtain statistically significant coefficients for the nonlinear terms. This may indicate that, if nonlinearity does exist in the relationship between size and growth, that the relationship is more complex than can be captured by the addition of quadratic and cubic terms.

[13] Note the instrument count includes the variables assumed to be exogenous, such as the year dummies.

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TABLE 1: Distribution of cities across countries in the sample

Country	Number of cities	Country	Number of cities
Austria	3	Japan	36
Belgium	4	Mexico	31
Denmark	1	Netherlands	5
Estonia	1	Norway	1
Finland	1	Portugal	2
France	15	Spain	8
Germany	24	Sweden	3
Italy	11	United States	72
Total			218

TABLE 2: Correlation between patents, GDP and population, 2008 (N = 218)

	Patents	GDP	Population
Patents	1.000		
GDP	0.833	1.000	
Population	0.797	0.939	1.000

TABLE 3: Top 10 cities with the largest number of patents in 2008

City	Population	Rank	Patents	Rank	GDP	Rank
Tokyo	34,482,744	1(1)	8,727.0	1(2)	1,316,049	1(-)
San Francisco	6,778,659	10(10)	5,138.2	2(1)	463,435	7(5)
Osaka	17,211,140	4(4)	4,451.1	3(4)	534,747	5(-)
San Diego	3,036,850	35(37)	2,689.3	4(10)	160,635	23(18)
Paris	11,529,670	7(7)	2,467.6	5(7)	575,983	4(3)
Boston	3,616,814	29(28)	2,207.5	6(3)	241,083	12(8)
New York	16,453,331	6(5)	2,001.7	7(6)	977,119	2(1)
Los Angeles	16,742,427	5(6)	1,957.7	8(5)	768,032	3(2)
Minneapolis	3,212,176	34(34)	1,672.5	9(11)	174,234	18(16)
Houston	5,363,803	16(17)	1,590.1	10(16)	323,819	9(7)

Notes: Figures in parentheses are ranks in 2000. (-) indicates that data was not available in the year 2000. GDP is in millions of US$.

TABLE 4: Descriptive statistics for population, GDP and patents

Variable	N	Mean	Std. Dev.
$\ln(pop)$	1,332	14.04	0.7547
$\ln(GDP)$	1,332	10.75	0.9210
$\ln(patent)$	1,332	4.906	1.3876

Notes: Statistics reported for a consistent sample of 148 cities for which complete data are available from 2000 to 2008.

TABLE 5: Zipf regressions for population, GDP and patents, by year

Year	(1) Population	(2) GDP	(3) Patents
2000	1.246	1.041	0.330
	(0.119)*	(0.121)	(0.032)**
2001	1.247	1.026	0.350
	(0.119)*	(0.107)	(0.033)**
2002	1.248	1.025	0.375
	(0.119)*	(0.107)	(0.036)**
2003	1.247	0.927	0.358
	(0.119)*	(0.089)	(0.034)**
2004	1.248	0.927	0.370
	(0.119)*	(0.089)	(0.035)**
2005	1.247	0.926	0.384
	(0.119)*	(0.089)	(0.037)**
2006	1.246	0.928	0.387
	(0.119)*	(0.089)	(0.037)**
2007	1.245	0.928	0.397
	(0.119)*	(0.089)	(0.038)**
2008	1.244	0.932	0.426
	(0.119)*	(0.089)	(0.041)**

Notes: † significant at 10%; * significant at 5%; ** significant at 1%. Statistical significance is in terms of the null hypothesis that the coefficient is equal to 1. N = 218 for all years in columns (1) and (3); N = 148 in 2000, N = 184 in 2001 and 2002, N = 217 in 2003 to 2007, and N = 218 in 2008 in column (2). The values reported are the values of α estimated using the Gabaix and Ibragimov (2011) approach in equation (3). Standard errors in parentheses are calculated using the Gabaix and Ibragimov (2011) approach.

TABLE 6: Transition probability matrices for population, GDP and patents, 2000-2008

2000	2008	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	Pop	**71.4**	25.0	3.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	GDP	**85.7**	14.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Patent	**86.4**	13.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2	Pop	25.9	**51.9**	18.5	3.7	0.0	0.0	0.0	0.0	0.0	0.0
	GDP	15.0	**65.0**	15.0	5.0	0.0	0.0	0.0	0.0	0.0	0.0
	Patent	13.6	**63.6**	22.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3	Pop	0.0	17.9	**53.6**	28.6	0.0	0.0	0.0	0.0	0.0	0.0
	GDP	0.0	20.0	**55.0**	15.0	10.0	0.0	0.0	0.0	0.0	0.0
	Patent	0.0	13.6	**36.4**	31.8	9.1	9.1	0.0	0.0	0.0	0.0
0.4	Pop	0.0	0.0	25.9	**48.2**	25.9	0.0	0.0	0.0	0.0	0.0
	GDP	0.0	0.0	25.0	**55.0**	20.0	0.0	0.0	0.0	0.0	0.0
	Patent	0.0	4.6	18.2	**18.2**	22.7	27.3	4.6	4.6	0.0	0.0
0.5	Pop	3.6	0.0	0.0	17.9	**67.9**	10.7	0.0	0.0	0.0	0.0
	GDP	0.0	0.0	5.0	25.0	**65.0**	5.0	0.0	0.0	0.0	0.0
	Patent	0.0	4.8	14.3	38.1	**28.6**	14.3	0.0	0.0	0.0	0.0
0.6	Pop	0.0	3.7	0.0	0.0	7.4	**66.7**	22.2	0.0	0.0	0.0
	GDP	0.0	0.0	0.0	0.0	4.8	**85.7**	9.5	0.0	0.0	0.0
	Patent	0.0	0.0	9.1	4.6	27.3	**31.8**	27.3	0.0	0.0	0.0
0.7	Pop	0.0	0.0	0.0	0.0	0.0	21.4	**64.3**	14.3	0.0	0.0
	GDP	0.0	0.0	0.0	0.0	0.0	10.0	**65.0**	25.0	0.0	0.0
	Patent	0.0	0.0	0.0	9.1	9.1	18.2	**36.4**	22.7	4.6	0.0
0.8	Pop	0.0	0.0	0.0	0.0	0.0	0.0	14.8	**81.5**	3.7	0.0
	GDP	0.0	0.0	0.0	0.0	0.0	0.0	25.0	**55.0**	20.0	0.0
	Patent	0.0	0.0	0.0	0.0	0.0	0.0	31.8	**54.6**	13.6	0.0
0.9	Pop	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.6	**89.3**	7.1
	GDP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	**75.0**	5.0
	Patent	0.0	0.0	0.0	0.0	0.0	0.0	0.0	18.2	**68.2**	13.6
1.0	Pop	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.4	**92.6**
	GDP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	**95.0**
	Patent	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.3	**85.7**

Notes: N = 275 for population, N = 202 for GDP, and N = 218 for patents. The number in each cell shows the probability of transitioning from one decile in 2000 to the corresponding decile in 2008. The values in **bold** are the percentages of cities that remain in the same decile between the two years.

TABLE 7: Test of Gibrat's Law

Variable Estimation method	(1) Population FE	(2) GDP FE	(3) Patents FE	(4) Population GMM	(5) GDP GMM	(6) Patents GMM
$\ln(population)_{t-1}$	0.919			0.969		
	(0.035)**			(0.350)**		
$\ln(GDP)_{t-1}$		0.897			0.654	
		(0.020)**			(0.317)*	
$\ln(patent)_{t-1}$			0.186			0.912
			(0.058)**			(0.137)**
R^2	0.99	0.90	0.31			
N	2,200	2,033	1,744	2,200	2,033	1,744
Cities	275	271	218	275	271	218
F-test $1 + \gamma = 1$	5.34	25.67	198.96	0.01	1.19	0.41
F-test p-value	0.022	0.000	0.000	0.930	0.276	0.522
Instruments				17	14	14
Hansen Test p-value				0.00	0.29	0.20
AB AR(1) Test p-value				0.94	0.06	0.00
AB AR(2) Test p-value				0.94	0.10	0.22
Principal Components				8	5	6
PCA R2				0.84	0.67	0.78
Kaiser-Meyer-Olkin				0.83	0.81	0.87

Notes: † significant at 10%; * significant at 5%; ** significant at 1%. All columns include unreported year and city fixed effects, and the sample covers the time period from 2001 to 2008. Estimation is via fixed effects with standard errors clustered by city in columns (1) to (3), and via the two-step Blundell-Bond (1998) System GMM with Windmeijer (2005) corrected standard errors in columns (4) to (6). The F-test of $1 + \gamma = 1$ is the test of Gibrat's Law of proportional growth. The Hansen test is the test of over-identifying restrictions. The Arellano and Bond tests (AB) are tests for serial correlation in the first-differenced errors, of orders 1 and 2. PCA R2 is the fraction of the variance explained by the principal components, and Kaiser-Meyer-Olkin is a measure of the sampling adequacy of the principal components.

TABLE 8: The determinants of patenting activity (dependent variable: $\ln(patent)_t$)

	(1)	(2)	(3)	(4)
Estimation method	GMM	GMM	GMM	GMM
$\ln(patent)_{t-1}$	0.936	0.973	0.958	0.927
	(0.080)**	(0.042)**	(0.048)**	(0.052)**
$\ln(pop)_t$	0.390	0.623	-0.152	-0.345
	(0.386)	(0.219)**	(3.014)	(0.429)
$\ln(GDP)_t$	0.393	0.525	0.397	0.430
	(0.195)*	(0.236)*	(0.217)†	(0.186)*
$\ln(CorePop)_t$			0.017	0.516
			(1.195)	(1.298)
$\ln(PopDens)_t$			0.063	0.500
			(3.200)	(0.244)*
$\ln(Poly)_t$			-3.963	0.678
			(34.333)	(3.120)
$\ln(Fragment)_t$			-0.672	-0.113
			(0.818)	(0.276)
Top 100 university			5.405	0.987
			(12.209)	(1.614)
N	1,671	1,671	1,671	1,671
Number of cities	218	218	218	218
Instruments	20	19	24	33
F test $1+\gamma=1$	0.64	0.41	0.78	1.96
F test p-value	0.424	0.525	0.378	0.163
Hansen Test p-value	0.13	0.11	0.27	0.18
AB AR(1) Test p-value	0.00	0.00	0.00	0.00
AB AR(2) Test p-value	0.30	0.31	0.32	0.33
Principal components	10	9	11	20
PCA R2	0.93	0.78	0.81	0.89
Kaiser-Meyer-Olkin	0.87	0.93	0.93	0.95

Notes: † significant at 10%; * significant at 5%; ** significant at 1%. All columns include unreported year and city fixed effects. Estimation is via the two-step Blundell-Bond (1998) System GMM with Windmeijer (2005) corrected standard errors. $CorePop$ is the concentration of population in the metropolitan core. $PopDens$ is population density. $Poly$ is the degree of polycentricity of the city. $Fragment$ is the degree of fragmentation of local government. Top 100 university is an indicator for whether there is a top-100 university in the city, as ranked by ARWU. The F-test of $1+\gamma=1$ is the test of Gibrat's Law of proportional growth. The Hansen test is the test of over-identifying restrictions. The Arellano and Bond tests (AB) are tests for serial correlation in the first-differenced errors, of orders 1 and 2. PCA R2 is the fraction of the variance explained by the principal components, and Kaiser-Meyer-Olkin is a measure of the sampling adequacy of the principal components.

FIGURE 1: Scatterplot of patent applications, population and real GDP, 2008 (N = 218)

Patent applications and population. Patent applications and real GDP.

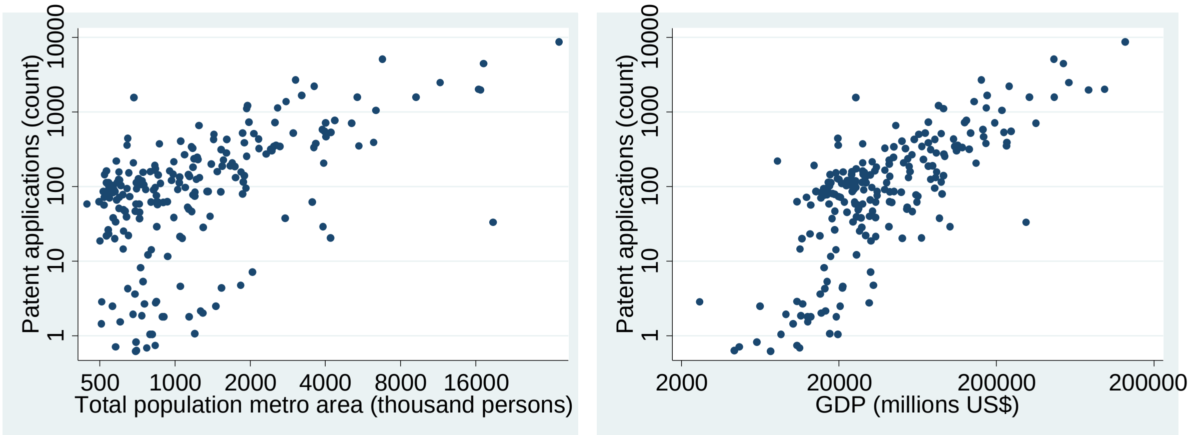


FIGURE 2: Coefficient of variation for population, GDP and patents, for a consistent sample of 148 cities, 2000-2008

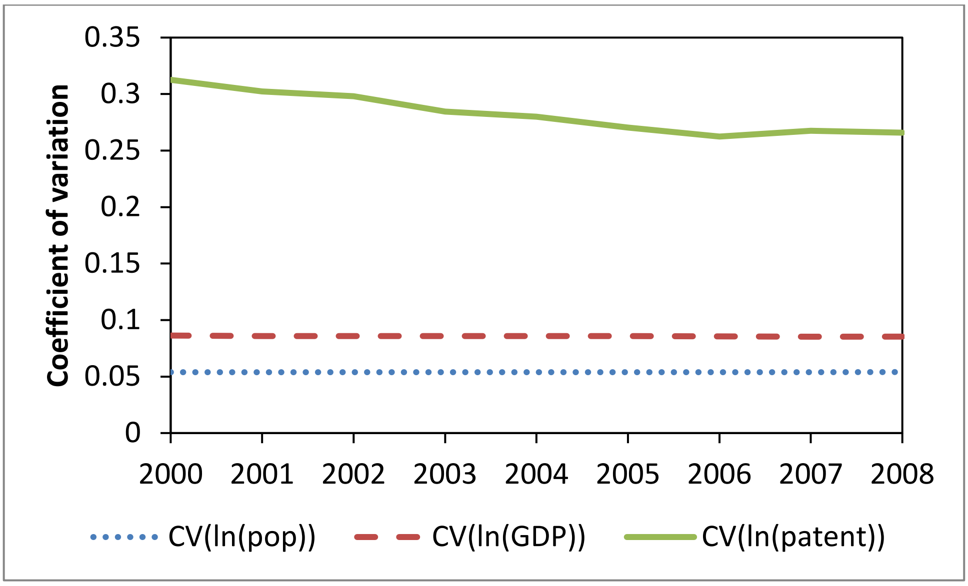
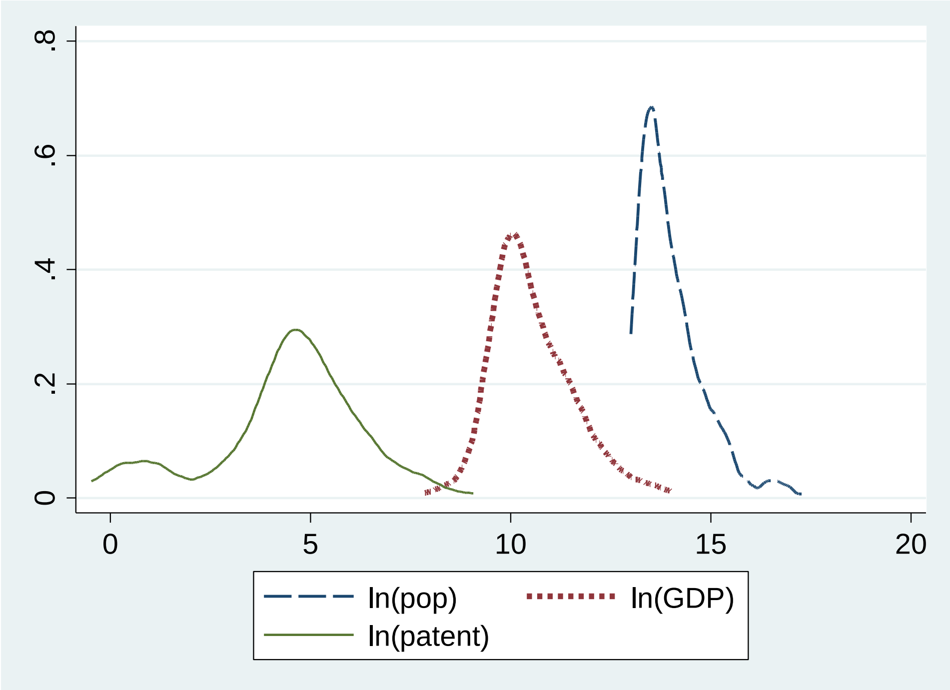


FIGURE 3: Kernel density functions for population, GDP and patents, for 2008, log scale



Notes: Epanechnikov kernel used. Bandwidth is the Silverman (1986) rule-of-thumb bandwidth. Bandwidth = 0.2254 for population, bandwidth = 0.3106 for GDP, bandwidth = 0.4393 for patents.

FIGURE 4: Zipf plots of population, patents and GDP, for 2008, log scale, normalised to the size of the largest city

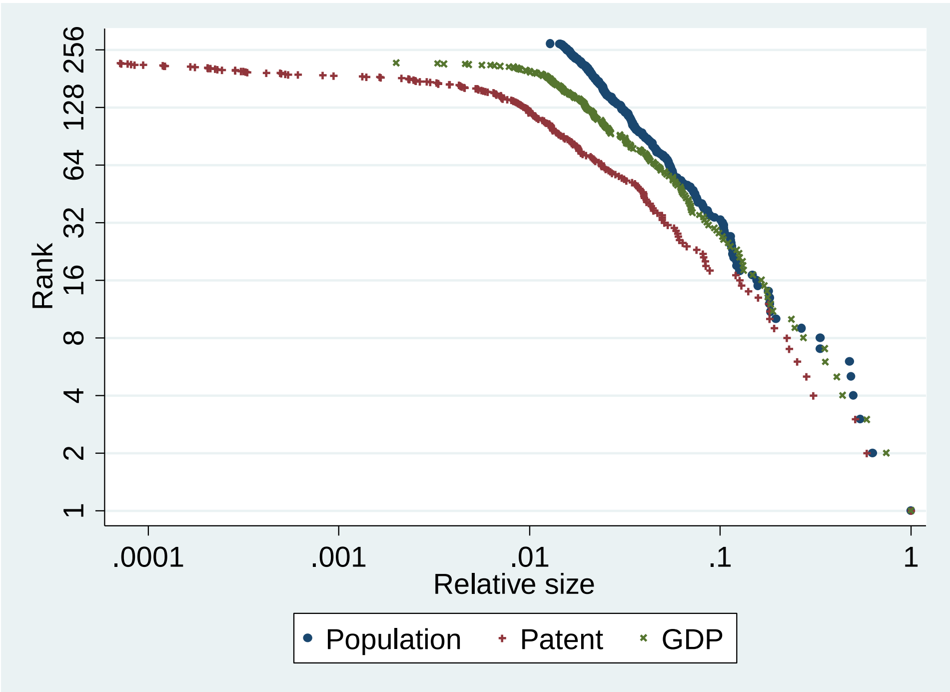
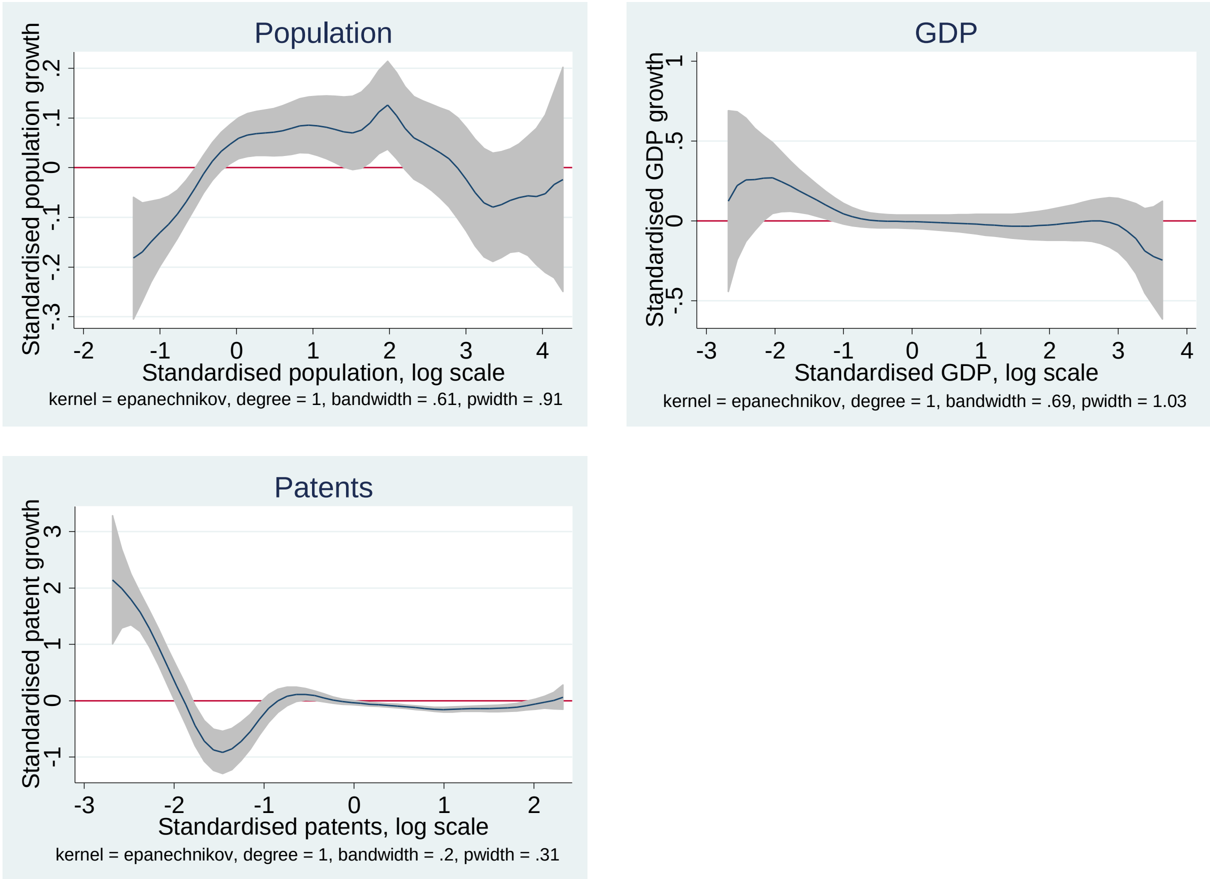


FIGURE 5: Nonparametric local linear estimates of the relationship between city size and city growth



Notes: The shaded area indicates the 95% confidence interval. Bandwidth indicates the bandwidth used for the smoothing, while pwidth = 1.5*bandwidth indicates the bandwidth used for the confidence interval. See the text for more details.

Appendix: Additional results for the determinants of patenting activity

In Section 6, the results were presented using the Blundell and Bond (1998) system GMM method, with the instruments used being the principal components of the GMM instruments, to avoid the problem of too many instruments (Roodman, 2009b). However, this may raise questions about the robustness of the inferences made. In this Appendix, we report two additional results to those reported in Section 6. First, as reported in Table A1, we use the standard GMM instruments instead of the principal components of the GMM instruments. Second, as reported in Table A2, we employ bootstrap resampling methods on the principal components estimates. One thousand replications were performed, with the sample drawn in each replication being a bootstrap sample of cities (i.e. a block bootstrap is used). The results reported in Table A2, in addition to the bootstrapped standard errors, also include a bias correction obtained from the difference between the bootstrap estimates and the GMM estimates reported in Table 8 (see, for instance, Cameron and Trivedi, 2005).

Comparing the results of Table A1 with those of Table 8, using the standard GMM instruments in Table A1 results in a large number of instruments; over 100 in columns (3) to (5). This may give rise to the problem of too many instruments. The use of the principal components of the GMM instruments in Table 8 results in similar performance of the Arellano and Bond (1991) tests, and superior performance of the Hansen test of overidentifying restrictions. In Table 8, we never reject the null hypothesis of overidentification at conventional significance levels, but in Table A1 we reject the null four out of five times. The coefficients on the lagged dependent variable are of similar orders of magnitude, although in Table A1 there is a slightly higher likelihood of rejecting the null hypothesis of Gibrat's Law. In addition, in Table 8 we are able to identify statistically significant coefficients for per capita GDP and population density, which we are unable to in Table A1. Overall, we interpret Table A1 as indicating that using the full set of GMM instruments may lead to incorrect inferences because of the presence of too many instruments, hence justifying the use of the principal components reduction of the instrument set in the text.

The results reported in Table A2 with bootstrap resampling are broadly comparable with those in Table 8, in terms of both the estimated coefficients, and the specification tests. We never statistically reject the Gibrat's Law hypothesis that $1 + \gamma = 1$. In column (5), which is comparable to column (4) in Table 8, GDP and population density are positively and significantly associated with patents. In addition, the presence of a top 100 university and how polycentric the city is, are now positively associated with patents. The positive association between the presence of a top university and patenting is perhaps unsurprising. That a more polycentric city is more innovative, controlling for other variables such as population density, may indicate that different districts specialise in different fields.

TABLE A1: The determinants of patenting activity (dependent variable: $\ln(patent)_t$): Full set of GMM instruments

Estimation method	(1) GMM	(2) GMM	(3) GMM	(4) GMM	(5) GMM
$\ln(patent)_{t-1}$	0.964	0.880	0.976	0.939	0.948
	(0.007)**	(0.054)**	(0.013)**	(0.033)**	(0.046)**
$\ln(pop)_t$		-0.108	0.071	-0.034	0.026
		(0.305)	(0.112)	(0.169)	(0.333)
$\ln(GDP)_t$		0.251	-0.033	0.120	0.004
		(0.223)	(0.064)	(0.147)	(0.231)
$\ln(CorePop)_t$				-0.125	0.164
				(0.241)	(0.497)
$\ln(PopDens)_t$				0.051	0.063
				(0.044)	(0.048)
$\ln(Poly)_t$				0.101	-0.161
				(0.226)	(0.497)
$\ln(Fragment)_t$				-0.019	0.041
				(0.046)	(0.065)
Top 100 university				0.082	0.277
				(0.204)	(0.264)
N	1,744	1,671	1,671	1,671	1,671
Number of cities	218	218	218	218	218
Instruments	35	37	109	112	154
F test $1+\gamma=1$	23.82	4.92	3.54	3.32	1.26
F test p-value	0.000	0.028	0.061	0.070	0.263
Hansen Test p-value	0.01	0.04	0.01	0.00	0.09
AB AR(1) Test p-value	0.00	0.00	0.00	0.00	0.00
AB AR(2) Test p-value	0.25	0.32	0.31	0.32	0.33

Notes: † significant at 10%; * significant at 5%; ** significant at 1%. All columns include unreported year fixed effects. Estimation is via the two-step Blundell-Bond (1998) System GMM with Windmeijer (2005) corrected standard errors. $CorePop$ is the concentration of population in the metropolitan core. $PopDens$ is population density. $Poly$ is the degree of polycentricity of the city. $Fragment$ is the degree of fragmentation of local government. Top 100 university is an indicator for whether there is a top-100 university in the city, as ranked by ARWU. The F-test of $1+\gamma=1$ is the test of Gibrat's Law of proportional growth. The Hansen test is the test of over-identifying restrictions. The Arellano and Bond tests (AB) are tests for serial correlation in the first-differenced errors, of orders 1 and 2.

TABLE A2: The determinants of patenting activity (dependent variable: $\ln(patent)_t$): Bootstrap resampling

	(1)	(2)	(3)	(4)	(5)
Estimation method	GMM	GMM	GMM	GMM	GMM
$\ln(patent)_{t-1}$	0.943	0.885	0.993	0.978	0.966
	(0.142)**	(0.125)**	(0.116)**	(0.136)**	(0.110)**
$\ln(pop)_t$		0.539	0.973	-0.248	-0.532
		(0.512)	(0.414)*	(1.215)	(0.334)
$\ln(GDP)_t$		0.343	0.610	0.351	0.330
		(0.224)	(0.209)**	(0.199)†	(0.185)†
$\ln(CorePop)_t$				-0.088	0.948
				(0.997)	(0.943)
$\ln(PopDens)_t$				-0.235	0.770
				(0.886)	(0.155)**
$\ln(Poly)_t$				-8.397	2.443
				(8.803)	(1.280)†
$\ln(Fragment)_t$				-1.319	-0.149
				(0.668)†	(0.251)
Top 100 university				10.871	2.086
				(6.193)	(1.214)†
N	1,744	1,671	1,671	1,671	1,671
Number of cities	218	218	218	218	218
Instruments	14	20	19	24	33
Test $1 + \gamma = 1$ p-value	0.690	0.357	0.952	0.875	0.762
Hansen Test p-value	0.20	0.13	0.11	0.28	0.18
AB AR(1) Test p-value	0.00	0.00	0.00	0.00	0.00
AB AR(2) Test p-value	0.22	0.30	0.31	0.32	0.33
Principal Components	6	10	9	11	20
PCA R2	0.78	0.93	0.78	0.81	0.89
Kaiser-Meyer-Olkin	0.87	0.87	0.93	0.93	0.95

Notes: † significant at 10%; * significant at 5%; ** significant at 1%. All columns include unreported year and city fixed effects. Estimation is via bootstrap resampling of the two-step Blundell-Bond (1998) System GMM with Windmeijer (2005) corrected standard errors, with 1,000 bootstrap replications. $CorePop$ is the concentration of population in the metropolitan core. $PopDens$ is population density. $Poly$ is the degree of polycentricity of the city. $Fragment$ is the degree of fragmentation of local government. Top 100 university is an indicator for whether there is a top-100 university in the city, as ranked by ARWU. The test of $1 + \gamma = 1$ is the test of Gibrat's Law of proportional growth. The Hansen test is the test of over-identifying restrictions. The Arellano and Bond tests (AB) are tests for serial correlation in the first-differenced errors, of orders 1 and 2. PCA R2 is the fraction of the variance explained by the principal components, and Kaiser-Meyer-Olkin is a measure of the sampling adequacy of the principal components.