Indivisibilities in the Ricardian model of trade

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JEL Classification: F11.
Keywords: Ricardian model; CES preferences; indivisible production; indivisible consumption.

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1. Introduction

This paper is about indivisibilities. Conventional economic analysis often assumes that goods and services are perfectly divisible. However, this may not be true in reality. On the consumption side, consumers can buy one unit of a good, or two units. But they usually cannot buy 1.5 units. Similarly, on the production side, a firm may set up a production facility that produces a certain level of output. To increase production, the firm may choose to set up a second production facility. But it often cannot, without difficulty, alter its output beyond its capacity constraint. This of course is related to the concept of the minimum efficient scale in the industrial organisation literature, and may be viewed as one of the key reasons for the existence of firms.

Consider the case of production indivisibility. Conventional models often assume that workers can divide their time costlessly between different activities. However, the existence of productivity gains from specialisation, or the presence of switching costs, means that most workers are not perfectly divisible. In reality, of course, workers may work part time, or be employed in more than one job; for simplicity the model will abstract from such possibilities. In the United Kingdom, despite the increase in part-time work since the 2008 financial crisis, in 2016 Q4, 73 percent of total employment is full time (with the remainder in part-time work), while 19.3 percent of all workers employed worked over 45 hours a week (this includes paid and unpaid overtime work), and only 3.5 percent of workers had second jobs (source: Office for National Statistics 2017). Hence, although there is evidence of some flexibility in the labour market, there also appears to be significant levels of indivisibility.

Similarly, an example of indivisibility in consumption is durable goods such as cars, which can often only be purchased (and consumed) in discrete quantities. Although durable goods may be shared or rented, renting or sharing is often more costly than (or is not a perfect substitute for) buying, so as before the model will abstract from such possibilities. Again in the United Kingdom, expenditure on durable goods (including transport equipment, furnishing and household goods, information processing equipment, equipment for outdoor recreation, and jewellery) amounted to 10.1 percent of total consumption expenditure in 2016 Q3, while semi-durable goods (including clothing and footwear, furnishing and household goods, games, toys and hobbies, and books) amounted to 10.3 percent of total consumption

(these figures correspond to 25.0 percent and 25.5 percent of total consumption expenditure on goods, respectively) (source: Office for National Statistics 2016). Although not all goods are indivisible, the figures suggest that a significant fraction is.

This paper develops a simple Ricardian model of international trade to analyse the effects of indivisibilities on both the production and consumption sides on the results of the model. It turns out that there are indeed large implications. If the conventional constant elasticity of substitution (CES) utility function is assumed, then the presence of either production or consumption indivisibility implies that a country may be completely specialised in its comparative advantage good, in autarky. In addition, consumption indivisibility implies that ex ante identical consumers may end up consuming different bundles of goods especially when international trade is allowed. This then has implications for the model's predictions on the volume of trade; all of these results depart from those of the standard Ricardian model.

Combining indivisibilities in both production and consumption yields additional insights. In particular, under certain conditions, having both types of indivisibilities is identical to having only consumption indivisibility. If these conditions do not hold, then the possibility also arises of heterogeneity in consumption in autarky. Both types of indivisibility give rise to welfare losses relative to the perfectly divisible case. International trade eliminates the welfare loss associated with production indivisibility, but not that associated with consumption indivisibility. Hence, if consumption indivisibility is a significant feature of the world, it suggests that we live in a second-best world, and therefore in principle there may be government policies which lead to welfare improvements compared to free trade (Lipsey and Lancaster, 1956).

The main results of the paper, although theoretical in nature, have empirical relevance. For instance, the result that complete specialisation occurs in autarky, implies that the gains from trade arise from an increase in the types of goods available for consumption. This source of the gains from trade has been documented empirically by, amongst others, Broda and Weinstein (2006). Similarly, the outcome that consumption may be heterogeneous for consumers with the same income level has been verified by Fisher et al (2015).

That CES preferences are assumed is going to be key in the analysis, especially for production indivisibility. Bhagwati (1967) showed that the proof of the theorem of

comparative advantage depends crucially on assumptions on consumer preferences. That there may be limits to the division of labour has been shown in several papers. Becker and Murphy (1992) showed that coordination costs may limit the gains from the division of labour. Related analyses can be found in Sobel (1992) and Kremer (1993), and in an international dimension in Francois (1990a, 1990b). Krishna and Yavas (2005) introduce consumption indivisibilities in a transition economy. To the best of our knowledge, none of this previous literature has directly addressed the implications of indivisibility in a standard trade model.

Also related is the large literature on international trade under external increasing returns (see especially the surveys by Helpman (1984), Helpman and Krugman (1985) Chapter 3, and Choi and Yu (2003)). In particular, Helpman and Krugman (1985) show that, to replicate the integrated equilibrium, production of the sector subject to (national) increasing returns must be concentrated in a single country; in other words, it is indivisible. Here, we address indivisibility in production directly, and also consider the implications of indivisibility in consumption.

This paper is perhaps closest in its approach to Cheng et al (2000), who introduce transaction costs into the Ricardian model, adopting Yang's (2001, 2003) infra-marginal approach. Similarly, in the present paper, the presence of indivisibilities means that analysing the model through direct comparison between alternative outcomes may be a more appropriate solution method than conventional marginal analysis. Nevertheless, the model we develop and the issues we address in this paper are different from those addressed in Cheng et al (2000).

Finally, in its approach of modifying a key assumption of the simple Ricardian model of trade, the present paper follows a recent literature. A highly selective and incomplete list includes Samuelson (2004) and Shachmurove and Spiegel (2013), who make use of the Ricardian model to analyse the impact of technological change. Goksel (2012) introduces financial constraints and non-homothetic preferences into the Krugman model of trade with monopolistic competition, while Lo (2014) develops a three-country Krugman-type model allowing for offshoring. By selectively modifying familiar models of international trade, each of these papers yields new insights, and this is the objective of the present paper.

The next section provides further background on the implications of indivisibilities, both theoretically and empirically. Section 3 develops the standard Ricardian model, which will serve as the benchmark for the remainder of the analysis. Section 4 considers indivisible production while Section 5 considers indivisible consumption. Section 6 combines both types of indivisibilities, while Section 7 provides some concluding comments.

2. Indivisibilities: Prior theory and evidence

It is easy to make the mistake of thinking that indivisibility is important only at the individual level, and that the implications of indivisibility disappear in a large economy with millions of firms and consumers. This is true for many of the implications of individual-level indivisibility; see Mas-Colell (1987) for a discussion. Yet a moment's thought shows that at least some of the implications of indivisibility may hold true irrespective of whether there are two or two million people in the economy (a similar point was made by, amongst others, Rogerson (1988)). For instance, if there are two million identical consumers, and one million units of a particular indivisible good, then despite the fact that the consumers are identical, one million consumers will consume one unit of the good, while the other million consumers will consume none of the good. In this example, indivisibility gives rise to heterogeneity in consumption amongst ex ante identical consumers. For additional discussion of the possibility and implications of economy-wide non-convexity, see Mas-Colell (1987).

There is in fact a large literature on the theoretical implications of indivisibilities in economics. Good surveys are provided by Baumol (1987) and Scarf (1990, 1994). Briefly, the literature emphasises the role of indivisibilities in leading to economies of scale and scope, and hence resulting in the inefficiency of marginal cost pricing (the second-best prices are some form of nonlinear pricing). Because indivisibility leads to non-convexity, conventional methods cannot be used to obtain the solution; instead, integer programming methods are required[1]. Yang (2001, 2003) takes a different tack, avoiding the intractability of integer programming by focussing on relatively simple models, making use of what he refers to as an infra-marginal approach. This combines the marginal approach with total cost-benefit analysis, and enables the analysis of models with discontinuous jumps in the endogenous variables. More recent theoretical work on indivisibilities is surveyed in Sonmez and Unver

[1] Integer programming is part of a class of intractable mathematical problems categorised as NP-complete.

(2011), who discuss applications to the housing market, kidney exchange, and school admissions.

Indivisibilities have also played a major role in other areas of economics. The idea of indivisibility is implicit in the widespread use of discrete choice models (i.e. probit and logit models) in microeconometrics, in applications ranging from decisions about mobility, marriage, fertility, occupational choice, and many others. Such models became popular in economics following McFadden (1974), who provided a link between the statistical model and consumer theory (see McFadden (2001) for further discussion). In macroeconomics, models with indivisible labour have been used to match the stylised fact of large fluctuations in hours worked relative to fluctuations in productivity (Hansen (1985), Rogerson (1988)). These stylised facts cannot be replicated in standard models with perfectly divisible labour. Along similar lines, Krishna and Yavas (2004) show that indivisible consumption goods may lead to endogenous business cycles.

There has been considerable empirical evidence on the importance of indivisibilities in the economy. One of the most prominent of these is the literature on roscas (Rotating and Credit Saving Associations). Roscas are informal microfinance institutions in which members pool their savings to buy durable, indivisible consumption goods (for instance, a bicycle). Besley et al (1993, 1994) developed the theory of roscas, while Besley and Levenson (1996) and Levenson and Besley (1996) provide empirical evidence on the role of roscas in durables consumption. Models of international trade have also been developed in which factors of production are immobile between regions of a country (Courant and Deardorff (1992)). Such regional factor immobility, which influences the pattern of international trade, may be viewed as a form of indivisibility across regions. This model has been analysed empirically by Debeare (2004) and Bernard et al (2010). Although Debeare (2004) finds little evidence that lumpiness of factors of production affects the trade pattern of Japan, the UK and India, Bernard et al (2010) show that lumpiness may be important in Mexico.

3. The model: Preliminaries

In this section we outline the standard Ricardian model of trade as the basis for our analysis of indivisibilities. There are two countries, Home and Foreign, and two goods, 1 and 2. Each

good is produced under perfect competition using labour as the only factor of production. Assume throughout the paper that all quantities can only take non-negative values. There are two workers in each country who share the same technology, which exhibits constant marginal product of labour. Production technologies take the following form:

Home: $Q_{1H} = AL_{1H}$ $Q_{2H} = L_{2H}$ (1)

Foreign: $Q_{1F} = L_{1F}$ $Q_{2F} = AL_{2F}$ (2)

Where $A > 1$ represents Home's comparative advantage in good 1 and Foreign's in good 2, and is assumed for simplicity to be identical between the two countries[2].

Preferences take the following constant elasticity of substitution (CES) form (where lower case letters denote per capita values, which will prove useful later):

$$u = \left[c_1^\theta + c_2^\theta\right]^{1/\theta}, \qquad 0 < \theta < 1 \tag{3}$$

We shall perform the analysis for the Home country; outcomes for the Foreign country are analogous. First consider the case of autarky. From the consumer's first order conditions and the zero profit conditions we have the relationship between prices and consumption in equilibrium:

$$\frac{P_{1H}}{P_{2H}} = \frac{1}{A} = \left(\frac{c_{1H}}{c_{2H}}\right)^{\theta-1} \tag{4}$$

Since total consumption of each good equals production in autarky, substituting from the production functions in equation (1) enables us to write down the relationship between the labour used in both goods:

$$L_{2H} = L_{1H}A^{\frac{\theta}{\theta-1}} \tag{5}$$

Substituting into the labour market clearing condition $L_{1H} + L_{2H} = L_H$, making use of the production functions (1) again, and noting that there are two workers/consumers in the country, gives per capita consumption of the two goods:

$$c_{1H} = A\left[1 + A^{\frac{\theta}{\theta-1}}\right]^{-1}, \qquad c_{2H} = A^{\frac{\theta}{\theta-1}}\left[1 + A^{\frac{\theta}{\theta-1}}\right]^{-1} \tag{6}$$

Since both of these expressions are strictly positive, the country will be incompletely specialised in autarky. Substituting these into the utility function (3) and simplifying gives Home's per capita utility under autarky:

[2] Here, as elsewhere in the paper, the use of simple functional forms has no effect on the qualitative results of the model, unless otherwise stated.

$$u_H^A = A\left(1 + A^{\frac{\theta}{\theta-1}}\right)^{\frac{1-\theta}{\theta}} \tag{7}$$

In free trade, each country will be specialised in its comparative advantage good, and export it to the other country in exchange for the other good. Hence, since we assume $A > 1$, and since the two countries have symmetric technologies and are identical in size, the per capita free trade utility level is:

$$u_H^{FT} = \left[2\left(\frac{A}{2}\right)^{\theta}\right]^{1/\theta} = 2^{\frac{1-\theta}{\theta}}A \tag{8}$$

Comparing equations (7) and (8), there are gains from trade; $u_H^{FT} > u_H^A$. The gains from trade arise because each country specialises in its comparative advantage good. Note as well the pattern of trade: each country will export the good in which it has a comparative advantage, and since the two countries are symmetric, each country will export half of its output, and the volume of trade (exports plus imports) is:

$$VT = 2A \tag{9}$$

In the sections below, to make the paper's argument more transparent, unless otherwise stated, we will assume that $A = 3$. If in addition we assume that $\theta = 0.7$, we can obtain numerical solutions to consumption and utility levels in both autarky and free trade (recall except for the volume of trade these are per capita values):

$$c_{1H}^A = 2.785 \qquad c_{2H}^A = 0.0715 \qquad u_H^A = 3.097 \tag{10}$$

$$c_{1H}^{FT} = c_{2H}^{FT} = 1.5 \qquad VT = 6 \qquad u_H^{FT} = 4.038 \tag{11}$$

These values will serve as useful benchmarks to compare with the results with indivisibilities.

4. Indivisible production

In this section we make one major change to the model in Section 3: the two workers in each country can each be employed in only one of the two sectors[3]. This may be due to productivity gains from specialisation, or the presence of switching costs. Formally, this involves modifying the production functions (1) and (2) as follows:

$$\text{Home:} \qquad Q_{1H} = AL_{1H} \qquad Q_{2H} = L_{2H} \qquad L_{1H}, L_{2H} \text{ integer} \tag{12}$$

$$\text{Foreign:} \qquad Q_{1F} = L_{1F} \qquad Q_{2F} = AL_{2F} \qquad L_{1F}, L_{2F} \text{ integer} \tag{13}$$

[3] Here, and in Section 5, one possible extension would be to endogenise the indivisibility. This possibility is left to future work.

There are three possible production structures for each country: (a) both workers produce good 1; (b) both workers produce good 2; (c) one worker produces good 1 and the other worker produces good 2[4].

Consider the case of the Home country in autarky (the case of the Foreign country follows analogously). Since Home has a comparative advantage in good 1, there are two possible production structures: (a) and (c) above (possibility (b) is strictly dominated by possibility (a)).

If Home is specialised in good 1 in autarky (possibility (a)), then we have:

$$Q_{1H} = 2A, \qquad c_{1H} = A, \qquad Q_{2H} = c_{2H} = 0 \tag{14}$$

$$u_1 = A \tag{15}$$

If Home produces both goods in autarky (possibility (c)), then we have:

$$Q_{1H} = A, \qquad c_{1H} = \frac{A}{2}, \qquad Q_{2H} = 1, \qquad c_{2H} = \frac{1}{2} \tag{16}$$

$$u_2 = \left[\left(\frac{A}{2}\right)^{\theta} + \left(\frac{1}{2}\right)^{\theta}\right]^{1/\theta} \tag{17}$$

Note that, regardless of the structure of production, the utility levels under autarky with indivisible labour are always lower than when there are no indivisibilities in equation (7). That is, the indivisibility leads to a loss of efficiency in the economy. Now, $u_1 > u_2$ if Assumption 1 holds:

Assumption 1: $A > \left(2^{\theta} - 1\right)^{-(1/\theta)}$.

This will be true provided the technology parameter A or the elasticity of substitution between goods θ is sufficiently large. That is, the labour productivity in the two goods is sufficiently different from each other, and/or the two goods are sufficiently substitutable in consumption. We assume that Assumption 1 holds for the remainder of this paper. This gives our first main result:

[4] Note here and in Section 5 the similarity of the solution method with the infra-marginal analysis of Yang (2001, 2003). Each possible outcome may be obtained through marginal analysis, and this is followed by a welfare comparison between the possible outcomes.

Proposition 1: If Assumption 1 holds, a country will be specialised in its comparative advantage good in autarky.

This result is new, since in the standard Ricardian model without indivisibilities in Section 3, in autarky a country will always produce both goods. When the country opens up to international trade, it will remain specialised in its comparative advantage good, and export it to the other country in exchange for the other good. Hence the free trade utility level remains as in equation (8) above with perfectly divisible workers. Because the indivisibility is on the production side, international trade eliminates the inefficiency caused by indivisibility, since the free trade production bundle is the same irrespective of whether or not there is indivisibility. Or, put another way, international trade leads to greater gains in the presence of production indivisibility.

Note as well that the source of the gains from trade is different from the traditional case discussed in Section 3. Here, the source of the gains from trade is that trade allows consumers in a country to consume both goods, compared to autarky in which they can only consume one good. In this sense the model is similar to the new trade theory of Krugman (1980), in which the gains from trade arise because trade allows consumers to consume a larger variety of goods than in autarky. We state this as Proposition 2:

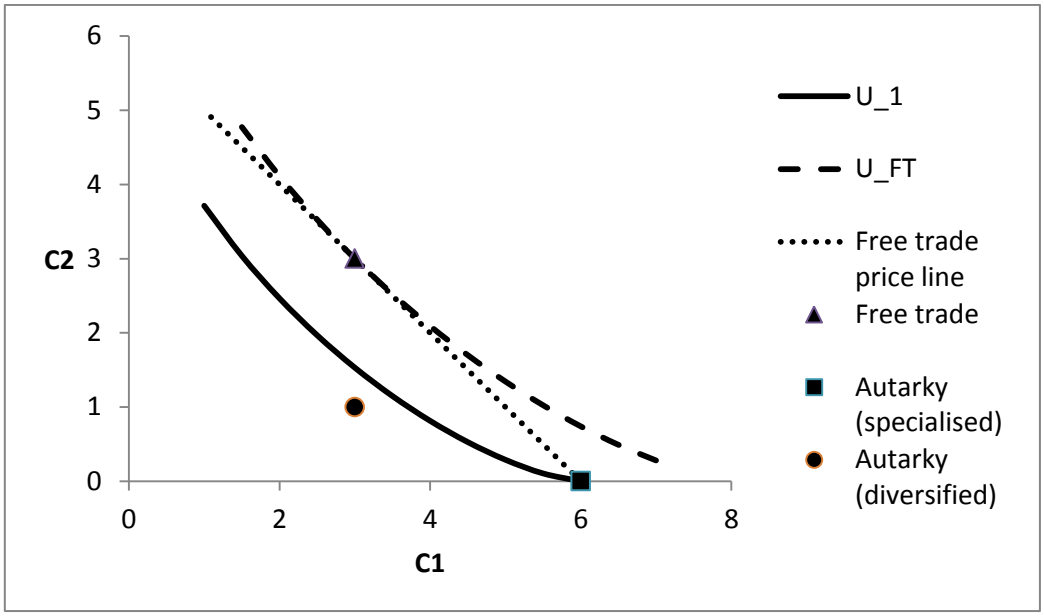
Proposition 2: If Assumption 1 holds, the gains from trade arise because trade enables consumers to consume more types of goods than in autarky.

Similarly to Krugman (1980), even in autarky countries endogenously specialise in a subset of the available goods. But differently from Krugman (1980), where the specialisation is due to increasing returns at the level of the firm, here countries specialise because of the presence of the indivisibility in production. An important corollary of Propositions 1 and 2 is that there is no change in the production structure when moving from autarky to free trade. Hence, no workers suffer even temporary unemployment as a result of trade liberalisation, and everyone in the economy gains from free trade[5]. In addition, if it is indeed the case in reality that a big part of the gains from trade arise from increasing product variety (see Broda and Weinstein

[5] However, see the Appendix for how the results may change if we make different assumptions about labour productivity in the production functions.

(2006) for evidence in the case of the United States), then this model provides an explanation of this from a Ricardian perspective.

Figure 1: The autarkic and free trade equilibria for Home when Assumption 1 holds.



An example of Proposition 1 is shown in Figure 1 for the Home country, where it is assumed that $A = 3$, and $\theta = 0.7$ (as in Section 3 above). Two indifference curves are drawn, one for autarky and one for free trade (national welfare is the sum of individual utility). The country obtains higher utility under autarky when it is completely specialised in its comparative advantage good than when it is diversified (produces both goods). Similarly, it obtains higher utility under free trade than under autarky. Note that there is no production possibility frontier, since the country cannot produce intermediate amounts of the two goods (workers cannot multi-task). Hence intermediate points between the diversified and specialised autarky points are not in the country's (autarkic) feasible set. There is however a free trade price line, along which the country can trade with the other country.

Numerically, given $A = 3$ and $\theta = 0.7$, we obtain the following values for consumption and utility in per capita terms under autarky with indivisible workers:

Specialised in good 1: $c_{1H} = 3$ $c_{2H} = 0$ $u_1 = 3$ (18)

Diversified production: $c_{1H} = 1.5$ $c_{2H} = 0.5$ $u_2 = 2.58$ (19)

Hence, as shown in Figure 1, being specialised in good 1 yields a higher level of utility than being diversified, for the parameter values chosen. Also, as has been noted above, comparing

these values to those of the standard model in equations (10) and (11), production indivisibility leads to a welfare loss in autarky, but not in free trade.

More generally, as illustrated in Figure 1, Proposition 1 arises because, with the CES utility function, the consumer can get positive utility even when he does not consume one of the two goods. We can rewrite the utility function as:

$$c_{2H} = \left(u_H^\theta - c_{1H}^\theta\right)^{1/\theta} \tag{20}$$

So if $c_{2H} = 0$, it must be that $u_H = c_{1H}$. This contrasts with the case of Cobb-Douglas utility where the consumer must consume positive amounts of both goods in order to get any utility, so complete specialisation under autarky is never a feasible outcome.

5. Indivisible consumption

In this section we restore divisibility of production, but introduce instead indivisible consumption. This may arise because some goods can only be purchased (and consumed) in discrete quantities. Suppose that one of the two goods is indivisible in consumption; without loss of generality, let this be good 1. Formally, this means modifying preferences (3) as follows:

$$u = \left[c_1^\theta + c_2^\theta\right]^{1/\theta}, \qquad 0 < \theta < 1, \qquad c_1 \text{ integer} \tag{21}$$

As before, we analyse the Home country; this time, we drop the country subscript to simplify notation. Consider first the case of autarky. Since the two workers/consumers are identical, utility maximisation results in both consumers seeking to consume the same bundle of goods. Since consumption of good 1 can take on only integer values, to solve for the autarkic equilibrium, start from the equilibrium without indivisibilities; then compare the utility obtainable from the two integer values of c_1 on either side of this equilibrium[6].

Define a *floor function* $\lfloor x \rfloor$ as the largest integer less than or equal to x, and a *ceiling function* $\lceil x \rceil$ as the smallest integer greater than or equal to x. Then, in general, letting the subscript NI stand for no indivisibilities, IC for indivisible consumption, and $\mathbb{N}_0$ as the set of natural numbers (non-negative integers), we have:

[6] Note that we cannot simply use the nearest integer function to obtain the equilibrium, since the marginal utility of consumption may be different between the two goods.

$$(c_1^A)_{IC} \in \mathbb{N}_0 \quad \text{such that} \quad (c_1^A)_{IC} = \lfloor (c_1^A)_{NI} \rfloor \quad \text{or} \quad (c_1^A)_{IC} = \lceil (c_1^A)_{NI} \rceil \tag{22}$$

$$(c_2^A)_{IC} = 1 - \left[\frac{(c_1^A)_{IC}}{A} \right] \tag{23}$$

$$(u^A)_{IC} = \max \left\{ [(c_1^A)_{IC}]^\theta + \left[\frac{A - (c_1^A)_{IC}}{A} \right]^\theta \right\}^{1/\theta} \tag{24}$$

Suppose as in the previous sections that $A = 3$ and $\theta = 0.7$. From equation (10), the optimal per capita consumption of good 1 without indivisibilities is $(c_1^A)_{NI} = 2.785$. From equation (22), when good 1 is indivisible, the two consumers may each choose to consume either 2 or 3 units of good 1. Consuming 2 units of good 1 yields $u_{c_1=2} = 2.86$, while consuming 3 units of good 1 yields $u_{c_1=3} = 3$; hence in this case the autarkic equilibrium exhibits complete specialisation in the comparative advantage good, as in the case with indivisible production[7]. More generally, it can be shown that, when there is indivisible consumption, if u_3 is defined as the utility under complete specialisation, and u_4 the utility under incomplete specialisation (in which the consumption bundle is as close as possible to complete specialisation, subject to the indivisibility constraint), then we have:

$$u_3 = A, \qquad u_4 = \left[(A-1)^\theta + \left(\frac{1}{A} \right)^\theta \right]^{1/\theta} \tag{25}$$

Complete specialisation will be the equilibrium in autarky, if $u_3 > u_4$. This will be true if Assumption 2 holds:

Assumption 2: $A > \left[(A-1)^\theta + (1/A)^\theta \right]^{1/\theta}$.

In general, the larger the values of A and θ, the more stringent is Assumption 2 relative to Assumption 1; both assumptions hold for our assumed parameter values $A = 3$ and $\theta = 0.7$. Note that the other results which follow from complete specialisation in autarky discussed in Section 4 also hold in this case. Similarly to the case of indivisibilities in production, in autarky indivisibilities in consumption lead to loss of efficiency relative to the perfectly divisible case, since consumers are restricted in the bundle of goods which they are able to

[7] This implies, given two consumers in the country, that we are comparing national consumption of good 1 of 4 units versus 6 units. One might be tempted to include a national consumption of 5 units of good 1 in the comparison. However, with indivisible consumption, national consumption of 5 units implies that one consumer consumes 3 units, while the other consumes 2 units, and it is shown in the text that consuming 3 units gives each consumer higher utility than consuming 2 units. This discussion emphasises the importance of focussing on per capita consumption and utility.

consume (recall from equation (10) the autarkic utility without indivisibilities is 3.097). The question is, does international trade remove the inefficiency as in the case of production indivisibilities?

It turns out not to be the case. Given the structure of the model, both countries are specialised in their respective comparative advantage goods in free trade. Since we assume $A = 3$, 6 units of each good will be produced in the world economy. However, there are 4 consumers, so if good 1 is indivisible in consumption, two of the four consumers will consume 1 unit of good 1, while the other two consumers will consume 2 units. Conversely, the consumers who have consumed 1 unit of good 1 will consume 2 units of good 2, since goods prices are the same and all consumers have the same income level. Without adding additional structure to the model there is no way to determine which consumers consume which bundle of goods. Thus it is possible that the two Foreign consumers together consume 4 units of the Home-produced good 1, so that the total volume of trade is 8 units – more than the volume of trade with perfectly divisible goods and workers. Of course it is equally possible that the two Foreign consumers together consume only 2 units of good 1, so that the total volume of trade is only 4 units. Since the utility obtained from consuming either bundle of goods is the same, in the presence of even very small trade costs, the latter outcome becomes the unique, utility-maximising solution.

If $\theta = 0.7$, per capita utility in free trade is 3.969, which is less than utility with perfect divisibility (from equation (11), this is 4.038); because the consumption indivisibility retains its bite in the presence of international trade, opening up the country to international trade does not eliminate the inefficiency associated with the indivisibility. Hence we can state:

Proposition 3: When there is indivisibility in the consumption of goods:

(a) If Assumption 2 holds, a country will be specialised in its comparative advantage good, in autarky.
(b) In the free trade equilibrium there may be heterogeneity in consumption and uncertainty in the volume of trade.
(c) International trade does not eliminate the inefficiency caused by consumption indivisibility.

Note that the heterogeneity in consumption in Proposition 3(b) arises because we have chosen $A = 3$; in general any value of A such that the output of the indivisible good, when divided by the number of consumers, does not yield an integer value, will generate this heterogeneity. On the other hand, both here and in the next section, values of A which generate integer values when output is divided by the number of consumers, will not result in consumption heterogeneity, since this implies that all consumers can consume the same bundle of goods despite the indivisibility. There is empirical evidence that consumers with the same income level, may choose to consume different amounts; see for example Fisher et al (2015) and the references therein[8].

6. Indivisible production and indivisible consumption

In this section we combine indivisibilities on both the consumption and production sides. Intuitively, since both types of indivisibility lead to inefficiencies, the combination of both should lead to even more inefficiencies. This intuition turns out to be true only in some cases; in other cases, consumption indivisibility appears to dominate the proceedings, with no additional impact of production indivisibility.

Start again with the case of autarky. Assume that both indivisibilities affect only good 1. On the production side, if Assumption 1 holds, then the Home economy specialises in its comparative advantage good 1, produces 6 units of the good since we assume $A = 3$, and each consumer consumes 3 units, obtaining utility equal to 3. On the other hand, if Assumption 1 does not hold, then Home will produce 3 units of good 1 and 1 unit of good 2. Good 2 is perfectly divisible in consumption. However, good 1 is not; the 3 units produced have to be divided between the two consumers, so one consumer will consume 2 units while the other will consume 1 unit. Therefore, if Assumption 1 does not hold, we may obtain heterogeneity in consumption across ex ante identical consumers even in autarky. This result is new, since in Section 5 above, with only consumption indivisibility, the possibility of consumption heterogeneity in equilibrium arises only when there is international trade, because without production indivisibility, in autarky a country can always produce what its consumers want to consume. As in Section 5, consumption heterogeneity is possible because

[8] Of course, we do not know if consumers with the same income level have the same preferences, as is assumed here.

we have chosen a value of A which does not yield an integer value when the output of the indivisible good 1 is divided by the number of consumers.

Next, consider international trade. Again each country will be specialised in its comparative advantage good. We get the same outcome as in Section 5 above: consumption may be heterogeneous even though consumers are identical ex ante. The results with both indivisible consumption and production are summarised by Proposition 4:

Proposition 4: When there is indivisibility in both the production and consumption of goods:

(a) In autarky, if Assumption 1 holds, the country will be specialised in its comparative advantage good, and consumption will be identical across consumers.

(b) In autarky, if Assumption 1 does not hold, the country will produce both goods, and there may be consumption heterogeneity across consumers.

(c) In free trade, the results are identical to those in Propositions 3(b) and 3(c).

Hence, when Assumption 1 holds, having indivisibility in both production and consumption is identical to having indivisibility in consumption alone. However, if Assumption 1 does not hold, then having both types of indivisibility may increase the degree of inefficiency in autarky relative to having only one type of indivisibility[9]. When international trade is allowed, the outcome collapses to that with indivisibility in consumption alone. So once again we can see that international trade can eliminate the inefficiency which arises from indivisibility in production, but not that which arises from indivisibility in consumption.

Some additional intuition on this last result is in order. In the Ricardian model presented in this paper, countries are completely specialised in production when trade is allowed; this is true whether or not there are indivisibilities in production[10]. As a result, international trade

[9] We have resisted the temptation to obtain numerical values for the case where Assumption 1 does not hold, since these values would not be directly comparable to the values in previous sections.

[10] Complete specialisation is an outcome of the assumption of constant marginal product of labour. In models with more than one factor of production (for instance, the Heckscher-Ohlin model) with diminishing marginal product of labour, complete specialisation is unlikely. In such models, it is conjectured that international trade will not allow the economy with indivisible production to replicate the outcome of the economy without indivisibilities, and hence trade may not eliminate the inefficiency from production indivisibility. This extension is left for future work.

with indivisible production alone replicates the free trade equilibrium without indivisibilities. However, trade does not overcome consumption indivisibility, as consumers cannot choose their optimal consumption bundle whether or not trade is allowed.

7. Conclusions

In this paper we have extended the standard Ricardian model of trade to consider the implications of indivisibilities in both production and consumption of goods. It turns out that such indivisibilities have large effects on the outcomes of the model. Indivisibilities in either production or consumption may give rise to complete specialisation even in autarky, while indivisibilities in consumption may give rise to consumption heterogeneity among ex ante identical consumers. Both forms of indivisibility lead to inefficiencies and lower welfare levels relative to the perfectly divisible case. International trade eliminates the inefficiency from production indivisibility, but not that from consumption indivisibility. This suggests, following the theory of the second best (Lipsey and Lancaster, 1956), that a policy which deviates from free trade may result in higher welfare.

The model developed in this paper is very special, and we have made specific assumptions regarding parameter values to clarify the analysis. Nevertheless, we believe the main results should hold in more general situations. As noted in the Introduction, indivisibilities in both production and consumption are an important part of the economy. What this paper has done is to show how we can analyse the implications of these indivisibilities in a simple model of international trade. Future work will consider refinements and generalisations of the model.

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Appendix: Relaxing the symmetry of production functions

In Section 4, Propositions 1 and 2 are obtained based on the assumption that the two countries have symmetric production functions. What if this is not the case? Suppose instead that the production functions with indivisible production (12) and (13) are replaced with the following, more general functions:

Home: $Q_{1H} = A_H L_{1H}$ $Q_{2H} = B_H L_{2H}$ L_{1H}, L_{2H} integer (A1)

Foreign: $Q_{1F} = A_F L_{1F}$ $Q_{2F} = B_F L_{2F}$ L_{1F}, L_{2F} integer (A2)

Where A_H, B_H, A_F and B_F are technology parameters. Then, suppose that:

$A_H = 3,$ $B_H = 1,$ $A_F = 9,$ $B_F = 2$ (A3)

With the other parameters as before. Now, both countries are better at producing good 1 than they are at producing good 2; that is, both countries have an "absolute advantage"[11] in producing good 1. However, Foreign now has a comparative advantage in producing good 1, since it has a lower opportunity cost: $(A_F/B_F) > (A_H/B_H)$. Both countries satisfy Assumption 1, which means that, in autarky, because of the production indivisibility, both countries will specialise in good 1, in which they have an "absolute advantage". When free trade is opened up between the two countries, both countries will remain specialised in good 1; that is, there will be no production of good 2, and no trade between the two countries. To see why, note that, since Assumption 1 is satisfied for both countries, utility maximisation with indivisible production implies that no consumer will want to consume any of good 2, and hence good 2 is never produced. Hence we have:

Proposition 5: In the presence of production indivisibility, if both countries have an "absolute advantage" in good 1, and Assumption 1 holds for both countries, opening up the two countries to free trade results in no international trade, and no change in the production bundle, consumption bundle, and welfare of the two countries.

The conditions underlying Proposition 5 are likely to hold when production of one good or service is much more efficient than production of another good or service. For instance, in principle there are many ways to travel long distances, but air travel is much more efficient

[11] "Absolute advantage" is in quotation marks, since Adam Smith's notion of absolute advantage relates to a country's technological superiority relative to another country, whereas our usage here relates to both countries' superiority in one good over another good.

(especially in terms of time) than the alternatives, so long distance travel is almost exclusively conducted by air.