The profit function system with output- and input- specific technical efficiency

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Abstract

In a recent paper Kumbhakar and Lai (2016) proposed an output-oriented non-radial measure of technical inefficiency derived from the revenue function. They proposed a closed skew-normal distribution for maximum likelihood estimation but they did not apply the model to data and their technique depends on multiple evaluations of multivariate normal integrals for each observation which can be very costly. In this paper we extend their approach to the profit function and we propose both input- and output-oriented non-radial measures of technical inefficiencies. Although the extension to the translog profit function is trivial many observations, in practice, may contain negative profits. For this reason we provide a nontrivial extension to the Symmetric Generalized McFadden (SGM) profit function. We propose and apply (to a large sample of US banks) Bayesian analysis of the SGM model (augmented with latent technical inefficiencies resulting in a highly nonlinear mixed effects model) using the integrated nested Laplace approximation.

Keywords: profit function; non-radial technical inefficiency; Symmetric Generalized McFadden form; integrated nested Laplace approximation; Bayesian analysis.

JEL classifications: C30, D22.

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1 Introduction

In this paper we build on Kumbhakar and Lai (2016) who developed an output-specific (vector) efficiency measure starting from the revenue function and using the envelope theorem to obtain the output shares. They have used the translog revenue function and, under the assumption that latent output-specific inefficiencies follow a multivariate half-normal distribution they obtained the likelihood function of the system of output shares (without the translog revenue function) using a closed skew-normal distribution. Kumbhakar and Lai (2016) did not take the model to the data. A difficulty that arises in their likelihood is that it requires multiple evaluations of multivariate normal integrals for each observation which can be very costly.

In this study we extend Kumbhakar and Lai (2016) to the profit function case in order to derive both output-specific and input-specific inefficiency measures. As the authors mention: "Although in the present model we consider only output slacks, the formulation can be extended to accommodate both input and output slacks in a profit maximizing model". This is true but certain nontrivial problems arise. In order to get rid of the awkward normalizing constants of the closed skew-normal distribution we propose a multivariate lognormal distribution for the latent input and output inefficiencies. Second, the extension of revenue to profit functions requires that all observations have positive profits which is rarely the case. Therefore, we adopt a Symmetric Generalized McFadden (SGM) profit function. We propose and apply (to a large sample of US banks) Bayesian analysis of the SGM model (augmented with latent technical inefficiencies resulting in a highly nonlinear mixed effects model) using the integrated nested Laplace approximation. To our knowledge this is the first study that analyzes the SGM profit function enforcing all regularity restrictions globally without calibrating certain parameters.

2 Model

We build on Kumbhakar and Lai (2016) to construct a profit system with both output- and input-oriented inefficiency. The vector of netputs is $z \in \Re^N$, assuming outputs are positive and inputs are negative. For simplicity $z_1, ..., z_M > 0$ are outputs and $z_{M+1}, ..., z_N < 0$ are inputs. Prices are $p \in \Re^N_+$. The objective of the firm is profit maximization:

$$\max_{z \in \Re^N} : \ p^\top z, \text{ s.t. } F(z^*) = 1, \tag{1}$$

where $z^* = \theta \odot z$, $\theta \in \Re^N$ with $\theta_1, ..., \theta_M \geq 1$ and $\theta_{M+1}, ..., \theta_N \leq 1$. The problem is equivalent to:

$$\Pi(p_*) = \max_{z \in \Re^N} : \ p_*^\top z, \text{ s.t. } F(z) = 1, \tag{2}$$

where $p_* = [p_n/\theta_n, n = 1, ..., N]$ in view of equations (2) in Kumbhakar and Lai (2016). Using the envelope theorem we have the netput demands in the form: $z = \frac{\partial \Pi(p_*)}{\partial p_*}$. Alternatively we have the shares: $\frac{\partial \log \Pi(p_*)}{\partial \log p_n^*} = \frac{p_n^* z_n}{\Pi(p_*)}, n = 1, ..., N$. Next we assume a translog profit function:

$$\log \Pi(p_*) = \beta_0 + \sum_{n=1}^{N} \beta_n \log p_n^* + \tfrac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} \log p_n^* \log p_m^*. \tag{3}$$

From the envelope theorem we obtain:

$$\frac{\partial \log \Pi(p_*)}{\partial \log p_n^*} = \beta_n + \sum_{m=1}^{n} \beta_{nm} \left(\log p_m - \log \theta_m\right), n = 1, ..., N. \tag{4}$$

Defining $\xi_n = -\log \theta_n, n = 1, ..., N$ we have the following system of equations:

$$
\begin{aligned}
\log \Pi(p_*) &= \beta_0 + \textstyle\sum_{n=1}^{N} \beta_n \log p_n + \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} \beta_{nm} \log p_n \log p_m + A(p,\xi) + v_0, \\
S_n &= \tfrac{p_n z_n}{\Pi} = \beta_n + \textstyle\sum_{m=1}^{n} \beta_{nm} \log p_m + \sum_{m=1}^{N} \beta_{nm}\xi_m + v_n,\ n = 1, ..., N-1,
\end{aligned} \tag{5}
$$

where

$$
A(p,\xi) = \sum_{n=1}^{N} \beta_n \xi_n + \sum_{n=1}^{N}\sum_{m=1}^{N} \beta_{nm} \log p_m \xi_n + \tfrac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} \beta_{nm}\xi_n\xi_m, \tag{6}
$$

and $\mathbf{v} = [v_0, v_1, ..., v_{N-1}]$ are error terms satisfying $\mathbf{v} \sim \mathcal{N}_N(O, \Sigma)$. For the $\xi_n s$ we have a multivariate half-normal distribution independently of $\mathbf{v}$ and prices:

$$
[\xi_1, ..., \xi_M, -\xi_{M+1}, ..., -\xi_N] \sim \mathcal{N}_N^{+}(O, \Omega) \tag{7}
$$

As $\sum_{n=1}^{N} S_n = 1$ we can omit the last share equation. To impose homogeneity of degree one in prices we can employ the usual parametric restrictions or redefine $p_n := p_n/p_1$ in which case the system in (5) takes the form:

$$
\begin{aligned}
\log \Pi(p_*) &= \beta_0 + \textstyle\sum_{n=2}^{N} \beta_n \log p_n + \frac{1}{2}\sum_{n=2}^{N}\sum_{m=2}^{N} \beta_{nm} \log p_n \log p_m + A(p,\xi) + v_0, \\
S_n &= \tfrac{p_n z_n}{\Pi} = \beta_n + \textstyle\sum_{m=2}^{n} \beta_{nm} \log p_m + \sum_{m=1}^{N} \beta_{nm}\xi_m + v_n,\ n = 1, ..., N-1,
\end{aligned} \tag{8}
$$

where

$$
A(p,\xi) = \sum_{n=1}^{N} \beta_n \xi_n + \sum_{n=1}^{N}\sum_{m=2}^{N} \beta_{nm} \log p_m \xi_n + \tfrac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} \beta_{nm}\xi_n\xi_m, \tag{9}
$$

Additionally, $\xi_1, ..., \xi_M \leq 0$ and $\xi_{M+1}, ..., \xi_N \geq 0$. Kumbhakar and Lai (2016) essentially consider *only* the system of the last N equations in (5) ignoring their revenue function because it is not linear in ξ. Ignoring, however, this information may be critical as the revenue or profit function provides significant identifying information.[1] Then they formulate the likelihood function from the system of share equations using properties of the closed skew-normal distribution. This involves evaluating multivariate normal integrals in $\Re^{N-1}$ for each observation which can be cumbersome and computationally non-trivial. The *entire system* in (5) can be estimated using Markov Chain Monte Carlo and especially efficient techniques developed in Kumbhakar and Tsionas (2004).

3 The Symmetric Generalized McFadden form and posterior analysis

If all profits are strictly positive we can proceed with the system in (5). In empirical applications, more often than not some observations have negative profit we have to proceed with a different functional form.[2] The Symmetric Generalized McFadden form (SGM) has been introduced by Diewert and Wales (1987) in the context of cost functions. As a profit function, the SGM takes the following form:

$$
\Pi(p^o) = \sum_{n=1}^{N} \beta_n p_n^o + \frac{1}{2}\frac{\sum_{n=1}^{N}\sum_{m=1}^{N} \beta_{nm} p_n^o p_m^o}{\sum_{n=1}^{N} \alpha_n p_n^o}, \tag{10}
$$

where $p_n^o = p_n + \theta_n$, $\forall n = 1, ..., N$ where $\theta_1, ..., \theta_M \geq 0$ and $\theta_{M+1}, ..., \theta_N \leq 0$. The SGM profit function is linear homogeneous in prices. Convexity can be imposed by restricting the $[\beta_{nm}]$ matrix to positive semidefinite (e.g. by using the Cholesky decomposition) and holds globally. From the envelope theorem we have the netput demands

[1] In principle identification of the one-sided component is not a problem, unless the distribution of the overall error term turns out to be nearly symmetric. In applications this may often be the case. Therefore, the inclusion of the profit (or revenue) function may become essential as it provided information, in nonlinear form, about the one-sided components.

[2] Some authors add a constant to profits so that all of them become positive. We do not follow this arbitrary practice here.

and, after introducing error terms we have the following system:

$$\begin{array}{c}\Pi(p^o)=\sum_{n=1}^N \beta_n p_n^o+\frac{1}{2}\frac{\sum_{n=1}^N\sum_{m=1}^N \beta_{nm}p_n^o p_m^o}{\sum_{n=1}^N \alpha_n p_n^o},+v_0,\\ z_n=\beta_n+\frac{\sum_{m=1}^N \beta_{nm}p_m^o}{S}-\frac{1}{2}\frac{\alpha_n\sum_{n=1}^N\sum_{m=1}^N \beta_{nm}p_n^o p_m^o}{S^2}+v_n, n=1,...,N,\end{array} \tag{11}$$

where $S=\sum_{m=1}^N \alpha_m p_m^o>0$, $\alpha_n\geq 0$, $\forall n=1,...,N$ and $\sum_{m=1}^N \beta_{nm}p_m^o=0$, $\forall n=1,...,N$.

Again, we assume $\mathbf{v}=[v_0,v_1,...,v_N]$ are error terms satisfying $\mathbf{v}\sim\mathcal{N}_{N+1}(O,\Sigma)$ and

$$\log\left[\theta_1,...,\theta_M,-\theta_{M+1},...,-\theta_N\right]\sim\mathcal{N}_N(\mu,\Omega), \tag{12}$$

where[3] $\mu\in\Re^{N+1}$. In "expanded" form the system is the following:

$$\begin{array}{c}\Pi=\sum_{n=1}^N \beta_n\left(p_n+\theta_n\right)+\frac{1}{2}\frac{\sum_{n=1}^N\sum_{m=1}^N \beta_{nm}(p_n+\theta_n)(p_m+\theta_m)}{\sum_{n=1}^N \alpha_n(p_n+\theta_n)},+v_0,\\ z_n=\frac{\partial\Pi}{\partial p_n^o}=\beta_n+\frac{\sum_{m=1}^N \beta_{nm}(p_m+\theta_m)}{\sum_{m=1}^N \alpha_m(p_m+\theta_m)}-\frac{1}{2}\frac{\alpha_n\sum_{n=1}^N\sum_{m=1}^N \beta_{nm}(p_n+\theta_n)(p_m+\theta_m)}{\left\{\sum_{m=1}^N \alpha_m(p_m+\theta_m)\right\}^2}+v_n,\ n=1,...,N.\end{array} \tag{13}$$

The SGM is quite complicated as a function of θ even after, possibly, dropping the profit function and keeping only the netput demands. In addition the θs must satisfy the following constraints:

$$\sum_{m=1}^N \alpha_m\left(p_m+\theta_m\right)>0,\ \sum_{m=1}^N \beta_{nm}=0,\ \forall n=1,...,N,\ \beta_{nm}=\beta_{mn},\ \forall n,m=1,...,N. \tag{14}$$

The first restriction in (14) will hold as long as θ_ns are "small enough" in the sense that $p_n+\theta_n>0$, $\forall n=1,...,N$. For $n=1,...,M$ this is not a problem but it can become a problem for $n=M+1,...,N$, i.e. for the inputs. We check for such problems after estimation. The remaining constraints in (14) are parametric restrictions that can be imposed easily through the prior. Contrary to standard practice, also recommended by Diewert and Wales (1987) we do not set the α_n equal to the means of netputs but we treat me as unknown parameters.

Suppose we have a set of observed data $\{\Pi_i,z_{in},n=1,...,N\},i=1,...,I$ so we have I observations on profits and netputs. Suppose

$$\mathbf{Y}_i=\left[\Pi_i,z_{i1},...,z_{iN}\right]',i=1,...,I,$$

and $\mathbf{Y}=\{\mathbf{Y}_i.i=1,...,I\}$. Write the system in (13) as follows:

$$\mathbf{Y}_i=\mathbf{f}(\mathbf{p}_i;\theta_i,\lambda)+v_i,\ i=1,...,I, \tag{15}$$

where $\mathbf{p}_i$ is the vector of prices for observation i. Given that we denote the parameter vector by $\lambda=\{\mu,\Sigma,\Omega,\alpha,\beta,[\beta_{nm}]\}$ the posterior distribution of the system in (13) is the following:

$$\pi(\lambda|\mathbf{Y})\propto|\Sigma|^{-I/2}\int_{\Re^{N+1}}\left\{\exp\left\{-\frac{1}{2}\sum_{i=1}^I\left[\mathbf{Y}_i-\mathbf{f}(\mathbf{p}_i;\theta_i,\lambda)\right]'\Sigma^{-1}\left[\mathbf{Y}_i-\mathbf{f}(\mathbf{p}_i;\theta_i,\lambda)\right]\right\}\prod_{i=1}^I p\left(\theta_i|\Omega\right)\right\}d\theta_i\pi(\lambda), \tag{16}$$

where $\pi\left(\theta_i|\Omega\right)$ is the prior of θ_i given by (12) and $\pi(\lambda)$ is the prior of parameters λ. The former is the following:

$$\pi\left(\theta_i|\Omega\right)\propto|\Omega|^{-1/2}\exp\left\{-\frac{1}{2}(\log\theta_i-\mu)'\Omega^{-1}(\log\theta_i-\mu)-\iota_{N+1}'\theta_i\right\}, \tag{17}$$

[3]The assumption of multivariate log-normality besides being quite flexible it avoids the presence of awkward integrating constants like the multivariate normal c.d.f which would, otherwise, pose certain obstacles to both maximum likelihood as well as posterior analysis using Monte Carlo techniques.

where ι_{N+1} is a vector of ones in $\Re^{N+1}$. Our prior for λ is flat over the domain defined by

$$\sum_{m=1}^{N} \beta_{nm} = 0,\ \forall n = 1, ..., N,\ \beta_{nm} = \beta_{mn},\ \forall n, m = 1, ..., N,$$

while for the covariance matrices we assume

$$\pi(\Omega) \propto |\Omega|^{-(N+1)/2},\ \pi(\Sigma) \propto |\Sigma|^{-(N+2)/2}$$

and $[\beta_{nm}]$ is positive semidefinite -a restriction that is enforced directly by reparametrizing using the Cholesky decomposition. As the multivariate integral in (16) is not available in closed form we consider the augmented posterior (augmented with $\{\theta_i\}$):

$$\begin{aligned}\pi(\lambda, \{\theta_i\}|\mathbf{Y}) \propto |\Sigma|^{-I/2}|\Omega|^{-I/2} \exp\left\{-\tfrac{1}{2}\textstyle\sum_{i=1}^{I} \left[\mathbf{Y}_i - \mathbf{f}(\mathbf{p}_i; \theta_i, \lambda)\right]' \Sigma^{-1} \left[\mathbf{Y}_i - \mathbf{f}(\mathbf{p}_i; \theta_i, \lambda)\right]\right\} \cdot \\ \exp\left\{-\tfrac{1}{2}\textstyle\sum_{i=1}^{I} \left[(\log\theta_i - \mu)'\Omega^{-1}(\log\theta_i - \mu) - \iota'_{N+1}\theta_i\right]\right\} \cdot \pi(\lambda).\end{aligned} \tag{18}$$

After integrating Σ and Ωout analytically we obtain:

$$\begin{aligned}\pi(\lambda, \{\theta_i\}|\mathbf{Y}) \propto |\textstyle\sum_{i=1}^{I} \left[\mathbf{Y}_i - \mathbf{f}(\mathbf{p}_i; \theta_i, \lambda)\right] \left[\mathbf{Y}_i - \mathbf{f}(\mathbf{p}_i; \theta_i, \lambda)\right]'|^{-(I+N+1)/2} \cdot \\ |\textstyle\sum_{i=1}^{I} \left[(\log\theta_i - \mu)(\log\theta_i - \mu)'\right]|^{-(I+N)/2} \cdot \exp\left\{-\iota'_{N+1} \textstyle\sum_{i=1}^{I} \theta_i\right\} \cdot \pi(\lambda).\end{aligned} \tag{19}$$

To explore the posterior in (19) we use the technique of integrated nested Laplace approximation (INLA) developed by Rue, Martino and Chopin (2009) which requires only first and second order derivatives of the log posterior.[4] INLA produces accurate approximations to marginal posterior densities of λ, $\{\theta_i\}$ and other functions of interest.

4 Data and empirical results

To illustrate the new methods we focus on a data set of a large number of U.S banks with 342,868 observations in total. We keep only banks in the highest decile of total assets. This panel and the variables employed are identical to those used by Koetter et al. (2012) and Restrepo-Tobon and Kumbhakar (2014). Specifically, as bank outputs, we use total loans and total securities. The input prices include the cost of fixed assets (expenditures on fixed assets divided by premises and fixed assets), the cost of labor (salaries divided by the number of full-time equivalent employees), and the cost of borrowed funds (interest expenses on deposits and interest expenses on fed funds divided by the sum of total deposits and fed funds purchased). We also include total assets, to control for bank size, and a time trend. For general studies of the banking industry we refer to Berger and Mester (1997, 2003), Hughes and Mester (1993, 1998), Feng and Serletis (2009), and Malikov, Kumbhakar and Tsionas (2016).

The empirical results are reported in Table 1 and Figure 1. From Table 1 we see that output inefficiencies average[5] 12.5% for loans and 8.5% for securities. Input inefficiencies are 3.5% for capital (fixed assets), 8.7% for labor and 12.8% for borrowed funds -a particularly large number indicating considerable slacks in borrowed funds. Over time there seem to be important increases of slacks in both loans (3.2%) as well as securities (1.5%) and practically no improvement at all in the front of input inefficiencies. Productivity growth is very small and averages 0.17% with sample standard deviation of 0.19%.

The sample distributions of output and input slacks are highly non-normal particularly for the slacks in loans

[4] These derivatives can be calculated relatively easily using analytical techniques.

[5] Instead of averaging across the sample we can present marginal posterior densities of the slacks for specific banks and / or years. We avoid this in the interest of space.

and securities. From Figure 2, sample distributions of temporal changes in output and input slacks show clearly that there has been very little, if at all, progress in terms of reducing slacks and inputs and, for the most part, if not exclusively, inefficiency has increased for both loans and securities. These findings clearly indicate certain structural weakness in the banking sector, at least for the period we examine here, and they are not incompatible with the "quiet life hypothesis" in banking.

Table 1. Posterior statistics of output- and input-oriented slacks

Notes: Posterior means and posterior standard deviations are computed from the marginal posterior densities of θ_is using the INLA approach, Rue, Martino and Chopin (2009). The reported statistics are sample averages of these statistics. Efficiency change measures the percentage change in efficiency over time.

	inefficiency		inefficiency change	
	sample mean	sample s.d.	sample mean	sample s.d.
OUTPUTS				
total loans	0.125	0.025	0.032	0.014
securities	0.085	0.017	0.015	0.009
INPUTS				
fixed assets	-0.035	0.011	-0.007	0.003
labor	-0.087	0.024	-0.005	0.004
borrowed funds	-0.128	0.036	0.002	0.002
OTHER STATISTICS				
productivity growth	0.0017	0.0019		

Figure 1. Sample distributions of output and input slacks

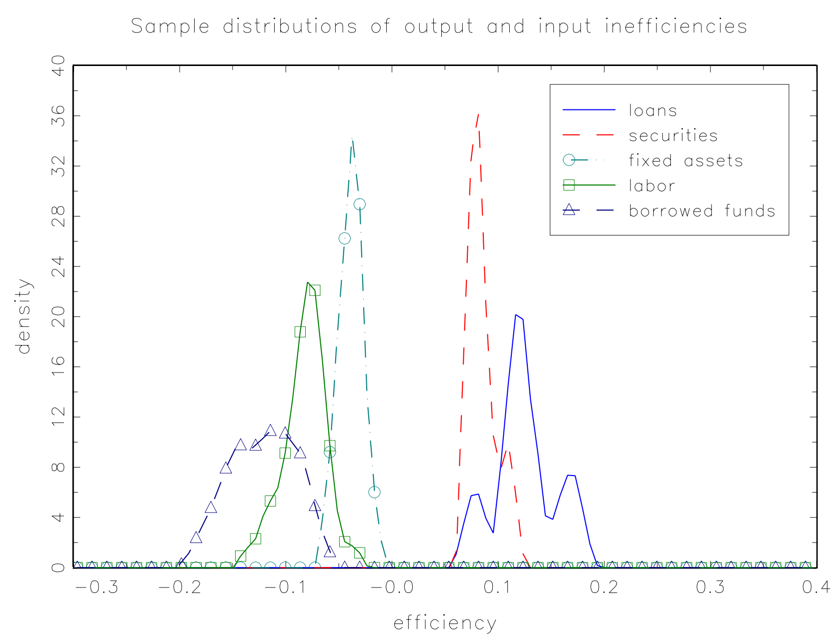
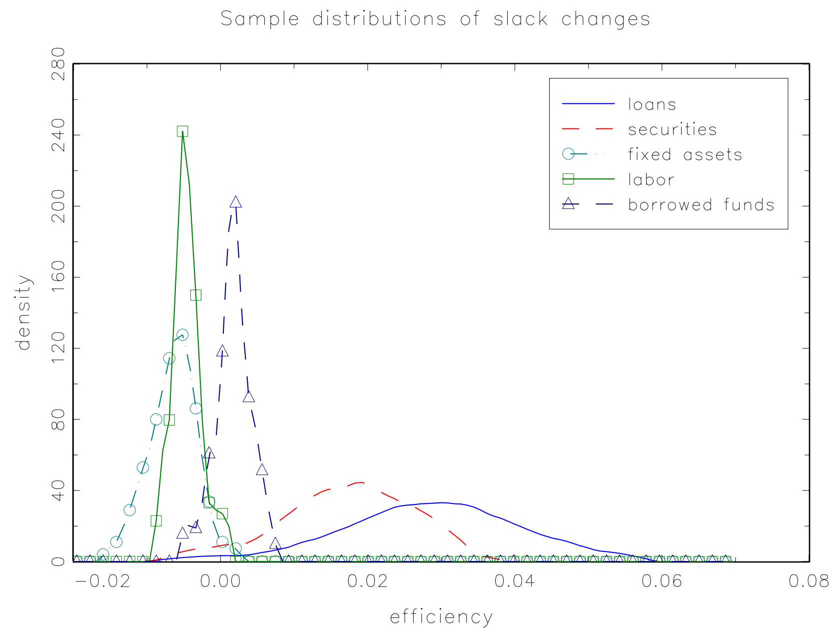


Figure 2. Sample distributions of temporal changes in output / input slacks



Conclusions

In this study we extend Kumbhakar and Lai (2016) to the profit function case in order to derive both output-specific and input-specific inefficiency measures. In order to get rid of the awkward normalizing constants of the closed skew-normal distribution we propose a multivariate lognormal distribution for the latent input and output slacks. We adopt a Symmetric Generalized McFadden (SGM) profit function. We propose and apply (to a large sample of US banks) Bayesian analysis of the SGM model. When augmented with latent technical slacks this results in a highly nonlinear mixed effects model. Posterior inferences are performed using the integrated nested Laplace approximation. To our knowledge this is the first study that analyzes the SGM profit function enforcing all regularity restrictions globally without calibrating certain parameters and, at the same time, delivers exact finite-sample estimates and posterior densities of input and output oriented slacks.

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