

Application of Non-Orthogonal Multiple Access in Cooperative Spectrum-Sharing Networks over Nakagami- m Fading Channels

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Abstract—This paper proposes a novel non-orthogonal multiple access (NOMA)-based cooperative transmission scheme for a spectrum-sharing cognitive radio network, whereby a secondary transmitter (ST) serves as a relay and helps transmit the primary and secondary messages simultaneously with employing NOMA signaling. This cooperation is particularly useful when the ST has good channel conditions to a primary receiver but lacks of the radio spectrum. To evaluate the performance of the proposed scheme, the outage probability and system throughput for the primary and secondary networks are derived in closed forms. Simulation results demonstrate the superior performance gains for both networks thanks to the use of the proposed NOMA-based cooperative transmission scheme. It is also revealed that NOMA outperforms conventional orthogonal multiple access and achieves better spectrum utilization.

Index Terms—Non-orthogonal multiple access (NOMA), cooperative spectrum-sharing, Nakagami- m fading.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) has been identified as a promising multiple access technique for the fifth generation (5G) mobile networks due to its superior spectral efficiency [1]–[3]. The rationale is that multiple users' signals are superimposed at the transmitter with different power allocation coefficients, and the user with better channel conditions decodes its own signal by employing successive interference cancellation (SIC). In [4], a cooperative NOMA scheme has been proposed based on user cooperation. Using a dedicated relay, the performance of cooperative relaying networks via NOMA has been analyzed in [5].

Cognitive radio networks (CRNs) have been a new paradigm to enable more efficient use of the radio spectrum [6], [7]. By introducing NOMA to CRN, the higher spectral efficiency can be potentially achieved. However, little research has combined NOMA with CRN in an effective way. Recently, the outage performance of NOMA in underlay CRNs has been characterized in [8] with applying stochastic geometry tools. A cooperative NOMA scheme for reliability enhancement in cognitive multicast networks has been proposed in [9].

In this paper, we investigate the application of NOMA to a cooperative spectrum-sharing CRN. Unlike the underlay CRN which sets interference limit to the primary network [8], we consider a cooperative scenario where the primary

and secondary networks actively seek opportunities to collaborate with each other. Compared with our prior work in [9], where the cooperative NOMA scheme slightly degrades the performance of the primary network, this paper proposes a win-win strategy. In particular, the secondary transmitter (ST) helps relay the information from the primary transmitter (PT) to the primary receiver (PR), meanwhile the ST utilizes the primary spectrum to serve its own secondary receiver (SR) for the compensation. This creates an incentive for the PT and ST to work together, and the ST is no longer constrained by low transmit power. The main contributions of this paper are three fold: First, we propose a novel NOMA-based cooperative transmission scheme to achieve better utilization of the spectrum, whereby the ST which has good channel conditions to the PR is recruited as a relay to assist the primary transmission, and in turn it can use the primary spectrum. To improve the spectral efficiency, the ST will adopt NOMA for the simultaneous primary and secondary transmissions. Consequently, an enhanced win-win situation can be achieved. Second, we derive closed-form expressions of the outage probability and the system throughput for both primary and secondary networks. To obtain further insights, asymptotic expressions for the outage probability are derived to validate the diversity order by the proposed scheme. Finally, numerical results are presented to demonstrate the effectiveness of the proposed scheme and the accuracy of the analytical results. Compared with conventional orthogonal multiple access (OMA)-based schemes, the performance in terms of the outage probability and the system throughput can be significantly improved.

II. SYSTEM MODEL

Consider a cooperative spectrum-sharing network consisting of the primary and secondary nodes, denoted by PT, PR, ST, and SR_i , $i \in \mathcal{K} = \{1, 2, \dots, K\}$, respectively. Each node is equipped with a single antenna and operates in a half-duplex mode. All channels are assumed to experience independent and identically distributed (i.i.d.) Nakagami- m fading. In particular, the channel gains from PT and ST to SR_i are denoted by $h_{p,i}$ and $h_{s,i}$. We have $E[|h_{p,i}|^2] = \Omega_{p,s}$ and $E[|h_{s,i}|^2] = \Omega_{s,s}$, whose fading parameter is m . The channel gains from PT to PR and ST are denoted by f_0 and f_1 , and the channel gain from ST to PR is denoted by f_2 . Similarly, we have $E[|f_0|^2] = \Omega_{f,0}$, $E[|f_1|^2] = \Omega_{f,1}$, and $E[|f_2|^2] = \Omega_{f,2}$, with fading parameter m . In this paper, we assume all channels follow quasi-static fading, i.e. the channel gains remain to be constant within each transmission block but vary independently between different blocks. Without loss of generality, it is assumed that the users' channel gains in the secondary network are ordered as $|h_{s,1}|^2 \leq \dots \leq |h_{s,K}|^2$. Let x_0 and x_i , $i \in \mathcal{K}$, denote the primary and secondary users' signals, with $E[x_0] = E[x_i] = 0$ and $E[|x_0|^2] = E[|x_i|^2] = 1$.

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The transmit power at PT and ST are denoted by P_p and P_s , and the background noise is modeled by a zero-mean, complex Gaussian variable with variance σ^2 .

As shown in Fig. 1 (a), the proposed cooperative NOMA-based transmission scheme consists of two phases. During the first transmission phase, x_0 is transmitted by PT, the received signal at PR is given by

$$y_{p1} = \sqrt{P_p}f_0x_0 + n_{p1} \quad (1)$$

and the received signal at ST is expressed as

$$y_s = \sqrt{P_p}f_0x_0 + n_s \quad (2)$$

where n_{p1} and n_s are the noises during the first transmission phase. The condition that ST is able to successfully decode x_0 is: $\rho_p|f_1|^2 \geq \psi_0$, where $\psi_0 = 2^{2R_0} - 1$ is the threshold for data decoding, R_0 is the target data rate of PR, and $\rho_p = \frac{P_p}{\sigma^2}$ is the transmit signal-to-noise ratio (SNR) at PT. At the end of this phase, ST tries to decode x_0 . If the decoding is successful, ST can regenerate x_0 and combine it with x_i , $i \in \mathcal{K}$, for relaying in the next NOMA transmission phase.

The received signal at SR_k is written as

$$y_{s,k1} = \sqrt{P_p}h_{s,k}x_0 + n_{k1} \quad (3)$$

where n_{k1} is the noise during the first transmission phase.

During the second transmission phase, conditioned on ST succeeding in decoding, ST transmits the superimposed signals $\sum_{i=0}^K \sqrt{\alpha_i}P_s x_i$ to PR and SR_i , where x_i contains the information required by SR_i for $i \in \mathcal{K}$ or PR for $i = 0$, and α_i denotes the power allocation coefficient of x_i , $i = 0, 1, \dots, K$. Due to $|h_{s,1}|^2 \leq \dots \leq |h_{s,K}|^2$ and the high priority of PR, we let $\alpha_0 \geq \alpha_1 \geq \dots \geq \alpha_K$ with $\sum_{i=0}^K \alpha_i = 1$ [8]. The received signals at PR during the second transmission phase are

$$y_{p2} = f_2 \sum_{i=0}^K \sqrt{\alpha_i}P_s x_i + n_{p2} \quad (4)$$

where n_{p2} is the noise during the second transmission phase. On maximal-ratio combining (MRC), the achievable data rate at PR can be expressed as

$$R_{p,1} = \frac{1}{2} \log_2 \left(1 + \rho_p |f_0|^2 + \frac{\rho_s |f_2|^2 \alpha_0}{\rho_s |f_2|^2 \sum_{i=1}^K \alpha_i + 1} \right) \quad (5)$$

where $\rho_s = \frac{P_s}{\sigma^2}$ is the transmit SNR at ST.

The observations at SR_k can be expressed as

$$y_{s,k2} = h_{s,k} \sum_{i=0}^K \sqrt{\alpha_i}P_s x_i + n_{k2} \quad (6)$$

where n_{k2} denotes the noise at SR_k during the second transmission phase. SIC will be carried out at SR_k to separate the multiplexed signals in the power domain. Specifically, SR_k will first detect SR_j 's signal, $j < k$, and then remove the signal from its observation in a successive manner [4]. While SR_j 's signal, $j > k$, will be treated as noise at SR_k . Recalling that x_0 has been received in the first transmission phase, then SR_k will use MRC to detect x_0 in the SIC process. Thus, the achievable data rate, i.e. $j = 0$, is expressed as

$$R_{0 \rightarrow k} = \frac{1}{2} \log_2 \left(1 + \rho_p |h_{p,k}|^2 + \frac{\rho_s |h_{s,k}|^2 \alpha_0}{\rho_s |h_{s,k}|^2 \sum_{i=1}^K \alpha_i + 1} \right). \quad (7)$$

The achievable data rate for SR_k to detect SR_j 's signal, $1 \leq j \leq k$, is given by

$$R_{j \rightarrow k} = \frac{1}{2} \log_2 \left(1 + \frac{\rho_s |h_{s,k}|^2 \alpha_j}{\rho_s |h_{s,k}|^2 \sum_{i=j+1}^K \alpha_i + 1} \right). \quad (8)$$

If $R_{j \rightarrow k} \geq R_j$ ($j = 0, \dots, K$), where R_j is the target data rate of x_j , then x_j can be decoded and removed by SR_k . This process will terminate until x_k can be successfully decoded at SR_k . The achievable data rate at SR_k , $k = 1, \dots, K - 1$, is

$$R_{k,1} = \frac{1}{2} \log_2 \left(1 + \frac{\rho_s |h_{s,k}|^2 \alpha_k}{\rho_s |h_{s,k}|^2 \sum_{i=k+1}^K \alpha_i + 1} \right). \quad (9)$$

Furthermore, the achievable data rate at SR_K can be obtained by $R_{K,1} = \frac{1}{2} \log_2(1 + \rho_s |h_{s,K}|^2 \alpha_K)$.

On the other hand, when ST fails to decode x_0 , it will remain silent in the second transmission phase. In this case, PR receives x_0 only from the first transmission phase. Thus, the achievable data rates at PR and SR_k are given by

$$R_{p,2} = \frac{1}{2} \log_2(1 + \rho_p |f_0|^2), \quad R_{k,2} = 0. \quad (10)$$

III. PERFORMANCE ANALYSIS

This section provides a comprehensive performance analysis of primary and secondary networks achieved by the proposed NOMA-based cooperative transmission scheme.

A. Primary Network

1) *Outage Probability*: The outage probability of the primary network with the proposed NOMA-based cooperative transmission scheme can be formulated as

$$P_p = \underbrace{\Pr(|f_1|^2 > \frac{\psi_0}{\rho_p}, R_{p,1} < R_0)}_{I_1} + \underbrace{\Pr(|f_1|^2 < \frac{\psi_0}{\rho_p}, R_{p,2} < R_0)}_{I_2}. \quad (11)$$

Particularly, I_1 is for the event that ST can decode the primary user's signal correctly but the achievable rate at PR is smaller than the target data rate; and I_2 is for the event that neither ST or PR can decode the primary user's signal.

The following theorem provides the outage probability for the primary network with our proposed scheme.

Theorem 1: The outage probability of the primary network with the proposed scheme can be approximated as (12), shown at the top of the next page, where N is a complexity-accuracy tradeoff parameter, $\varpi = \min(\psi_0, \frac{\alpha_0}{\sum_{i=1}^K \alpha_i})$, $\theta_n = \cos(\frac{2n-1}{2N}\pi)$, $b_n = \frac{(\theta_n+1)\varpi}{2}$, and $f_Y(y)$ is defined in Appendix A.

Proof: See Appendix A. ■

2) *Asymptotic Analysis*: We define the system SNR as $\bar{\rho} = \rho_p = \mu \rho_s$ with $\mu = \rho_p / \rho_s$. With the help of $e^x \simeq 1 - x$ for a sufficiently large system SNR, i.e. $\bar{\rho} \rightarrow \infty$, the asymptotic expression for (12) is given by

$$P_p \stackrel{\bar{\rho} \rightarrow \infty}{\simeq} \frac{1}{\bar{\rho}^{2m}} \left(\frac{\varpi \pi}{2N} \sum_{n=1}^N c_n d_n + \chi_m \right) \quad (13)$$

where $\chi_m = \frac{(m\psi_0)^{2m}}{(\Omega_{f,0}\Omega_{f,1})^m(m!)^2}$, $c_n = \sqrt{1 - \theta_n^2} \left(\frac{\psi_0 - b_n}{\Omega_{f,0}} \right)^m / m!$, and $d_n = \frac{m^m (b_n / (\alpha_0 - \sum_{i=1}^K \alpha_i))^{m-1}}{(\mu \Omega_{f,2})^m \Gamma(m)}$. Therefore, it is clear from (13) that a diversity order of $2m$ is achieved for the primary network, thanks to the cooperative NOMA transmission.

3) *Outage Throughput*: In this paper, the transmitter sends information at a fixed rate and the throughput is determined by evaluating the outage probability. This implies that the receiver has to decode the signal block by block and thus it is important to study the outage throughput for practical implementations, which is given by

$$\tau_p = (1 - P_p)R_0. \quad (14)$$

$$\begin{aligned}
 P_p \approx F_Y(\varpi) - \frac{\varpi\pi}{2N} \sum_{n=1}^N \sum_{s=0}^{m-1} \sum_{r=0}^{m-1} \sqrt{1-\theta_n^2} \exp\left(-\frac{m\psi_0}{\rho_p\Omega_{f,1}}\right) \exp\left(\frac{m(b_n-\psi_0)}{\rho_p\Omega_{f,0}}\right) \frac{(m\psi_0/\rho_p\Omega_{f,1})^s}{s!} \frac{(m(\psi_0-b_n))^r}{r!(\rho_p\Omega_{f,0})^r} f_Y(b_n) \\
 + \left(1 - \exp\left(-\frac{m\psi_0}{\rho_p\Omega_{f,1}}\right)\right) \sum_{s=0}^{m-1} \frac{(m\psi_0/\rho_p\Omega_{f,1})^s}{s!} \exp\left(-\frac{m\psi_0}{\rho_p\Omega_{f,0}}\right) \sum_{l=0}^{m-1} \frac{(m\psi_0/\rho_p\Omega_{f,0})^l}{l!}.
 \end{aligned} \quad (12)$$

B. Secondary Network

1) *Outage Probability*: The outage event for SR_k is defined as follows. First, we denote $\mathcal{O}_{k,j} \triangleq \{R_{j \rightarrow k} < R_j\}$ as the event that SR_k cannot detect SR_j 's signal, $0 \leq j \leq k$, and $\mathcal{O}_{k,j}^c$ as the complementary set of $\mathcal{O}_{k,j}$. When SR_k decodes x_0 successfully, the outage probability can be expressed as

$$\tilde{P}_k = 1 - \Pr\left(\mathcal{O}_{k,0}^c \cap \dots \cap \mathcal{O}_{k,k}^c\right). \quad (15)$$

Since SR_k will first adopt MRC to detect the primary user's signal (i.e. $j=0$), the event $\mathcal{O}_{k,0}^c$ is given by

$$\mathcal{O}_{k,0}^c \triangleq \left\{ \rho_p |h_{p,k}|^2 + \frac{\rho_s |h_{s,k}|^2 \alpha_0}{\rho_s |h_{s,k}|^2 \sum_{i=1}^K \alpha_i + 1} > \psi_0 \right\} \quad (16)$$

and the event $\mathcal{O}_{k,j}^c$, $1 \leq j \leq k$, is formulated as

$$\mathcal{O}_{k,j}^c \triangleq \left\{ \frac{\rho_s |h_{s,k}|^2 \alpha_j}{\rho_s |h_{s,k}|^2 \sum_{i=j+1}^K \alpha_i + 1} > \psi_j \right\} \quad (17)$$

where $\psi_j = 2^{2R_j} - 1$, $j \in \mathcal{K}$. In addition, note that the event $\mathcal{O}_{K,K}^c$ can be expressed as $\mathcal{O}_{K,K}^c \triangleq \{\rho_s |h_{s,K}|^2 > \psi_K\}$.

Theorem 2: \tilde{P}_k can be approximated as (18), shown at the top of the next page, where N is a complexity-accuracy trade-off parameter, $\phi_n = \cos(\frac{2n-1}{2N}\pi)$, $\tau_k = \frac{K!}{(K-k)!(k-1)!}$, $\epsilon(\psi_k^*) = \frac{\psi_0 - \alpha_0 \psi_k^*}{\rho_p - \rho_p(1+\psi_k^* \sum_{i=1}^K \alpha_i)}$, $\psi'_0 = \frac{\psi_0 - \rho_p(\phi_n+1)\epsilon(\psi_k^*)/2}{\alpha_0 - (\psi_0 - \rho_p(\phi_n+1)\epsilon(\psi_k^*)/2) \sum_{i=1}^K \alpha_i}$, $f_{|h_{p,k}|^2}(z)$ and ψ_k^* are defined in Appendix B.

Proof: See Appendix B. ■

The outage probability of SR_k with the proposed NOMA-based cooperative transmission scheme is given by

$$P_k = \Pr(\rho_p |f_1|^2 > \psi_0) \tilde{P}_{out,k} + \Pr(\rho_p |f_1|^2 < \psi_0). \quad (19)$$

Substituting (18) and (28) into (19), with the help of Theorem 2, a closed-form expression for P_k can be readily obtained.

2) *Asymptotic Analysis*: Using the same rationale with (13), an asymptotic expression for (19) can be expressed as

$$P_k \stackrel{\rho_s \rightarrow \infty}{\simeq} \frac{1}{\rho_s^{mk}} \left(\tau'_k \zeta_m + \frac{\epsilon(\psi_k^*) \pi}{2N} \sum_{n=1}^N e_n \delta_m \right) \quad (20)$$

where $\tau'_k = \frac{K!}{(K-k)!(k-1)!}$, $\zeta_m = \left(\frac{m\psi_k^*}{\mu\Omega_{s,s}}\right)^{mk}$, $\delta_m = \left(\frac{m\psi'_0}{\mu\Omega_{s,s}}\right)^{mk}$, and $e_n = \sqrt{1 - \phi_n^2}$. It is seen from (20) that a diversity order of mk is achieved at SR_k . An intuitive explanation is that if all the previous $(k-1)$ ordered channels fail, it declares an outage at the k th user, which ensures a diversity order of mk .

3) *Outage Throughput*: Based on the analytical results for the outage probability of the secondary network, the outage throughput with the fixed transmission rates is

$$\tau_s = (1 - P_1)R_1 + \dots + (1 - P_K)R_K. \quad (21)$$

C. OMA-Based Schemes for the Spectrum-Sharing CRN

This subsection closes with a discussion of the OMA-based schemes for the considered spectrum-sharing CRN.

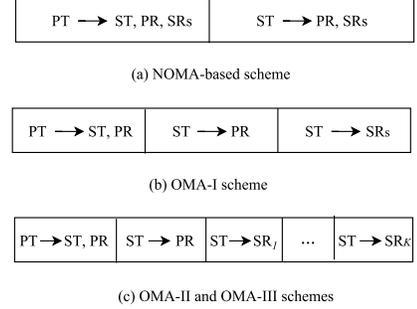


Fig. 1. Transmission illustration of NOMA-based and OMA-based schemes.

1) *OMA-I Scheme*: Similar to the spectrum leasing scheme [6], a fraction of $1 - \xi$ of the phase is used for the cooperative transmission of the primary user's information, and the remaining ξ of the phase is dedicated to the secondary transmissions by employing NOMA, as shown in Fig. 1 (b). Therefore, the achievable data rate at PR can be expressed as

$$R_p^{\text{OMA-I}} = \frac{1-\xi}{2} \log_2 \left(1 + \rho_p |f_0|^2 + \rho_s |f_2|^2 \right). \quad (22)$$

The achievable data rate at SR_k after SIC is obtained as

$$R_k^{\text{OMA-I}} = \xi \log_2 \left(1 + \frac{\rho_s |h_{s,k}|^2 \beta_k}{\rho_s |h_{s,k}|^2 \sum_{i=k+1}^K \beta_i + 1} \right) \quad (23)$$

where β_i , $i \in \mathcal{K}$, denotes the power allocation coefficients.

2) *OMA-II Scheme*: By using conventional OMA (i.e. time-division multiple access) for the secondary transmissions, the OMA-II scheme can be revised as follows. The first $1 - \xi$ of the phase is same with the OMA-I scheme. The remaining ξ of the phase is divided equally into K time slots, as shown in Fig. 1 (c). The achievable data rate at SR_k during the k -th time slot is written as

$$R_k^{\text{OMA-II}} = \frac{\xi}{K} \log_2 \left(1 + \rho_s |h_{s,k}|^2 \right). \quad (24)$$

3) *OMA-III Scheme*: An opportunistic scheme is adopted in the remaining ξ of the phase for the secondary access¹, as depicted in Fig. 1 (c). The k th user with the better channel conditions is allocated with a larger weight coefficient ω_k , and the achievable data rate of the secondary network is

$$R_k^{\text{OMA-III}} = \omega_k \xi \log_2 \left(1 + \rho_s |h_{s,k}|^2 \right). \quad (25)$$

Moreover, the first $1 - \xi$ of the phase is similar to the OMA-I and OMA-II schemes. Note that we can use (22)–(25) to derive the outage probability and the system throughput achieved by the OMA schemes. These results can be used as benchmarks to compare with our NOMA-based cooperative transmission scheme in the next section.

¹This comes from exploiting the fluctuations of the fading channels, i.e. allocating more resources to the user with better channel conditions, instead of equally distributing the resources as in the OMA-II scheme.

$$\begin{aligned} \tilde{P}_k \approx & \tau_k \sum_{i=0}^{K-k} \frac{(-1)^i}{k+i} \binom{K-k}{i} \left(\left(1 - \exp\left(-\frac{m\psi_k^*}{\rho_s \Omega_{s,s}}\right) \sum_{l=0}^{m-1} \frac{(m\psi_k^*/\rho_s \Omega_{s,s})^l}{l!} \right)^{k+i} \exp\left(-\frac{m\epsilon(\psi_k^*)}{\Omega_{p,s}}\right) \sum_{r=0}^{m-1} \frac{(m\epsilon(\psi_k^*)/\Omega_{p,s})^r}{r!} \right. \\ & \left. + \frac{\epsilon(\psi_k^*)\pi}{2N} \sum_{n=1}^N \sqrt{1-\phi_n^2} \left(1 - \exp\left(-\frac{m\psi_0'}{\rho_s \Omega_{s,s}}\right) \sum_{l=0}^{m-1} \frac{(m\psi_0'/\rho_s \Omega_{s,s})^l}{l!} \right)^{k+i} f_{|h_{p,k}|^2} \left(\frac{(\phi_n+1)\epsilon(\psi_k^*)}{2} \right) \right). \end{aligned} \quad (18)$$

IV. NUMERICAL EXAMPLES

In this section, simulation results are presented to validate the efficiency of the NOMA-based cooperative transmission scheme. Without loss of generality, it is assumed that $\Omega_{f,0} = \Omega_{p,s} = 0.1$ and $\Omega_{f,1} = \Omega_{f,2} = \Omega_{s,s} = 0.2$. The power allocation coefficients are $\alpha_0 = 0.6$, $\alpha_1 = 0.3$ and $\alpha_2 = 0.1$. The Gaussian-Chebyshev parameter is chosen as $N = 10$, since it generally provides a close approximation of the outage probability, and the phase allocation factor ξ of the OMA-based schemes is set to $1/3$ for the illustration purpose².

In Fig. 2, the outage performance of the primary network is shown as a function of the system SNR with different fading parameter m , where the target data rate is $R_0 = 0.5$ bps/Hz. It is observed that the proposed NOMA-based scheme achieves a lower outage probability than the OMA-based schemes for the primary network. The reason is that our proposed scheme can serve the primary and secondary users simultaneously without additional time/bandwidth, which effectively improves the spectrum utilization. It is also observed that all the curves (with the same the fading parameter m) have the same slope, due to the use of cooperative transmission for the primary network. Furthermore, it is worth pointing that the approximated results in (12) match perfectly the simulations, and the derived asymptotic results in (13) are very tight at high SNRs.

Fig. 3 provides the outage performance of the secondary network versus ρ_s . The target data rates are set the same as $R_0 = R_1 = R_2 = 0.5$ bps/Hz, and the power coefficients for OMA-I scheme are $\beta_1 = 0.8$, $\beta_2 = 0.2$. As expected, our proposed scheme outperforms the OMA-based schemes for most cases, and the diversity order at SR_i is a function of its ordered channels, which is consistent to (20). Interestingly, note that the OMA-I scheme achieves a lower outage probability than the proposed scheme for SR_1 . This can be explained by the fact that compared with the proposed scheme, more power is allocated to SR_1 by the OMA-I scheme (i.e. $\beta_1 = 0.8$ for the OMA-I scheme and $\alpha_1 = 0.3$ for the proposed scheme), and the available transmit power dominates the outage probability in this case. This indicates the significance of the appropriate power allocation coefficients for NOMA.

Fig. 4 depicts the system throughput for the primary and secondary networks (i.e. $\tau_p + \tau_s$) based on different data rate settings. Clearly, our proposed scheme achieves the higher system throughput due to its lower outage probability for both networks. Note also that there exist throughput ceilings in the high-SNR regime, since the outage probability is approaching to zero and in this case, the system throughput is determined only by the target data rates of all users. Furthermore, the

system throughput by the proposed scheme increases with an increase in the target data rates, as seen by the red and black lines. However, when $R_0 = 1$ bps/Hz, the system throughput of the proposed scheme is lowered and becomes worse than the OMA-based schemes. An intuitive explanation is that with the fixed power coefficients, the received SNR to detect x_0 cannot support the target threshold at SR_i with a larger R_0 , and thus leads to the failure of the detection for x_1 and x_2 . As a result, an outage will be declared at SR_1 and SR_2 , which in return reduces the system throughput. Therefore, it is important to select the target data rates in designing NOMA schemes.

V. CONCLUSION

In this paper, a novel NOMA-based cooperative transmission scheme has been proposed for the spectrum-sharing CRN. The rationale behind is that the primary network shares the available spectrum with the secondary network in exchange for the cooperation, and thus the secondary transmitter is willing to transmit the primary message as well as its messages with employing NOMA signaling. This ensures that multiple users can be served simultaneously and the overall spectrum can be better utilized. Thus, an enhanced win-win situation for both networks can be achieved. Analytical expressions of the outage probability and the system throughput have been derived in closed forms. Substantial performance gains have been shown using the NOMA-based scheme than the conventional OMA-based schemes.

APPENDIX A: PROOF OF THEOREM 1

If X is Nakagami- m distributed with an integer fading parameter m , then its cumulative density function $F_X(x)$ can be expressed as [5]

$$F_X(x) = 1 - \exp\left(-\frac{mx}{\Omega_x}\right) \sum_{l=0}^{m-1} \frac{(mx/\Omega_x)^l}{l!}. \quad (26)$$

Then, we can rewrite I_1 in (11) as

$$\begin{aligned} I_1 \stackrel{(a)}{=} & \underbrace{\Pr\left(|f_1|^2 > \frac{\psi_0}{\rho_p}\right)}_{I_{11}} \\ & \times \underbrace{\Pr\left(\rho_p |f_0|^2 + \frac{\rho_s |f_2|^2 \alpha_0}{\rho_s |f_2|^2 \sum_{i=1}^K \alpha_i + 1} < \psi_0\right)}_{I_{12}} \end{aligned} \quad (27)$$

where step (a) is due to the fact that all channels are independent. With the help of (26), I_{11} is readily derived as

$$I_{11} = \exp\left(-\frac{m\psi_0}{\rho_p \Omega_{f,1}}\right) \sum_{s=0}^{m-1} \frac{(m\psi_0/\rho_p \Omega_{f,1})^s}{s!}. \quad (28)$$

²With an additional constraint, i.e. the minimum rate requirement of PR, the value of ξ can be optimized for the OMA schemes. The consideration of such optimization is beyond the scope of this paper.

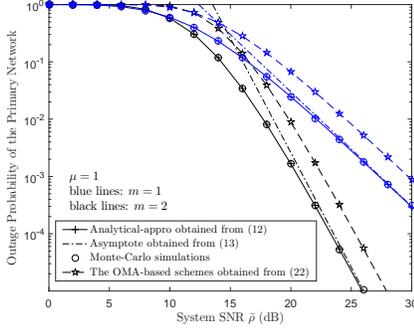


Fig. 2. Outage probability of the PR.

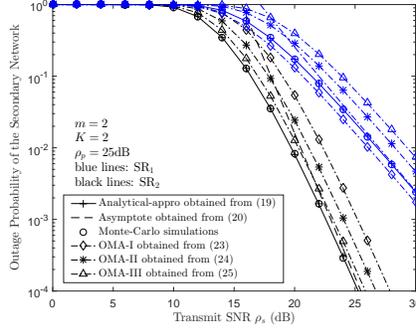


Fig. 3. Outage probability of the SRs.

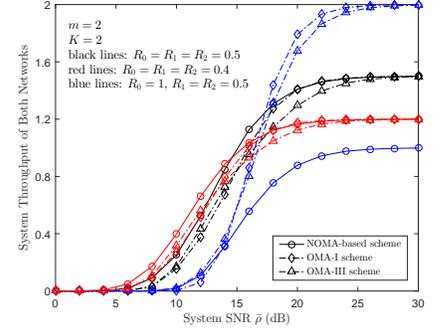


Fig. 4. System throughput of both networks.

Denote $Y = \frac{\rho_s |f_2|^2 \alpha_0}{\rho_s |f_2|^2 \sum_{i=1}^K \alpha_i + 1}$, then I_{12} can be calculated as

$$\begin{aligned} I_{12} &= \int_0^\infty \Pr\left(|f_0|^2 < \frac{\psi_0 - y}{\rho_p}\right) f_Y(y) dy \\ &= F_Y(\infty) - \int_0^\infty \exp\left(-\frac{m(\psi_0 - y)}{\rho_p \Omega_{f,0}}\right) \\ &\quad \times \sum_{r=0}^{m-1} \frac{(m(\psi_0 - y)/\rho_p \Omega_{f,0})^r}{r!} f_Y(y) dy. \end{aligned} \quad (29)$$

However, it is challenging to obtain a closed-form expression for (29). In this case, by changing of variable $y = \frac{(x+1)\varpi}{2}$ and using Gaussian-Chebyshev quadrature [10, eq. (25.4.38)], we can approximate (29) as

$$\begin{aligned} I_{12} &\approx F_Y(\varpi) - \frac{\varpi\pi}{2N} \sum_{n=1}^N \sum_{r=0}^{m-1} \sqrt{1 - \theta_n^2} \\ &\quad \times \exp\left(\frac{m(b_n - \psi_0)}{\rho_p \Omega_{f,0}}\right) \frac{(m(\psi_0 - b_n))^r}{r!(\rho_p \Omega_{f,0})^r} f_Y(b_n) \end{aligned} \quad (30)$$

where $F_Y(\varpi)$ can be easily obtained by using (26). From (30), $f_Y(y)$ can be derived as

$$\begin{aligned} f_Y(y) &= \frac{m^m (y/\rho_s (\alpha_0 - y \sum_{i=1}^K \alpha_i))^{m-1}}{\Omega_{f,2}^m \Gamma(m)} \\ &\quad \times \exp\left(-\frac{my}{\rho_s \Omega_{f,2} (\alpha_0 - y \sum_{i=1}^K \alpha_i)}\right) \end{aligned} \quad (31)$$

which is conditioned on $\alpha_0 > y \sum_{i=1}^K \alpha_i$. By substituting the above results into (27), a closed-form for I_1 is obtained.

In addition, I_2 in (11) can be calculated as

$$\begin{aligned} I_2 &= \Pr\left(|f_1|^2 < \frac{\psi_0}{\rho_p}\right) \Pr\left(|f_0|^2 < \frac{\psi_0}{\rho_p}\right) \\ &= \left(1 - \exp\left(-\frac{m\psi_0}{\rho_p \Omega_{f,1}}\right) \sum_{s=0}^{m-1} \frac{(m\psi_0/\rho_p \Omega_{f,1})^s}{s!}\right) \\ &\quad \times \left(1 - \exp\left(-\frac{m\psi_0}{\rho_p \Omega_{f,0}}\right) \sum_{l=0}^{m-1} \frac{(m\psi_0/\rho_p \Omega_{f,0})^l}{l!}\right). \end{aligned} \quad (32)$$

Now, combining I_1 and I_2 with (11), after some mathematical simplifications, Theorem 1 can be readily proved.

APPENDIX B: PROOF OF THEOREM 2

When $j = 0$, we can rewrite (16) as

$$\mathcal{O}_{k,0}^c \triangleq \left\{ |h_{s,k}|^2 > \frac{\tilde{\psi}_0}{\rho_s} \right\} \quad (33)$$

where $\tilde{\psi}_0 = \frac{\psi_0 - \rho_p |h_{p,k}|^2}{\alpha_0 - (\psi_0 - \rho_p |h_{p,k}|^2) \sum_{i=1}^K \alpha_i}$. It is assumed that $\alpha_0 > (\psi_0 - \rho_p |h_{p,k}|^2) \sum_{i=1}^K \alpha_i$, otherwise, an outage will be declared at SR_k .

When $1 \leq j \leq k$, (17) can be rewritten as

$$\mathcal{O}_{k,j}^c \triangleq \left\{ |h_{s,k}|^2 > \frac{\tilde{\psi}_j}{\rho_s} \right\} \quad (34)$$

where $\tilde{\psi}_j = \frac{\psi_j}{\alpha_j - \psi_j \sum_{i=j+1}^K \alpha_i}$, by assuming a necessary condition as $\alpha_j > \psi_j \sum_{i=j+1}^K \alpha_i$. Denote $\psi_k^* = \max\{\tilde{\psi}_1, \dots, \tilde{\psi}_k\}$, the outage probability in (15) can be reformulated as

$$\begin{aligned} \tilde{P}_{out,k} &= 1 - \Pr\left(|h_{s,k}|^2 > \frac{\max\{\tilde{\psi}_0, \psi_k^*\}}{\rho_s}\right) \\ &= \underbrace{\int_{\epsilon(\psi_k^*)}^{\infty} \Pr\left(|h_{s,k}|^2 < \frac{\psi_k^*}{\rho_s}\right) f_{|h_{p,k}|^2}(x) dx}_{I_3} \\ &\quad + \underbrace{\int_0^{\epsilon(\psi_k^*)} \Pr\left(|h_{s,k}|^2 < \frac{\tilde{\psi}_0}{\rho_s}\right) f_{|h_{p,k}|^2}(x) dx}_{I_4}. \end{aligned} \quad (35)$$

By using binomial expansion and order statistics [11], I_3 can be further calculated as

$$\begin{aligned} I_3 &= \tau_k \sum_{i=0}^{K-k} \frac{(-1)^i}{k+i} \binom{K-k}{i} \left(F_{|\tilde{h}_{s,k}|^2}\left(\frac{\psi_k^*}{\rho_s}\right)\right)^{k+i} \\ &\quad \times \left(1 - F_{|h_{p,k}|^2}(\epsilon(\psi_k^*))\right) \end{aligned} \quad (36)$$

where $|\tilde{h}_{s,k}|^2$ denotes the unordered channel gain. However, it is challenging to attain an exact closed form for I_4 . In the following, the Gaussian-Chebyshev quadrature [10, eq. (25.4.38)] is used to find an approximation, shown by

$$\begin{aligned} I_4 &\approx \frac{\epsilon(\psi_k^*) \pi \tau_k}{2N} \sum_{n=1}^N \sum_{i=0}^{K-k} \frac{(-1)^i}{k+i} \binom{K-k}{i} \sqrt{1 - \phi_n^2} \\ &\quad \times \left(F_{|\tilde{h}_{s,k}|^2}\left(\frac{\tilde{\psi}_0}{\rho_s}\right)\right)^{k+i} f_{|h_{p,k}|^2}\left(\frac{(\phi_n + 1)\epsilon(\psi_k^*)}{2}\right) \end{aligned} \quad (37)$$

in which $f_{|h_{p,k}|^2}(z)$ is derived readily as

$$f_{|h_{p,k}|^2}(z) = \frac{m^m z^{m-1}}{\Omega_{p,s} \Gamma(m)} \exp\left(-\frac{mz}{\Omega_{p,s}}\right). \quad (38)$$

Hence, combining the foregoing results with (35), one can prove Theorem 2 straightforwardly.

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