Brief paper

Freeway Traffic Estimation Within Particle Filtering Framework

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Abstract

This paper formulates the problem of real-time estimation of traffic state in freeway networks by means of particle filtering framework. A particle filter (PF) is developed based on a recently proposed speed-extended cell-transmission model of freeway traffic. The freeway is considered as a network of components representing different freeway stretches called *segments*. The evolution of the traffic in a segment is modelled as a *dynamic stochastic system*, influenced by states of neighbour segments. Measurements are received only at boundaries between some segments and averaged within possibly irregular time intervals. This limits the measurement update in the PF to only these time instants when a new measurement arrives, with possibly many state updates in between consecutive measurement updates. The PF performance is validated and evaluated using synthetic and real traffic data from a Belgian freeway. An Unscented Kalman filter is also presented. A comparison of the particle filter with the Unscented Kalman filter is performed with respect to accuracy and complexity.

Keywords: Bayesian estimation, particle filtering, macroscopic traffic models, stochastic systems, unscented Kalman filter

1 Motivation

Traffic state estimation and prediction is of paramount importance for on-line road traffic management, efficiency and safety. Vehicular traffic is characterised with highly nonlinear behaviour (Helbing, 2002), with many interactions between vehicles, and high complexity which makes this problem challenging. This behaviour can be described by *macroscopic* models (Hoogendoorn and Bovy, 2001; Kotsialos et al., 2002; Helbing, 2002) that are suitable for real-time problems in view the fact that they represent the *average* traffic behaviour in terms of aggregated variables (flow, density and speed at different locations). Most papers dealing with recursive traffic state estimation apply the Extended Kalman filter (EKF) to such macroscopic models. For example (Wang and Papageorgiou, 2005) proposes an EKF to estimate the unknown parameters and states of a stochastic version of the macroscopic freeway traffic flow model METANET (Papageorgiou and Blosseville, 1989) of freeway traffic. These estimators have all the advantages and disadvantages of the EKF technique: presumably computationally cheap, but relying on a linearisation of the state and measurement models which can cause filter divergence. A powerful and scalable approach has recently been developed, known under different names such as *particle filters* (PFs) (Doucet et al., 2001; Ristic et al., 2004; Chen, 2003) and *bootstrap method* (Gordon et al., 1993). All information about the states of interest can be obtained from the conditional distribution of the state given the past observations and the dynamics of the system. It approximates the posterior density function of the state by an empirical histogram obtained from samples generated by a Monte Carlo simulation. Particle filtering allows to cope with uncertainties and nonlinearities of different kinds, nonGaussian noises and hence is suitable for the traffic estimation problem.

In the present paper we formulate the traffic estimation problem within this Bayesian framework and develop a particle filter for freeway traffic flow estimation. This is an extension and generalisation of the results reported in (Mihaylova and Boel, 2004). The structure of the PF fits well to the compositional traffic networks, and it allows for parallelisation for different segments.

In (Sun et al., 2003) a solution to highway traffic estimation is proposed by a sequential Monte Carlo algorithm, the so-called mixture Kalman filtering. First-order traffic models represent the network, i.e. only the traffic density is modelled, distinguishing between the *free-flow mode* and *congestion mode*. In contrast to (Sun et al., 2003), the traffic in the present paper is described

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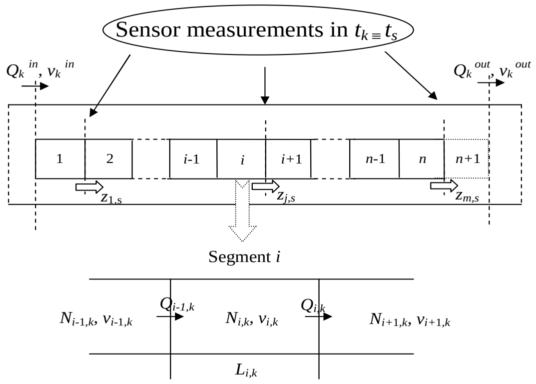

Fig. 1. Freeway segments and measurement points. Q_i is the average number of vehicles crossing the boundary between segments i and $i+1$, N_i and v_i are respectively the average number of vehicles and speed within segment i.

by a second-order macroscopic model, and we develop a particle filter that estimates both the traffic density and speed. The traffic is described by the recently developed model (Boel and Mihaylova, 2006) that is an extension to the cell-transmission model (Daganzo, 1994). The freeway network is modelled as a sequence of segments (Fig. 1). Sensors are available only at some boundaries between segments. Technological limitations (such as limited bandwidth of communication channels) force one to average these measurements over regular or irregular time intervals before they are transmitted to the centre where the Bayesian update of the conditional densities is carried out (for all segments concurrently).

We investigate the PF performance with respect to accuracy and complexity and we compare it with another method suitable for traffic flow estimation, the Unscented Kalman filter (UKF) (Julier and Uhlmann, 2004; Wan and van der Merwe, 2001). The UKF is a derivative-free estimation method, that has proven to outperform the EKF. The UKF can be viewed as a method to approximate the first two moments of the state: the mean and covariance. Unlike the EKF, the UKF does not require calculation of Jacobians and Hessians. Deterministic sampling approach is used to calculate the mean and covariance, by the so-called sigma points. Compared with the EKF's first-order accuracy, the estimation accuracy of the UKF is to the third order (Taylor series expansion) for any nonlinearity. The EKF requires calculation of derivatives for all traffic segments which for the traffic problem with interconnected components is quite complicated. Moreover, divergence problems are not excluded. The UKF is much easier to implement and more accurate. However, the UKF often encounters the ill-conditioned problem of the covariance matrix in practice (though theoretically it is positive semi-definite). Methods for enhancing the numerical properties of the UKF (e.g. as based on singular-value decomposition) can overcome these numerical instabilities (Chen, 2003).

The added values and innovative aspects of this paper as compared to previous investigations include:

1. A general stochastic macroscopic traffic flow model is presented together with measurement equations, suitable for a PF real-time estimation and prediction.

2. We demonstrate that PF can be efficiently and easily implemented for large compositional models and sparse measurements, received *synchronously* or *asynchronously* at intervals, bigger than the state-update sample time. The developed approach is general and applicable to freeway networks with different topologies.

3. We compare the PF performance with respect to an UKF. We show that the PF estimates are more accurate than those of the UKF, nevertheless the PF is more computationally expensive.

The outline of the paper is as follows. Section 2 presents a stochastic macroscopic traffic model for freeway stretches and a model for real-time traffic measurements. Bayesian formulation of the traffic estimation problem is given in Section 3. A PF framework for traffic state estimation is developed in Section 4 which takes advantage of the compositional traffic model. Section 5 describes the UKF for traffic estimation that is compared with the developed PF. The PF performance is evaluated in Section 6. Conclusions and future research issues are highlighted in Section 7.

2 Freeway Traffic Flow Model

2.1 Compositional Macroscopic Traffic Model

Traffic states are estimated consecutively at discrete time instants $t_1, t_2, \ldots, t_k, \ldots$, possibly *asynchronously*, based on all the incoming information up to the current time transmitted by sensors to the filter. The overall state vector $\boldsymbol{x}_k = (\boldsymbol{x}_{1,k}^T, \boldsymbol{x}_{2,k}^T, \ldots, \boldsymbol{x}_{n,k}^T)^T$ at time t_k consists of local state vectors $\boldsymbol{x}_{i,k} = (N_{i,k}, v_{i,k})^T$, where $N_{i,k}$, $[veh]$, is the number of vehicles counted in segment $i \in \mathcal{I} = \{1, 2, \ldots n\}$, and $v_{i,k}$, $[km/h]$, is their average speed. The traffic state evolution is described by the system of equations

$$\boldsymbol{x}_{1,k+1} = f_1(Q_k^{in}, v_k^{in}, \boldsymbol{x}_{1,k}, \boldsymbol{x}_{2,k}, \boldsymbol{\eta}_{1,k}), \quad (1)$$

$$\boldsymbol{x}_{i,k+1} = f_i(\boldsymbol{x}_{i-1,k}, \boldsymbol{x}_{i,k}, \boldsymbol{x}_{i+1,k}, \boldsymbol{\eta}_{i,k}), \quad (2)$$

$$\boldsymbol{x}_{n,k+1} = f_n(\boldsymbol{x}_{n-1,k}, \boldsymbol{x}_{n,k}, Q_k^{out}, v_k^{out}, \boldsymbol{\eta}_{n,k}), \quad (3)$$

where f_i is specified by the traffic model, Q_k^{in} is the number of vehicles entering segment 1 during the interval $\Delta t_k = t_{k+1} - t_k$ with average speed v_k^{in}, Q_k^{out} is the outflow leaving a 'fictitious' segment $n+1$, with an average speed v_k^{out}. $\boldsymbol{\eta}_k$ is a disturbance vector, reflecting random fluctuations and the effect of modelling errors in the state evolution. Note that Q_k^{in}, v_k^{in}, and Q_k^{out}, v_k^{out} are respectively, inflow and outflow *boundary variables*. They *are not* traffic states and are not estimated. They are supplied by the traffic detectors. Hence, a chain of interconnected segments is considered, together with their boundary conditions.

In this paper the general state-space description (1)-(3) takes a particular form of the recently developed compositional stochastic macroscopic traffic model (Boel and Mihaylova, 2006). This speed-extended cell-transmission model describes the complex traffic behaviour with *forward* and *backward* propagation of traffic perturbations and is suitable for large networks and for distributed processing. The forward and backward traffic perturbations were characterised by (Daganzo, 1994) through deterministic sending and receiving functions where piecewise affine representations are used. In (Boel and Mihaylova, 2004; Boel and Mihaylova, 2006) *speed-dependent* random sending and receiving functions are introduced that represent also the evolution of the average speed in each segment. The model is given in concise form as Algorithm 1.

The *sending function* $S_{i,k}$ for each segment i, having length L_i, is calculated by (4). $S_{i,k}$ represents the vehicles that "intend to leave" segment i within Δt_k. The *receiving function* $R_{i,k}$ (6). expresses the maximum number of vehicles that are allowed to enter segment $i+1$. In (6) $N^{max}_{i+1,k}$ characterises the *maximum number of vehicles* that can simultaneously be present in segment $i+1$ in Δt_k. $N^{max}_{i+1,k}$ depends on the available space, L_{i+1} time the number of lanes $\ell_{i+1,k}$, in segment $i+1$, on the average length A_ℓ of vehicles, the average speed $v_{i+1,k}$ and the time distance t_d between two vehicles (in order to allow safe driving).

The evolution of $N_{i,k+1}$ is governed by the principle of conservation of vehicles (9). The traffic density $\rho_{i,k+1}$, $[veh/km/lane]$, is given by (10). The anticipated density $\rho^{antic}_{i,k+1}$ is then obtained as a weighed average between the density of segment i and segment $i+1$, (11). This corresponds to the drivers' tendency usually to look ahead when they change their speed. The average vehicle speed $v_{i,k+1}$ is a function of the 'intermediate' speed $v^{interm}_{i,k+1}$, calculated in step 5 of Algorithm 1, and of the equilibrium speed satisfying a speed-density relation $v^e(\rho^{antic}_{i,k+1})$ (Kotsialos et al., 2002).

Design traffic parameters are: the free-flow speed v_{free}, the critical density ρ_{crit} (density below which the interactions between vehicles will be negligible), the density in jam, ρ_{jam}, above which the vehicles do not move, and the minimum vehicle speed v_{min}, the parameters $\alpha \in (0,1]$, $0 < \beta^I < \beta^{II}$, a threshold density value $\rho_{threshold}$. Other details for the model can be found in (Boel and Mihaylova, 2004; Boel and Mihaylova, 2006) where this extended cell-transmission model has been validated both against the well established METANET model (Papageorgiou and Blosseville, 1989; Kotsialos et al., 2002), and over real traffic data.

Algorithm 1. The compositional traffic model.

1. *Forward wave* : for $i = 1, 2, \ldots, n$

$$S_{i,k} = max\left(N_{i,k}\frac{v_{i,k}.\Delta t_k}{L_i} + \eta_{S_i,k},\ N_{i,k}\frac{v_{min}.\Delta t_k}{L_i}\right) \quad (4)$$

and set $Q_{i,k} = S_{i,k}$. (5)

2. *Backward wave* : for $i = n, n-1, \ldots, 1$

$$R_{i,k} = N^{max}_{i+1,k} - N_{i+1,k} + Q_{i+1,k}, \quad (6)$$

where $N^{max}_{i+1,k} = (L_{i+1}\ell_{i+1,k})/(A_\ell + v_{i+1,k}t_d)$.

$$if \quad S_{i,k} < R_{i,k}, \quad Q_{i,k} = S_{i,k}, \quad (7)$$

$$else \quad Q_{i,k} = R_{i,k}, \quad v_{i,k} = Q_{i,k}L_i/(N_{i,k}\Delta t_k), \quad (8)$$

3. Update the number of vehicles inside segments, for $i = 1, 2, \ldots, n$

$$N_{i,k+1} = N_{i,k} + Q_{i-1,k} - Q_{i,k}, \quad (9)$$

4. Update the density, for $i = 1, 2, \ldots, n$

$$\rho_{i,k+1} = N_{i,k+1}/(L_i\ell_{i,k+1}), \quad (10)$$

$$\rho^{antic}_{i,k+1} = \alpha\rho_{i,k+1} + (1-\alpha)\rho_{i+1,k+1}. \quad (11)$$

5. Update of the speed, for $i = 1, 2, \ldots, n$

$$v^{interm}_{i,k+1} = \begin{cases} \frac{v_{i-1,k}Q_{i-1,k} + v_{i,k}(N_{i,k}-Q_{i,k})}{N_{i,k+1}}, & \text{for } N_{i,k+1} \neq 0, \\ v_f, & \text{otherwise}, \end{cases}$$

$$v^{interm}_{i,k+1} = max(v^{interm}_{i,k+1}, v_{min}),$$

$$v_{i,k+1} = \beta_{k+1}v^{interm}_{i,k+1} + (1-\beta_{k+1})v^e(\rho^{antic}_{i,k+1}) + \eta_{v_i,k+1},$$

where

$$\beta_{k+1} = \begin{cases} \beta^I, & \text{if } |\rho^{antic}_{i+1,k+1} - \rho^{antic}_{i,k+1}| \geq \rho_{threshold}, \\ \beta^{II} & \text{otherwise}. \end{cases}$$

2.2 Measurement Model

Sensors (magnetic loops, video cameras, radar detectors) are located at boundaries between some segments. Usually, measurements are collected at the entrance and at the exit of the considered stretch of the road, at the on-ramps and off-ramps, etc.

Let us consider m sensors along the stretch. Traffic states are measured at discrete time instants. The overall measurement vector $\boldsymbol{z}_s = (\boldsymbol{z}^T_{1,s}, \boldsymbol{z}^T_{2,s}, \ldots, \boldsymbol{z}^T_{m,s})^T$ at time t_s consists of local measurement vectors $\boldsymbol{z}_{j,s} = (Q_{j,s}, v_{j,s})^T$, where $j \in \mathcal{J} = \{1, 2, \ldots, m\}$. $Q_{j,s}$ is the noisy measurement of the number of vehicles crossing the boundaries between the corresponding segment i and segment $i+1$ during the time interval $\Delta t_s = t_{s+1} - t_s$, and $v_{j,s}$ is the measured mean speed of these vehicles. The intervals Δt_s are typically several times longer than the intervals Δt_k between q successive state update steps, i.e. $\Delta t_s = q\Delta t_k$. Given the measurement equation

$$\boldsymbol{z}_s = \boldsymbol{h}(\boldsymbol{x}_s, \boldsymbol{\xi}_s), \quad (12)$$

the distribution $p(\boldsymbol{x}_0)$ of the initial state vector $\boldsymbol{x}_0$, the state update model (1)-(3) with noises $\boldsymbol{\eta}$, $\boldsymbol{\xi}$, the traffic estimation problem can be formulated within Bayesian framework. We consider the following particular form for equation (12)

$$\boldsymbol{z}_{j,s} = \begin{pmatrix} \bar{Q}_{j,s} \\ \bar{v}_{j,s} \end{pmatrix} + \boldsymbol{\xi}_{j,s}, \tag{13}$$

where $\bar{Q}_{j,s}$ is the sum of the number of vehicles (calculated by the state model) crossing the boundary between segments i and $i+1$ within the interval Δt_s, and $\bar{v}_{j,s}$ is their average speed. Although Gaussian distributions of the noise vector $\boldsymbol{\xi}_{j,s} = (\xi_{Q_j,s},\ \xi_{v_j,s})^T$ in (13) has been used previously in the literature (Wang and Papageorgiou, 2005), we propose a more realistic noise model. This is another advantage of the PF: the knowledge of the noise distributions can be utilised for a better state tracking. Based on statistical analysis of different sets of traffic data we found that there are two kinds of measurement errors in the counted vehicles by the video cameras: errors due to *false detections*, $\xi_{Q_j^{false},s}$, and errors due to *missed vehicles*, $\xi_{Q_j^{missed},s}$. Hence, the measurement error in the observation equation (12) resp. (13) is of the form

$$\xi_{j,s} = Q_{j,s}^{err} = Q_{j,s}^{false} - Q_{j,s}^{missed}, \tag{14}$$

where the number of the vehicles that a detector j missed is denoted by $Q_{j,s}^{missed}$, and the number of the false detections by $Q_{j,s}^{false}$. Analysis of data from video cameras has shown that $Q_{j,s}^{false}$ and $Q_{j,s}^{missed}$ can both be considered independent Poisson random variables with parameters λ_1 and λ_2. Based on our analysis we estimate $\lambda_1 + \lambda_2 = 2$, $\lambda_1 = 4/3$, $\lambda_2 = 2/3$. Then the PDF of the measurement noise $\xi_{Q_i,s}$ is a convolution of the form

$$p(Q_{j,s}^{err} = \nu_{i,s}^{err}) = \sum_{\nu_{i,s}^{missed}=0}^{\infty} \frac{\lambda_1^{(\nu_{i,s}^{err}+\nu_{i,s}^{missed})} e^{-\lambda_1}}{(\nu_{i,s}^{err}+\nu_{i,s}^{missed})!} \cdot \frac{\lambda_2^{\nu_{i,s}^{missed}} e^{-\lambda_2}}{\nu_{i,s}^{missed}!}. \tag{15}$$

Eq. (15) represents the PF likelihood function of the observations over the counted number of vehicles. We assume that the speed noises $\xi_{v_j,s}$ are Gaussian. Under the assumption that the vehicle counts are statistically independent from the speed measurements, the entire likelihood $p(\boldsymbol{z}_s|\boldsymbol{x}_s)$ given the state $\boldsymbol{x}_s$ is the product of the likelihood of the measured counts with the likelihood of the measured speeds.

3 Bayesian Estimation of Traffic Flows

Bayesian estimation evaluates the *posterior probability density function* (PDF) $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ of the state vector $\boldsymbol{x}_k$ up to time instant t_k given a set $\boldsymbol{Z}^k = \{\boldsymbol{z}_1, ..., \boldsymbol{z}_k\}$ of sensor measurements available at time t_k. Within the recursive Bayesian framework (Ristic et al., 2004), the conditional density function $p(\boldsymbol{x}_k|\boldsymbol{Z}^{k-1})$ of the state $\boldsymbol{x}_k$ given a set of measurements $\boldsymbol{Z}^{k-1}$ is recursively updated according to

$$p(\boldsymbol{x}_k|\boldsymbol{Z}^{k-1}) = \int_{\mathbb{R}^{n_x}} p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) p(\boldsymbol{x}_{k-1}|\boldsymbol{Z}^{k-1}) d\boldsymbol{x}_{k-1}, \tag{16}$$

$$p(\boldsymbol{x}_k|\boldsymbol{Z}^k) = \frac{p(\boldsymbol{z}_k|\boldsymbol{x}_k) p(\boldsymbol{x}_k|\boldsymbol{Z}^{k-1})}{p(\boldsymbol{z}_k|\boldsymbol{Z}^{k-1})}, \tag{17}$$

where $p(\boldsymbol{z}_k|\boldsymbol{Z}^{k-1})$ is a normalising constant. Therefore, the recursive update of $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ is proportional to

$$p(\boldsymbol{x}_k|\boldsymbol{Z}^k) \propto p(\boldsymbol{z}_k|\boldsymbol{x}_k) p(\boldsymbol{x}_k|\boldsymbol{Z}^{k-1}). \tag{18}$$

The state prediction step (16) and the measurement update step (17) use respectively the conditional density functions $p(\boldsymbol{z}_k|\boldsymbol{x}^k)$ and $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ that are defined by the model from Section 2.1.

4 Particle Filtering for Freeway Traffic

Evaluating (16)-(18) is computationally very expensive. The particle filter technique (Gordon et al., 1993; Doucet et al., 2001) provides an approximate solution to (16)-(18) by a discrete-time recursive update of the *posterior* PDF $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ of the state given the measurements The particle filter approximates $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ by the empirical histogram corresponding to a collection of M *particles (samples)* $\{\boldsymbol{x}_k^{(l)}\}_{l=1}^M$. To each particle l a weight $w_k^{(l)}$ is assigned at time t_k (the sum of these weights must be normalised to 1). The weight and the value of all particles together define a histogram that approximates the conditional density function of the state vector $\boldsymbol{x}_k$. After the arrival of a new observation vector $\boldsymbol{z}_s$, the particle filter updates the weights according to (18). The cloud of particles evolves with time and depending on the observations, so that the particles represent with sufficient accuracy the true PDF of the state (Doucet et al., 2001). A *resampling* procedure introduces variety in the particles, by eliminating the particles with small weights and by replicating particles with larger weights.

The traffic estimation problem has particularities distinguishing it from other estimation problems: *i)* the *limited* amount of available data from traffic detectors. The number of traffic variables to be estimated is much larger than the number of the traffic variables that are directly observed, and this "interpolation" is an essential contribution to the freeway traffic estimation task. *ii)* the state estimates are highly dependent on the inflow Q^{in}, v^{in} and random Q^{out}, v^{out} boundary variables.

The likelihood function $p(\boldsymbol{z}_k|\boldsymbol{x}_k)$ is calculated from (13) only when a measurement arrives, using the predicted state values and the known measurement noise density

Algorithm 2. A Particle Filter for Traffic Estimation.

```
I. Initialisation: k = 0
For l = 1,...,M, generate samples {x_0^(l)} from
    the initial distribution p(x_0) and
    initial weights w_0^(l) = 1/M.
End For
II. For k = 1,2,...,
(1) Prediction step:
For l = 1,...,M, sample x_k^(l) ~ p(x_k|x_{k-1}^(l))
     according to (4)-(11) for segments between
     two boundaries where measurements arrive
End For
(2) Measurement processing step (only for t_k ≡ t_s,
    on boundaries between the segments where
    measurements are available) compute the weights
For l = 1,...,M
   w_s^(l) = w_{s-1}^(l) p(z_s|x_s^(l)),
End For
 where the likelihood p(z_s|x_s^(l))
 is calculated by the model (13)
from section 2.2.
For l = 1,...,M
       Normalise the weights: ŵ_s^(l) = w_s^(l) / Σ_{l=1}^M w_s^(l).
End For
(3) Output: x̂_s = Σ_{l=1}^M ŵ_s^(l) x_s^(l),
(4) Selection step (resampling) only for t_k ≡ t_s:
    Multiply/ Suppress samples x_s^(l) with high/ low
    importance weights ŵ_s^(l), in order to obtain M
    random samples approximately distributed accor-
    ding to p(x_s^(l)|Z^s), e.g. by residual resampling.
* For l = 1,...,M, set w_s^(l) = ŵ_s^(l) = 1/M,  End For
(5) k ← k + 1 and return to step (1).
```

function $p(\boldsymbol{\xi}_s)$. The cloud of weighted particles representing the posterior conditional PDF, is used to map integrals to discrete sums: $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$ is approximated by

$$\hat{p}(\boldsymbol{x}_k|\boldsymbol{Z}^k) \approx \sum_{l=1}^{M} \tilde{w}_k^{(l)} \delta(\boldsymbol{x}_k - \boldsymbol{x}_k^{(l)}), \tag{19}$$

where δ is the delta-Dirac function and $\tilde{w}_k^{(l)}$ are the normalised weights of the posterior conditional PDF. New weights are calculated putting more weight on particles that are important according to the posterior probability density function (19). The random samples $\{\boldsymbol{x}_k^{(l)}, l = 1, 2, \dots, M\}$ are drawn from $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$. It is often impossible to sample from the posterior density function $p(\boldsymbol{x}_k|\boldsymbol{Z}^k)$. However, this difficulty is circumvented by making use of the importance sampling from a known *proposal distribution* $\pi(\boldsymbol{x}_k|\boldsymbol{Z}^k)$. The transition prior is the most popular choice of the proposal distribution (Wan and van der Merwe, 2001): $\pi(\boldsymbol{x}_k|\boldsymbol{Z}^k) = p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$, which in our solution to the traffic problem is the traffic state model. Algorithm 2 presents the PF developed in this paper.

5 An Unscented Kalman Filter for Traffic Flow Estimation

Other algorithms for approximating the posterior state PDF have been introduced. The Unscented Kalman filter (UKF) relies on the unscented transformation (Julier and Uhlmann, 2004; Wan and van der Merwe, 2001), a method for calculating the statistics of a random variable which undergoes a nonlinear transformation. Consider propagating a random variable $\boldsymbol{x}$ (with dimension n_x) through a nonlinear transformation $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$. Assume that $\boldsymbol{x}$ has mean $\hat{\boldsymbol{x}}$ and covariance matrix $\boldsymbol{P}$. To calculate the statistics of $\boldsymbol{y}$, a matrix $\boldsymbol{\mathcal{X}}$ of $2.n_x+1$ sigma points $\mathcal{X}_i$ is formed. These sigma points are propagated through the time update. To compute the measurement update step, we propagate these sigma points through the measurement function $\boldsymbol{h}$ and we get transformed points $\mathcal{Z}_{i,k/k-1}$ that form the matrix $\boldsymbol{Z}_{k/k-1}$. Similarly to the Kalman filter, the Kalman gain $\boldsymbol{K}$, the state estimate $\hat{\boldsymbol{x}}$ and the corresponding covariance matrix $\boldsymbol{P}$ are updated by (20)-(22). The UKF equations are given as Algorithm 3. We implemented the UKF using an augmented state vector concatenating the original state and the noise variables: $\boldsymbol{x}_k^a = (\boldsymbol{x}_k^T,\ \boldsymbol{\eta}_k^T,\ \boldsymbol{\xi}_k^T)^T$ (Wan and van der Merwe, 2001). The corresponding matrix with sigma points is $\boldsymbol{\mathcal{X}}^a = ((\boldsymbol{\mathcal{X}}^x)^T,\ (\boldsymbol{\mathcal{X}}^\eta)^T,\ (\boldsymbol{\mathcal{X}}^\xi)^T)^T$. Unlike the PF, the sigma points of the UKF are deterministically chosen so that they exhibit certain properties, e.g. have a given mean and covariance. The UKF is formulated for Gaussian distributions of the noises, whereas the PF has the advantage to work with arbitrary distributions.

6 Particle Filter Performance Evaluation

6.1 Investigations with Synthetic Data

The PF performance is evaluated versus the UKF over of freeway stretch of 4 $[km]$ consisting of eight segments, having periods of congestion. The data are generated by the compositional model (Boel and Mihaylova, 2006) with independent measurement noises for different runs and with different initial state conditions. The congestion is due to variations in the inflow Q_k^{in} and outflow Q_k^{out} (shown in Fig. 2) within the period 1.12 h - 1.7 h and due to the fall in the speed v_k^{out} within the interval 2.4 h-2.65 h. The measurements are generated, by adding measurement noises to the counted number of vehicles $Q_{i,k}$ and to the speed $v_{i,k}$ for segments 1 and 8. These measurements are used in the filters also as inflow/

Algorithm 3. Unscented Kalman Filter Equations.

I. Initialise with:

$\hat{\boldsymbol{x}}_0 = E[\boldsymbol{x}_0]$, $\boldsymbol{P}_0 = E[(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)^T]$, $\hat{\boldsymbol{x}}_0^a = E[\boldsymbol{x}_0^a]$,

$\boldsymbol{P}_0^a = E[(\boldsymbol{x}_0^a - \hat{\boldsymbol{x}}_0^a)(\boldsymbol{x}_0^a - \hat{\boldsymbol{x}}_0^a)^T] = \mathrm{diag}\{\boldsymbol{P}_0, \boldsymbol{P}_{\boldsymbol{\eta}}, \boldsymbol{P}_{\boldsymbol{\xi}}\}$

For $k = 1, 2, \dots,$

II. Calculate sigma points :

$$\boldsymbol{\mathcal{X}}_{k-1}^a = [\hat{\boldsymbol{x}}_{k-1}^a,\ \hat{\boldsymbol{x}}_{k-1}^a + \gamma\sqrt{\boldsymbol{P}_{k-1}^a},\ \hat{\boldsymbol{x}}_{k-1}^a - \gamma\sqrt{\boldsymbol{P}_{k-1}^a}],$$

where $\sqrt{\boldsymbol{P}_{k-1}^a}$ is a Cholesky factor, $\gamma = \sqrt{n_x + \lambda}$,

$\lambda = \alpha^2(n_x + \kappa) - n_x$, $1 \le \alpha \le 1e-4$, $\kappa = 3 - n_x$

III. Time update :

$$\boldsymbol{\mathcal{X}}_{k/k-1}^x = \boldsymbol{f}(\boldsymbol{\mathcal{X}}_{k-1}^x, \boldsymbol{\mathcal{X}}_{k-1}^\eta),$$

$$\hat{\boldsymbol{x}}_{k/k-1} = \sum_{i=0}^{2n_x} W_i^{(m)} \mathcal{X}_{i,k/k-1}^x,$$

$$\boldsymbol{P}_{k/k-1} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathcal{X}_{i,k/k-1}^x - \hat{\boldsymbol{x}}_{k/k-1}][\mathcal{X}_{i,k/k-1}^x - \hat{\boldsymbol{x}}_{k/k-1}]^T,$$

$$\boldsymbol{Z}_{k/k-1} = \boldsymbol{h}(\boldsymbol{\mathcal{X}}_{k/k-1}^x, \boldsymbol{\mathcal{X}}_{k-1}^\xi),$$

$$\hat{\boldsymbol{z}}_{k/k-1} = \sum_{i=0}^{2n_x} W_i^{(m)} \mathcal{Z}_{i,k/k-1},$$

IV. Measurement update equations:

$$\boldsymbol{P}_{\boldsymbol{z}_k\boldsymbol{z}_k} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathcal{Z}_{i,k/k-1} - \hat{\boldsymbol{z}}_{k/k-1}][\mathcal{Z}_{i,k/k-1} - \hat{\boldsymbol{z}}_{k/k-1}]^T,$$

$$\boldsymbol{P}_{\boldsymbol{x}_k\boldsymbol{z}_k} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathcal{X}_{i,k/k-1}^x - \hat{\boldsymbol{x}}_{k/k-1}][\mathcal{Z}_{i,k/k-1} - \hat{\boldsymbol{z}}_{k/k-1}]^T,$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{\boldsymbol{x}_k\boldsymbol{z}_k} \boldsymbol{P}_{\boldsymbol{z}_k\boldsymbol{z}_k}^{-1}, \quad (20)$$

$$\hat{\boldsymbol{x}}_{k/k} = \hat{\boldsymbol{x}}_{k/k-1} + \boldsymbol{K}_k(\boldsymbol{z}_k - \hat{\boldsymbol{z}}_{k/k-1}), \quad (21)$$

$$\boldsymbol{P}_{k/k} = \boldsymbol{P}_{k/k-1} - \boldsymbol{K}_k \boldsymbol{P}_{\boldsymbol{z}_k\boldsymbol{z}_k} \boldsymbol{K}_k^T, \quad (22)$$

where the weights are: $W_0^{(m)} = \lambda/(n_x + \lambda)$,

$W_0^{(c)} = \lambda/(n_x + \lambda) + (1 - \alpha^2 + \beta)$,

$W_i^{(m)} = W_i^{(c)} = 1/2(n_x + \lambda)$, $i = 1, \dots, 2n_x$.

outflow boundary conditions (for the state model). The augmented state vector is $\boldsymbol{x}_k = (\boldsymbol{x}_{1,k}^T, \boldsymbol{x}_{2,k}^T, \dots, \boldsymbol{x}_{8,k}^T)^T$, i.e. $i = 1, 2, \dots, 8$, and the measurement vector $\boldsymbol{z}_s = (\boldsymbol{z}_{1,s}^T, \boldsymbol{z}_{8,s}^T)^T$. The per minute aggregated measurements are supplied to the PF and UKF as would be the case with real data. The state prediction is performed also at each intermediate state update time step. We are estimating the states of all segments between two measurements as one augmented state vector.

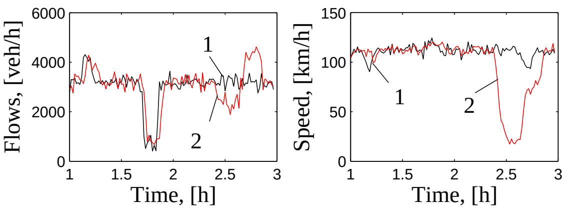

Fig. 2. Boundary conditions: 1 - in, 2 - out

The filters' performance is evaluated by *Root mean square errors* (*RMSEs*) $\epsilon(\hat{x}_{i,k}) = [\frac{1}{r}\sum_{i=1}^{r}(\varepsilon_{i,k})^T(\varepsilon_{i,k})]^{1/2}$, for the state errors, $\varepsilon_{i,k} = x_{i,k} - \hat{x}_{i,k}$, over r independent Monte Carlo runs, with respect to density, speed and flow. The initial particles for the PF are randomly generated by adding Gaussian noise to the actual states. Table 4 gives the parameters of the state model. The evolution of the flow and speed in time (for one realisation) are given in Figures 3 and 4. We see the backward wave on the evolution of the speed and flow in time. The flow-density and the speed-flow diagrams have the typical bell-shaped forms. The filter performance is evaluated for $r = 100$ independent Monte Carlo runs. RMSEs calculated with $M = 200$ for segments 1, 5, and 8 are presented in Fig. 5.

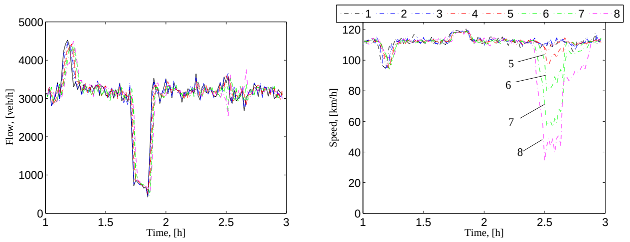

Fig. 3. Diagrams based on the PF and UKF estimated states

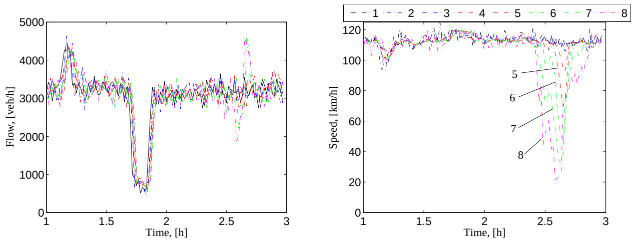

Fig. 4. Diagrams based on the PF and UKF estimated states

We see the influence of the backward wave on these RMSEs. We observe that the RMSE values in segment 1 are smaller than their values in the intermediate segment 5 (it is also due to the fact that there are no sensor data in this segment). According to these results the PF estimates are more accurate than the UKF estimates. However, the PF complexity is more computationally expensive than the UKF. The complexity of the PF is proportional to the the number of particles, times the dimension of the overall state vector, $M.n_x$, whilst the complexity of the UKF is proportional to the number $2.n_x + 1$ of sigma points. Note that n_x is equal to the number of segments n times the number of states 2 in a segment. We calculated the ratio between the PF and UKF computational time and it is: 2.8 (with $M = 100$ particles), 5.45 (with $M = 200$), 15 (with $M = 500$).

The PF more accurate performance compared to the UKF performance can be explained with the fact that the PF approximates the state PDF function, whereas the UKF propagates only the first two moments.

We have also a case with 12.5 km road length (25 segments) where we used the PF with 350 particles and we obtained accuracy comparable to the accuracy with 4 kilometers (with 200 particles). In general, the number of necessary particles is increasing with the increased number of states for reaching a certain accuracy, but not very much. It is difficult to characterise in general the PF accuracy and complexity because they highly depend on the road structure and the traffic conditions.

Table 4. Parameters of the traffic model in the PF

$v_{free} = 120\ [km/h]$, $v_{min} = 7.4\ [km/h]$
$\rho_{crit} = 20.89\ [veh/km/lane]$, $\rho_{jam} = 180\ [veh/km]$
$\alpha = 0.65$, $\beta_{k+1} = \begin{cases} 0.25, & \text{if} \quad \lvert\rho^{antic}_{i+1,k+1} - \rho_{i,k+1}\rvert \geq 2, \\ 0.75, & \text{otherwise.} \end{cases}$
$\Delta t_i = 10\ [sec]$, $t_d = 2\ [sec]$, $L_i = 0.5\ [km]$, $i = 1, ..., 8$,
$M = 200$ particles, $t_d = 2\ [sec]$, $A_\ell = 0.01\ [km]$, $\ell_i = 3$
$cov\{\eta_{S_{i,k}}\} = (0.03 N_{i,k} v_{i,k} \Delta t_k / L_i)^2\ [veh]^2$
$cov\{\eta_{Q_i}\} = 1^2\ [veh]^2$, $cov\{\eta_{v_i}\} = 3.5^2\ [km/h]^2$
$cov\{\xi_{Q_i}\} = 1^2\ [veh]^2$, $cov\{\xi_{v_i}\} = 5^2\ [km/h]^2$

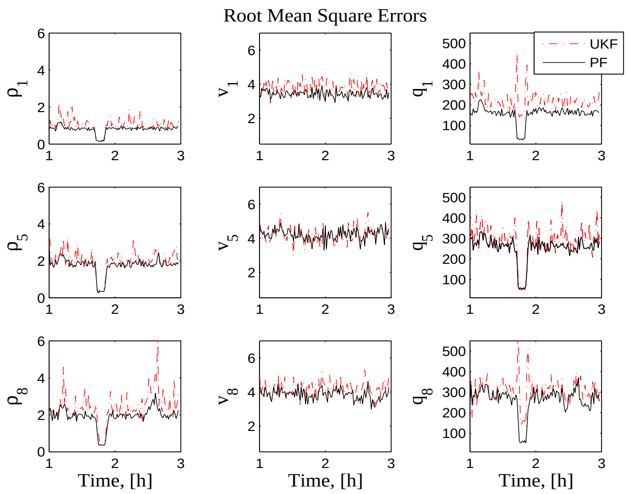

Fig. 5. PF and UKF RMSEs of the density (for all lanes) $[veh/km]$, speed $[km/h]$ and flow $[veh/h]$ of segments 1, 5 and 8 (with $M = 200$ for the PF)

6.2 Application of the Particle Filter to Traffic Data from E17 Freeway in Belgium

The PF performance has also been evaluated with real data, over a stretch of E17 (between CLOF and CLOA on Fig. 6) freeway between the cities of Ghent and Antwerp, subject to frequent congestions. Measurement data are available from video cameras installed at location CLOA, CLOB, CLOD, CLOE, and CLOF, including the total number of vehicles that cross the sensor location during each one minute interval, and the average speed of these vehicles during that one minute interval. We tested the PF and UKF using data measured from September, 2001 from 6.4 [h] a.m. till 10.6 [h] a.m. . This period includes heavy congestions.

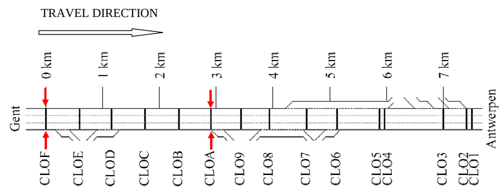

Fig. 6. Schematic representation of the segmentation of the E17 case study freeway. The labels CLOF to CLO1 indicate the locations of the traffic measurement cameras. The vertical arrows indicate the location of the used measurements.

Table 5. Model parameters

$v_{free} = 120\ [km/h]$, $v_{min} = 7.4\ [km/h]$
$\beta_{k+1} = \begin{cases} 0.3, & \text{if} \quad \lvert\rho^{antic}_{i+1,k+1} - \rho_{i,k+1}\rvert \geq 2, \\ 0.7, & \text{otherwise.} \end{cases}$
$L_1 = L_2 = L_3 = 0.6\ [km]$, $L_4 = L_5 = 0.5\ [km]$
$\Delta t_i = 10\ [sec]$, $t_d = 1.5\ [sec]$, $A_\ell = 0.01\ [km]$
$\rho_{crit} = 20.89\ [veh/km/lane]$, $\rho_{jam} = 180\ [veh/km]$
$M = 100$ particles, $\alpha = 0.65$
Gaussian noises $\eta_{S_{i,k}}$, $\eta_{v_{i,k}}$ with covariances:
$cov\{\eta_{S_{i,k}}\} = (0.035 N_{i,k} v_{i,k} \Delta t_k / L_i)^2\ [veh]^2$
$cov\{\eta_{Q_i}\} = 1^2\ [veh]^2$, $cov\{\eta_{v_i}\} = 3.5^2\ [km/h]^2$,
$cov\{\xi_{Q_i}\} = 1^2\ [veh]^2$, $cov\{\xi_{v_i}\} = 5^2\ [km/h]^2$

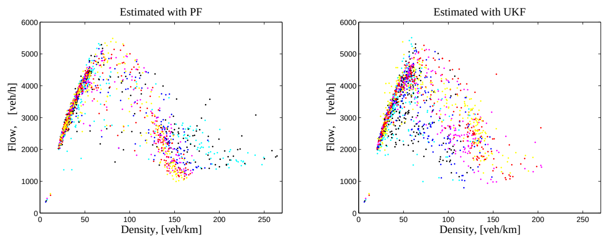

Fig. 7. Diagrams based on the PF and UKF estimated states

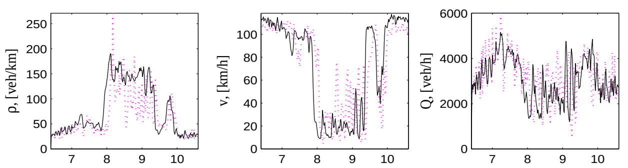
Fig. 8. PF estimated states (solid line) versus measured states in CLOD (dashed line)

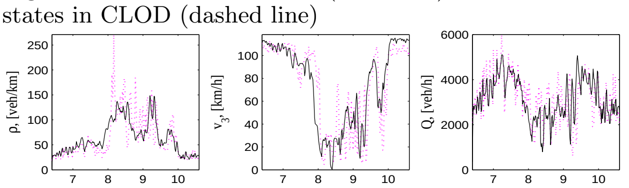
Fig. 9. UKF estimated states (solid line) versus measured states in CLOD (dashed line)

The data are available from two sensors installed at from CLOF and CLOA (Fig. 6). The link CLOF to CLOA contains an off-ramp towards and an on-ramp from a parking lot, but we assume that the flow of vehicles using this parking lot is negligible so that the conservation equation (9) remains valid in the state prediction step. The parameters of the models and of the filters are given in Table 5. The filters generate estimates of the state of each segment in a link, and also of the speed and density (and hence also of the flow) at each boundary between segments. Figure 7 presents flow-density diagrams plotted based on the estimates. The bell-shaped diagram shows nicely that the estimated states indeed have properties as one can expect for traffic data. These estimates of the density, speed, and flow at the boundaries are compared with the measured data in the intermediate segment boundary CLOD (Figs. 8 and 9).

7 Conclusions and Open Issues

This paper formulates the freeway traffic flow estimation within Bayesian recursive framework. A particle filter is developed using traffic and observation models with aggregated variables. The traffic is modelled by a recently developed stochastic compositional traffic model with interconnected states of neighbour segments. The PF and UKF performance is investigated and validated by simulated data and by real traffic data from a Belgian freeway. Both the results with simulated and real traffic data confirm that the PF provides accurate tracking performance, better than the UKF. Both the PF and the UKF are suitable methods for real-time traffic estimation, and both are easy to implement because of the fact that they do not require linearisation. The estimation approach presented is straightforward, general, easily executable to freeway and urban networks, with different topologies, with any number of sensors, with regularly or irregularly received data in space and in time. Both methods are suitable for distributed realisation and parts of them – for parallel computations.

Both the PF and UKF can be used for on-line traffic control strategies, e.g. within the model predictive control framework (Sun et al., 2003; Hegyi, 2004). One could interpret the results of this paper as follows. Particle filtering can successfully estimate and predict the state of all segments of a road link using only observations on the inflow and the outflow of the link. This suggests that it will be possible to obtain efficient filters in large networks if a few intermediate measurements of the flow are available, and moreover it suggests that these filters for a large network will be nicely decomposable.

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