Have applications of continuous rainfall-runoff simulation realised the vision for process-based flood frequency analysis?

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1 Abstract

Keith Beven was amongst the first to propose and demonstrate a combination of conceptual rainfall-runoff modelling and stochastically-generated rainfall data in what is known as the "continuous simulation" approach for flood frequency analysis. The motivations included the potential to establish better links with physical processes and to avoid restrictive assumptions inherent in existing methods applied in design flood studies. Subsequently attempts have been made to establish continuous simulation as a routine method for flood frequency analysis, particularly in the UK. The approach has not been adopted universally, but numerous studies have benefitted from applications of continuous simulation methods. This paper asks whether industry has yet realised the vision of the pioneering research by Beven and others. It reviews the generic methodology and illustrates applications of the original vision for a more physically-realistic approach to flood frequency analysis through a set of practical case studies, highlighting why continuous simulation was useful and appropriate in each case. The case studies illustrate how continuous simulation has helped to offer users of flood frequency analysis more confidence about model results by avoiding (or exposing) bad assumptions relating to catchment heterogeneity, inappropriateness of assumptions made in (UK) industry-standard design event flood estimation methods and the representation of engineered or natural dynamic controls on flood flows. By implementing the vision for physically-realistic analysis of flood frequency through continuous simulation, each of these examples illustrates how more relevant and improved information was provided for flood risk decision-making than would have been possible using standard methods. They further demonstrate that integrating engineered infrastructure into flood frequency analysis, and assessment of environmental change are also significant motivations for adopting the continuous simulation approach in practice.

2 Introduction

Building on advances in hydrological modelling made in the 1970s, in particular the physically-based TOPMODEL concepts (Beven and Kirkby, 1979), Keith Beven (1986, 1987) was amongst the first hydrologists to demonstrate what is now described as a "continuous simulation" (CS) approach for flood frequency analysis. This paper examines how the original vision to "provide more physically-based techniques for prediction of flood frequency characteristics" (Beven, 1987) has been realised, and what lessons can be learned from applications in practice.

Probabilistic methods have underpinned the design and economic analysis of flood mitigation measures for over 50 years (Benson, 1968). In a flood frequency analysis, peak river flows are regarded as a random variable, Y, and the probability of the flow not exceeding a given value $Y = y$ is

$$G_Y(y) = \int_0^y g_Y(y)\, dy \qquad (1)$$

where $g_Y(y)$ is a probability density function describing Y. The temporal scale is usually defined such that $G_Y(y)$ represents the annual probability, and the return period in years is

$$\tau(y) = \left[1 - G_Y(y)\right]^{-1}. \qquad (2)$$

Flood frequency analysis is often combined with hydraulic modelling to calculate flood extents or water levels that inform planning or engineering designs. In economic appraisal of flood protection works, it is useful to integrate possible flood damages over the distribution of hydrological events to estimate the expected damage,

$$\mu_Z = \int_y z(y)\ g_Y(y)\ dy\,, \qquad (3)$$

where $z(y)$ is the damage corresponding to the event $Y = y$. Sometimes the damage z_α associated with a small tail probability $\Pr(z > z_\alpha) = \alpha$ is required, especially in applications relating to flood insurance. A robust description of $g_Y(y)$ is therefore important for a range of flood risk management decisions.

Furthermore, the damaging effects of flooding may be controlled not only by the peak flows, y, but also by other features of the flow regime, in particular the volume flowing onto the floodplain during an event, or temporal aspects such as the speed of onset of flooding or the duration of flow above a threshold. By modelling the full flow hydrograph, it is possible in the CS approach to extract not only the distribution of peaks, $g_Y(y)$, but also these related features, which can be referred to as a coherent vector of connected variables, $\mathbf{Y}$, whose joint density $g_{\mathbf{Y}}(\mathbf{y})$ is made accessible through CS, informing decisions such as dam safety, (Blazkova and Beven, 2004), flood protection design or operational planning.

Hence, there are four central motivations for adopting a CS approach. By exploiting conceptual understanding of catchment hydrology through rainfall-runoff modelling, the aim is to improve confidence in estimates of $\mathbf{g}_{\mathbf{Y}}(\mathbf{y})$ in the following circumstances:

1) for events more extreme than previously observed (extrapolation into the tails of $\mathbf{g_Y}(\mathbf{y})$),
2) for analysis in ungauged catchments (to overcome lack of observations),
3) to assess impacts of environmental change on $\mathbf{g_Y}(\mathbf{y})$,
4) to assess the joint probability distribution of a set of features of the flow regime, such as flood volumes or duration of flows above a threshold $\mathbf{g_Y}(\mathbf{y})$.

The four motivations are explored in more detail below and through the subsequent case studies. In many cases, there are further advantages to adopting a CS approach, specifically overcoming the need to make simplifying assumptions about the combined probabilities of extreme events and their antecedent conditions, and also to avoid the highly simplified, artificial temporal storm profiles that are often required in conventional design event methods, but which may not adequately capture precipitation sequences of relevance to a particular application (both points discussed by Calver et al., 1999 as considered in national CS research in the UK).

2.1 Extrapolation to extreme events

In assessing flood risk it is important to model the tail probabilities robustly, especially since peak flow distributions can be heavy-tailed (El Adlouni et al., 2008, Strupczewski et al., 2011). This is difficult where river flow measurements are limited, which is often a problem encountered in practice.

Sometimes further information may be gained from historical or paleoflood evidence (e.g. Stedinger and Cohn, 1986; Kjeldsen et al., 2014c). More often, additional information from the recent past can be gained from weather data. Classically, design event models based on unit hydrograph theory have been applied to transform information about rainfall distributions into estimates of extreme river flows. A more direct connection between the distribution functions of rainfall and river flows was proposed by Eagleson (1972), introducing the derived distributions concept using a statistical model for the joint distribution of rainfall intensity and duration, combined with kinematic runoff routing. In this pioneering work some assumptions about runoff production were needed to integrate analytically over the rainfall events. However, Eagleson's rainfall model, with the addition of a storm inter-arrival time distribution, served as the basis for the CS studies of Beven (1986, 1987). Further developments are discussed later.

In Beven (1987), the underlying hypothesis was that "as basin scale increases, changes in catchment geomorphology and other characteristics may be expected to lead to changes in the dynamics of flood runoff…". Research building on this early investigation of the physical controls on flood frequency has provided useful insights (e.g. Robinson and Sivapalan, 1997) and there is some evidence of physical scaling laws that may help in understanding how to extrapolate robustly into the tail of the flood flow distribution (e.g. Gupta, 2004, Bernardara et al., 2008). Some of the motivating science cited by Beven was the development of geomorphological network theories in hydrology (Rodríguez-Iturbe and Valdés, 1979, Córdova and Rodríguez-Iturbe , 1983, Gupta and Waymire, 1983). However, hydrological theory has not coalesced into a definitive, unified set of equations that can be applied with generality to make predictions about flood risk in all practical circumstances. Indeed, the search continues for fundamental theories to explain the evolution and dynamics of hydro-geomorphological systems (Beven, 2015).

Hence, a variety of models with different representations of physical processes are still in use for hydrological applications in practice, as exemplified in the case studies below. Many applications of CS, whilst seeking physically robust models of extreme hydrological events, place even greater emphasis on the influence of flood mitigation infrastructure systems. Where these systems are complex, adaptive, or involve storage that is sensitive to variation in (spatial and temporal) rainfall sequences then a CS approach is beneficial because it "allows study of the system under varying operational scenarios" (Bras et al., 1985), as explored in the Bentley Ings and Medway case studies later in this paper.

2.2 Extrapolation to ungauged catchments

Numerous studies have explored the application of CS for ungauged catchments through regression or region-of-influence parameter transfers, including in Europe (Blazkova and Beven, 1997, Sefton and Howarth, 1998, Calver et al., 1999, Seibert, 1999, Xu, 1999, Viviroli et al, 2009), the USA (Abdulla and Lettenmaier, 1997, Kokkonen et al., 2003), Australia (Post and Jakeman, 1999), Japan (Yokoo et al., 2001) and Africa (Servat and Dezetter, 1993). Reviews include Kay et al. (2006), Parajka et al., (2005) and Razavi and Coulibaly (2013). Despite this intense activity, and renewed impetus from the IAHS international decade on Predictions in Ungauged Basins (Sivapalan et al., 2013), prediction in ungauged catchments remains an important source of uncertainty in CS flood frequency analysis (Lamb and Kay, 2004, Wagener and Wheater, 2006)

2.3 Change analysis

A related motivation for CS applications is modelling the effects of changes in climate and land use on flood frequency. In the case of land use change, a physically-based modelling approach appeals because of the potential to link model parameters to physical change scenarios. This is closely related to parameterising a model for an ungauged catchment, with the additional problem that whilst land use can have a localised influence on runoff (Verstraeten and Poesen, 1999), it has proven difficult to detect changes in the distribution of peak flows at catchment scale (O'Connell et al., 2007, Beven et al., 2008, Lins and Cohn, 2011). This issue is compounded by uncertainties in translating between real, physical changes in a catchment and the conceptual parameters of a hydrological model (see Beven, 1989, 1995, 2000, 2001).

Changes in climate can affect all input variables to a hydrological model over a range of time scales, from annual or seasonal temperature changes to shifts in the temporal distribution of rainfall (Westra et al., 2014). Models that account continuously for water balance changes can, in principle, integrate over multiple scales. Hence CS is an effective tool for climate change impacts analysis (see River Lossie case study below) and has been applied in support of national guidance on climate change and flood risk in the UK (Reynard et al., 2001).

3 Methods

A general view of the CS approach is that it replaces the distribution function $G_{\mathbf{Y}}$ with an empirical distribution function $H_{\mathbf{Y}}$, such that

$$G_{\mathbf{Y}}(\mathbf{y}) \approx H_{\mathbf{Y}}(\mathbf{y}) = H_{\mathbf{Y}}(\mathbf{y} \mid M[\Theta_M, \mathrm{B}_M(t)]) \qquad (4)$$

where flow events are outputs of a hydrological model, M, which contains a vector of parameters, Θ_M, and a vector of time-varying boundary conditions, $\mathrm{B}_M(t)$. In practice, H_Y may also be replaced by estimates for the parameters of $G_{\mathbf{Y}}$ inferred from modelled time series outputs from M.

The inputs to the hydrological model may be derived directly from measurements if the hydrological model is simply applied to exploit weather records that are longer than gauged flow records. More often though, B_M will be time series of synthetic data generated using a weather model,

$$B_M = W(t, \Theta_W), \qquad (5)$$

where Θ_W is a vector of model parameters and t is time. The model W then becomes a key stochastic component, allowing the hydrological system to be modelled deterministically through M, although Θ_M and Θ_W may also be sampled from distributions for the purposes of uncertainty analysis (see Lossie case study). Further epistemic uncertainty could be explored by allowing for multiple plausible weather models, whilst assumptions about future change can be expressed through time-varying model parameters.

3.1 Catchment models

In principle M in equation (4) could be any hydrological model. Boughton and Droop (2003), Lamb (2005) and Beven (2011) reviewed models applied in CS studies. Typically these have been conceptual storage or transfer function models with some physical interpretation, chosen for their parsimonious structures to aid parameterisation and to achieve fast run times. Some CS studies have used more explicitly physics-based catchment models, such as the IHDM (Calver and Cameraat, 1993) and SHETRAN (Kilsby et al., 1998). The Probability Distributed Model (PDM, Moore, 1985, 2007, Moore and Bell, 2002) has been used in several applications, including case studies reviewed later in this paper, and within a gridded flow routing scheme for national applications in the UK for climate change impacts assessment (Bell et al., 2007).

It will be seen that representation of infrastructure can be an important motivation for CS applications, requiring additional models for floodplain storage, particularly structures that are operated adaptively to manage flood events such as sluices and gates. These systems can often be represented in detail in hydraulic river modelling software (e.g. packages assessed by Environment Agency, 2013), as shown in two case studies below.

An understanding of the physical influences on flood risk in a catchment may, in principle, be gained through studying the sensitivity of the derived distribution of flood peaks, or associated variables, to perturbation of model parameters. However, where calibration data are available, studies of model uncertainty have shown that interactions between parameters in the model M may lead to difficulty in identifying definitive "best estimates" (see, for example, the River Lossie case study below, Lamb, 1999, Blazkova and Beven, 2009, and many examples cited by Beven and Binley, 2014). Franchini et al. (2000) found that parameter interactions could be important not only within the rainfall-runoff model M, but also between parameters of hydrological and weather models (W, see below) when combined in a CS analysis, particularly with respect to the curvature of the flood frequency growth curve (i.e. the effects of the shape and scale parameters of an extreme value distribution fitted to g_Y).

3.2 Weather models

Building on Eagleson's (1972) assumption of an exponential joint distribution for rain intensity and duration, the models that are applied to operationalise the function W in equation (5) have since grown more complex. In general, the emphasis is on precipitation, which tends to dominate over other atmospheric boundary conditions. Cox and Isham (1994) identified three types of rainfall model that can be applied: empirical statistical models, dynamic meteorological models and conceptual stochastic models informed by physical reasoning. However, for climate change impacts analysis or in catchments where large soil moisture deficits occur, a CS approach can be useful in representing the evolving soil moisture state. Ideally, *W* therefore represents a model for coherent sets of weather variables or processes, including snow accumulation and melt where necessary.

For moderately large catchments the rainfall in a CS model should ideally be spatially and temporally varying, although point rainfall models have been found to be at least as good in permeable catchments where long residence times smooth the effects of rainfall patterns (Defra, 2006). Recently advances have been made in spatial-temporal modelling (Defra, 2006, Serinaldi and Kilsby, 2014, Burton at al., 2008, Blanc et al., 2012, Hashemi et al., 2000). Assumptions of spatial uniformity in early CS studies reflected a lack of statistical techniques or data processing capacity to work with large, spatial data sets. Now there are fewer restrictions in this respect, and advances in statistical theory relating to multivariate extremes (Davison et al., 2012, Heffernan and Tawn, 2004) allow for spatial-temporal analysis over large scales (Lamb et al, 2010, Keef et al. 2013), and multi-site rainfall simulators are already being applied in CS studies (for example see Falter et al., 2015 and Hundecha and Merz, 2012).

It is now also becoming possible to consider using dynamic atmospheric weather models (numerical weather prediction, NWP) as an input to a CS flood analysis. For example, recent climate change attribution studies have shown the feasibility of running extremely large ensembles or 100,000 or more seasonal time series simulations of flood events with a linked atmospheric and hydrological modelling chain (Kay et al., 2011).

One of the most challenging aspects of CS in practice may be the choice and implementation of a suitable sampling scheme for generation of stochastic weather data, especially in combination with analysis of uncertainty in weather and hydrological model parameters. The precise approaches to be implemented will vary depending on the structure of the models used in any given CS analysis. Lamb (2005, Figure 8 therein) illustrated an approach for the calibration of a rainfall-runoff model in the context of a CS flood frequency analysis, while Kjeldsen et al. (2014b) discuss more complex methods that have been applied for assessment of uncertainty in a CS study based on the application of fuzzy rules within a Bayes Monte Carlo framework.

4 Applications: case studies

Four case studies from catchments in the UK (Figure 1) are presented to demonstrate practical applications of CS for flood frequency analysis and illustrate the progression in operational hydrology from Beven's (1986, 1987) early concepts of a single rainfall model driving a single hydrological model to later ideas on uncertainty (e.g. Beven and Binley, 1992), and other developments such as spatially-varying stochastic rainfall modelling, which was not readily available during early work on CS. The selection of case studies is intended to be illustrative, not exhaustive, and is necessarily a small sub-set of applications in practice, many of which will be unpublished consultancy projects. The case studies were chosen to illustrate how CS has met requirements to:

1. overcome a lack of data in a hydrologically complex catchment,
2. integrate spatially-distributed hydrological responses with adaptively-managed flood management infrastructure,
3. account for uncertainty in hydrological modelling and future climate,
4. investigate land management changes and their role in flood mitigation.

Each case study illustrates specific applications of the CS approach, implemented through methodological innovations relative both to industry-standard flood estimation methods or the CS approach outlines by Beven (1987). The case study applications and associated innovations are summarised in Table 1.

4.1 Sparse data and hydrological complexity: Bentley Ings

4.1.1 Motivations

Situated on clay drift to the east of Bentley, near Doncaster, South Yorkshire, Bentley Ings is a low-lying area of mixed urban and arable land affected by mining subsidence and draining via pumps into the River Don. The study was motivated by a need to assess the required capacity of Bentley pumping station, draining a catchment of 41 km^2, to help manage flood risk (Figure 2). Bentley Ings occupies around 10% of the total catchment, the rest being limestone slopes to the west. Recharge to the limestone can migrate beyond the topographic catchment, emerging in springs at the base of scarp slopes further west. However, a greater proportion of the rainfall may follow the topographic drainage network during flood conditions. Around 18% of the catchment is urbanised.

The CS application here followed a similar approach to Beven (1987) by linking a point stochastic rainfall model and a hydrological model, but with modifications appropriate to the complex catchment situation. Flood frequency analysis is rarely straightforward in highly permeable, heavily urbanised or pumped catchments (Robson and Reed, 1999; Webster, 1999; Kjeldsen, 2010). All three problems are present at Bentley Ings. There are no permanent river gauging stations in the catchment and transfer of flood frequency information (Kjeldsen et al, 2014a) from elsewhere is problematic since there are unlikely to be any gauged catchments with a similar combination of complicating factors. A statistical approach to estimation of $G_{\boldsymbol{Y}}(y)$ from gauged peak flow data was therefore not possible, and rainfall-runoff modelling was required.

In the UK, the recommended method at the time was the event-based Revitalised Flood Hydrograph rainfall-runoff model (Kjeldsen et al., 2005), a unit-hydrograph approach not well-suited to very permeable or urban catchments. In general, the assumptions of design event rainfall-runoff methods are difficult to support on complex catchments where it is difficult to identify what conditions are most likely to cause flooding. A CS approach was applied to avoid strong assumptions about the size, shape and duration of the design flood event.

4.1.2 Methodology

Three models were developed and linked together: a stochastic rainfall model, a deterministic conceptual model to convert the rainfall into runoff, and a deterministic hydraulic model to route the runoff through the Bentley Ings drainage channels and calculate water levels. The approach taken was to gather flow and rainfall data from temporary instruments, operated for a short period, with the aim of learning enough to calibrate the hydrological model and help condition the rainfall simulations.

Rainfall was simulated as a stochastic point-process (Bartlett-Lewis rectangular pulses, BLRPM, Onof and Wheater, 1993) using a hybrid of the gamma distribution model of Onof and Wheater (1994) and the Generalised Pareto model of Cameron et al. (2001). Faulkner and Wass (2005) describe how the model selects an initial pulse intensity from a gamma distribution, and if this exceeds a threshold u then the intensity (constrained to be greater than *u*) is re-sampled from a Generalised Pareto distribution. Using the threshold improves the representation of extreme short-duration rainfall (Cameron et al., 2001).

The nine rainfall model parameters were initially calibrated to the statistical characteristics of hourly rainfall measurements then adjusted, by trial and error, to give a close match to the mean and variance of local extreme rainfalls for rainfall durations of one to three days, as characterised by the Flood Estimation Handbook (FEH) design rainfall statistics (Faulkner, 1999). The FEH rainfall frequency analysis pooled data from a large number of gauges to extend rainfall growth curves to long return periods, and thereby provide more reliable estimates than from analysis at a single local gauge. The calibration objective was to reproduce the features of rainfall in the Bentley catchment that are important for producing high flows in the low-lying watercourses. Separate parameter sets were obtained for summer and winter.

Three temporary flow gauges (Figure 2) were installed and operated for up to eight months. The flow data were used to calibrate PDM (Probability Distributed Moisture, Moore, 2007) rainfall-runoff models for three sub-catchments, accounting for 78% of the catchment. Figure 3 is an example calibration fit, showing reasonable simulation of low flows and the highest peak flows, although some over-responsiveness for small events. Parameters for three smaller ungauged sub-catchments in the impermeable Bentley Ings area were transferred from a gauged analogue catchment 40km away, with similar lacustrine clay geology.

River channels and structures were represented in a hydraulic model using the one-dimensional (1D, spatially) Saint-Venant equations, and floodplains using the two-dimensional (2D, spatially) shallow water equations, with dynamic linking between these two components.

Eight thousand years of rainfall and flow data were simulated. For efficiency, a hierarchical approach was taken in which the 150 largest events (defined in terms of peak flow on the largest sub-catchment) were then abstracted for detailed hydraulic modelling. It was assumed that the events from the 8,000-year simulation that would result in the 100 highest water levels throughout the hydraulic model domain would be contained within this set of 150. The resulting peak water levels at every model node were ranked, and design water levels for return periods of interest estimated using Gringorten (1963) plotting positions.

4.1.3 Results

The stochastic rainfall model gave good agreement with important characteristics of local rainfall; average annual totals matched to within 1% and the 100 year return period rainfall from the FEH was matched to within 3% for 1- and 3-day durations (Figure 4).

There is no long-term river flow record with which to compare the simulated peak flows. Alternative methods for ungauged catchment flood estimation (Kjeldsen et al, 2008) gave much lower design flows. During its short 7-month period of operation, one flow logger recorded a peak flow more than twice the estimate of the 2-year flood obtained by regression on catchment properties. Although the flow record was too short to draw any statistically significant conclusions, the finding did cast serious doubt on the results of conventional flood estimation methods which do not account for the uniqueness of the catchment.

The way in which the CS modelling contributed to understanding of the flood hazard in the Bentley catchment is illustrated in Figure 5. The symbols show the location of nodes in the 1D hydraulic model network. At each node there is potential for a different simulated flood event to yield the estimated 100-year return period water level. The ten events that do so at the largest number of model nodes are distinguished by colours on the figure. The labels of each event give an indication of its severity in terms of peak flow and maximum 12–hour volume. The ranks provided in each label are calculated using the simulated annual maximum flows at the sub-catchment that provides the largest amount of runoff. Note how for some events the relative severity of the peak flow can be quite different from that of the volume. The resilience of planned flood mitigation measures can therefore be tested against a range of possible hydrological scenarios, each consistent in some way with a 100-year return period event, whereas without a CS analysis many of these scenarios would not be identified.

Some spatial consistency is evident in the results, although on some watercourses there are numerous changes in the composition of the design event, indicating variation in the sensitivity over different parts of the drainage network to different aspects of the simulated events, such as peak flow on the various tributaries, volume of flow, and timing of tributary response.

In comparison with other methods, CS modelling was perceived by the operating authorities to have provided a greater degree of confidence that the design flows and water levels were representative of processes in the catchment, thus strengthening the basis of the refurbishment plans for the pumping station.

Bentley Ings is an example of one of the primary motivations for CS, extrapolation to ungauged catchments. Naturally some uncertainties remained, including concerns over the calibration of the PDM rainfall-runoff models to a short, and relatively dry, flow record. It is possible that runoff generation processes change in wetter conditions, as reported in other permeable catchments (Midgley and Taylor, 1995; Bradford and Croker, 2007). The use of a spatially uniform rainfall model was considered justifiable given the relatively small catchment area. However, interpretation of the results and planning of future work would have been more straightforward if uncertainty had been quantified explicitly.

4.2 Flood risk management infrastructure: The Medway

4.2.1 Motivation

This case study illustrates a natural progression from the early vision for CS by drawing together multiple rainfall-runoff models, with spatially varying rainfall inputs, and hydraulic modelling, in a complex catchment. The River Medway in Kent, SE England, is one of four main tributaries (with the Eden, Teise and Beult) draining a catchment of around 1,300km^2 (Figure 6). As a commitment under the European Flood Directive (2007/60/EC), areas of flood risk had to be mapped, in turn driving a hydrological assessment with the following objectives:

- to provide “design” flows for flood mapping,
- to support option development for future flood alleviation schemes, including floodplain storage,
- to support future local flood risk assessments.

The catchment accumulates substantial soil moisture deficits over the summer. Runoff is therefore sensitive to antecedent conditions over a range of time scales. Response times vary spatially, making it difficult to specify one, unique “design storm” for a flood event model. These heterogeneities lead to complex patterns of flooding. For example, a record-breaking flood on the Eden barely registered high flows on the Beult, yet in December 2013 high-ranking flow events were observed on all tributaries.

A large in-line flood storage area (FSA) at Leigh controls flow from 500km^2 of the Eden and Medway catchments, protecting Tonbridge and the Middle Medway from flooding. Extensive open floodplain in middle reaches provides additional natural attenuation.

The controlled and natural storage make inundation extents sensitive to volumes as well as peak flow rates. Probabilistic analysis is complicated by the FSA's adaptive operating rules; large events are regulated at a higher flow than smaller ones, with decisions made on the basis of a 24-hour-ahead forecast. The variation in relative timing of responses in the tributaries, contributing variable proportions of the total Medway flow, further challenges assumptions made in industry-standard UK design event models (Kjeldsen, 2005), because of the need to represent temporal variability in hydrodynamic model boundary conditions. The catchment is large and responsive enough that application of a spatially uniform rainfall return period would fail to represent variation across the tributaries. Assuming a fixed event rainstorm profile would risk failing to capture the susceptibility of the storage areas to multi-peaked events. A statistical analysis of observed floods was thwarted by their short record and changes in storage area operation over time.

The above factors all motivated the development of a CS model. The study took place within the context of requirements from the commissioning agency to avoid piecemeal design flow estimation by deriving a consistent design flow dataset, applicable to the whole catchment. The CS approach inherently provided this by integrating the time and space variability in flood-producing conditions.

4.2.2 Methodology

CS on the Medway required two river models. One was a linked 1D-2D hydraulic model to simulate all flood pathways and give detailed flood extents. Run times were several days per event for this model. The second was a simplified whole catchment model designed to route rainfall to flow accurately, at all risk areas, without simulating inundation extents in detail. Run times for this model were minutes per storm. It was used to identify events of specified probabilities that then formed boundary conditions for the detailed 1D-2D model. The whole catchment model comprised nine PDM sub-catchment rainfall-runoff models and 1D hydraulic models (similar to the Bentley Ings case, see above). The hydraulic models were calibrated using data at flow gauges on the Eden at Vexour, the Medway at Colliers Land Bridge, the Teise at Stonebridge and the Beult at Stilebridge.

The modelling included adaptive rules for operation of gates controlling discharge from the Leigh FSA as a function of predicted inflows. These were checked against the actions of operational staff, using their own modelling tools, to demonstrate that the CS model would choose the same course of action as the operators. Figure 7 shows model fits for peak inflows and event volumes in the FSA after calibration against all events between 1999 and 2014.

Synthetic rainfall was initially simulated through a spatially-varying stochastic daily rainfall model (GLIMCLIM, Chandler 2002), fitted to data at 15 rain gauges in the catchment. As in Bentley Ings, calibration included adjustments to match estimates of extreme rainfall from pooled UK data (Faulkner, 1999), for accumulations longer than 24 hours.

Simulated daily rain was disaggregated to a 5,000-year series of hourly data for each sub-catchment by assuming the same hourly temporal profile in each sub-catchment. Rainfall totals could therefore vary between catchments per event, but had the same dimensionless time profile. The within-day rainfall profiles were generated by the same sub-daily point process model used for the Bentley study, fitted to data from Weirwood gauge (see Figure 6). The simplified whole catchment model was run for the full 5,000-year hourly simulation, and a sub set of 3,175 events was selected by identifying flow annual maxima from PDM simulations with return periods of greater than three years at any of the four calibration flow gauges.

4.2.3 Results

Simulated rainfall totals agreed well at the rain gauge sites with the pooled national FEH model over a wide range of durations, for return periods in the range 25 to 100 years (Figure 8 shows example results at Weirwood). Estimation uncertainty obscures the difference between the models for more extreme events. In most cases, the hybrid stochastic model and FEH rainfall models both suggest slightly higher frequencies for a given rainfall accumulation than obtained from a plotting position estimate using the rain gauge observations. This may be due to the short period of observed record (less than 15 years). Daily observations, available for a longer period at Weirwood, suggested slightly higher estimates for extreme rainfall depths than those derived from the shorter hourly record.

Flow outputs from the catchment model are compared with observed annual maximum (AMAX) flows and fitted frequency distributions in Figure 9. There is close agreement between the CS estimates and the gauged data on the Medway, especially when compared with confidence intervals (calculated using the resampling method of Faulkner and Jones, 1999). Vexour is an exception where the most extreme (rank 1) observed AMAX is the 1968 flood, which skews the fitted distribution and the confidence intervals. The CS results are not impacted by this outlier and highlight the extreme magnitude of that event. Downstream of the Medway-Eden confluence the gauged AMAX distribution is more aligned with the CS results. The final plot illustrates the benefits of CS by providing an estimate of the flood frequency distribution $G_y(y)$ downstream of the flood storage area, incorporating its adaptive operating rules.

The application of CS provided the first robust assessment of the standard of protection for Tonbridge offered by the FSA, based on explicit simulation of the operation of the Leigh flood gates. It achieved the goal of developing a consistent set of catchment-wide design flows and addressed the concerns raised about inappropriate assumptions in more typical flood frequency analysis methods used within the industry.

The use of CS enabled the consultant to address this complex natural and engineered catchment system by, in effect, testing the resilience to flood risk over a wide range of plausible flood event and operational scenarios within a probabilistic simulation. Uncertainties about input data, modes and outputs were constrained and quantified by comparisons with reliable observations where they existed (simulated rainfall was compared to observations and national statistical models, CS flood frequency curves at model boundaries were compared to observed peak flows). For the Medway, the CS approach eliminated poor assumptions of uniform rainfall profiles in design-event or standard statistical flood frequency methods and gave the user a more coherent, catchment-wide flood risk analysis than would otherwise have been possible.

4.3 Uncertainty about future climate: River Lossie, Scotland

4.3.1 Motivation

The 216 km^2 River Lossie catchment in the north east of Scotland has a long history of flooding. A flood in 2002, the largest since 1829, caused widespread damage, notably in the main settlement of Elgin. Plans for a Flood Protection Scheme (FPS) at Elgin were already underway following events in 1997 and 2000. The 2002 event highlighted the importance of robust estimation of extreme flows. The sequence of three major events in five years also raised questions over stationarity assumptions in flood frequency analysis and the possible effects of climate change. Coincidentally, publication of the UK Climate Impact Change Programme scenarios in 2002 (UKCIP02, Hulme et al., 2002) presented a new opportunity for exploring climate change impacts.

By continuously accounting for soil moisture changes, and therefore simulating both wet and dry antecedent conditions (e.g. Quinn and Beven, 1993), CS was the most appropriate approach to exploring flood frequency analysis under climate change scenarios. Cameron (2006) built upon earlier work by Blazkova and Beven (1997) and Cameron et al. (1999 and 2000) to apply CS to this data-limited catchment. The analysis applied the Generalised Likelihood Uncertainty Estimation (GLUE) methodology of Beven and Binley (1992) to quantify uncertainties associated with rainfall-runoff model parameter estimation, and UKCIP02 scenarios to represent uncertainty about future climate. The study therefore draws together elements of CS as originally envisaged by Beven (1986, 1987), including use of TOPMODEL to simulate runoff.

4.3.2 Methodology

A stochastic rainfall model was used to drive TOPMODEL generating 2,000 years of hourly data initially under baseline climatic conditions. The rainfall model was a variant of the Cameron et al. (1999) approach, using duration classes to allow dependence between mean storm intensity and duration (where mean storm intensity, duration and storm inter-arrival times were sampled from empirical density functions derived from observed rainfall data). This particular study focussed upon floods with return periods of up to 1 in 200 years and it was determined that, for this purpose, there was a sufficiently large sample within the observed rainfall data to allow the model to generate storms of sufficient magnitude with no parameterisation required (Cameron, 2006). Uncertainty in the rainfall model was therefore not explicitly considered in this particular application but has been considered in other studies (Cameron et al., 1999 and 2000).

Uncertainty about the TOPMODEL simulations was assessed by weighting randomly sampled parameter sets according to the agreement of the corresponding simulations with a flood frequency curve and flow duration curve derived from gauged data. The two comparisons were used to identify parameter sets judged to be hydrologically realistic, as detailed in Table 2. It can be seen that there is a fairly wide range in the parameter values illustrating the difficulty in parameter identifiability as discussed by Beven and Binley (1992) and Beven (1993). Likelihood weights were rescaled over the simulations in order to produce a cumulative sum of 1.0. The weighted empirical distribution function of the discharge estimates was then constructed for each simulated AMAX flow, and used to extract flow estimates for cumulative likelihoods of 0.025, 0.5, and 0.975. This allowed 95% uncertainty bounds and a median estimate to be derived (Figure 10).

UKCIP02 scenarios for “Low”, “Medium-Low”, “Medium-High” and “High” emissions were modelled for the 2080s (a 30 year period centred on the 2080s; monthly output was available for these scenarios). The scenario data represented changes in monthly temperature ranging from 1.29 °C (for January, "Low emissions") to 4.09 °C (for September, "High Emissions"). Percentages changes in monthly rainfall ranged from -31% to + 24% (for July and January, respectively from the "High Emissions" scenario). For a given scenario, the percentage rainfall changes for a given month were applied directly to the stochastic rainfall series which had been generated under current climatic conditions with an hourly timestep. Potential evapotranspiration (PET) was perturbed by applying changes derived from a Thornthwaite PET series (based on the UKCIP02 temperature changes) to the TOPMODEL PET parameter, which controls the amplitude of an assumed sinusoidal PET series.

4.3.3 Results

Results for baseline conditions and for the “Medium-High” scenario are shown in Figure 10. It can be seen that there is reasonable coverage of observed AMAX flow data and a frequency curve derived from pooled statistical analysis of AMAX data from hydrologically similar catchments under the current climatic conditions. Figure 11 shows distributions for the estimated 200-year flow (a key parameter for planning controls in Scotland) under current climatic conditions and the four UKCIP02 scenarios. The results illustrate that although there is an overlap in the uncertainty bounds between the current and future climate scenarios, particularly for lower magnitude flood events (up to about 60 m^3/s), there is also an increase in flood magnitude overall in the future scenarios in line with emissions levels.

A further example in Table 3 illustrates the estimated change in 50th percentile estimate of peak flow for a variety of return period events under the four climate change scenarios (the 50th percentile is shown for illustrative purposes and it is recognised that there is a degree of overlap in the uncertainty bounds across the scenarios, as noted in the discussion of Figure 11 above). The largest increase (14% under the "High" scenario for the 200-year event) is less than the 20% figure that was recommended at the time as a sensitivity test in flood risk planning guidance (see SEPA, 2015), suggesting that the guidance was precautionary in this particular case.

This study demonstrated that CS could be applied as an appropriate methodology for analysis of flood frequency to support the planning of mitigation measures, with explicit analysis of uncertainty through multiple climate change scenarios and Bayesian analysis of model uncertainties.

4.4 Land use and flood risk: Holnicote, Devon

4.4.1 Motivation

Flood management agencies in the UK and elsewhere interested in the use of natural processes as flood management measures (WWF, 2002; Scottish Government, 2009; Environment Agency, 2012), sometimes known as "Natural Flood Management" (NFM). To help in developing an evidence base, the National Trust, a charity that owns large areas of countryside in England, chose a pair of demonstration catchments in SW England, the Aller Water and Horner Water, to test NFM techniques. Their headwaters include uplands on Exmoor and steep wooded gullies leading to lower-lying areas of grassland and arable agriculture. Most of the land lies in the Holnicote Estate, owned by the National Trust.

The demonstration study, described by Defra (2015), included river flow and water quality monitoring before and after installation of NFM measures (blocking of upland drains, creation of floodplain storage, installation of woody debris dams, leaky weirs and wet woodland floodplain to attenuate flows), along with hydrological and hydraulic modelling of some measures.

CS was used to represent the hydrological effects of altering agricultural practices. The motivation was to explore how soil condition and soil management in the Holnicote Estate could affect rapid runoff generation, and hence frequency of river flooding. The principal reason for choosing CS was the desire to use a rainfall-runoff model whose parameters could be altered to reflect land management changes in sub-catchments, especially with respect to soil characteristics. It was thought desirable to select a technique that continually accounted for soil moisture, so that the effects of changes could be represented over a long period, accounting for any long-term persistence and encompassing a wide range of flow conditions.

A secondary motivation was to estimate flood frequency downstream of the confluence of the two rivers, to aid in hydraulic modelling of the impacts of flood meadow restoration.

4.4.2 Methodology

The methodology was similar to that described for the Bentley Ings study, using the BLRPM rainfall model calibrated against local raingauge data and FEH frequency statistics. The main aims in calibration were to reproduce extreme rainfall depths, principally for a duration around 3 hours, which is critical in the Holnicote catchments, and to allow for seasonal variation.

PDM rainfall-runoff models were calibrated to flow data from one permanent gauge and one temporary project gauge. Both were calibrated using data from the baseline period only, i.e. before installation of the NFM measures. A 2000-year simulation enabled estimation of design flows from simulated AMAX flows for return periods up to 100 years. The rainfall-runoff models were then re-run with parameters altered to simulate changes in soil condition from good to severely degraded. The way in which the soil might become degraded varies according to land use. For example, in moorland areas, degradation was taken as resulting from loss of peatm whereas on improved grassland it might result from increased stocking density leading to over-compaction, or conversion to intensive arable production.

The parameters altered were those that represent the process of infiltration and generation of rapid runoff, as guided by a set of empirical formulae linking PDM parameters to physical catchment characteristics (Calver et al., 2005).

4.4.3 Results

It was possible to achieve a close match to the statistical characteristics of extreme rainfall over the critical duration for the catchment (3 hours). Figure 12 shows rainfall frequency distributions from five alternative parameter sets of the stochastic model, in comparison with the FEH statistics.

Similarly, the empirical distribution function of peak flows, $H_Y(y)$, estimated from the CS model on the Horner Water was in good agreement (within 20%) with a flood frequency distribution estimated directly from the 31 years of gauged AMAX flows. Figure 13 is a frequency plot of the simulated annual maximum flows alongside the observed annual maximum flows and a Generalised Logistic distribution estimated using L-moments (Hosking and Wallis, 1997). The figure also provides an indication of the sensitivity of the derived flood frequency relationship to two key parameters of the PDM rainfall-runoff model, showing how changes in assumed soil characteristics lead to steeper or shallower flood growth curves. These effects would be indistinguishable from sampling uncertainty in the FEH analysis, being enclosed by the 95% confidence limits of the flood frequency distribution estimated from the gauged data, but may reflect real sensitivity to catchment change.

As a result of the modelled changes in land management, peak flows were found to increase by 6-9% over a range of return periods relevant to management of flood risk in the catchment, from quantiles around the 5-year return period (corresponding to a low level agricultural flood bank) through to 75 years or greater, of relevance for property protection.

Despite the good performance of CS modelling in representing baseline conditions, there was considerable uncertainty in linking changes in soil management practice to changes in parameters of the PDM models. Notwithstanding this uncertainty, the results of the modelling were helpful in informing the debate about the future management of the Holnicote Estate and assist in the targeting the implementation of appropriate NFM measures across the landscape.

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5 Discussion

The case studies illustrate how the vision for more physically-based flood frequency analysis has been realised in practice, both in dealing with hydrological complexity and incorporating engineered water management systems (Bentley Ings and Medway), especially those that respond adaptively. The Lossie and Holnicote cases illustrate applications of CS for change analysis, tackling, in different ways, potential climate (Lossie) or catchment (Holnicote) changes.

With CS models that encapsulate conceptual understanding of physical systems, it is possible to extract various relevant features of the simulated flows ($y|M$ in equation 4), such as flood volumes, timing or duration. This was important, in all of the case studies, to establish confidence in the flood frequency analysis by comparison with observations or alternative models to confirm that the component parts were fit for purpose. Such corroboration need not be restricted to flood flows; for example, simulations can be usefully compared with gauged data using flow duration curves (Lossie case study, see also Lamb, 1999). Checks on the rates of rise and timing of multiple peak flow events, as in the Medway example, help establish confidence in a model.

CS methods are appealing for climate change impacts analysis, as discussed in the Lossie case study but, in practice, projected changes in peak river flows are often applied as change factors to adjust results from other estimation methods. Hence there has been partial uptake of CS for change analysis, but also a combination of models being applied in particular flood management decisions. This points to a need for analysis approaches that can handle multiple models and multiple outputs. Indeed, this is not a new situation in that multiple alternative methods already co-exist in industry guidance for flood estimation.

With CS there is much greater scope for catchment-wide outputs and a more complete, realistic description of events than with more conventional methods. The description of H_Y can be multivariate, rather than uniquely defined in terms of only one variable such as peak flow (see Gräler et al., 2013). This should permit closer links between hydrological frequency analysis and the real problems posed in applications by simulating coherent sets of outputs that support the analysis of decision variables, which may be metrics such as μ_Z (equation 3) that integrate over multiple aspects of flood risk, rather than univariate measures such as peak water level at a single point in space.

6 Management of uncertainty

CS first emerged as a method of building more explicit physical realism into a probabilistic flood analysis. It may also be regarded more generally as a method of constraining uncertainty in the estimation of the distribution function $\mathrm{G}_{\mathbf{Y}}(\mathbf{y})$, or decision variables that depend on it.

In an empirical analysis of gauged flow data, uncertainty in $\mathrm{G}_{\mathbf{Y}}(\mathbf{y})$ would most obviously be considered in terms of sampling error, and its effect on estimators for coefficients in a parametric model for $\mathrm{G}_{\mathbf{Y}}(\mathbf{y})$. The suitability of any particular choice of model for $\mathrm{G}_{Y}(\boldsymbol{y})$ or univariate distributions within it (especially the distribution of peak flows) would typically be considered qualitatively by hydrologists, through an assessment of the assumptions of stationarity, independence in flood peak data and representativeness.

CS allows for a more comprehensive and structured approach to managing uncertainties that stem from errors in accounting for physical processes in a catchment system. Even if a CS model must inevitably be a simplified view of reality, the focus on processes enables these important epistemic uncertainties to be exposed and constrained, or least acknowledged explicitly. In the above case studies, the identification of epistemic uncertainties could also be framed in rather less theoretical terms as the avoidance of unrealistic assumptions, something that was found to encourage constructive discussion with end-users.

Quantification of uncertainty presents some difficulties with CS because the complex model chains and data sets are not amenable for analytical calculation of confidence intervals or other measures of uncertainty. The Lossie case study (and references therein) illustrates how uncertainty may be assessed using sampling-based approaches. Explicit representation of temporal dependence in rainfall, at increasing scales, remains a source of uncertainty that should be accounted for, especially for systems with long natural response times (e.g. groundwater catchments) or where flood storage induces sensitivity to runoff volumes, and hence the temporal distribution of inputs. As catchment system models grow in scale, the robust representation of spatial and temporal variabilities will become increasingly important.

Software tools for CS lag behind more established flood frequency analysis methods, certainly in the UK if not elsewhere. The lack of convenient, packaged software tools may impede routine application of CS within industry, but may also be advantageous in that applications tend to be developed as catchment-specific models.

7 Conclusions and forward look

CS is often, in practice, a more expensive option for flood frequency analysis than the application of statistical approaches to infer $G_{\mathbf{Y}}(\mathbf{y})$ from gauged data, or the application of event-based models derived from unit hydrograph theory and regionalised runoff or "loss model" coefficients. Extra work is needed to assemble data sets, including time series evapotranspiration data (typically not required for other flood estimation methods), to calibrate and test stochastic rainfall models, as well as the rainfall-runoff model, and to run long simulations.

Most importantly, there are few, if any, standardised data sets and tools to support the parameterisation of CS models and rainfall inputs. In the UK research was undertaken to develop national procedures (Calver et al., 1999, Defra, 2005), but was not driven through into day-to-day practice (with access to data resources and modelling tools being a limitation). An application of CS requires time, effort and skill to establish suitable model parameterisations. In view of the additional costs, the end-user has to believe that CS will deliver benefits that justify the cost. In the authors' experience, the benefits accrue through increased confidence gained by the avoidance or mitigation of unrealistic assumptions, and by permitting a more coherent, multivariate probabilistic view than available from conventional hydrological frequency analysis methods.

In early applications of CS the challenges of managing large data sets and model simulations were a constraint. Now computational and data storage constraints are rapidly diminishing, although with increasingly rich data come new needs to manage complex data sets and meta-data. Approaches based on CS should benefit from wider availability of conditioning data such as open data portals (e.g. SWITCH- ON Consortium, 2014, CUASHI, 2015). The analysis of uncertainties continues to demand research about methods for sampling and integration over multiple dimensions, including uncertainty in chains of models (or "model fusions") that link separate components for weather generation, runoff modelling, river and floodplain hydraulics and infrastructure (Beven and Lamb, 2014).

As applications scale up geographically it becomes more important to consider the fundamental sources of spatial-temporal variation that give risk to the flood risk. Advances in weather data analysis and modelling, both statistical and dynamical models, have already moved beyond the spatially univariate, rainfall intensity-duration distributions of Eagleson (1972) and Beven (1986, 1987). New models and data need to be accessible for hydrologists in a way that can be linked to hydrological models, and adapted for uncertainty analysis.

The problem definition in applications of CS has, in the authors' experience, often been expressed in terms of hydrological or hydraulic variables such as peak flows or water levels. There remains a need to integrate simulated flood scenarios into decision analysis, and the richer hydrological outputs that may be generated by CS models offers great potential to do this, and also increases the complexity of the analysis required in order to obtain the most value from the simulations.

The case studies presented here exemplify how the original vision to “provide more physically-based techniques for prediction of flood frequency characteristics” has been realised successfully. In particular, they illustrate how process-based reasoning underlying Beven (1986, 1987) and others’ research has been integrated with river basin management infrastructure, also one of the early motivations for CS in engineering studies (Bras et al, 1985).

What lessons can be learned from applications in practice? The CS approach helps flood risk managers to be confident about probabilistic analysis in situations where the catchment hydrology challenges assumptions in event-based models, where fitting statistical distributions to gauged data is unfeasible or restrictive (in terms of the target variables), and where there is sufficient risk exposure to justify investing time and resources in specialist modelling. Where the above conditions are not met, it remains unlikely that a CS approach will be commissioned by flood management agencies. It could be said, therefore, that CS remains a method applied in special cases, and by implication that the vision for “more physically-based techniques” for flood frequency analysis has been realised selectively.

Applications, at least in the UK, are constrained by the absence of standardised data resources or models for CS. However even were these resources to exist, it seems likely, given the focus on processes in the CS approach, that they would be applied as a foundation for more locally-tuned models, echoing concepts of “uniqueness of place” (Beven, 2000) in environmental modelling. As data and tools continue to evolve, it could be that more flood hydrology studies seek to view their catchments as unique.

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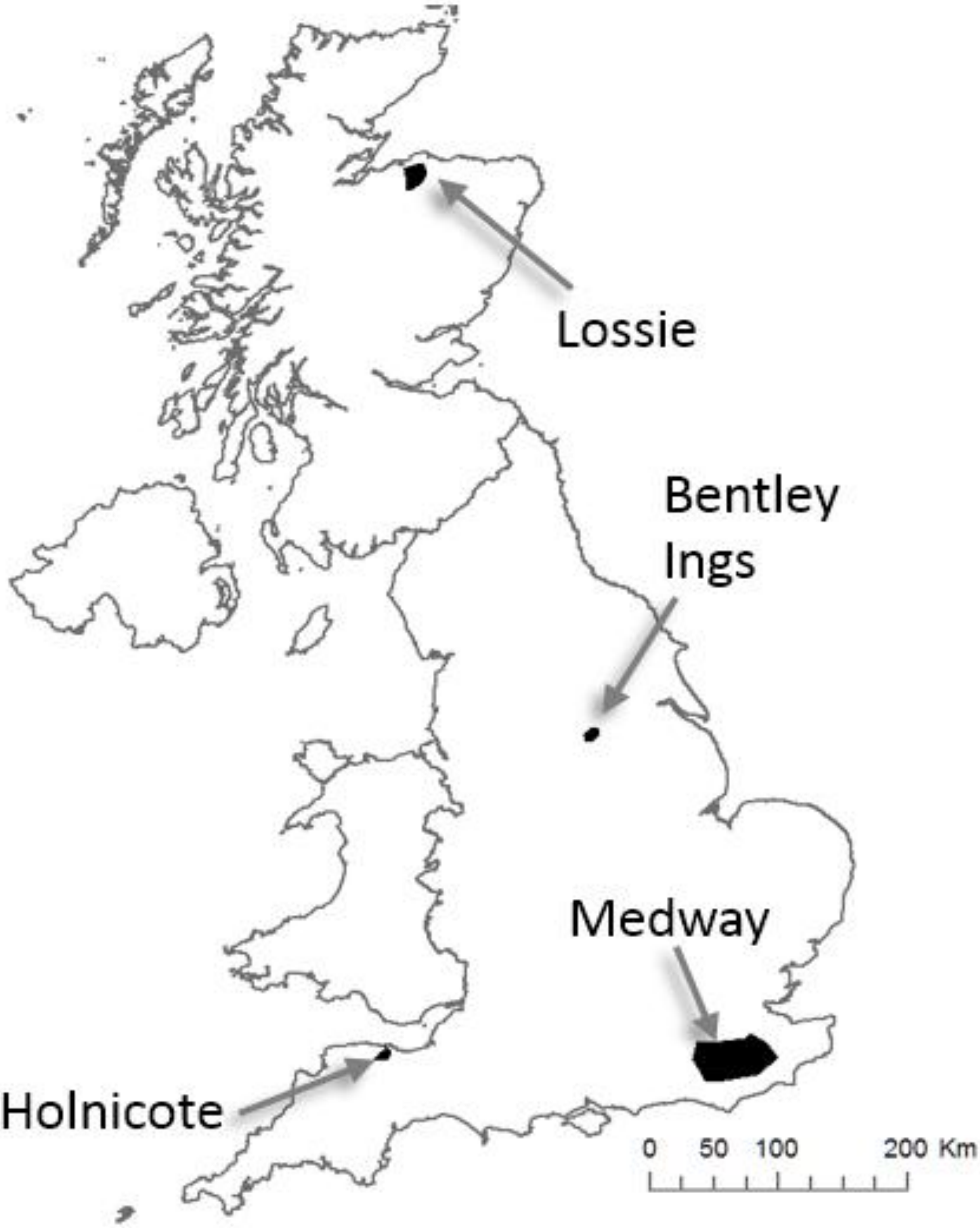

Figure 1. Location of selected case studies in the UK

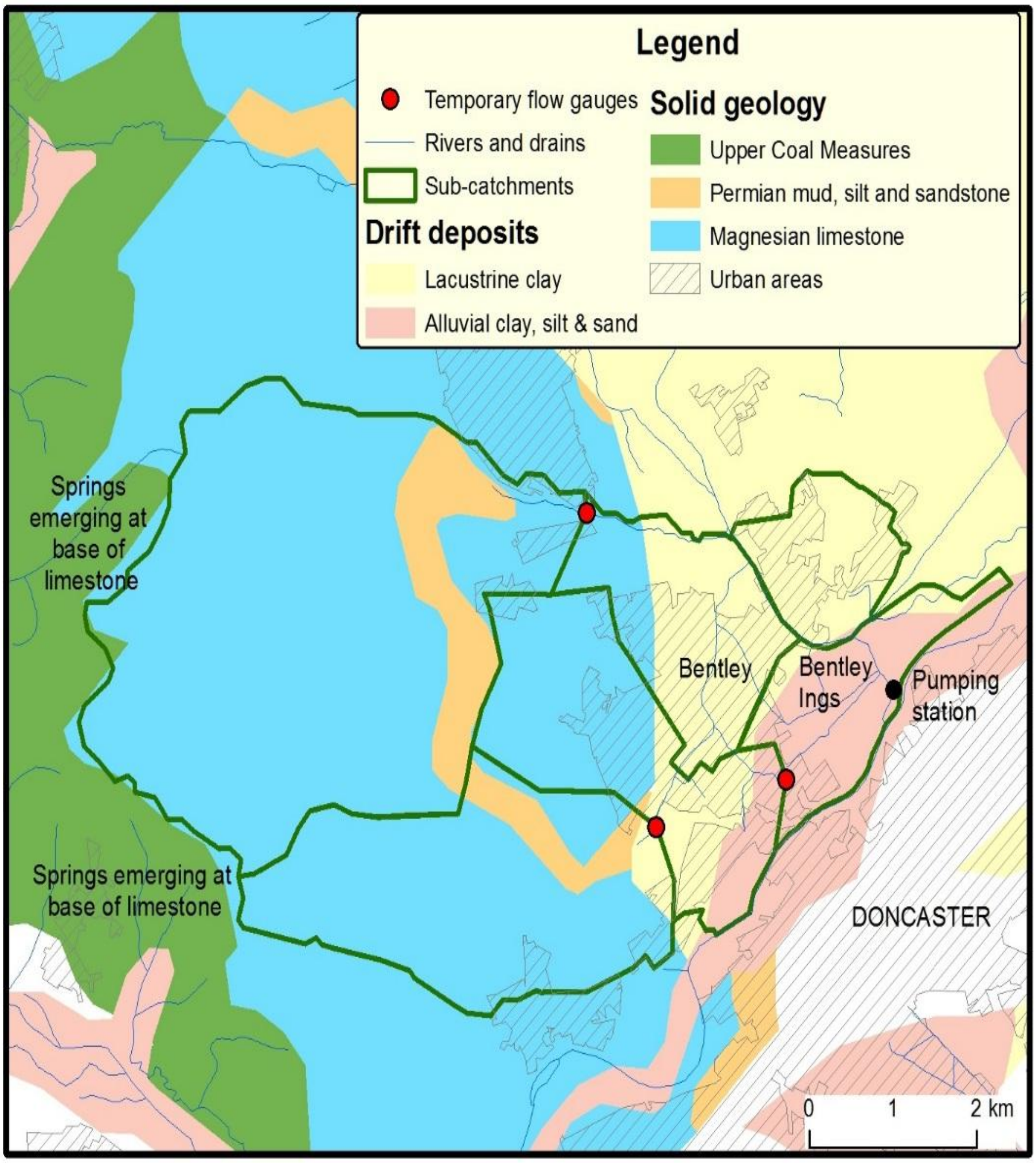

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Figure 2: Catchments draining to Bentley Ings, UK.

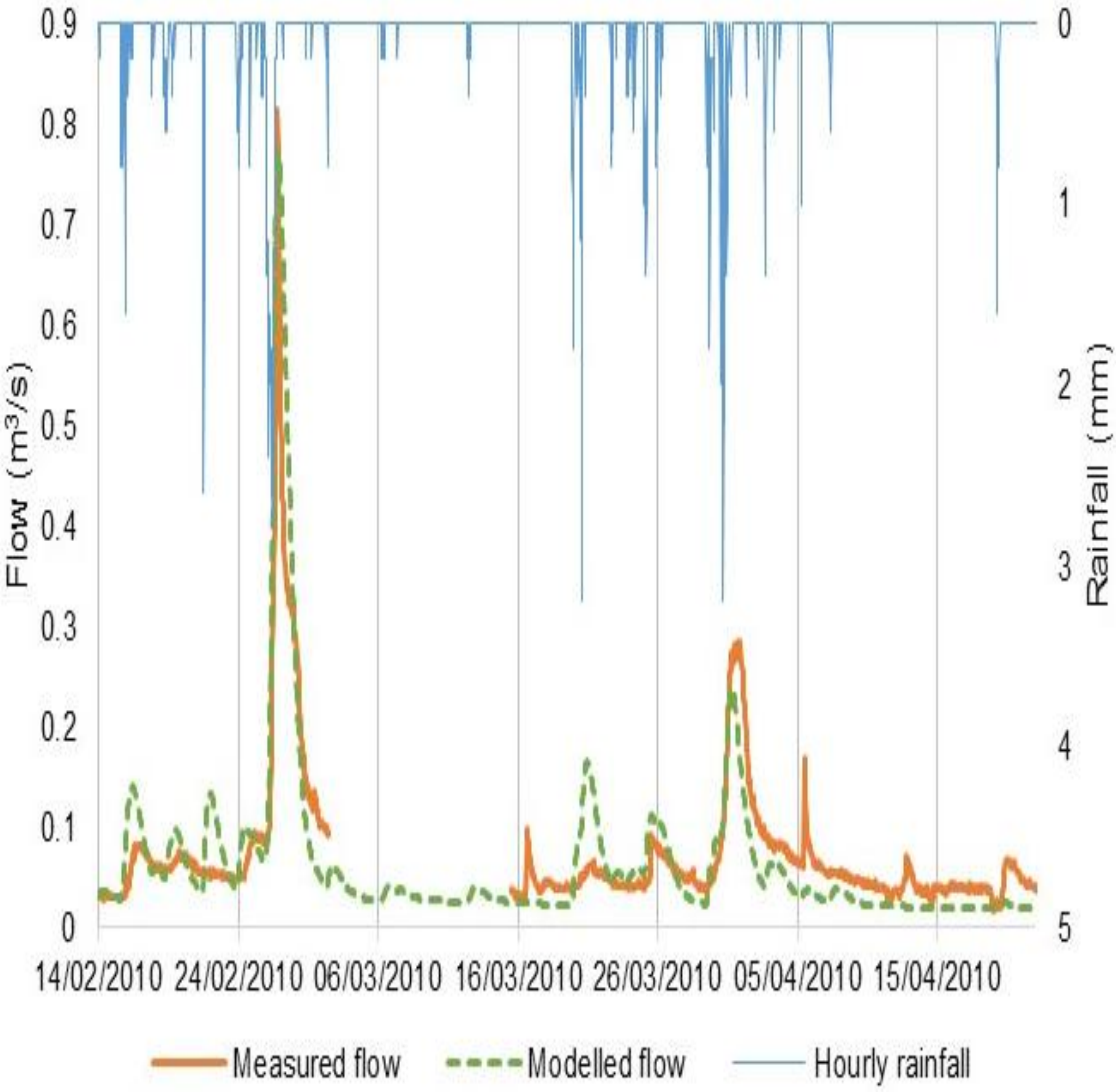

Figure 3: Example PDM calibration for the northernmost sub-catchment at Bentley Ings

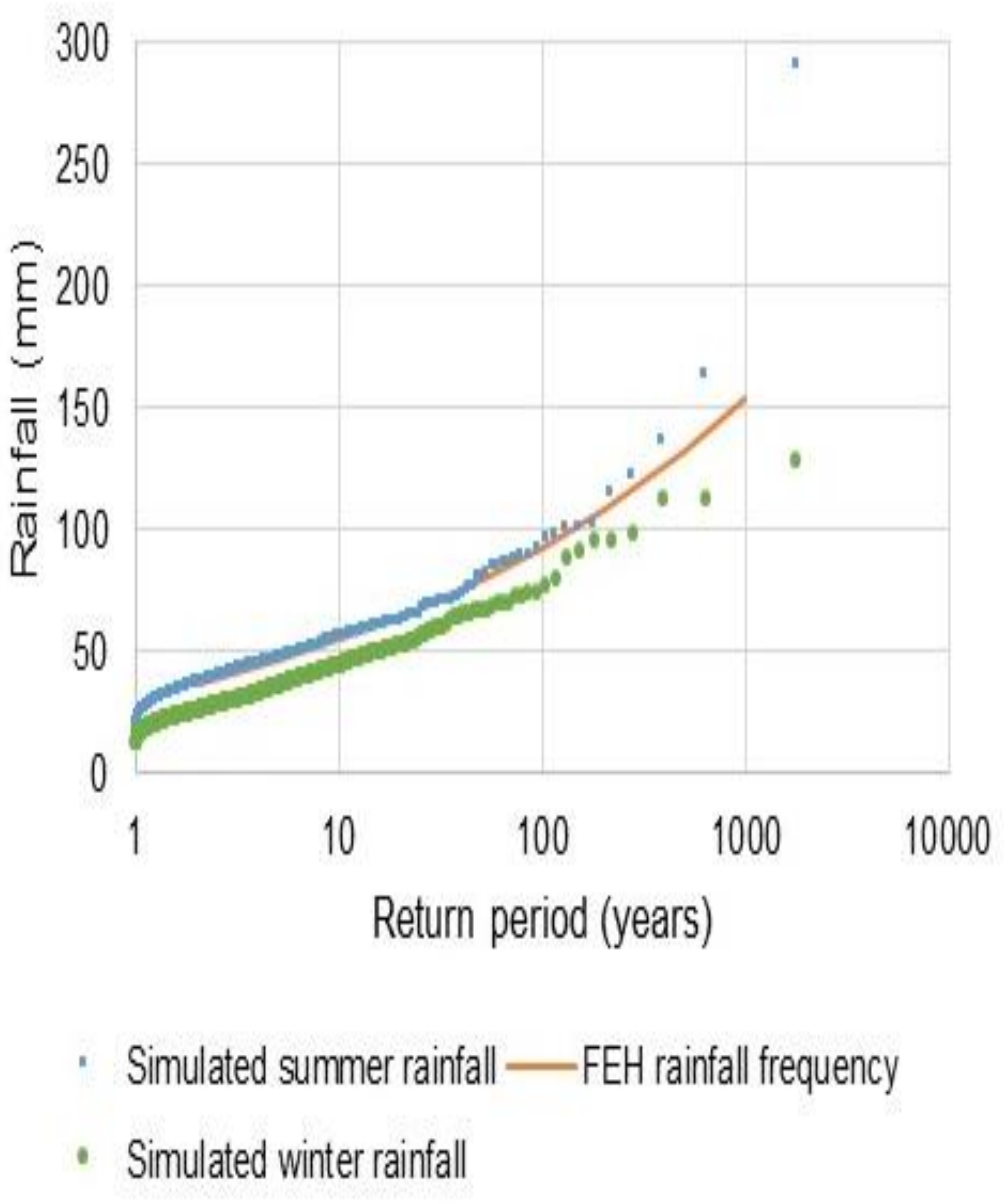

Figure 4: Frequency plot of simulated annual maximum rainfalls (24-hour duration): comparison with FEH rainfall frequency distribution

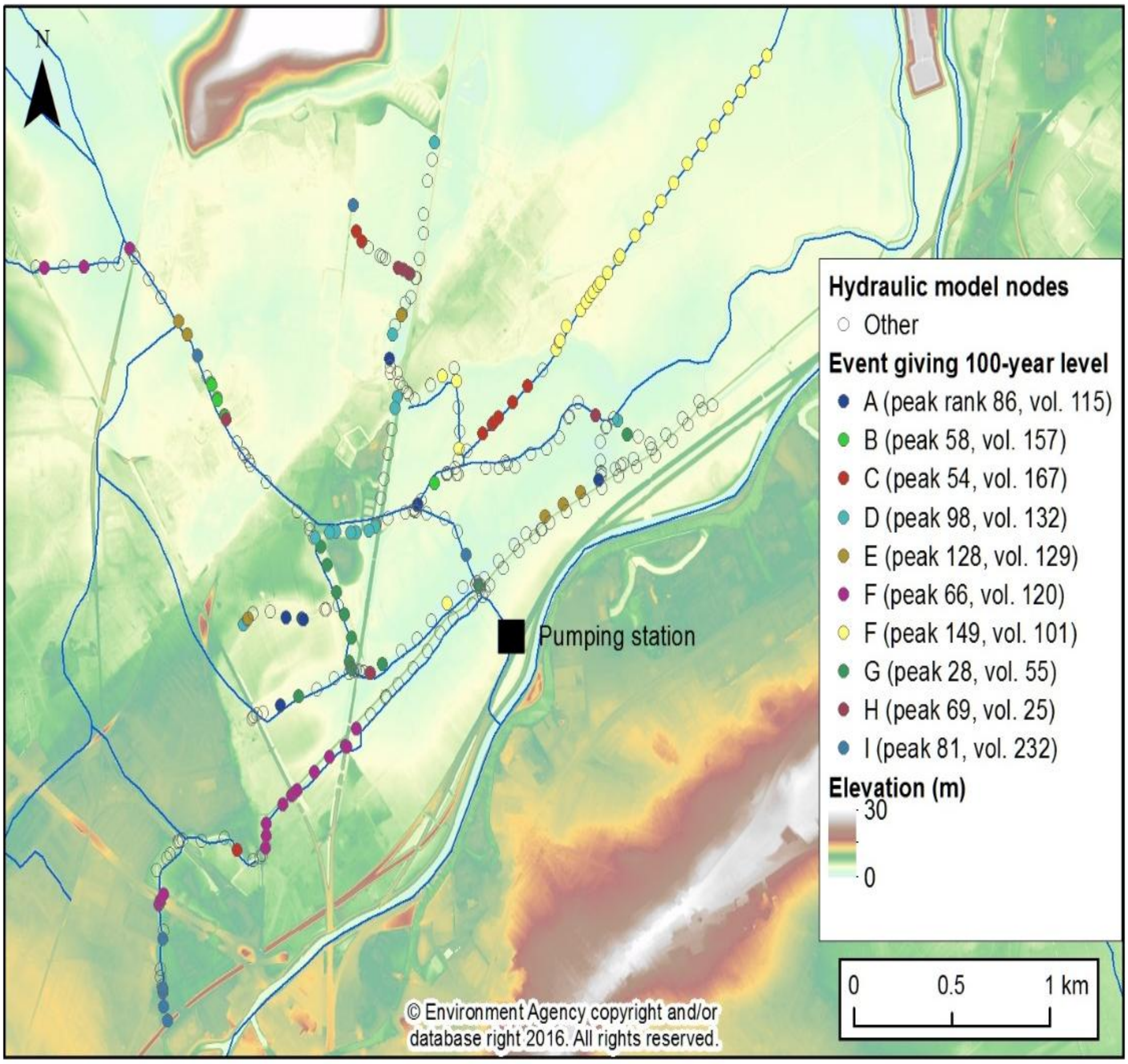

Figure 5: Range of simulated annual maximum floods giving rise to the 100-year return period water level at each node of the hydraulic model. The labels of each event indicate the ranks of the peak flow and the 12-hour maximum volume simulated for one of the PDM sub-catchments, 1 being the highest rank and 8000 the lowest.

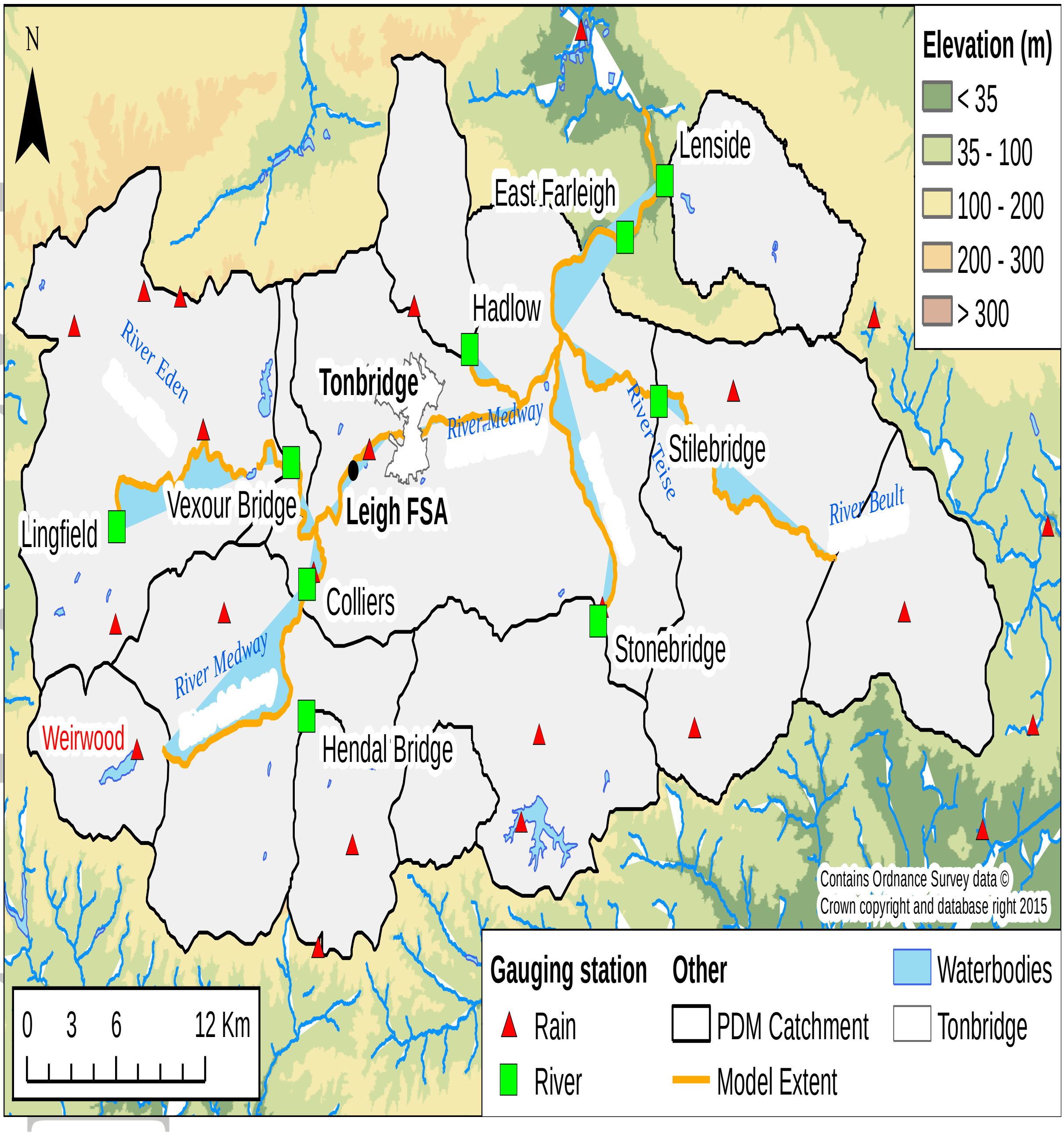

Figure 6: Medway catchment

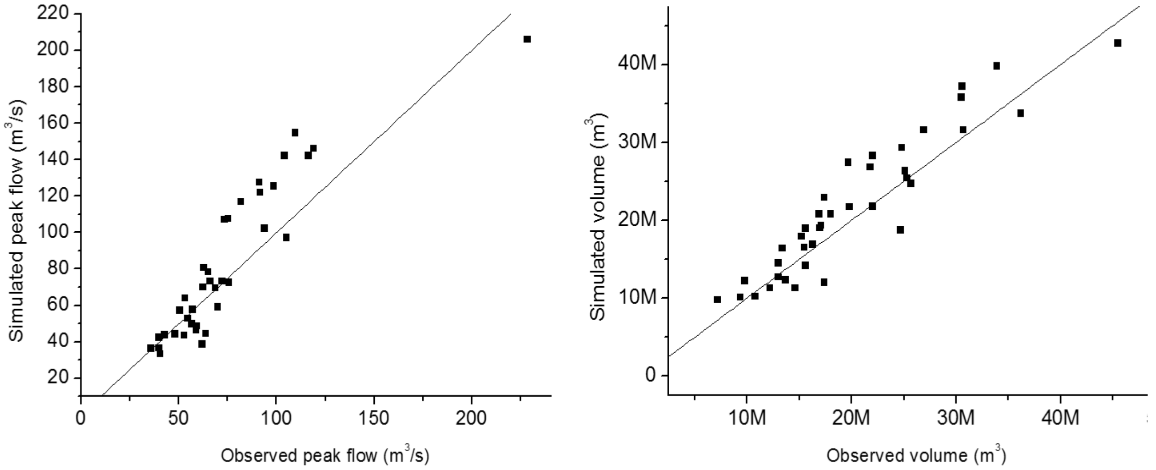

Figure 7: Simulated and observed peak inflows (left) and event volume (right) to the Leigh flood storage area

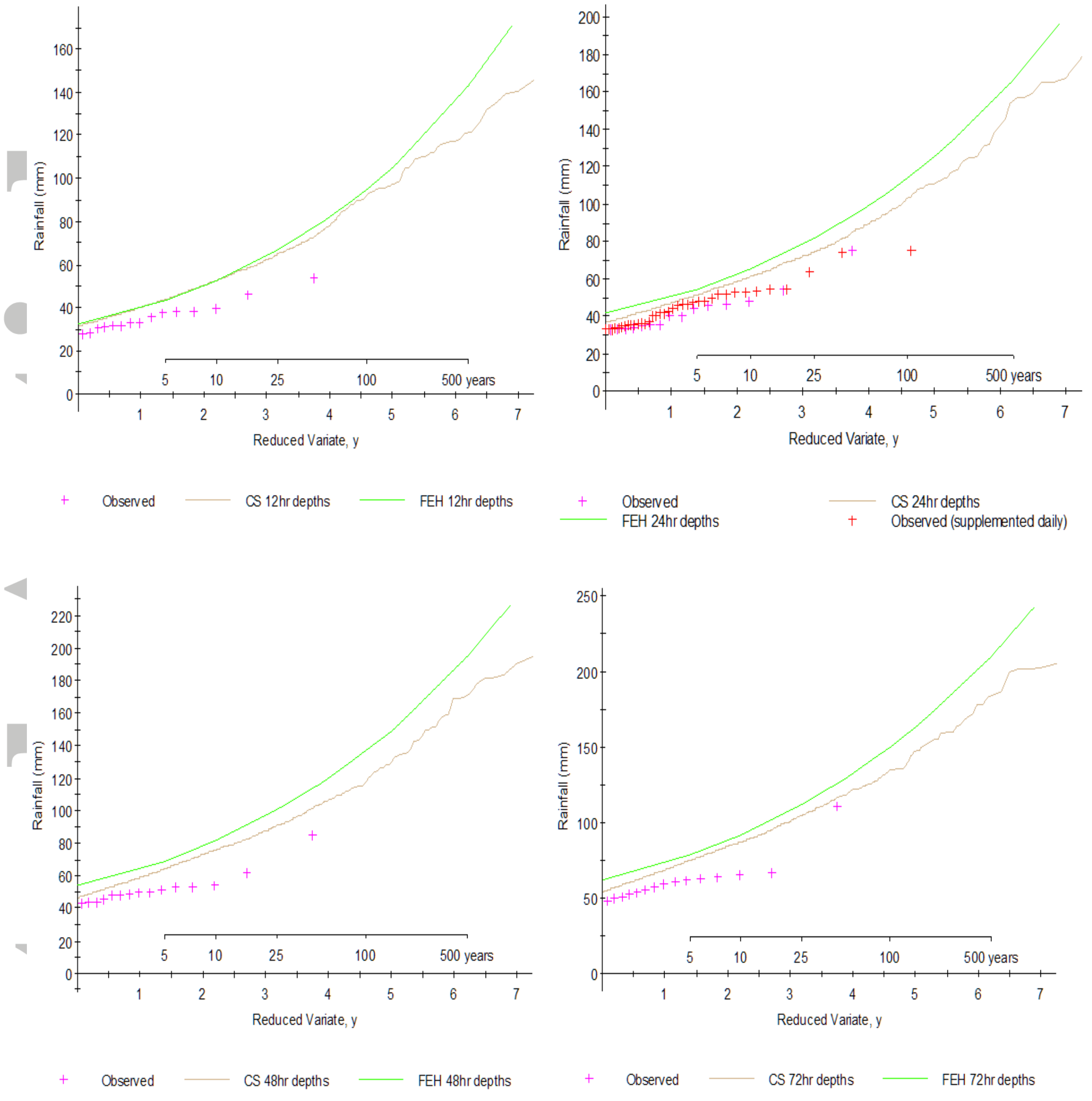

Figure 8: FEH and CS model rainfall frequency curves at Weirwood Reservoir plotted against observed annual maxima (AMAX) (1999-2014) for different durations. At the 24 hour duration a longer AMAX series (red crosses) has been assembled using multiple local daily gauges (1955-2014)

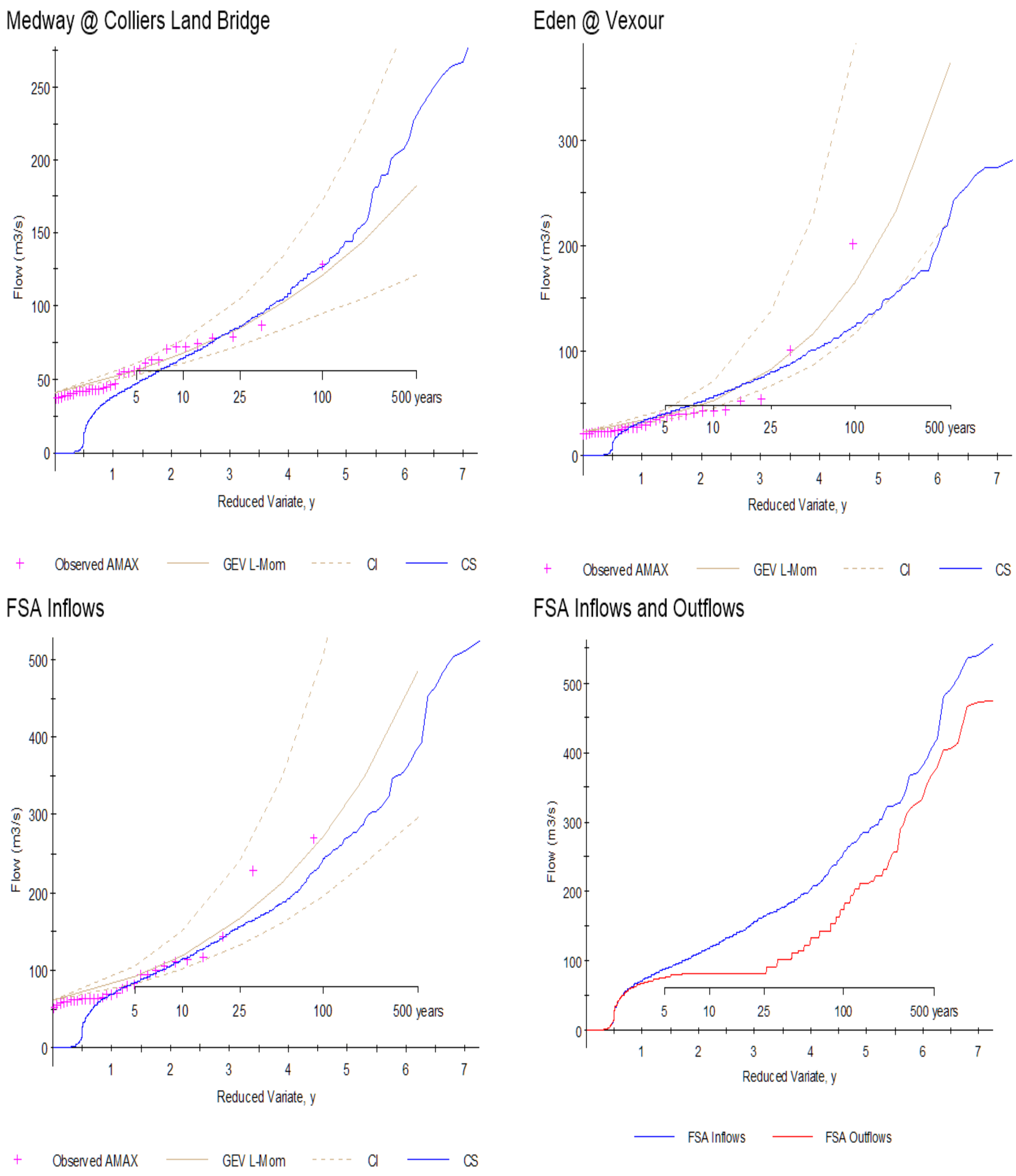

Figure 9: Comparison of CS and gauged (observed) river flow annual maxima (AMAX), plotted using Gringorten positions with Generalised Extreme Value curves with confidence limits established by bootstrapping fitted to the observations

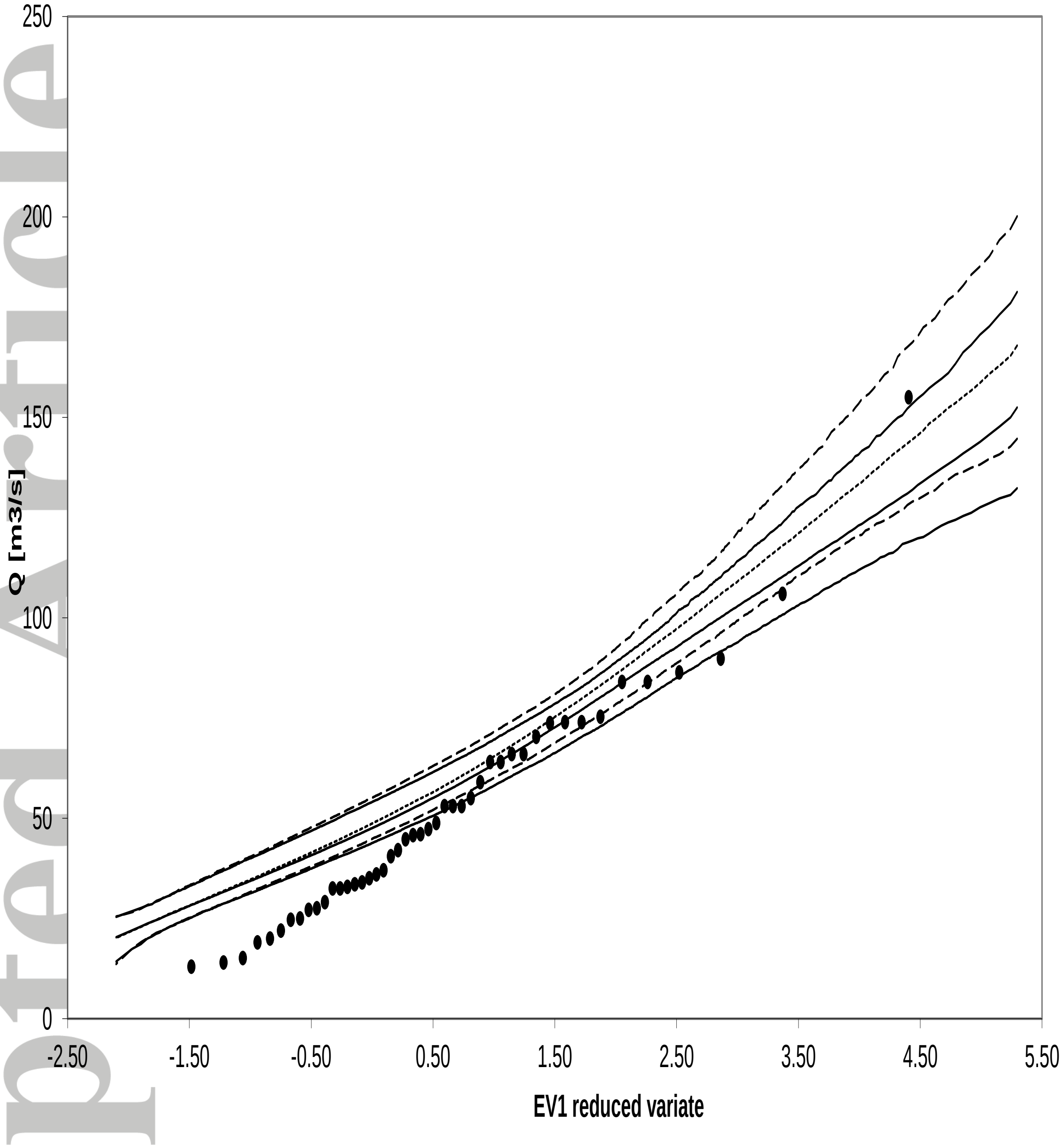

Figure 10: Median and 95% likelihood weighted uncertainty bounds for the baseline climate conditions (solid lines) and climate change (dotted lines) under the "Medium-High" UKCIP02 climate change scenario. Observed annual maxima flow data are shown as circles.

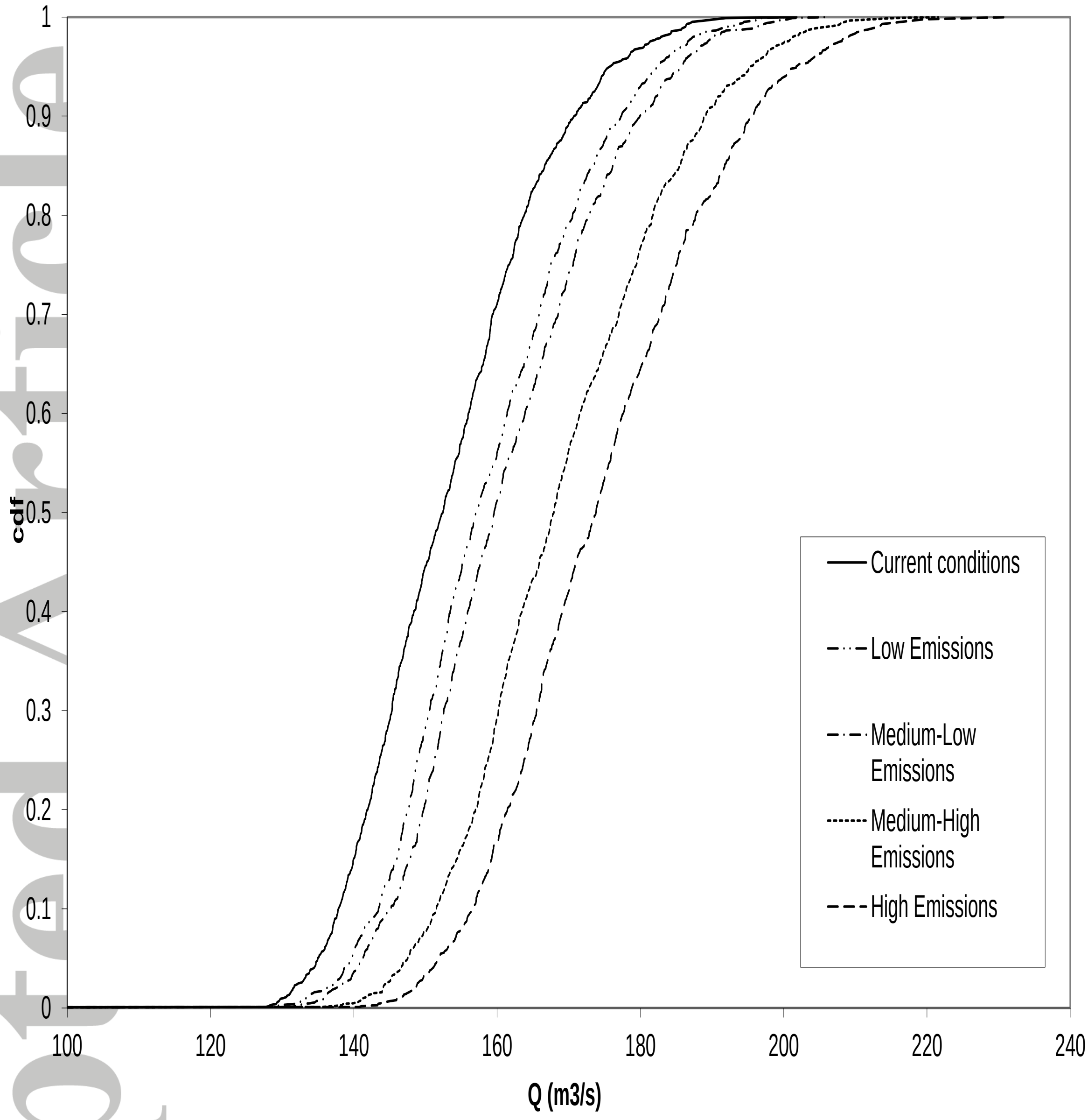

Figure 11: CDFs calculated for the 200 year event under baseline climatic conditions and under climate change scenarios

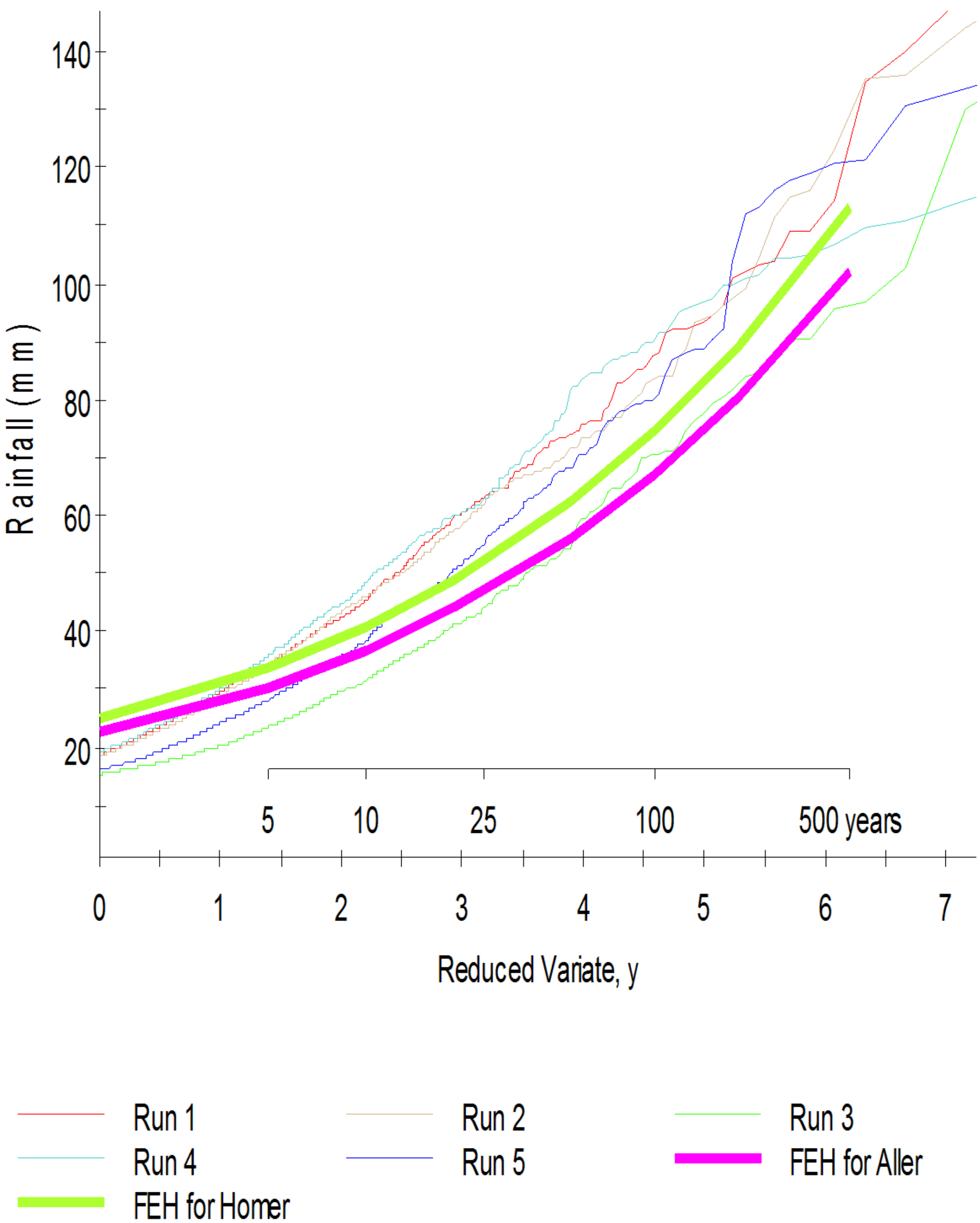

Figure 12: Frequency plot of annual maximum rainfalls (3-hour duration): comparison with FEH rainfall frequency distributions, with five alternative parameter sets for the stochastic model

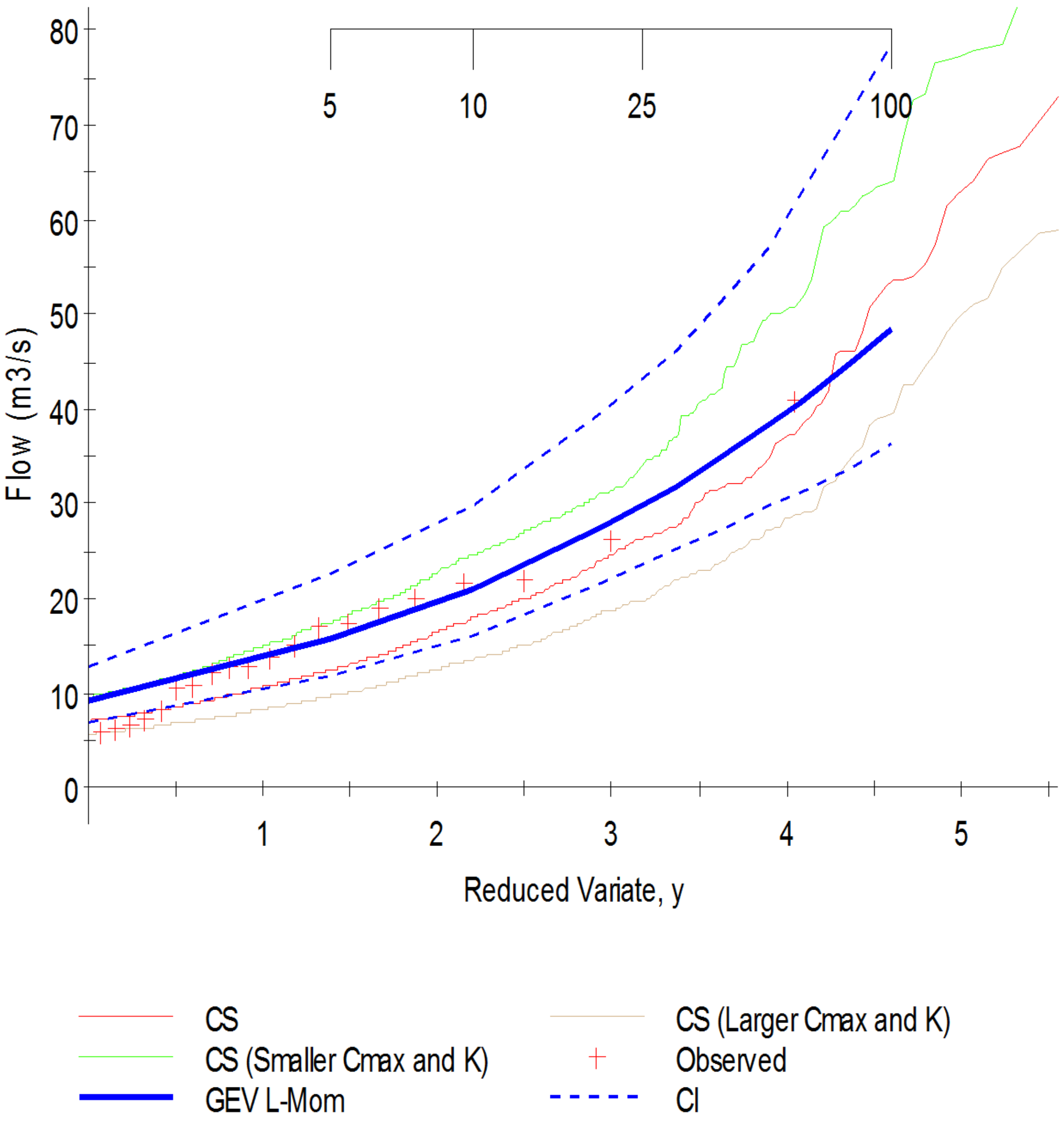

Figure 13: Comparison of simulated and observed annual maximum flows and fitted flood frequency distribution for the Horner Water at West Luccombe

Table 1. Summary of selected application case studies.

	Catchment features and study motivation	Models used	Innovations
Bentley Ings	41 km^2 lowland pumped catchment, permeable geology. Objective to assess required pump capacity for flood management.	Point process rainfall model. Three PDM rainfall-runoff models linked to 1D/2D hydraulic system model.	Multi-distribution rainfall model, conditioned on standard national marginal analysis for extreme rainfall quantiles. Hydrological calibration based on temporary flow gauges added confidence relative to standard models for ungauged catchments. Hierarchical sampling procedure to make efficient use of computationally expensive hydraulic model.
Medway	1,300 km^2 catchment containing significant engineered flood attenuation infrastructure and sensitive to both seasonal and short-term variation in forcing. Requirement for coherent basin-scale analysis for risk mapping and assessment of flood management options.	Spatially-varying stochastic daily rainfall model. Point process local sub-daily rainfall models. 9 PDM hydrological models. Detailed 1D/2D hydraulic system model combined with efficient 1D river network routing model.	Use of spatial stochastic rainfall model at daily scale and disaggregation via local point process models is an advance over Beven (1987) approach and industry-standard practice. Use of efficient 1D routing model to identify important events in stochastic simulation allows practical application of complex 1D/2D system model with long run times.
Losise	216 km^2 upland, relatively impermeable catchment. Requirement for climate change analysis.	Point stochastic rainfall model based on multiple distribution functions. TOPMODEL hydrological model.	Application of Bayesian Monte Carlo method (GLUE) to assess uncertainty in projected changes in flood peaks, accounting for uncertainty in rainfall and hydrological model parameters.
Holnicote	Pair of 18-22 km^2 responsive catchments draining steep headwaters into arable lowland floodplains.	Point process stochastic rainfall model. PDM hydrological model.	Adjustment of model parameters to represent changes in land management.

Table 2: Parameter ranges of the 1000 realistic TOPMODEL parameter sets, defined as those which yielded both a weighted sum of absolute errors of 12 m^3/s or less (when evaluated against the observed flood peaks) and a value of 18.5 or less on a chi-squared test between the simulated and observed flow duration curves. These evaluations were made through running TOPMODEL with observed rainfall.

Parameter	**Range**
m (recession)	0.0304 to 0.0450 m
DTH1 (effective drained porosity)	0.0011 to 1.0000
SRMAX (maximum root zone storage)	0.0130 to 0.0348 m
T_0 (transmissivity)	0.6701 to 7.9565 (log scale)
STDT (standard deviation from transmissivity)	0.8054 to 9.9877 (log scale)

Table 3: Flood flows (*Q*) estimated for return periods (*T*) of up to 200 years as derived from the median model simulation under current climatic conditions and for each of the four UKCIP02 climate change scenarios for the 2080s. Percentage changes between the flood flows estimated under the current climate and under the climate change scenarios are also shown (to the nearest whole percentage).

	Return period, T (years)											
	10		**25**		**50**		**75**		**100**		**200**	
Scenario	*Q* (m^3s^{-1})	*%*	*Q* (m^3s^{-1})	*%*	*Q* (m^3s^{-1})	*%*	*Q* (m^3s^{-1})	*%* .	*Q* (m^3s^{-1})	*%*	*Q* (m^3s^{-1})	*%*
Current	88		107		121		129		136		151	
Low	89	*1*	109	*2*	125	*3*	134	*4*	141	*4*	156	*3*
Medium-Low	89	*1*	110	*3*	126	*4*	136	*5*	143	*5*	160	*6*
Medium-High	92	*5*	114	*7*	131	*8*	142	*10*	150	*10*	168	*11*
High	93	*6*	117	*9*	135	*12*	146	*13*	154	*13*	172	*14*