Weighted Sum Throughput Maximization in Heterogeneous OFDMA Networks

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***Abstract*—We formulate the resource allocation in the downlink of heterogeneous orthogonal frequency division multiple access (OFDMA) networks. Our main objective is to maximize the system sum throughput subject to service and system constraints, including maximum transmit power, quality of service and per-user subchannel allocation. Due to the inter-cell interference the corresponding optimization problem is, in fact, nonconvex, that cannot be solved using standard convex optimization techniques. Here we propose an algorithm based on local search method and use of penalty function to approximate the formulated constrained optimization problem by an unconstrained one. To approximate a global optimal, we set escaping procedure from critical point based on constraint function conditions. The result shows that the proposed method might achieve optimum conditions by a hybrid of split and shared spectrum allocation. Numerical analysis indicates that the proposed algorithm outperform the other conventional methods in the scenario of high level of inter-cell interference with high number of users. In the case of small number of users, we further observe that the proposed method performs better than equal power allocation method (EPA). Moreover, the proposed method approximates the global optimum by considering channel gain and inter-cell interference with a fast rate of convergence.**

I. Introduction

According to market report [1], mobile subscriptions all over the world grew around 7 % per year during Q1 2014. Over the same period, mobile broadband subscriptions grew even faster at a rate of 35 % per year, reaching 2.3 billion. Smart phones dominate the mobile phones selling in Q1 2014 at around 65 %. Moreover, data usage per subscription continued to grow, which is dominated by video (40%) and followed by social network (10%) (in 2013). These factors have contributed to a 65 % growth of mobile data traffic in the period of Q1 2013 and Q1 2014. This report noted that most data traffic is generated indoors by users, either from indoor solutions or by outdoor solutions that provides radio access for indoor users. One solution to increase capacity and coverage is heterogeneous networks that complements existing networks with small cells.

Femtocell is a small and underlying cell in heterogeneous networks. The network is designed to cover indoor or very small areas and connected to main cellular network through internet backbone provided by a user. Moreover, these kinds of networks can be randomly deployed by users without centralized network coordination in many aspects such as frequency and location plan, maximum transmit power adjustment or time access scheduling [2]. Considering the flexibility, economical aspects and market trend, it might be the most cellular networks that co-exist with larger existing cellular networks in the future, such as macrocell or microcell. These situations make femtocells have a potency of interfering adjacent femtocells and the main macrocell networks. Instead of improving network performance, the presence of interference in heterogeneous networks can dismiss the expectation of cellular providers as well as their subscribers to have the performance improved.

Because of inter-cell interference, sum rate optimization in multi-cells is a nonconvex problem [3]. There are a number of researches with different approaches to solve these kinds of problems. Currently, the widely used strategies to solve the problem are using convex optimization approach to solve nonconvex problems [4]. To achieve the maximum capacity of the secondary service for heterogeneous networks, [5] develops a number of access strategies for spectrum sharing, i.e. overlay, underlay and mixed. In cognitive radio, the secondary service is the service being provided for users with less priority for spectrum access. Using an approach of Jensen's inequality [4] to simplify the problem and subsequently solve it using Lagrange duality, this method is simple and achieves the capacity that is close to the maximum achievable capacity of the secondary service. However, this work focuses on the secondary network. It does not maximize the total capacity of heterogeneous networks.

To optimize data rate in digital subscriber line systems, [6] develops distributed power control based on iterative water-filling technique. In this paper, interference channel is modelled as a non-cooperative game. The method can be implemented distributively without centralized control. It results in competitive optimal power allocation by offering opportunity to negotiate the best use of power and frequency between two edges of the system.

To maximize the throughput of heterogeneous networks, [7] proposes spectrum splitting-based cognitive interference management in two-tier LTE networks using a Monte Carlo simulation. The results were achieved by allocating transmit power, frequency spectrum and time slot based on pilot signals from base-stations (BSs) and control channel information. Power is assigned to each subchannel equally. Subchannels are allocated separately to each tier network by considering the best gain and the best trial number, instead of the optimal one, of subchannels for each BS. Thus the method is still away from optimal result.

In this paper, we elaborates our proposed method of optimal resource allocation in OFDMA heterogeneous networks. We consider maximum transmit power and quality of service (QoS) constraints to maximize sum throughput of heterogeneous networks. As the optimization problem is non-linear and nonconvex [3] that cannot be solved using standard convex method [4], we propose an approximation using a local search strategy which considers global optimal condition for critical point escaping procedure [8]. As optimal power allocation at fading channel assumes average power constraint [9], we approximate to solve the problem using local search method

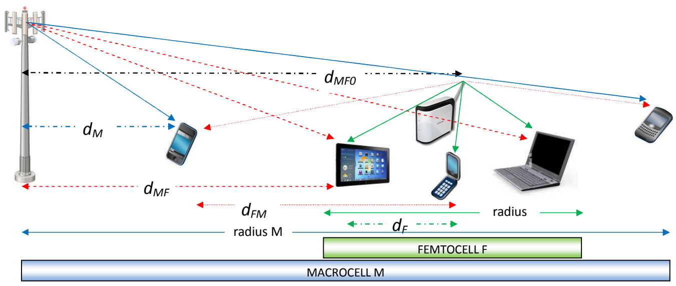

Fig. 1. System model.

to find the greatest lower bound of the objective function by assuming average power allocation in each subchannel, which is the spectrum and power allocation for each BS in heterogeneous networks.? ***

The remaining of this paper is organized as follows. Section II presents System Model and Problem Formulation. Section III elaborates the proposed method, i.e. Optimum Spectrum and Power Allocation. Results and Analysis are revealed in Section IV. And Section V. concludes this paper.

II. System Model and Problem Formulation

A. System Model

In this work, a downlink sectorized heterogeneous OFDMA cellular networks is considered. To ease identification, analysis and solving the problem, networks are modelled in one dimension as having been done in [3]. However, the model still captures main aspects of the real problem in heterogeneous cellular networks as described in Fig. 1. The radius of coverage areas is 500 m for macrocell (r_M) and 40 m for femtocell (r_F). Same numbers of user terminals (UTs), k_M for macrocell and k_F for femtocell, are uniformly distributed in each cell. These networks share the same spectrum. System parameters are presented in Table I. The data rate (bits per second) of UT-k in cell A on subchannel n is:

$$R_{k^A}^{A,n} = B \times log_2 \left(1 + \frac{P_n^A G_{k^A}^{A,n}}{N_0 B + P_n^B G_{k^A}^{B,n}} \right), \tag{1}$$

where k^A is the selected UT of cell A. P_n^A and P_n^B are the power transmitted on subchannel n by cell A and cell B, respectively. $G_{k^A}^{A,n}$ and $G_{k^A}^{B,n}$ denote the channel gain from serving-BS A and interfering-BS B, respectively, to UT-k of cell A on subchannel n. For propagation path losses, a free space [9] and 3GPP's path-loss models [11] are used.

B. Problem Formulation

In this paper, we propose our method to maximize sum throughput of heterogeneous wireless OFDMA networks (2) under a number of constraints, i.e. (3) to (6). The optimization variables are the set of allocated power at each subchannel of each BS. [12] has showed that the maximum data rate of an

TABLE I. System Parameters

Symbol	Parameter (Unit)	Value
f_c	carrier frequency (GHz)	2
B_{sc}	freq. bandwidth per-subchannel (kHz)	180
N_{sc}	number of subchannels	25
N_0	thermal noise density (W/Hz)	$5.556 \cdot 10^{-21}$
fd	channel fading per-subchannel	Rayleigh
fs	channel fading in all spectrum	frequency selective
L_w	wall penetration loss (dB)	10 - 20
P_{tot}^M	macro base-station (BS) total power (dBm)	48
P_{tot}^F	femto-BS total power (dBm)	30

OFDMA system is achieved when each subcarrier is allocated to one UT with the best channel gain on that subcarrier. However, in heterogeneous networks, performing only the same approach above to each network may not lead the best capacity because of the interference. To optimize the capacity of these networks, in addition to the best channels of allocated users, resource allocation also need to consider the channels among adjacent interfering networks. Thus, power allocation in heterogeneous networks must consider properly both high transmit power for high capacity and interference avoidance to adjacent interfered networks caused by this resource allocation. The constrained optimization problem can be formulated as follows:

$$f(P_n^M, P_n^F) = \max_{P_n^M, P_n^F} \sum_{k}^{K} \sum_{n \in \mathcal{N}_k} w_k^{M,n} R_k^{M,n} + \sum_{k'}^{K'} \sum_{n \in \mathcal{N}_{k'}} w_{k'}^{F,n} R_{k'}^{F,n}, \tag{2}$$

subject to power constraints:

$$C_1: \quad \sum_n P_n^M \le P_{tot}^M, \qquad \forall n \in N, \tag{3}$$

$$\sum_n P_n^F \le P_{tot}^F, \qquad \forall n \in N,$$

$$C_2: \quad P_n^M \ge 0, P_n^F \ge 0, \quad \forall n \in N, \tag{4}$$

subject to QoS and subchannel allocation constraints:

$$C_3: \quad \frac{P_n^M G_k^{M,n}}{N_0 B + P_n^F G_{kM}^{F,n}} - \gamma_{th} \ge 0, \qquad \forall n \in N, \tag{5}$$

$$\frac{P_n^F G_k^{F,n}}{N_0 B + P_n^M G_{kF}^{M,n}} - \gamma_{th} \ge 0, \qquad \forall n \in N,$$

$$C_4: \quad N_k^M \cap N_{k'}^M = \varnothing, N_k^F \cap N_{k'}^F = \varnothing, \quad \forall k \neq k', \tag{6}$$

where M and F are indexes for macro and femto cells, respectively. P_n^A is the allocated power on subchannel n of cell A. $w_k^{A,n} \in [0, 1]$ is the weight of UT-k of cell A on subchannel n. $R_k^{A,n}$ is the data rate of UT-k of cell A on subchannel n. P_{tot}^A is the total power of cell A. γ_{th} is the signal-to-interference-plus-noise ratio (SINR) threshold; the input parameter that is imposed by the desired QoS level. N_k^M and $N_{k'}^F$ are allocated subchannels to UT-k and UT-k' in macro and femto cells, respectively. It is assumed that channel states have been known prior to resource allocation.

We consider the optimization problem as weighted sum throughput maximization problem which evaluates power and QoS constraints as weighted factors for each network. The objective function is not linear and not concave in (P_n^M, P_n^F), because of the presence of the inter-cell interference term [3]. Thus the problem cannot be solved by standard convex optimization method [4]. However, nonlinear optimization problem can be solved using different approaches that involve some compromises. One of them is global optimization [4]. To improve the efficiency of the global search, [8] proposes the usage of a local search at each iteration. [13] describes the usage of a mathematical apparatus to make possible to escape a local solution. This approaches helps finding the global solution in game equilibrium problems, hierarchical optimization problems, and other nonconvex optimization problems. In this paper, we propose an optimal resource allocation method for heterogeneous networks based on a *local search* method. As this method is suitable for unconstrained optimization problem and finding a local minimum of an objective function [4], it needs a modification to solve the constrained global optimization problem. We use a *penalty function* method to approximate a constrained optimization problem using an unconstrained one [10]. To approximate the global optimum, we set an escaping procedure from critical point based on constraint function conditions.

III. Optimum Spectrum and Power Allocation

A. Proposed Method

In this section, we propose an optimal spectrum and power allocation algorithm (OSPA) for OFDMA heterogeneous networks based on local search and penalty function methods. Radio resources are allocated to the best gain channels among all UTs' for each subchannel of each cell. We use a local search strategy and set critical point escaping procedure based on some constraint functions. By setting the proper step size matrix ($\mathbf{A}$), then we have an equation for variable updating.

$$\mathbf{X}^{k+1} = \mathbf{X}^{k} - \mathbf{A} \circ \boldsymbol{\nabla} f(\mathbf{X}^{k}) \tag{7}$$

where $\mathbf{X}$ is a matrix of variables of the objective function, i.e. the power allocated for each subchannel. k is the iteration index. $\boldsymbol{\nabla} f$ is the gradient of the objective function (2), not the variable updating function (7), which is used as a multiplier of iterative searching of the allocated power in each subchannel of each cells. $\circ$ is the Hadamard product operator. $\mathbf{A}$ is a step size matrix that obtained as follows.

$$A_{nk} = \begin{cases} \epsilon \cdot J_{nk}/\nabla_{nk} f, & \text{if } \nabla_{nk} f > 0. \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

where n and k are the indexes of the subchannel and the cellular network, respectively. $A_{nk} \in \{\mathbf{A}\}$, is the element of the step size matrix $\mathbf{A}$. ϵ is a small value constant. $J_{nk} \in \{\mathbf{J}\}$. $\mathbf{J}$ is an n by k matrix of ones. $\nabla_{nk} f \in \{\boldsymbol{\nabla} f\}$.

The penalty function $\mathcal{V}_n^A(\mathbf{X})$ is a function that is designed for relieving the impact of power allocation on subchannel n of cell A whose constraints are violated. We develop this function based on constraint formulas as follows.

$$\mathcal{C}_{n,1}^{A} = \frac{P_{tot}^{A}}{N} - P_{n}^{A}, \qquad \forall n \in N, \tag{9}$$

$$\mathcal{C}_{n,2}^{A} = \frac{P_n^A \cdot G_{kA}^{A,n}}{N_0 B + P_n^B \cdot G_{kA}^{B,n}} - \gamma_{th}, \qquad \forall n \in N, \tag{10}$$

$$\boldsymbol{\mathcal{C}} = \text{Transpose}\left\{\boldsymbol{c}_1^A, \boldsymbol{c}_1^B, \boldsymbol{c}_2^A, \boldsymbol{c}_2^B\right\} \tag{11}$$

where $\mathcal{C}_{n,1}^{A}$ and $\mathcal{C}_{n,2}^{A}$ are the values of constraint functions of cell A on subchannel n above, i.e. (9) and (10). $\boldsymbol{c}_k^A = \{\mathcal{C}_{1,k}^A, \mathcal{C}_{2,k}^A, \cdots \mathcal{C}_{N,k}^A\}$, a vector of constraint function values of cell A, which $k \in \{1, 2\}$ is the index of constraint functions above. $\boldsymbol{\mathcal{C}}$ is a matrix of constraint function values.

Step size vector ($\boldsymbol{\delta}$) of the penalty function is set to gradually vanish power allocation are subchannels whose constraints are violated; so the rate of convergence is set faster than $\mathbf{A}$ (8). The step size vector ($\boldsymbol{\delta}$) is obtained as follows.

$$\boldsymbol{\delta} = \left|\overline{\boldsymbol{\mathcal{C}}_0}\right| / N, \tag{12}$$

where

$$\boldsymbol{\mathcal{C}}_0 = \begin{cases} \mathcal{C}_{n,m}^{A/B}, & \mathcal{C}_{n,m}^{A/B} < 0, \forall n \in N, \forall m \in [1, 2], \\ 0, & \text{otherwise.} \end{cases}$$

Then penalty function multiplier is set as follows.

$$\boldsymbol{\Omega} = \begin{cases} \beta \cdot |\boldsymbol{\nabla} f_{neg}| \div \boldsymbol{\Omega}_2, & \text{if } \nabla f < 0. \\ 1, & \text{otherwise.} \end{cases} \tag{13}$$

where

$$\nabla f_{neg} = \begin{cases} \nabla f, & \text{if } \nabla f < 0; \nabla f_{neg} \in \{\boldsymbol{\nabla} f_{neg}\}. \\ 0, & \text{otherwise; } \nabla f \in \{\boldsymbol{\nabla} f\}. \end{cases}$$

$$\boldsymbol{\Omega}_2 = |\min(\boldsymbol{\nabla} f_{neg})| \otimes \boldsymbol{J}.$$

β is a number being set to make penalty function gradually eliminates allocated power on subchannels whose constraints are violated. $\otimes$ is the Kronecker product operator. $\boldsymbol{J}$ is an N element vector of ones.

And the penalty function is obtained as follows.

$$\mathcal{V}(\mathbf{X}) = \text{Transpose}\left\{\boldsymbol{\rho}^A + \boldsymbol{\mu}^A, \boldsymbol{\rho}^B + \boldsymbol{\mu}^B\right\}, \tag{14}$$

where

$$\boldsymbol{\rho}^A = \begin{cases} -\delta_1^A \cdot \frac{P_{tot}^A}{N} \cdot \boldsymbol{c}_1^A, & \text{if } \mathcal{C}_{n,1}^A < 0. \\ 0, & \text{otherwise.} \end{cases}$$

$$\boldsymbol{\mu}^A = \begin{cases} -\delta_2^A \cdot \frac{P_{tot}^A}{N} \cdot \boldsymbol{c}_2^A, & \text{if } \mathcal{C}_{n,2}^A < 0. \\ 0, & \text{otherwise.} \end{cases}$$

$\delta_k^A,\ k \in \{1, 2\}, \delta_k^A \in \{\boldsymbol{\delta}\}$, is a step size variable for cell A.

Then (7) will be rewritten as follows.

$$\mathbf{X}^{k+1} = \mathbf{X}^{k} - \mathbf{A} \circ \boldsymbol{\nabla} f(\mathbf{X}^{k}) - \boldsymbol{\Omega} \circ \mathcal{V}(\mathbf{X}^{k}). \tag{15}$$

Stopping condition is set to approach the global optimum by considering constraint functions as follows.

$$0 \le \mathcal{C}_{n,1}^{A/B} \le \frac{P_{tot}^{A/B}}{N}, \quad \forall n \in N, \tag{16}$$

$$\mathcal{C}_{n,2}^{A/B} \ge -\gamma_{th}, \quad \forall n \in N, \tag{17}$$

$$\frac{\Delta f}{f} \le \epsilon, \tag{18}$$

where f is the objective function as presented in (2).

B. Algorithm Summary

In general, the proposed method is summarized as follows.

1) Initially, for each subchannel of each network, the best channel of all users is selected and power allocation is set equally.
2) Transmit power of each subchannel of each BS is reduced iteratively using local search method (7) till optimum power allocation for interferencing cells is achieved while maintain the global optimum objective.
3) For subchannels with violated constraints, power reduction is set faster using penalty function.
4) At the end of an iteration cycle, spectrum allocation for both networks can be a hybrid of split and shared spectrum.

Hence, the algorithm can be written as follows.

Initialization:
$P_{tot}^M, P_{tot}^F, P_n^M, P_n^F, d_{MF0}, N_k^M, N_k^F, channel\_type$;
$(d_M, d_F, d_{MF}, d_{FM}) \leftarrow$ load *distance_vector*;
$(G^M, G^F, G^{MF}, G^{FM}) \leftarrow$ generate *channel_gain*;
$\max_k (G_k^{Mn}, G_k^{Fn}, G_k^{MFn}, G_k^{FMn}), \forall n \in N, \forall k \in K$
$\leftarrow$ find the best gain of each subchannel;
$f(P_k^{M,n}, P_{k'}^F, n) \leftarrow$ set the *objective function* (2);
$\boldsymbol{\nabla} f \leftarrow$ set the *gradient function*;
$\boldsymbol{\mathcal{C}} \leftarrow$ set *constraint functions and matrix* (9 - 11);
while NOT stopping condition **do**
  $\mathbf{A} \leftarrow$ set the step size matrix (8);
  Calculate the penalty function: $\boldsymbol{\delta}, \boldsymbol{\Omega}$ and $\mathcal{V}(\mathbf{X})$ (12 - 14)
  Update $\mathbf{X}^{k+1}$; (15)
  Evaluate variable bounds,
  e.g. $P \ge 0, \sum P_n \le P_{tot}$;
  $count(N_{sc}^M, N_{sc}^F)$;
  $set(P_{tot}^{M,n}, P_{tot}^{F,n})$
  Evaluate stopping conditions (16 - 18)
end while

IV. Numerical Results and Analyses

In this section, we present the results of the proposed method using numerical analysis to find the optimum result for each iteration cycle. And then repeat the algorithm for different network configurations to get the average final results. We compare and analyse the performance of the proposed algorithm with the following algorithms:

- Multicells iterative water-filling (IWF) algorithm [6]: An optimal multi-channel power allocation method that is implemented in distributed manner.
- Equal power allocation (EPA): Total transmit power is divided and distributed evenly into all subchannels.
- Split spectrum allocation (SSA): Total spectrums is divided equally for each cell.

We use Friis' free space and 3GPP's path loss channel models. The average sum throughput is obtained by simulating the method in a number of repetitions that parameters, i.e. UT's positions, are set randomly.

Fig. 2 shows the average sum throughput of OSPA with different scenarios when γ_{th} is selected differently. The different scenarios are distances between two cells (d_{MF0}) and channel models, i.e free space and 3GPP's path losses. k^M and k^F are 6 UTs for each network. The other parameters are the same as described above. The figure shows that the different value of γ_{th} affects to the different average sum throughput and the different peak rate for each scenarios. OSPA with d_{MF0} 150 m in free space path loss channel model reaches a peak rate at γ_{th} 8 dB. Whereas, OSPA with d_{MF0} 250 m in free space path loss reaches a peak rate at γ_{th} 6 dB. It reveals that OSPA with the appropriate selection of γ_{th} can optimize average sum throughput of heterogeneous networks in free space channel model. When using 3GPP's channel model, wall penetration loss is assigned. This kind of path loss can reduce interference power significantly from outside cells that depend on wall material. However, when implemented in 3GPP's channel model with d_{MF0} 150 m, OSPA has decreasing trend for the increasing of γ_{th}. It reveals that this method is not suitable to optimize the throughput of heterogeneous networks in low interference condition.

Fig. 3 shows average sum throughput of heterogeneous networks with varied number of users. d_{MF0} is 150 m. Path loss channel model is free space. In this figure, the proposed method (OSPA with γ_{th} 8 dB) is compared with IWF, EPA and SSA. In general, sum throughput of all methods increases with increasing number of UTs. The proposed method outperforms EPA for all number of users. OSPA allocates transmit power in each subchannel of each cell by iteratively reducing the power of each cell to reduce inter-cell interference and to avoid violated constraints. Using this approach, OSPA occupies the best subchannels and releases the worse ones, which lets the other BS to occupy. Whereas, EPA distributes transmit power equally to each subchannel. Using EPA, high gain inter-cell subchannels will interfere to adjacent BS; while the low ones reduce power efficiency.

Comparing to IWF, OSPA has two different conditions. Small user number decreases the probability of finding high gains of selected subchannels. In this case, OSPA underperforms IWF. Water-filling power allocation, the core algorithm of IWF, is built by assuming Gaussian channel with no interference power [9]. It allocates more power to high gain channels, less power to low gain channels, and no power to channels which results in lower SINR compared to the threshold. In this case, IWF allocates power optimally to each subchannel based on water-filling algorithm. Whereas, OSPA approaches optimum point by reducing transmit power in each subchannel using same rate and higher reducing rate for subchannels with violated constraints. It makes OSPA underperforms IWF in subchannels with greatly varied gains. For high user number, when systems allocate resources using best gains of channels policy, it increases the probability of finding subchannels with moderately varied gains. When implemented in interference environment, especially in multi channels whose gains moderately vary, IWF will look for optimal equilibrium between all BSs using competition approach [6]. Speed convergence of this method is paid off by loss of optimal point. Meanwhile, OSPA approximates optimum conditions iteratively, gradually and in parallel for all subchannels and multicells. Thus, OSPA outperforms IWF in multichannel heterogeneous networks with high number of users.

Comparing to SSA, OSPA has two different conditions. For small user number, OSPA gets fewer throughput than SSA, but more throughputs for high users. SSA selects the best half spectrums for macrocell and leaves the best of rest spectrums for femtocell. SSA maximizes subchannel occupation since there is no interference in occupied subchannels; while OSPA allocates resources in each subchannel by considering channel gain and interference. For high user number, the probability of finding the high gain subchannels is higher. These conditions enable OSPA to select better channel state, i.e. high gain subchannels and low interference power, and to allocate resources more optimum than SSA.

Fig. 4 shows the portion of allocated power over the total (maximum) power of each network in one iteration cycles. d_{MF0} is 150 m. Propagation channel model is free space path loss. γ_{th} is 8 dB. The number of UTs is 9 units. At the end of the iteration cycle, it shows that both networks allocate less than the maximum power allocated to each of them. Fig. 5 shows the sum throughput of the proposed method at one

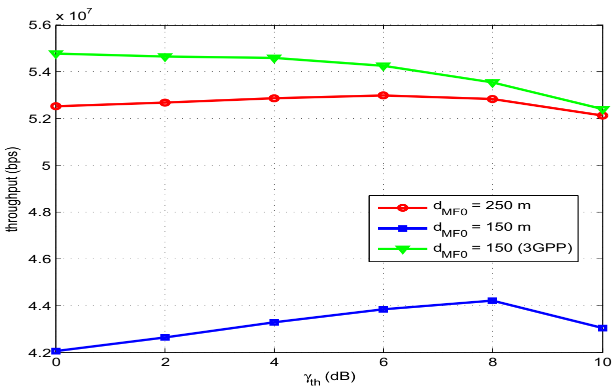

Fig. 2. OSPA with varied threshold.

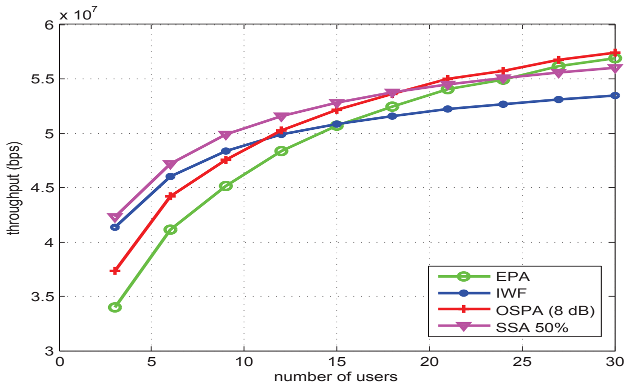

Fig. 3. Average sum throughput of heterogeneous networks with varied user number.

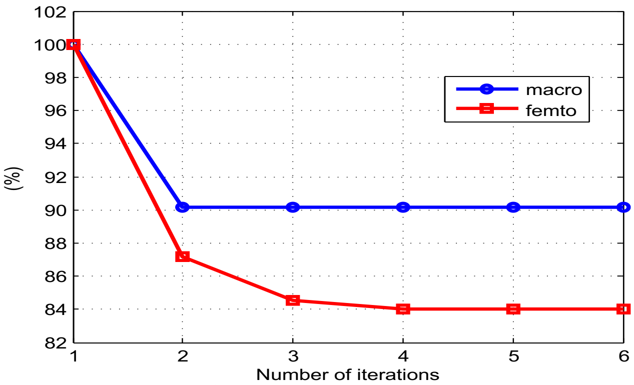

Fig. 4. The portion of allocated power over the maximum power of each network in one iteration cycle.

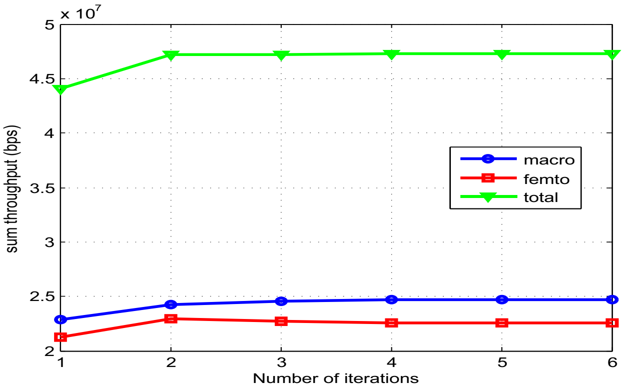

Fig. 5. Sum throughput of OSPA method at one iteration cycles.

iteration cycles. The simulation scenario follows the previous one. If compared to Fig. 5, Fig. 4 shows that decreasing transmit power from iteration step 1 to 2 results in increasing throughput for both macro and femto networks. For step 2 to 4, decreasing transmit power in femto leads to slightly decreasing sum throughput, but it increases the macro sum throughput though its transmit power remains unchanged. It reveals that proper power allocation in each subchannel of each cell leads to decreasing interference power as well as increasing sum throughput of each network. For step 4 to 6, transmit power of both networks remain unchanged. Power allocation of each subchannel of both networks has achieved equilibrium point in these steps. It shows that the proposed method has achieved local optima of power allocation for each network. To conclude, the proposed method achieves optimum points, i.e. optimal power allocation in each network, by considering channel gain and inter-cell interference.

In Fig. 5, sum throughput of each network slightly increase for step 1 to 2, which leads to significant increase in total sum throughput. It reveals that little increase of sum throughput in each cell could result in significant increase in total sum throughput. For step 2 to 4, sum throughput in macrocell is a slightly increase; but a slightly decrease in femtocell. Meanwhile, total sum throughput remains unchanged for these steps. It reveals both networks seek equilibrium out for these steps. For step 4 to 6, which is the stopping point for the iteration cycle, sum throughput of each network achieves a steady state condition. It leads to the same condition for total networks. It reveals that the system has achieved equilibrium points and also approximates the global optimum of the objective function. Moreover, the proposed method has a fast rate of convergence that shown by a small steps to stop.

V. Conclusion

In this paper, our investigation on sum throughput maximization in downlink heterogeneous OFDMA networks has been elaborated. The proposed method approximates the global optimum using a local search and a penalty function methods iteratively and simultaneously through power allocation for each subchannel of heterogeneous networks. Using the proposed method, optimum conditions might be achieved by a hybrid of split and shared spectrum allocation, which also might be achieved by IWF. IWF achieves optimum by iteratively allocate resources of each network using water-filling algorithm after getting channel state information; while the proposed method achieves optimum by finding out equilibrium of equal power allocation in each subchannel of each network and set less or even no power for violated subchannels. In high-interference environment, the proposed method with the right selection of γ_{th} achieves higher throughput than the other conventional methods for high number of users. For small user number, the method can achieve higher throughput than EPA. Moreover, the proposed method approximates the global optimum by considering channel gain and inter-cell interference with a fast rate of convergence.

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