# Minimizing CO2e Emissions by Setting a Road Toll

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## Abstract

The main purpose of this paper is to develop a bi-level pricing model to minimize the CO2e emissions and the total travel time in a small road network. In the lower level of the model, it is assumed that users of the road network find a dynamic user equilibrium which minimises the total costs of those in the system. For the higher level of the model, different road toll strategies are applied in order to minimize the CO2e emissions. The model has been applied to an illustrative example. It shows the effects on traffic flows, revenues, total time and CO2e emissions for different numbers of servers collecting tolls and different pricing strategies over a morning peak traffic period. The results show that the CO2e emissions produced can be significantly affected by the number of servers and the type of toll strategy employed. The model is also used to find the best toll strategy when there is a constraint on the revenue that is required to be raised from the toll and how this affects the emissions produced. Further runs compare strategies to minimize the CO2e emissions with those that minimize total travel time in the road system. In the illustrative example, the results for minimizing CO2e emissions are shown to be similar to the results obtained from minimizing the total travel time.

Keywords: Bi-level pricing model; Minimizing CO2e Emissions; Road Toll

## 1. Introduction

With the growth of road traffic, the problem of traffic congestion attracts increasing concern from the public, academic researchers and government authorities. A road toll is one policy that could reduce traffic congestion and improve the quality of the air conditions. Road tolls have become a well-researched topic in transportation planning. Road toll pricing is about charging money for access onto a road/specific area at certain times or for certain road users.

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The road toll will influence the usage of the road system for different departure times and choices of alternative routes. So different road toll strategies will change the traffic patterns and choosing a suitable and practical tolling strategy is important to reduce the fuel emissions in the whole road network.

Existing methods are available for modelling road toll pricing, but most of them focus on optimizing the total travel time or its relevant costs. There is little research on how to apply these models for a different objective such as minimizing emissions. Therefore, the research described in this paper aims to fill this gap.

The rest of the paper is organized as follows: the next section provides a literature review of previous work about road toll pricing. Section 3 describes the road toll pricing problem and how to apply it to minimize CO2e emissions and total travel time. Section 4 presents an illustrative case study and in the following section the computational results obtained through the proposed methods are discussed. The last section presents conclusions and directions for further research.

## 2. Literature Review Related to Road Toll Pricing

Road tolls are implemented in different cities around the world, such as Stockholm, London, Singapore and many other places. The road toll can be not only for a single road but also for an area. In some places the road toll is set as a constant while in other places it may be set as a variable charge which is different at different times of the day. There are two main types of method that are used to model the effect of road tolls: the first is to use marginal cost pricing theory and the other is to use a bi-level pricing model.

(Pigou, 1952) was the first to suggest that vehicles using congested roads should bear a tax equal to the difference between marginal social and marginal private cost. This suggestion has often been repeated and explored. (Walters, 1961) applies the theory of marginal cost pricing to estimate an efficient system of taxation for a network of highways. He suggests that efficient taxes (marginal private cost) should be equal to marginal social cost and proposes a mixture of gasoline taxes, mileage taxes, and congestion tolls.

(Dafermos, 1973) applies marginal cost pricing theory to decide the toll patterns for multiclass-user transportation networks and formulates the link-toll and path-toll collection problems.

(Smith, 1979) presents a small example to illustrate marginal cost theory, where vehicles using congested roads should bear a tax equal to the difference between marginal social and marginal private cost. The paper proves that if the cost and demand functions satisfy certain weak smoothness conditions then the marginal cost taxation of a transportation network is optimal, where the objective of the model is to maximize the benefit (revenue).

(Olof, 1997) discusses some important external costs associated with road transport in urban areas. The paper uses the marginal cost model to calculate the road toll and also discusses the speed-flow relationship, showing that the maximum flow is obtained at a certain speed level, which is equal to (or higher than) the actual speed at the peak hour. The relationship between fuel consumption and speed is described and the paper goes on to illustrate the optimal tax by considering the marginal private cost (own pollution) and marginal social cost (pollution for others, fuel consumption for others and time losses for others).

(Wie and Tobin, 1998) states that there are generally two classes of congestion pricing model in the literature: one is based on a static analysis and the other is based on a dynamic analysis. The first class of congestion pricing models assumes the general traffic network to be at a steady-state condition at all times and thus travel demands and costs are not time varying. The second class of congestion pricing models is dynamic in that travel demands and costs vary over time and thus congestion tolls need to be time varying. These two types of dynamic congestion pricing model are based on the theory of marginal cost pricing. The first model is appropriate for situations where commuters have the ability to learn the best route choices through day-to-day explorations on a network with arc capacities and travel demands that are stable from day to day. The second model is appropriate for situations where commuters optimize their routing decisions each day on a network with arc capacities and travel demands that fluctuate significantly from day to day.

(Wardrop, 1953) discusses some theories about road traffic research. Wardrop's principle of user equilibrium is introduced which assumes that traffic will tend to settle down into an equilibrium situation in which no driver can reduce his journey time by choosing a new route and then the driver has no incentive to improve their route. This principle is the theoretical basis for the bi-level pricing model. It describes a condition where the road network settles down into equilibrium at a particular time. Using the principle of equilibrium, an assumption is made that all drivers have the same perfect knowledge of routes in the network, and that they all seek to minimise the cost of travel subject to every other driver doing the same.

(Yan and Lam, 1996) presents some developments in model formulation and solution procedures for the congestion road pricing problem under queuing network equilibrium conditions. It describes a bi-level model of a leader-follower type, where the system manager is the leader and the network users are the followers. The lower-level problem is a queuing network equilibrium model that describes users' route choice behaviours under conditions of both queuing and congestion for a given link toll pattern. The model assumes a fixed travel demand pattern and in the light of any toll decision, the road users make their route choice decisions in a user-optimal manner.

The upper-level problem determines the toll pattern to optimize system performance, while taking into account the users' reactions in response to alternative road tolls. There are several alterative choices for the objectives, such as to minimize the total network cost, to maximize total revenue, or to maximize the ratio of the total revenue to total cost. Sensitivity analysis is used to provide the derivatives of link flows and queuing delays with respect to link tolls and hence indicates the "direction" in which the queuing network equilibrium pattern will move if the toll pattern is changed. The model helps to determine optimal road tolls such that total travel time is minimized or total toll revenue is maximized and the paper also provides a small example to show how the algorithm works. The bi-level pricing model is being applied to coordination of tunnel toll patterns in the Hong Kong road network.

(Labbé et al., 1998) describes the road toll problem as a bi-level problem. The paper proves that it is an NP-hard problem. It is a relatively early paper which makes some assumptions to simplify reality. Firstly, it assumes no dispersion of traffic along the routes of the network. Secondly, it assumes that the value-of-time parameter is uniform throughout the user population, and that, given the choice between two paths of equal cost, the users always select the one with the highest toll. These assumptions imply Dynamic User Equilibrium (DUE) is not achieved, where no user can unilaterally reduce their origin to destination travel time (or cost). The major contribution of the paper is to describe the bi-level framework for optimal motorway pricing.

(Joksimovic, 2007) designs a bi-level optimization problem in which the upper level describes the network performance with chosen toll levels while the lower level describes the dynamic network model including user-specific route and departure time choice and the dynamic network loading. The lower level of the model tries different combinations of departure time and route choices until it achieves a DUE. This model will be discussed in more detail in Section 3.

(Stewart and Ge, 2014) illustrates the feasibility of determining low–revenue toll sets to reduce the total cost of a network under the Dynamic User Equilibrium principle. It presents a model formulation and framework for dynamic congestion charging. It also presents two small examples (one network with 2 links, 2 paths and one network with 4 links, 3 paths) on the within-day scale. Algorithms for DUE iterate between two components: the dynamic network loading and the route choice or path reassignment. (Stewart and Ge, 2014) uses the theory from (Chow, 2007) to find the dynamic system optimum, where the total, rather than the individual, travel cost of all travellers through the network is minimized. (Chow, 2007) provides the necessary conditions and the sensitivity analysis for dynamic system optimizing flow.

(Ge et al., 2014) observe that when the congestion charge is applied, two undesired demand peaks have been observed. One is just before the start time of congestion charging and the other is at the end of it. These two peaks are defined as "temporal boundary effects". The other problem is that a traveller would rather stay away from a charging zone than pay congestion charging tolls, which causes undesired congestion on those roads or paths on the edge of the charging zone. This is called a "spatial boundary effect". In (Ge et al., 2014), three types of tolls are applied. The first is a constant toll across the charging period. The second type allows the toll to increase linearly from zero to a maximum level and then decrease linearly to zero. The third is that the toll rises linearly from zero up to a maximum level, stays flat for a period and then falls linearly to zero. The paper shows the constant toll results in both temporal and spatial boundary effects which are undesired. The multiple step tolling schemes reduce both temporal and spatial boundary effects. Eliminating the spatial issues would require an alternative scheme design other than a simple cordon. Adjusting the length of a charging period or the start and/or end times of the charging period lead to mitigation of the temporal boundary effect. Single and multi-step tolls are discussed in the academic literature including (Lindsey et al., 2010), (van den Berg, 2012), etc. The single step or flat toll leads to temporal and spatial boundary effects, which alter the effectiveness and efficiency of congestion charging.

(Saleh and Farrell, 2005) suggests a model where travellers reaching their destination within given arrival time windows will incur no schedule delay cost, which helps to resolve the boundary effects due to congestion charging. It incorporates reschedule flexibility by allowing travellers to get to their destination earlier or later than a normal work start time.

In the model introduced in Section 3, the main objective is to minimise the greenhouse gas effects of the traffic by minimising the CO2e emissions produced. The emissions are calculated as being proportional to the fuel consumed. Different models have been proposed for estimating the fuel used by vehicles travelling on roads. A discussion of the different types of model can be found in (Eglese and Bektaş, 2014) and (Demir et al., 2011) provides a quantitative comparison of a number of such models. Some models relate fuel consumption to speed using regression techniques and making assumptions about the load carried and other factors. An example of this type of model is one published by the European Commission in the MEET report described by (Hickman et al., 1999). Other models are suitable for calculating the instantaneous rate of fuel consumption under particular conditions for a given type of vehicle. An example of this type of model is the Comprehensive Modal Emissions Model (CMEM) described by (Scora and Barth, 2006). In this study, the relationship between the speed of the vehicles and fuel consumption is determined using the report from the (Department of Transport, 2009).

In the next section, a bi-level pricing model is developed based on Joksimovic's model (Joksimovic, 2007). Joksimovic's model is very similar to the one described in Stewart and Ge's model (Stewart and Ge, 2014) , but there are some differences in detail:

- The demand in Joksimovic's model is fixed and uses the model to spread out the demand over time. A time varying travel demand function was used in Stewart and Ge (2014). The traffic flow increases from zero to its peak and then declines gradually.
- The network loading methods are different. In Joksimovic's model, the travel time is determined by the traffic flow. There are two parts in Stewart and Ge's model ((Stewart and Ge, 2014)). One is the flow-density and the other is the traffic conversion equation. The capacity of the path determines the traffic flow.
- Joksimovic's model uses a time-varying road toll to achieve the objectives, but in Stewart and Ge's model, a fixed toll or bell-shaped tolls are applied.

In a later paper involving the same authors, (Ge et al., 2014) discuss the negative temporal and spatial boundary effects arising from implementing congestion charging and examines how to design the congestion charging in order to resolve them. Discussion of temporal and spatial boundary effects is absent in Joksimovic's model.

# 3. Bi-level pricing model

## 3.1 Model Definition

The design of our model follows the approach described in (Joksimovic, 2007), which was introduced in the previous section.

There are three components in Joksimovic's model: they are dynamic network loading (DNL), the route and departure time choice (RDC) model, and the road-pricing (RP) model. These three components interact with each other to find the solution of the dynamic toll design problem. The total demand for traffic to be carried by the network is a constant and the numbers of each type of road user are predefined. Different road users are assigned different values of time (VOT). Road Tolls can be set to vary at different times of the day or to be constant during some period of the day. The travel time for a link may depend on the loading of the network, so the model is dynamic. The demand in terms of traffic volume is assigned to different time periods of the day. Setting the preferred departure time and preferred arrival time affects the choices of starting times for the vehicles. The model assumes the change of the road tolls will not change the total traffic volume in the network, but road users may choose another starting time or another route to reduce the travel cost when the road toll is applied. Road users may not change from one type to another. The effect of applying a road toll to high traffic volume time periods is to reduce the traffic volume in those time intervals. The traffic volume within each time period is controlled by the preferred departure time (PDT) and preferred arrival time (PAT) for each set of users. PDT and PAT are single points of time, which could lead to temporal and spatial boundary effects. Following this approach, our bi-level pricing model consists of two tiers. The upper level of the model is the authority to decide the level of toll. The lower level of the model is for the users to choose their routes and departure times reacting to the change of toll level. The lower level of the model searches for a solution until it achieves the Dynamic User Equilibrium (DUE) for the current toll levels.

Figure 1 shows the Joksimovic model flow chart. The DNL will generate the traffic flow pattern including the travel time based on the road network inputs (e.g. the travel time for each link for a given traffic flow) and the total demand of the road network for different types of road users. The RP decides the levels of toll to apply in each time period to generate different types of tolling scheme. The RDC collects information from RP and DNL to calculate the total cost for each departure time and each route chosen. Then in the RDC, the users' choices of route and departure time are modified in order to react to the change of the road toll. For each set of road tolls, the system iterates making changes to the users' choices of route and departure time until it achieves the Dynamic User Equilibrium (DUE). Dynamic User Equilibrium is reached in this model where no user can unilaterally reduce their origin to destination travel cost. For situations where the toll is not fixed, but can be varied, then a further set of iterations applies different toll levels in order to find the set that optimises the model goal or upper level objective, and then the result is produced.

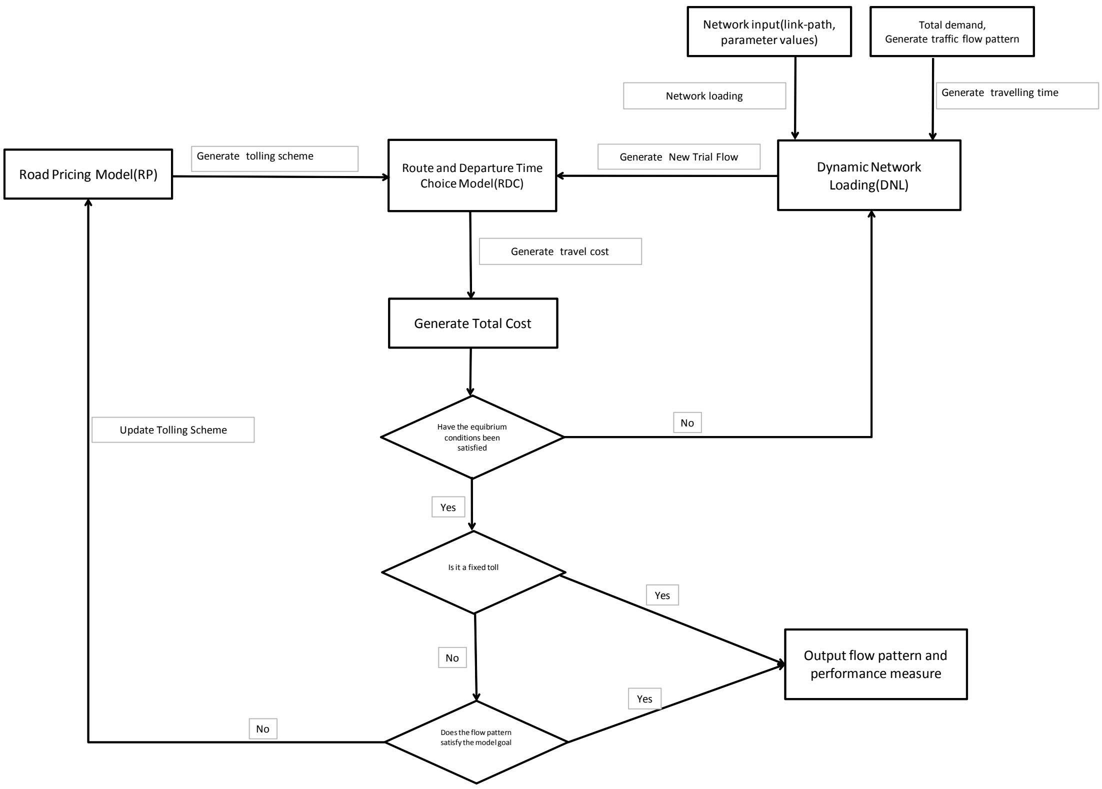


Figure 1 Framework for Joksimovic's model

Compared with the model presented in (Joksimovic, 2007), the model presented in this paper introduces some changes as described below:

- The traffic volume does not affect the travelling time. The only delay comes from the queue for paying the road toll when the vehicle is going across a tolled road. In this case, queuing theory is applied in the model to find the delay and the details are given at the end of this section describing the model. Otherwise we assume that all vehicles can travel in free flow traffic conditions.

- The upper-level objective of our model is changed to determine toll sets to reduce the total CO2e emissions of a network under the dynamic user equilibrium principle. The fuel consumption functions are incorporated into our model.
- Multiple step road tolls and variable road tolls are used to test whether the temporal and spatial boundary effects are reduced. Multiple levels of road toll are specified to investigate how the traffic will be affected by the road toll.
- Reschedule flexibility as described in (Saleh and Farrell, 2005) is used in our model. The preferred arrival time is replaced by a time interval between the earliest arrival time and latest arrival time and providing a traveller gets to their destination within that time interval then no penalty is incurred.

## 3.2 Assumptions

Some assumptions are included in the model as applied to the illustrative example introduced in the next section.

- Only two alternative routes are available in the experiment. All vehicles in the area are assumed to have the same origin and destination.
- The total cost includes fuel cost, driver cost (VOT), road toll and penalties for not reaching the destination in time.
- Three types of pricing strategy are applied. The first is the constant toll strategy, the second is the step toll strategy and the last is a variable toll strategy.
- The traffic flows are distributed to different time periods by using the logit model. In the logit model, the road user only considers the travel cost.
- Total demand of the traffic is a constant. In other words, the demand will not decrease when the toll is high and will not increase even if the toll is low.
- All vehicles have the same characteristics and are charged the same road toll.

## 3.3 Total Cost Calculation

The calculation for the total cost is shown in Formula (1). The expression provides the total cost for user group m taking path p between an origin r and destination s, starting at time k. The total cost includes value of time, fuel cost, road toll and penalties for violating the soft time windows. The model covers a particular time period including the morning rush hour (6am-12noon). Preferred earliest arrival times (PEAT) and preferred latest arrival times (PLAT) are set for each user group. These are soft time window constraints and can be violated with penalty determined by the parameters $\beta$ and $\gamma$, where $\beta$ is the penalty for deviation from PEAT and $\gamma$ is the penalty for deviation from PLAT. $\gamma$ is set bigger than $\beta$ as it is preferable to arrive early than to be late. The PEAT and PLAT are the main cause for the traffic to be more concentrated at particular times of the day. Different groups of users have different values of time (VOT), which are determined by the parameter $a$. $\theta$ is the road toll and $\tau$ is total travel time.

$$\begin{aligned} c_{pm}^{rs}(k) &= a_m \tau_{pm}^{rs}(k) + \theta_{pm}^{rs}(k) \\ &+ \beta_m * \max(PEAT_m^{rs} - (k + \tau_{pm}^{rs}(k)), 0) + \gamma_m * \max((k + \tau_{pm}^{rs}(k) - PLAT_{pm}^{rs}, 0) + \text{fuel cost} \end{aligned} \quad (1)$$

### Travel Time Calculation

The travel time of the route ($\tau_a(t)$) is decided by the free flow travel time plus the queuing time. $\tau_a^0$ is the free flow travel time for route a. $q_a(t)$ is the queuing time in period t for route a. Formula (2) shows the calculation for the travel time.

$$\tau_a(t) = \tau_a^0 + q_a(t) \quad (2)$$

### Fuel Consumption

The fuel consumption of a vehicle depends on many factors. In this model we focus on the relationship between the speed of the vehicle and the fuel efficiency measured in g/km. A typical relationship is shown in Figure 2 which applies for a HGV diesel rigid >32 t EURO 5 vehicle. Once the fuel consumption for a journey has been calculated, the CO2e emissions and fuel cost can be determined by using specific conversion factors.

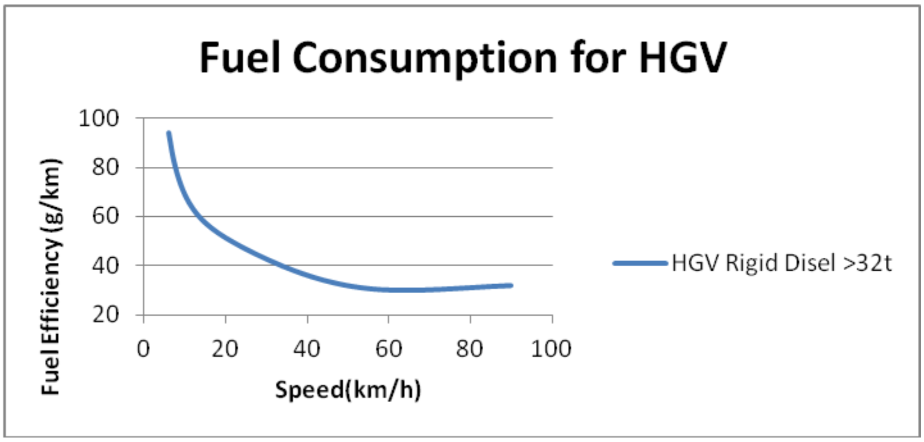


Figure 2 Fuel Consumption for HGV Diesel Rigid >32 t EURO 5

## Logit Model

The logit model is used to simulate the users' choices of the route and departure time. The logit model depends on the total cost of the users for the journey. The result of the logit model is a percentage representing the proportion of users from a user group that select a particular route and the sum of all the results for different time periods is equal to 1. We assume the users are only concerned about their costs (c). $\mu$ is the scale parameter of the logit model.

$$\psi_{pm}^{rs}(k) = \frac{\exp(-\mu c_{pm}^{rs}(k))}{\sum_{p \in P^{rs}} \sum_{k \in K} \exp(-\mu c_{pm}^{rs}(k))} \tag{3}$$

## Delay due to congestion

Generally, delays may be caused due to road capacity restrictions and congestion. For example, in the peak time interval, the traffic demand may exceed the road capacity which leads to congestion. But in our illustration which will be described in section 4, the only delay is that due to queuing to pay the toll. We are going to use a simple queuing (M/M/c) model to find the extra delay (service time + queuing time) for a given arrival rate, given the service rate and number of servers. The service times and arrival times follow a negative exponential distribution, so it is a standard queuing model. The following section gives the formula for calculating the queuing time.

***Definition***

$\lambda$ = Arrival rate

$\mu$ =Service Rate

$\rho = \lambda / \mu$

c= Number of Service Channels

***In the M/M/c case (random arrival, random service, and c service channels)***

$\frac{\rho}{c}$ must be < 1.0

The probability of having zero vehicles ( $p_0$ ) in the system is

$$P_0 = \left[ \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!(1-\rho/c)} \right]^{-1} \tag{4}$$

Expected average queue length is calculated as in Formula (5)

$$E(m) = P_0 \frac{\rho^{c+1}}{cc!} \frac{1}{(1-\rho/c)^2} \tag{5}$$

Expected average number in the system is calculated as in Formula (6)

$$E(n) = E(m) + \rho \tag{6}$$

Expected average total time is calculated as in Formula (7)

$$E(v) = E(n) / \lambda \tag{7}$$

# 4. Description of illustrative example

In this section, the bi-level pricing model is applied to a simple example based on the road network in the north east of England including the Humber Bridge for the purpose of illustration. It does not include real traffic data for this region. We compare different combinations of the number of servers and road tolling strategies.

We assume there are only two routes between Scunthorpe and Hull. One uses the tolled route across the Humber Bridge; the other is a longer journey to avoid the toll and is referred to as the non- tolled route.

Figure 3 shows the simplified routes, where R represents Scunthorpe and S represents Hull; Route 1 is longer than Route 2 and Route 1 takes a longer time than Route 2 when the traffic flows freely. Details are given in Table 1. The distance for Route 1 is 81.1km and the distance for Route 2 is 38.5km. The road toll is only applied to Route 2. Different values of the road toll are explored to achieve the optimal solution that minimises the CO2e emissions. Different levels of road toll can be applied during different time periods. There are two types of user with two different values of time. The users with higher VOT represent people making the journey in order to work at their place of employment in Hull and who want to reach the destination early in the day. The users with lower VOT represent people making the journey for other purposes such as shopping or tourism and can be more flexible for their arrival time.

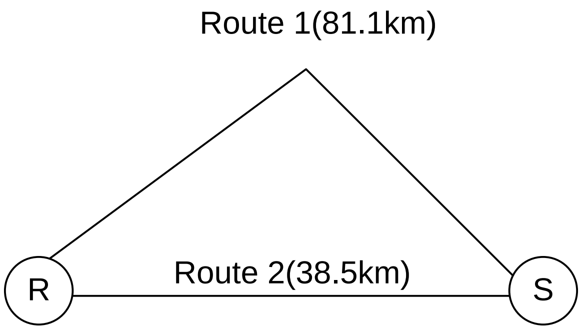


Figure 3 Simplified Routes

The travel time of the route ($\tau_a(t)$) is decided by the free flow travel time plus any queuing time on the bridge to pay the toll if Route 2 is used as introduced in Formula (2). The free flow traffic time for Route 1 is longer than Route 2, but using Route 1 does not require paying any road toll. q(t) is the queuing time on the bridge at time period t for traffic using Route 2.

In this simplified illustrative example, the time required for the longer Route 1 is constant and unaffected by the number of vehicles using the route. However the time required for Route 2 is affected by the additional time spent queuing in order to pay the toll and so will be affected by the number of toll booths available and the number of vehicles using the route. In practice, these queues may be reduced by methods such as the use of vehicle recognition technology to allow vehicles that have registered to pay the toll to bypass the toll booths, but such options are not included in this illustration.

In the experiments that follow, the emphasis will be on the effects of using different numbers of servers to collect the tolls and the toll pricing strategies. The number of servers will affect the length of the queue and hence the time required for Route 2. Travellers will be taking this into account along with the amount they must pay for using Route 2 to decide whether to adjust the time of their journey or to take the longer Route 1.

| | Parameter | notation | value |
|---|---|---|---|
| 1 | Free flow travel time | $\tau_a^0$ | Route 1 = 75 min |
| | | | Route 2 = 42 min |
| 2 | Queuing time at the bridge | *q(t)* | Route 1 = 0 |
| | | | Route 2(see detail in queuing Formula ) |

Table 1 Link Travel Time Function Parameters

Formula (1) is used to calculate the total cost of an individual user. Table 2 shows the parameters for calculating total cost in this case study.

| | Parameter | notation | value | unit |
|---|---|---|---|---|
| 1 | Preferred earliest arrival time | PEAT | 2(8am-9am) | |
| 2 | Preferred latest arrival time lower bound | PLAT | 4(10am-11am) | |
| 3 | Number of departure time intervals | K | 6(6am-12am) | |
| 4 | Penalty for early deviation from PEAT (for both groups) | $\beta$ | 8 | £/hour |
| 5 | Penalty for late deviation from PLAT(for both groups) | $\gamma$ | 20 | £/hour |
| 6 | VOT group 1(User 1) | $a_1$ | 0.08 | £/min |
| 7 | VOT group 2(User 2) | $a_2$ | 0.25 | £/min |
| 8 | Demand group 1 | $d_1$ | 7615 | |
| 9 | Demand group 2 | $d_2$ | 1161 | |
| 10 | Scale parameter(logit model) | $\mu$ | 0.35 | |
| 11 | Length of Route 1 | $l_1$ | 81.1 | km |
| 12 | Length of Route 2 | $l_2$ | 38.5 | km |

Table 2 Parameters for Calculating Total Cost

## 4.1 Road Toll Pricing Strategies

The following road toll pricing strategies are tested in the experiment:

1) Constant Road Toll: the road toll is set as a constant for all time periods. Three values

(£1.5, £3, £5) can be selected.

2) Step Toll: We compare with a "congestion charge-like" scheme (multiple steps toll pricing), where the charges start at 0, go to a fixed level C (£1.5, £3, £5), and back to zero in a day. Different levels of C are used for comparison. C can be £1.5, £3 or £5. We assume the tolls will be implemented from 7am to 9am.
3) Variable Toll: the road toll is set as a time varying road toll. Four levels of road toll (£0, £1.5, £3, £5) can be chosen in each time period, which corresponds to 1 hour in this experiment. We define the variable toll without the £0 toll option as VT1. We define the variable toll including the £0 toll option as VT2. A comparison of the traffic flows between the two types of variable toll (VT1 and VT2) is discussed in a later section.

In order to obtain the optimal solution, tests were run for each combination of possibilities and the best outcome was presented. The following section will discuss different solutions for different objectives.

# 5. Experimental results

In this section, we carry out some experiments with different objectives. Different types of strategies are used for controlling the toll price and the results are compared. Firstly, we minimize the total CO2e emissions as an objective under different toll price strategies.

## 5.1 Minimum Total CO2e emissions

The number of servers is increased in order to see how it affects the objective and the traffic flows. The results for the minimum total CO2e emissions experiments are consolidated in Table 3. The minimum total CO2e emissions for all pricing strategies with the same server numbers are highlighted in bold in Table 3. The variable toll (VT2) always gives the best solutions. This is not unexpected as variable toll VT2 includes the other strategies as special cases.

When the number of servers is small, the payment of the toll will deter some users and prevents the queue from being too long. If the queue is too long, the waiting time can be very high and a large amount of CO2e emissions are generated because of the low average speed.

In the variable toll cases, as the number of servers increases, the toll charges can be reduced because the queues will be smaller and fewer travellers will be deterred from using the shorter route. Generally, as the number of servers increases, the total revenue still increases as the increased number of vehicles using the tolled route more than compensates for the lower tolls charged. However, when 10 servers are used for the variable toll policies VT1 and VT2, the revenue is decreased compared to using 7 servers, as the increase in traffic flow is not enough to compensate for the reduced toll charges. In particular, for toll policy VT2 the best solution for 10 servers gives a revenue of zero, because the toll applied is £0 at all times. In practice there would be no point in arranging to collect a toll of £0, but the result indicates that with an objective of minimizing CO2e emissions, there may be cases where it is best not to impose toll charges.

However, if we exclude the £0 toll option from the variable toll as in VT1, then VT1 is not always the best choice. Sometimes, the step tolls give the best solutions. VT1 does not contain the same toll settings as the step tolls, because the £0 toll option is not allowed within VT1.

We have the following findings from the experiment:

- As the number of servers increases, total travelling time and CO2e emissions decrease for all experiments.
- The multiple steps toll pricing strategy and time varying toll pricing strategy always give better solutions than the constant toll pricing strategy.
- In this illustrative experiment, we can see a big improvement in terms of total travel time when the number of servers increases from 5 to 7 for the constant tolls. For the variable toll strategies, the biggest improvement in terms of total travel time occurs when the number of servers is increased from 3 to 5.
- Higher VOT users are more likely to use the tolled road.

| | | No. of Servers | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 3 | 5 | 7 | 10 |
| Constant toll 1.5 | Revenue(£) | 744 | 4677 | 12666 | 13120 | 13138 |
| | Travel time(min) | 687825 | 684951 | 607785 | 386786 | 369396 |
| | *CO2e emissions(kg)* | *673735* | *602221* | *437727* | *337346* | *331372* |
| Constant toll 3 | Revenue(£) | 1467 | 11925 | 24957 | 26190 | 26256 |
| | Travel time(min) | 682835 | 752249 | 602413 | 386928 | 369633 |
| | *CO2e emissions(kg)* | *673172* | *599662* | *439055* | *337861* | *331660* |
| Constant toll 5 | Revenue(£) | 2405 | 19615 | 40415 | 43450 | 43675 |
| | Travel time(min) | 678797 | 691019 | 592428 | 387467 | 370222 |
| | *CO2e emissions(kg)* | *672607* | *585348* | *441787* | *339188* | *332371* |
| Step toll 1.5 | Revenue(£) | 424 | 3567 | 8790 | 12406 | 12843 |
| | Travel time(min) | 685915 | 686322 | 584258 | 382862 | 369390 |
| | *CO2e emissions(kg)* | *672821* | *593842* | *425647* | *336043* | *331368* |
| Step toll 3 | Revenue(£) | 843 | 7257 | 17418 | 23988 | 25269 |
| | Travel time(min) | 685306 | 768566 | 560295 | 378804 | 369613 |
| | *CO2e emissions(kg)* | *672745* | *604150* | *416839* | *335125* | *331640* |
| Step toll 5 | Revenue(£) | 1395 | 11885 | 28625 | 37910 | 40460 |
| | Travel time(min) | 683944 | 778456 | 530753 | 373746 | 370139 |
| | *CO2e emissions(kg)* | *672561* | *604643* | *404988* | *334038* | *332282* |
| Variable (£1.5,£3,£5) | Revenue(£) | 1675 | 9704 | 18031 | 22617 | 13138 |
| | Travel time(min) | 679554 | 650655 | 423191 | 375944 | 369396 |
| | *CO2e emissions(kg)* | *672166* | *576147* | *365857* | *335037* | *331372* |
| Variable(0,£1.5,£3,£5) | Revenue(£) | 1977 | 8782 | 12639 | 28642 | 0 |
| | Travel time(min) | 679352 | 614018 | 420748 | 374856 | 369254 |
| | *CO2e emissions(kg)* | **671461** | **565983** | **359640** | **333044** | **331201** |

Table 3 Results for Minimizing Total CO2e emissions

## Comparing the Traffic Flow for Different Pricing Strategies

The traffic flow distribution is investigated in this section. We take the five servers for all pricing strategies as an example. The traffic distribution for the tolled road is shown in Table 4 and the traffic distribution for the non-tolled road is shown in Table 5. The entries in the tables are the number of vehicles per hour. It shows that the variable toll attracts more users to use the tolled road and results in a smoother traffic flow in different time periods compared with a constant toll and step toll pricing strategy. VT2 attracts more road users and a smoother traffic flow than VT1 for the tolled road.

The results in Tables 4 and 5 are illustrated in Figure 4.

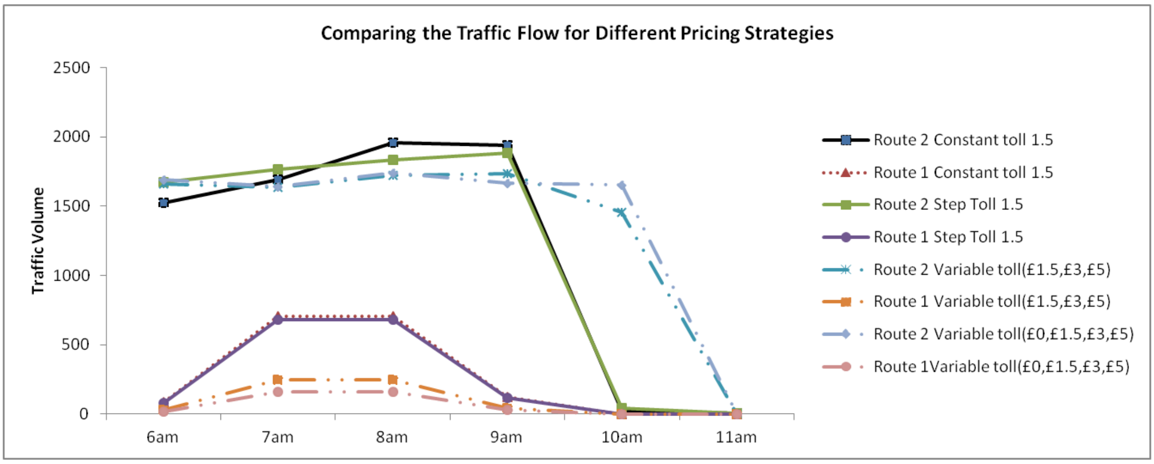


Figure 4 – Traffic Flow for Different Pricing Strategies

| **Route 2** | **6am-7am** | **7am-8am** | **8am-9am** | **9am-10am** | **10am-11am** | **11am-12noon** | **Total** |
|---|---|---|---|---|---|---|---|
| Constant toll 1.5 | 1524 | 1694 | 1955 | 1937 | 27 | 4 | 7141 |
| Step Toll 1.5 | 1670 | 1767 | 1835 | 1883 | 41 | 8 | 7204 |
| Variable toll(£1.5,£3,£5) | 1659 | 1635 | 1723 | 1735 | 1454 | 0 | 8206 |
| Variable toll(£0,£1.5,£3,£5) | 1691 | 1640 | 1740 | 1666 | 1656 | 0 | 8390 |

Table 4 Tolled Road Traffic Data

| **Route 1** | **6am-7am** | **7am-8am** | **8am-9am** | **9am-10am** | **10am-11am** | **11am-12noon** | **Total** |
|---|---|---|---|---|---|---|---|
| Constant toll 1.5 | 86 | 708 | 708 | 122 | 0 | 0 | 1624 |
| Step Toll 1.5 | 83 | 681 | 681 | 118 | 0 | 0 | 1563 |
| Variable toll(£1.5,£3,£5) | 29 | 245 | 245 | 42 | 0 | 0 | 561 |
| Variable toll(£0,£1.5,£3,£5) | 19 | 163 | 163 | 28 | 0 | 0 | 373 |

Table 5 Non Tolled Road Traffic Data

## Comparing the Variable Toll Strategies

A more detailed investigation is made for VT1 and VT2. In this section, we only discuss VT1 and VT2 to see the impact of having £0 in the variable toll strategy and the detailed information for the road toll in each time period. Table 6 shows the traffic distribution for the tolled road and Table 7 shows the traffic distribution for the non-tolled road. The values of the road toll are shown in brackets under the number of vehicles.

The travel time will be affected by the queue, so travellers will need to take that into account when selecting their departure time. The preferred arrival time is from 8am to 11am which is shown in italic in Table 6. The travel time for the free flow time of the tolled route is 42 minutes. The preferred departure time is about 1 hour ahead of the preferred arrival time range. So, the preferred departure time is from 7am to 10am which is the peak time for the traffic. If there are enough toll servers, most of the drivers will choose the tolled road and depart earlier in order to avoid the penalty of arriving late. It makes the traffic very busy in the peak time. In order to reduce the queue length, higher tolls are always implemented in peak time periods (from 7am to 10am). Very few users use the tolled road from 10am-12noon. As a result, the toll is relatively low from 10am to 12noon.

The number of servers decides the queue length which impacts users' travel time. When there are enough toll servers, almost all the vehicle would decide to use the tolled road. When the number of servers is 5, more than 90% of the users choose the tolled road. When the number of servers increases to 7, more than 99% of the users choose the tolled road. Increasing the number of servers allows the users to take the tolled road during a more concentrated time period. The length of the peak time will reduce from four hours (from 6am to 11am) to three hours (7am to 10am). Increasing the number of servers improves the capacity to handle the queue and makes the bridge more popular.

| No. of Servers | Toll Type | 6am-7am | 7am-8am | *8am-9am* | *9am-10am* | *10am-11am* | 11am-12noon | Total |
|---|---|---|---|---|---|---|---|---|
| 1 | Variable toll (£1.5,£3,£5) | 101 (1.5) | 47 (3) | *101 (3)* | *69 (1.5)* | *89 (3)* | 22 (1.5) | 429 |
| | Variable toll (£0,£1.5,£3,£5) | 102 (3) | 27 (1.5) | *99 (1.5)* | *77 (1.5)* | *91 (0)* | 14 (3) | 411 |
| 3 | Variable toll (£1.5,£3,£5) | 806 (1.5) | 731 (5) | *792 (5)* | *735 (5)* | *41 (5)* | 19 (1.5) | 3124 |
| | Variable toll (£0,£1.5,£3,£5) | 730 (3) | 753 (3) | *806 (1.5)* | *753 (3)* | *577 (1.5)* | 29 (0) | 3648 |
| 5 | Variable toll(£1.5,£3,£5) | 1659 (1.5) | 1635 (5) | *1723 (1.5)* | *1735 (1.5)* | *1454 (1.5)* | 0 (0) | 8206 |
| | Variable toll (£0,£1.5,£3,£5) | 1691 (0) | 1640 (3) | *1740 (3)* | *1666 (1.5)* | *1656 (0)* | 0 (1.5) | 8393 |
| 7 | Variable toll (£1.5,£3,£5) | 713 (1.5) | 2549 (5) | *2526 (3)* | *2705 (5)* | *199 (1.5)* | 0 (0) | 8692 |
| | Variable toll (£0,£1.5,£3,£5) | 957 (0) | 2300 (5) | *2550 (3)* | *2635 (3)* | *267 (0)* | 0 (0) | 8709 |
| 10 | Variable toll (£1.5,£3,£5) | 92 (1.5) | 1534 (1.5) | *3554 (1.5)* | *3554 (1.5)* | *25 (1.5)* | 0 (1.5) | 8759 |
| | Variable toll (£0,£1.5,£3,£5) | 93 (0) | 1535 (0) | *3556 (0)* | *3556 (0)* | *25 (0)* | 0 (0) | 8765 |

Table 6 Tolled Road Traffic Distribution

The free flow travel time for the non-tolled road is 75 minutes, so the users should start their journeys earlier than if they used the tolled road. From Table 7, we find that the non-tolled road shows a similar traffic pattern to the tolled road. Most users will use the non-tolled road from 7am -10am. There is no traffic on the non-tolled road at all from 10am to 12noon. Because there is a heavy late penalty cost, the users depart earlier to avoid the penalty.

| **No. of Servers** | **Toll Type** | **6am-7am** | **7am-8am** | ***8am-9am*** | ***9am-10am*** | ***10am-11am*** | **11am-12noon** | **Total** |
|---|---|---|---|---|---|---|---|---|
| 1 | Variable toll (£1.5,£3,£5) | 444 | 3632 | *3632* | *631* | *0* | 0 | 8339 |
| | Variable toll (£0,£1.5,£3,£5) | 445 | 3639 | *3639* | *632* | *0* | 0 | 8355 |
| 3 | Variable toll (£1.5,£3,£5) | 300 | 2458 | *2458* | *427* | *0* | 0 | 5643 |
| | Variable toll (£0,£1.5,£3,£5) | 272 | 2229 | *2229* | *387* | *0* | 0 | 5117 |
| 5 | Variable toll (£1.5,£3,£5) | 29 | 245 | *245* | *42* | *0* | 0 | 561 |
| | Variable toll (£0,£1.5,£3,£5) | 19 | 163 | *163* | *28* | *0* | 0 | 373 |
| 7 | Variable toll (£1.5,£3,£5) | 4 | 33 | *33* | *5* | *0* | 0 | 75 |
| | Variable toll (£0,£1.5,£3,£5) | 3 | 26 | *26* | *4* | *0* | 0 | 59 |
| 10 | Variable toll (£1.5,£3,£5) | 0 | 4 | *4* | *0* | *0* | 0 | 8 |
| | Variable toll (£0,£1.5,£3,£5) | 0 | 2 | *2* | *0* | *0* | 0 | 4 |

Table 7 Non Tolled Road Traffic Distribution

From Table 6, zero tolls are used in all time periods for VT2 with 10 servers which generate the best solutions. However, in practice, the bridge administration team may take into account financial considerations, such as maintenance fees, administration fees, and paying back any loan for constructing the bridge. This means the £0 toll may not be an option any more. A trade-off between revenue and total CO2e emissions may be preferred leading to the best solution subject to a constraint on the amount of money collected. For example, if we set the minimum revenue to be £20,000, then the best solution is the step tolls strategy (£3). Table 8 summarizes the best strategy to be applied for each level of minimum revenue. However these strategies will result in higher CO2e emissions than the optimal solution.

| Minimum Revenue | Best Strategy |
|---|---|
| £20,000 | Step Toll (£1.5) with 10 servers |
| £30,000 | Variable Toll with 7 servers |
| £40,000 | Step Toll (£5) with 10 servers |

Table 8 Best Strategy for each level of Minimum Revenue

## 5.2 Minimizing Total Travel Time

The objective of the model is changed to minimize the total travel time and the results are shown in Table 9. The minimum total travel times for all pricing strategies with the same number of servers are highlighted in bold. The variable toll strategy (VT2) always gives the best solutions, as expected because the other strategies are special cases of VT2. Using the objective of minimizing total travel time gives similar solutions to use of the objective of minimizing the CO2e emissions. Comparing the results from using the two objectives, the difference in terms of the total CO2e emissions is less than 1%. Although travel time is not proportional to CO2e emissions, it appears in this example that a system that minimises the total travel time for the users is close to one that is optimal in terms of minimising CO2e emissions.

| | | No. of Servers | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 3 | 5 | 7 | 10 |
| Constant toll £1.5 | Revenue | 700 | 4677 | 10711 | 13120 | 13138 |
| | *Travel time* | *686689* | *684951* | *552107* | *386786* | *369396* |
| | CO2e emissions | 674426 | 602221 | 449686 | 337346 | 331372 |
| Constant toll £3 | Revenue | 1467 | 11925 | 21264 | 26190 | 26256 |
| | *Travel time* | *682835* | *752249* | *543664* | *386928* | *369633* |
| | CO2e emissions | 673172 | 599662 | 447860 | 337861 | 331660 |
| Constant toll £5 | Revenue | 2405 | 19615 | 40415 | 43450 | 43675 |
| | *Travel time* | *678797* | *691019* | *592428* | *387467* | *370222* |
| | CO2e emissions | 672607 | 585348 | 441787 | 339188 | 332371 |
| Step toll £1.5 | Revenue | 424 | 3567 | 8227 | 12406 | 12843 |
| | *Travel time* | *685915* | *686322* | *503683* | *382862* | *369390* |
| | CO2e emissions | 672821 | 593842 | 426998 | 336043 | 331368 |
| Step toll £3 | Revenue | 843 | 5085 | 17415 | 23988 | 25269 |
| | *Travel time* | *685306* | *700114* | *560243* | *378804* | *369613* |
| | CO2e emissions | 672745 | 621342 | 416925 | 335125 | 331640 |
| Step toll £5 | Revenue | 1395 | 12030 | 28625 | 37910 | 40460 |
| | *Travel time* | *683944* | *777033* | *530753* | *373746* | *370139* |
| | CO2e emissions | 672561 | 605978 | 404988 | 334038 | 332282 |
| Variable (£1.5, £3, £5) | Revenue | 996 | 12732 | 18031 | 35216 | 13138 |
| | *Travel time* | *673587* | *640963* | *423191* | *375944* | *369396* |
| | CO2e emissions | 672489 | 590653 | 365857 | 335357 | 331372 |
| Variable (0, £1.5, £3, £5) | Revenue | 654 | 8782 | 12639 | 27055 | 0 |
| | *Travel time* | **671662** | **614018** | **420748** | **373095** | **369254** |
| | CO2e emissions | 672490 | 565983 | 359640 | 333318 | 331201 |

Table 9 Results for Minimizing Total Travel Time

## 5.3 Minimizing Time without considering Fuel Cost

The users do not always have perfect information about the travelling cost. In this section, the user is assumed to have no information about the fuel cost when they consider their routes and departure times. They only consider the VOT, road toll and penalties for not reaching the preferred arrival time period. This would affect their choices of the routes and departure times which result in changing traffic patterns in the road network. The results of minimizing the total travelling time experiment are shown in Figure 5. The table on the left contains the results for the users without considering fuel cost. The table on the right contains the results considering fuel cost. The left table has higher CO2e emissions and more total travel time but less revenues than the right table for the same conditions in most, though not all, cases. The size of the difference increases as the number of servers increases to seven. The result shows the availability of information is an important factor when attempting to optimize the tolling strategy for a road network. Without considering the fuel cost, more users prefer to drive a longer distance by using the non-tolled road. At the same time, the queuing time for the toll road is reduced.

| | | No. of Servers | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 3 | 5 | 7 | 10 |
| Constant toll £1.5 | Revenue | 603 | 3600 | 7023 | 8620 | 8620 |
| | *Travel time* | *677270* | *616733* | *511158* | *469360* | *469360* |
| | CO2e emissions | 674084 | 600125 | 495960 | 451458 | 451458 |
| Constant toll £3 | Revenue | 1203 | 6843 | 12567 | 14130 | 14130 |
| | *Travel time* | *674470* | *620803* | *526990* | *503032* | *503032* |
| | CO2e emissions | 673538 | 605117 | 515540 | 492674 | 492674 |
| Constant toll £5 | Revenue | 1735 | 10435 | 16615 | 16615 | 16615 |
| | *Travel time* | *657009* | *608093* | *549807* | *549807* | *549807* |
| | CO2e emissions | 670358 | 604539 | 548131 | 548131 | 548131 |
| Step toll £1.5 | Revenue | 424 | 3567 | 6822 | 8454 | 8454 |
| | *Travel time* | *685915* | *686322* | *509496* | *468730* | *468730* |
| | CO2e emissions | 672821 | 593842 | 494506 | 450740 | 450740 |
| Step toll £3 | Revenue | 810 | 6648 | 12048 | 13743 | 13743 |
| | *Travel time* | *674798* | *624629* | *522893* | *501345* | *501345* |
| | CO2e emissions | 673965 | 600056 | 512788 | 490643 | 490643 |
| Step toll £5 | Revenue | 1305 | 10320 | 15950 | 15950 | 15950 |
| | *Travel time* | *671912* | *601174* | *545620* | *545620* | *545620* |
| | CO2e emissions | 672452 | 594618 | 543341 | 543341 | 543341 |
| Variable (£1.5, £3, £5) | Revenue | 824 | 4866 | 11676 | 8620 | 8620 |
| | *Travel time* | *649522* | *595802* | *511068* | *469360* | *469360* |
| | CO2e emissions | 667750 | 600738 | 495995 | 451458 | 451458 |
| Variable (0, £1.5, £3, £5) | Revenue | 1035 | 2919 | 9066 | 0 | 0 |
| | *Travel time* | **648683** | **590322** | **508547** | **443188** | **438645** |
| | CO2e emissions | 667442 | 591607 | 492015 | 416372 | 415041 |

| | | No. of Servers | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 3 | 5 | 7 | 10 |
| Constant toll £1.5 | Revenue | 700 | 4677 | 10711 | 13120 | 13138 |
| | *Travel time* | *686889* | *684951* | *552107* | *386786* | *369396* |
| | CO2e emissions | 674426 | 602221 | 449686 | 337346 | 331372 |
| Constant toll £3 | Revenue | 1467 | 11925 | 21264 | 26190 | 26256 |
| | *Travel time* | *682835* | *752249* | *543664* | *386928* | *369633* |
| | CO2e emissions | 673172 | 599662 | 447860 | 337861 | 331660 |
| Constant toll £5 | Revenue | 2405 | 19615 | 40415 | 43450 | 43675 |
| | *Travel time* | *678797* | *691019* | *592428* | *387467* | *370222* |
| | CO2e emissions | 672607 | 585348 | 441787 | 339188 | 332371 |
| Step toll £1.5 | Revenue | 424 | 3567 | 8227 | 12406 | 12843 |
| | *Travel time* | *685915* | *686322* | *503683* | *382862* | *369390* |
| | CO2e emissions | 672821 | 593842 | 426998 | 336043 | 331368 |
| Step toll £3 | Revenue | 843 | 5085 | 17415 | 23988 | 25269 |
| | *Travel time* | *685306* | *700114* | *560243* | *378804* | *369613* |
| | CO2e emissions | 672745 | 621342 | 416925 | 335125 | 331640 |
| Step toll £5 | Revenue | 1395 | 12030 | 28625 | 37910 | 40460 |
| | *Travel time* | *683944* | *777033* | *530753* | *373746* | *370139* |
| | CO2e emissions | 672561 | 605978 | 404988 | 334038 | 332282 |
| Variable (£1.5, £3, £5) | Revenue | 996 | 12732 | 18031 | 35216 | 13138 |
| | *Travel time* | *673587* | *640963* | *423191* | *375944* | *369396* |
| | CO2e emissions | 672489 | 590653 | 365857 | 335357 | 331372 |
| Variable (0, £1.5, £3, £5) | Revenue | 654 | 8782 | 12639 | 27055 | 0 |
| | *Travel time* | **671662** | **614018** | **420748** | **373095** | **369254** |
| | CO2e emissions | 672490 | 565983 | 359640 | 333318 | 331201 |

Figure 5 Results for minimizing time without considering fuel cost

# 6. Conclusions and Further Work

The bi-level pricing model has been developed to solve a problem of setting the tolling strategy and the number of toll servers for the new objectives of minimizing CO2e emissions and minimizing total travel time. The bi-level pricing model is applied to an illustrative example based on the road network containing the Humber Bridge, where different combinations of the number of servers and road tolling strategies are compared. Minimizing CO2e emissions gives similar results and traffic patterns as minimizing travel time and for both these objectives we found that the multiple steps toll pricing strategy and time varying toll pricing strategy always give better solutions than the constant toll pricing strategy. As the number of servers increases, total traveling time and CO2e emissions decrease. Overall, the best solution is for a variable road toll with 10 servers, but for this solution, no revenue is collected from the toll, so if there is a constraint on the minimum revenue that is needed from the tolled route, then a solution providing a trade-off between revenue and total travel time or total CO2e emissions may be preferred.

In practice, the constant road toll and steps tolls are easily applied. The variable road toll strategy is hard to apply in practice because it is difficult to determine in advance what will be the best toll values to apply at different times and how this will affect traffic flows. This is the main reason why constant road tolls and step road tolls are widely applied all over the world. However, technology advance is leading to more vehicles being equipped with real time traffic information via GPS devices. This means that drivers can react more quickly to changing traffic conditions and also means that those managing a tolled road have much better access to traffic flow information. So a variable road toll strategy may be easier to implement successfully in the future.

The availability of information influences the decisions of the drivers. For example, if the drivers do not have information about the fuel cost, they may choose longer distance routes in order to prevent paying the road toll. This makes the traffic on the non-tolled route more congested and increases $CO_2e$ emissions and total travel time.

The model and simple illustrative example have demonstrated how a tolling strategy can be used to influence traffic patterns and to reduce $CO_2e$ emissions. However for this approach to be used in practice further considerations would be needed:

i) The set of users would have to be expanded to cover the main origins and destinations of travelers who might consider using the tolled road. For each origin-destination pair, the vehicles would need to be divided into more subsets representing users who have different values of time and also different types of vehicles, such as cars, vans and heavy goods vehicles which have different relationships for the way the $CO_2e$ emissions depend on speed.

ii) As well as considering the increased time required when waiting in a queue to pay a toll, the capacities of the roads on each route should also be considered so that reduced speed due to congestion is also included in the model.

iii) The model could also be enhanced by allowing the total volume of traffic to be influenced by the costs and time required for the journeys as has been done in some of the other studies mentioned in the literature review.

iv) As mentioned previously, smart-payment technology can be used to reduce the need to queue to pay tolls and should be considered for any new scheme.

The most important future work worthwhile to be undertaken is to develop a road toll pricing model for a larger network, such as the London Congestion Charge Zone. When the network becomes larger, more factors need to be considered. We may not only consider the varying traffic pattern in the zone but also the traffic condition on the boundary of the zone and the area outside the zone. Drivers may go around the congestion charge zone in order to avoid paying the charge, with the effect of reducing the traffic in the zone but possibly exacerbating the congestion outside the zone. This is another area worthy of further research.

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