



# Classical and quantum initial conditions for Higgs inflation



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## ABSTRACT

We investigate whether Higgs inflation can occur in the Standard Model starting from natural initial conditions or not. The Higgs has a non-minimal coupling to the Ricci scalar. We confine our attention to the regime where quantum Einstein gravity effects are small in order to have results that are independent of the ultraviolet completion of gravity. At the classical level we find no tuning is required to have successful Higgs inflation, provided the initial homogeneity condition is satisfied. On the other hand, at the quantum level we obtain that the renormalization for large non-minimal coupling requires an additional degree of freedom, unless a tuning of the initial values of the running parameters is made. In order to see that this effect may change the predictions we finally include such degree of freedom in the field content and show that Starobinsky's  $R^2$  inflation dominates over Higgs inflation.

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## 1. Introduction

Inflation [1–3] is perhaps one of the most natural ways to stretch the initial quantum vacuum fluctuations to the size of the current Hubble patch, seeding the initial perturbations for the cosmic microwave background (CMB) radiation and large scale structure in the universe [4] (for a theoretical treatment, see [5]). Since inflation dilutes all matter it is pertinent that after the end of inflation the universe is filled with the right thermal degrees of freedom, i.e. the Standard Model (SM) degrees of freedom (for a review on pre- and post-inflationary dynamics, see [6]). The most economical way to achieve this would be via the vacuum energy density stored within the SM Higgs, whose properties are now being measured at the Large Hadron Collider (LHC) [7,8]. Naturally, the decay of the Higgs would create all the SM quarks and leptons observed within the visible sector of the universe. Albeit, with just alone SM Higgs and minimal coupling to gravity, it is hard to explain the temperature anisotropy observed in the CMB radiation without invoking physics beyond the SM.<sup>1</sup>

However, a very interesting possibility may arise within the SM if the Higgs were to couple to gravity non-minimally – such as in

the context of extended inflation [10], which has recently received particular attention after the Higgs discovery at the LHC in the context of Higgs inflation [11]. By tuning this non-minimal coupling constant,  $\xi$ , between the Ricci scalar of the Einstein–Hilbert term and the SM Higgs, it is possible to explain sufficient amount of e-folds of inflation and also fit other observables such as the amplitude of temperature anisotropy and the spectral tilt in the CMB data. Indeed, this is very nice and satisfactory, except that the non-minimal coupling,  $\xi$ , turns out to be very large (at the classical level  $\xi \sim 10^4$ ) in order to explain the CMB observables. This effectively redefines the Planck's constant during inflation, and invites new challenges for this model, whose consequences have been debated vigorously in many papers, such as [12].

One particular consequence of such large non-minimal coupling is that there is a new scale in the theory,  $\bar{M}_{\text{Pl}}/\sqrt{\xi}$ , lower than the standard reduced Planck mass,  $\bar{M}_{\text{Pl}} \approx 2.435 \times 10^{18}$  GeV. Typically inflation occurs above this scale, the Higgs field takes a vacuum expectation value (VEV) above  $\bar{M}_{\text{Pl}}/\sqrt{\xi}$  in order to sustain inflation sufficiently. In fact, the inflaton potential, in the Einstein frame, approaches a constant plateau for sufficiently large field values. Effectively, the inflaton becomes a flat direction, where it does not cost any energy for the field to take any VEV beyond this cut-off.

Given this constraint on the initial VEV of the inflaton and the new scale, we wish to address two particularly relevant issues concerning the Higgs inflation model [11], one on the classical front and the other on the quantum front.

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<sup>1</sup> Within supersymmetry it is indeed possible to invoke the flat direction composed of the Higgses to realize inflation with minimal gravitational interaction, see [9], which can explain the current CMB observations.

I. Classically, a large VEV of the inflaton does not pose a big problem as long as the initial energy density stored in the inflaton system, in the Einstein frame, is below the cut-off of the theory. Since, the potential energy remains bounded below this cut-off, the question remains – what should be the classical initial condition for the kinetic energy of the inflaton?

A priori there is no reason for the inflaton to move slowly on the plateau, therefore the question we wish to settle in this paper is what should be the range of phase space allowed for a sustainable inflation to occur with almost a flat potential? The aim of this paper is to address this classical initial condition problem.<sup>2</sup> Here we strictly assume homogeneity of the universe from the very beginning; we do not raise the issue of initial homogeneity condition required for a successful inflation; this issue has been discussed earlier in a generic inflationary context in many classic papers (see [15,16]). In our paper, instead we look into the possibility of initial phase space for a *spatially flat* universe, and study under what pre-inflationary conditions Higgs inflation could prevail.

II. At quantum level, the original Higgs model poses a completely different challenge. A large  $\xi$  will inevitably modify the initial action. One may argue that there will be quantum corrections to the Ricci scalar,  $R$ , such as a Higgs-loop correction – leading to a quadratic in curvature action, i.e.  $R + \alpha R^2$  type correction, where  $\alpha$  is a constant, whose magnitude we shall discuss in this paper. The analysis is based on the renormalization group equations (RGEs) of the SM parameters and the gravitational interactions. By restricting for simplicity the study to operators up to dimension 4, the RGE analysis will yield a gravitational action that will become very similar to the Starobinsky type inflationary model [17].<sup>3</sup>

One of the features of theories with curvature squared terms is that there are extra degrees of freedom involved in the problem, besides the SM ones and the graviton. There is another scalar mode arising from  $R^2$ , which will also participate during inflation. The question then arises when this new scalar degree of freedom becomes dominant dynamically, and play the role of an inflaton creating the initial density perturbations?

The aim of this paper will be to address both the classical and quantum issues.

We briefly begin our discussion with essential ingredients of Higgs inflation in Section 2, then we discuss the classical pre-inflationary initial conditions for Higgs inflation in Section 3. In this section, we discuss both analytical in Subsection 3.1, and numerical results in Subsection 3.2. In Section 4, we discuss the quantum correction to the original Higgs inflation model, i.e. we discuss the RGEs of the Planck mass in Subsection 4.1, SM parameters in Subsection 4.2, and the gravitational correction arising due to large  $\xi$  in Subsection 4.3, respectively. We briefly discuss our results and consequences for inflation in Subsection 4.4, before concluding our paper.

## 2. The model

Let us define the Higgs inflation model [11]. The action is

$$S = \int d^4x \sqrt{-g} \left[ \mathcal{L}_{\text{SM}} - \left( \frac{\bar{M}_{\text{Pl}}^2}{2} + \xi |\mathcal{H}|^2 \right) R \right], \quad (1)$$

where  $\mathcal{L}_{\text{SM}}$  is the SM Lagrangian minimally coupled to gravity,  $\xi$  is the parameter that determines the non-minimal coupling between the Higgs and the Ricci scalar  $R$ , and  $\mathcal{H}$  is the Higgs doublet. The part of the action that depends on the metric and the Higgs doublet *only* is

$$\int d^4x \sqrt{-g} \left[ |\partial \mathcal{H}|^2 - V - \left( \frac{\bar{M}_{\text{Pl}}^2}{2} + \xi |\mathcal{H}|^2 \right) R \right], \quad (2)$$

where  $V = \lambda(|\mathcal{H}|^2 - v^2/2)^2$  is the Higgs potential and  $v$  is the electroweak Higgs VEV. We take a sizable non-minimal coupling,  $\xi > 1$ , because this is required by inflation as we will see.

The non-minimal coupling  $-\xi |\mathcal{H}|^2 R$  can be eliminated through the *conformal* transformation

$$g_{\mu\nu} \rightarrow \Omega^{-2} g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{2\xi |\mathcal{H}|^2}{\bar{M}_{\text{Pl}}^2}. \quad (3)$$

The original frame, where the Lagrangian has the form in (1), is called the Jordan frame, while the one where gravity is canonically normalized (obtained with the transformation above) is called the Einstein frame. Therefore, the scalar-tensor action for the metric and the physical Higgs mode  $\phi \equiv \sqrt{2} |\mathcal{H}|^2$  is (after the conformal transformation)

$$S_{\text{st}} = \int d^4x \sqrt{-g} \left[ K \frac{(\partial \phi)^2}{2} - \frac{V}{\Omega^4} - \frac{\bar{M}_{\text{Pl}}^2}{2} R \right], \quad (4)$$

where  $K \equiv (\Omega^2 + 6\xi^2 \phi^2 / \bar{M}_{\text{Pl}}^2) / \Omega^4$ . The non-canonical Higgs kinetic term can be made canonical through the (invertible) field redefinition  $\phi = \phi(\chi)$  defined by

$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2 \phi^2 / \bar{M}_{\text{Pl}}^2}{\Omega^4}}, \quad (5)$$

with the conventional condition  $\phi(\chi = 0) = 0$ . One can find a closed expression of  $\chi$  as a function of  $\phi$ :

$$\begin{aligned} \chi(\phi) = \bar{M}_{\text{Pl}} \sqrt{\frac{1 + 6\xi}{\xi}} \sinh^{-1} \left[ \frac{\sqrt{\xi(1 + 6\xi)} \phi}{\bar{M}_{\text{Pl}}} \right] \\ - \sqrt{6} \bar{M}_{\text{Pl}} \tanh^{-1} \left[ \frac{\sqrt{6\xi} \phi}{\sqrt{\bar{M}_{\text{Pl}}^2 + \xi(1 + 6\xi) \phi^2}} \right]. \end{aligned} \quad (6)$$

Thus,  $\chi$  feels a potential

$$U \equiv \frac{V}{\Omega^4} = \frac{\lambda(\phi(\chi)^2 - v^2)^2}{4(1 + \xi \phi(\chi)^2 / \bar{M}_{\text{Pl}}^2)^2}. \quad (7)$$

Let us now recall how slow-roll inflation emerges. From (5) and (7) it follows [11] that  $U$  is exponentially flat when  $\chi \gg \bar{M}_{\text{Pl}}$ , which is the key property to have inflation. Indeed, for such high field values the slow-roll parameters

$$\epsilon \equiv \frac{\bar{M}_{\text{Pl}}^2}{2} \left( \frac{1}{U} \frac{dU}{d\chi} \right)^2, \quad \eta \equiv \frac{\bar{M}_{\text{Pl}}^2}{U} \frac{d^2U}{d\chi^2} \quad (8)$$

are guaranteed to be small. Therefore, the region in field configurations where  $\chi > \bar{M}_{\text{Pl}}$  (or equivalently [11]  $\phi > \bar{M}_{\text{Pl}} / \sqrt{\xi}$ ) corresponds to inflation. We will investigate whether successful slow-roll inflation emerges also for large initial field kinetic energy in the next section. Here we simply assume that the time derivatives are small.

All the parameters of the model can be fixed through experiments and observations, including  $\xi$  [11,21].  $\xi$  can be obtained by requiring that the measured power spectrum [4],

<sup>2</sup> Some single monomial potentials and exponential potentials exhibit a classic example of late time attractor where the inflaton field approaches a slow roll phase from large initial kinetic energy, see [13,14].

<sup>3</sup> In principle, large  $\xi$  may also yield higher derivative corrections up to quadratic in order, see [18], and also higher curvature corrections, but in this paper, we will consider for simplicity the lowest order corrections. We will argue that the  $\alpha R^2$  is necessarily generated unless one is at the critical point of Refs. [19,20] or invokes a fine-tuning on the initial values of the running parameters.

$$P_R = \frac{U/\epsilon}{24\pi^2 \bar{M}_{\text{pl}}^4} = (2.14 \pm 0.05) \times 10^{-9}, \quad (9)$$

is reproduced for a field value  $\phi = \phi_b$  corresponding to an appropriate number of e-folds of inflation [21]:

$$N = \int_{\phi_{\text{end}}}^{\phi_b} \frac{U}{\bar{M}_{\text{pl}}^2} \left( \frac{dU}{d\phi} \right)^{-1} \left( \frac{d\chi}{d\phi} \right)^2 d\phi \approx 59, \quad (10)$$

where  $\phi_{\text{end}}$  is the field value at the end of inflation, that is

$$\epsilon(\phi_{\text{end}}) \approx 1. \quad (11)$$

For  $N = 59$ , by using the classical potential we obtain

$$\xi = (5.02 \mp 0.06) \times 10^4 \sqrt{\lambda}, \quad (N = 59) \quad (12)$$

where the uncertainty corresponds to the experimental uncertainty in Eq. (9). Note that  $\xi$  depends on  $N$ :

$$\xi = (4.61 \mp 0.06) \times 10^4 \sqrt{\lambda}, \quad (N = 54) \quad (13)$$

$$\xi = (5.43 \mp 0.06) \times 10^4 \sqrt{\lambda}. \quad (N = 64) \quad (14)$$

This result indicates that  $\xi$  has to be much larger than one because  $\lambda \sim 0.1$  (for precise determinations of this coupling in the SM see Refs. [22,23]).

### 3. Pre-inflationary dynamics: classical analysis

Let us now analyze the dynamics of this classical system in the homogeneous case without making any assumption on the initial value of the time derivative  $\dot{\chi}$ . We will assume that the universe is sufficiently homogeneous to begin inflation.

In the Einstein frame  $S_{\text{st}}$  is given by:

$$S_{\text{st}} = \int d^4x \sqrt{-g} \left[ \frac{(\partial\chi)^2}{2} - U - \frac{\bar{M}_{\text{pl}}^2}{2} R \right], \quad (15)$$

where  $U$  is the Einstein frame potential given in Eq. (7).

Let us assume a universe with three dimensional translational and rotational symmetry, that is a Friedmann–Robertson–Walker (FRW) metric

$$ds^2 = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right], \quad (16)$$

with  $k = 0, \pm 1$ .

Then the Einstein equations and the scalar equations imply the following equations for  $a(t)$  and the spatially homogeneous field  $\chi(t)$

$$\ddot{\chi} + 3H\dot{\chi} + U' = 0, \quad (17)$$

$$\frac{\dot{a}^2 + k}{a^2} - \frac{\dot{\chi}^2 + 2U}{6\bar{M}_{\text{pl}}^2} = 0, \quad (18)$$

$$\frac{k}{a^2} - \dot{H} - \frac{\dot{\chi}^2}{2\bar{M}_{\text{pl}}^2} = 0. \quad (19)$$

where  $H \equiv \dot{a}/a$ , a dot denotes a derivative with respect to  $t$  and a prime is a derivative with respect to  $\chi$ . Notice that Eq. (17) tells us that  $\chi$  cannot be constant before inflation unless  $U$  is flat. From Eqs. (17) and (18) one can derive (19), which is therefore dependent.

Thus, we have to solve the following system with initial conditions

$$\begin{cases} \dot{\Pi} + 3H\Pi + U' = 0, & \Pi(\bar{t}) = \bar{\Pi}, \\ \dot{\chi} = \Pi, & \chi(\bar{t}) = \bar{\chi}, \\ \dot{a}^2 + k = \frac{a^2}{6\bar{M}_{\text{pl}}^2} (\Pi^2 + 2U), & a(\bar{t}) = \bar{a}, \end{cases} \quad (20)$$

where  $\bar{t}$  is some initial time before inflation and  $\bar{\chi}$ ,  $\bar{\Pi}$  and  $\bar{a}$  are the initial conditions for the three dynamical variables. In the case  $k = 0$  the previous system can be reduced to a single second order equation. Indeed, by setting  $k = 0$  in Eq. (18) and inserting it in Eq. (17), one obtains

$$\ddot{\chi} + \sqrt{\frac{3\dot{\chi}^2 + 6U}{2\bar{M}_{\text{pl}}^2}} \dot{\chi} + U' = 0, \quad (k = 0). \quad (21)$$

This equation has to be solved with two initial conditions (for  $\chi$  and  $\dot{\chi}$ ). The initial condition for  $a$  is not needed in this case as its overall normalization does not have a physical meaning for  $k = 0$ .

We confine our attention to the regime where quantum Einstein gravity corrections are small:

$$U \ll \bar{M}_{\text{pl}}^4, \quad \dot{\chi}^2 \ll \bar{M}_{\text{pl}}^4, \quad \frac{|k|}{a^2} \ll \bar{M}_{\text{pl}}^2 \quad (22)$$

such that we can ignore the details of the ultraviolet (UV) completion of Einstein gravity.<sup>4</sup> However, we do not always require to be initially in a slow-roll regime. The first and second conditions in (22) come from the requirement that the energy–momentum tensor is small (in units of the Planck scale) so that it does not source a large curvature; the third condition ensures that the three-dimensional curvature is also small. The first condition is automatically fulfilled by the Higgs inflation potential, Eq. (7): the quartic coupling  $\lambda$  is small [25,22,23] and the non-minimal coupling  $\xi$  is large (see Eq. (12)). The second and third conditions in (22) are implied by the requirement of starting from an (approximately) de Sitter space, which is maximally symmetric; therefore we do not consider them as a fine-tuning in the initial conditions. In de Sitter we have to set  $k = 0$  and  $\dot{H} = 0$ , which then implies  $\dot{\chi} = 0$  from Eq. (19). Notice also that we cannot start from an exact de Sitter, given Eq. (17): the potential  $U$  is almost, but not exactly flat in the large field case (see Eq. (7)).

In order for the Higgs to trigger inflation sooner or later one should have a slow-roll regime, where the kinetic energy is small compared to the potential energy,  $\dot{\chi}^2/2 \ll U$ , and the field equations are approximately

$$\frac{\dot{a}^2 + k}{a^2} \approx \frac{U}{3\bar{M}_{\text{pl}}^2}, \quad \dot{\chi} \approx -\frac{1}{3H} U', \quad (\text{slow-roll equations}). \quad (23)$$

The conditions for this to be true are

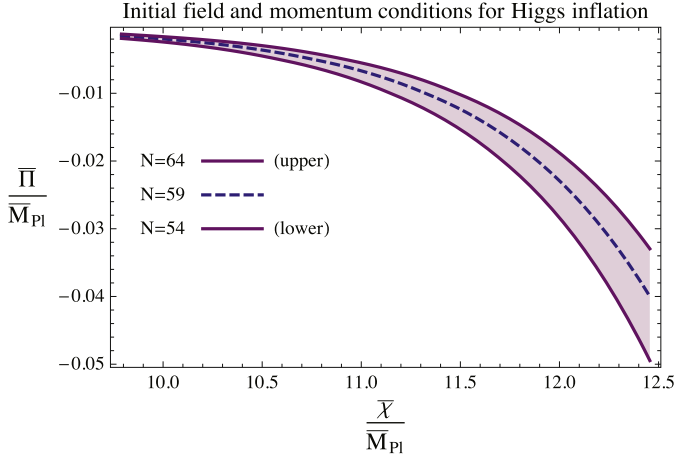
$$\dot{\chi}^2 \ll 2U, \quad |\dot{\chi}| \ll 3|H\dot{\chi}| \quad (\text{slow-roll regime}). \quad (24)$$

We will use these conditions rather than the standard  $\epsilon \ll 1$  and  $\eta \ll 1$  as we do not assume a priori a small kinetic energy.

#### 3.1. Analytic approximations in simple cases

Let us assume, for simplicity, that the parameter  $k$  in the FRW metric vanishes, i.e. a spatially flat metric, and consider the case  $\dot{\chi}^2 \gg U$ , such that the potential energy can be neglected compared to the kinetic energy. In this case, combining Eqs. (18) and (19) gives

<sup>4</sup> The conditions in (22) may not be necessary in scenarios where gravitational interactions are softened at energies much below  $\bar{M}_{\text{pl}}$  and remain small at and above  $\bar{M}_{\text{pl}}$  [24]. However, here we do not want to rely on any specific quantum gravity theory.



**Fig. 1.** Initial conditions  $\bar{\chi}$  and  $\bar{\Pi}$  for the Higgs field and its momentum  $\Pi \equiv \dot{\chi}$  respectively. The thickness of the lines corresponds to  $2\sigma$  uncertainty in the value of the power spectrum, Eq. (9).

$$\dot{H} + 3H^2 + \frac{2k}{a^2} = 0, \quad (\dot{\chi}^2 \gg U), \quad (25)$$

which for spatially flat curvature,  $k = 0$ , leads to

$$H(t) = \frac{\bar{H}}{1 + 3\bar{H}(t - \bar{t})}, \quad (\dot{\chi}^2 \gg U, k = 0), \quad (26)$$

where  $\bar{H} \equiv H(\bar{t})$ . By inserting this result into Eq. (19), we find

$$\dot{\chi}^2 = \frac{6\bar{M}_{\text{Pl}}^2 \bar{H}^2}{[1 + 3\bar{H}(t - \bar{t})]^2}, \quad (\dot{\chi}^2 \gg U, k = 0), \quad (27)$$

that is the kinetic energy density scales as  $1/t^2$  by taking into account the time dependence of  $H$ . This result [15] tells us that an initial condition with large kinetic energy is attracted towards one with smaller kinetic energy, but it also shows that dropping the potential energy cannot be a good approximation for arbitrarily large times. Moreover, notice that Eqs. (26) and (27) imply

$$\ddot{\chi} = -3H\dot{\chi} \quad (28)$$

so the dynamics is *not* approaching the second condition in (24). Therefore, the argument above is not conclusive and we need to solve the equations with  $U$  included in order to see if the slow-roll regime is an attractor.

### 3.2. Numerical studies

We studied numerically the system in (20) assuming  $k = 0$ ; this case is realistic and is the simplest one: it does not require an initial condition for  $a$ . We found that even for an initial kinetic energy density  $\bar{\Pi}^2$  of order  $10^{-3}\bar{M}_{\text{Pl}}^4$  (which we regard as the maximal order of magnitude to have negligibly small quantum gravity), one should start from an initial field value  $\bar{\chi}$  of order  $10\bar{M}_{\text{Pl}}$  to inflate the universe for an appropriate number of e-folds, i.e.  $N = 59$ . This value of  $\bar{\chi}$  is only one order of magnitude bigger than the one needed in the ordinary case,  $\bar{\Pi}^2 \ll U(\bar{\chi}) \sim 10^{-10}\bar{M}_{\text{Pl}}^4$ , where the initial kinetic energy is much smaller than the potential energy.

Fig. 1 presents these results more quantitatively. There the initial conditions for  $\bar{\Pi}$  have been chosen to be negative because positive values favor slow-roll even with respect to the case where the initial kinetic energy is much smaller than the potential energy: this is because the potential in Eq. (7) is an increasing function of  $\chi$  for  $\chi \gg v$ .

We conclude that at the classical level Higgs inflation does not suffer from a worrisome fine-tuning problem for the initial conditions.

## 4. Quantum corrections

The theory in Eq. (1) is not renormalizable. This means that quantum corrections  $\Delta\Gamma$  at a given order in perturbation theory can generate terms that are not combinations of those in the classical action  $S$ . In formulae the (quantum) effective action is given by:

$$\Gamma = S + \Delta\Gamma \quad (29)$$

where  $S + \Delta\Gamma$  cannot generically be reproduced by substituting the parameters in  $S$  with some renormalized quantities.

A UV completion requires the existence of additional degrees of freedom that render the theory renormalizable or even finite. Much below the scale of this new physics, the effective action can be approximated by an expansion of the form

$$\Delta\Gamma = \int d^4x \sqrt{-g} (\delta\mathcal{L}_2 + \delta\mathcal{L}_4 + \dots) \quad (30)$$

where  $\delta\mathcal{L}_n$  represents a combination of dimension  $n$  operators, having given dimension 0 to the metric  $g_{\mu\nu}$  and neglecting  $v$  as it is very small compared to inflationary energies.

We consider the one-loop corrections generated by all fields of the theory, both the matter fields and gravity. Our purpose is to apply it to inflationary and pre-inflationary dynamics. We approximate  $\Delta\Gamma$  by including all operators up to  $n = 4$ :

$$\Delta\Gamma \approx \int d^4x \sqrt{-g} (\delta\mathcal{L}_2 + \delta\mathcal{L}_4). \quad (31)$$

This is the simplest approximation that allows us to include the dynamics of the Higgs field and possess scale invariance at high energies and high Higgs field values (up to running effects). We have

$$\delta\mathcal{L}_2 = -\frac{\delta\bar{M}_{\text{Pl}}^2}{2} R \quad (32)$$

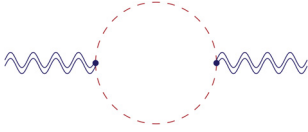
$$\delta\mathcal{L}_4 = \alpha R^2 + \beta \left( \frac{1}{3} R^2 - R_{\mu\nu} R^{\mu\nu} \right) + \delta Z_{\mathcal{H}} |\partial\mathcal{H}|^2 - \delta\lambda |\mathcal{H}|^4 - \delta\xi |\mathcal{H}|^2 R + \dots \quad (33)$$

where for each parameter  $p_c$  in the classical action we have introduced a corresponding quantum correction  $\delta p$  and the dots represent the additional terms due to the fermions and gauge fields of the SM. Notice that we have added general<sup>5</sup> quantum corrections that are quadratic in the curvature tensors as they are also possible dimension 4 operators. These are parameterized by two dimensionless couplings  $\alpha$  and  $\beta$ .

Our purpose is now to determine the RGEs for the renormalized couplings

$$p = p_c + \delta p$$

as well as for the new couplings  $\alpha$  and  $\beta$  generated by quantum corrections. Indeed the RGEs encode the leading quantum corrections. We will use the dimensional regularization (DR) scheme to regularize the loop integrals and the modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme to renormalize away the divergences. This as usual leads to a renormalization scale that we denote with  $\bar{\mu}$ .



**Fig. 2.** The leading loop diagram that generates the  $R^2$  term in the effective action. The dashed lines correspond to the Higgs field, while the external double lines represent gravitons.

#### 4.1. One-loop RGE of the Planck mass

In the absence of the dimensionful parameters besides  $\bar{M}_{\text{Pl}}$ , the only possible contributions to the RGE of  $\bar{M}_{\text{Pl}}$  are the rainbow and the seagull diagram contributions to the graviton propagator due to gravity itself: the rainbow topology is the one of Fig. 2, while the seagull one is obtained by making the two vertices of Fig. 2 coincide without deforming the loop.

The seagull diagram vanishes as it is given by combinations of loop integrals of the form

$$\int d^d k \frac{k_\mu k_\nu}{k^2 + i\epsilon}, \quad \int d^d k \frac{1}{k^2 + i\epsilon}, \quad (34)$$

where  $d$  is the space-time dimension in DR. These types of loop integrals vanish in DR. The rainbow diagram does not contribute to the RGE of  $\bar{M}_{\text{Pl}}$  either. The reason is that each graviton propagator carries a factor of  $1/\bar{M}_{\text{Pl}}^2$  and each graviton vertex carries a factor of  $\bar{M}_{\text{Pl}}^2$  (because the graviton kinetic term  $-\bar{M}_{\text{Pl}}^2 R/2$  is proportional to  $\bar{M}_{\text{Pl}}^2$ ): the rainbow diagram has two graviton propagators and two vertices, therefore this contribution is dimensionless and cannot contribute to the RGE of a dimensionful quantity. We conclude that  $\bar{M}_{\text{Pl}}$  does not run in this case. This argument assumes that the graviton wave function renormalization is trivial, which we have checked to be the case at the one-loop level at hand.

#### 4.2. One-loop RGEs of SM parameters

At small classical backgrounds  $\phi$ , having neglected  $v$  all SM parameters are dimensionless and thus cannot receive contributions from loops involving graviton propagators (that carry a factor of  $1/\bar{M}_{\text{Pl}}^2$ ). Therefore, the SM RGEs apply and can be found (up to the three-loop level) in a convenient form in the appendix of Ref. [23]. The effect of the classical background is to suppress loops involving the physical Higgs fluctuation through a factor  $s$  which depends on  $\xi\phi^2/\bar{M}_{\text{Pl}}^2$  such that  $s \approx 1$  for small  $\xi\phi^2/\bar{M}_{\text{Pl}}^2$  and  $s \approx 1/(1+6\xi)$  for large  $\xi\phi^2/\bar{M}_{\text{Pl}}^2$  [30]. The RGEs with such  $s$ -insertions can be found in [19] and have to be solved by choosing  $\phi \sim \bar{\mu}$ .

#### 4.3. One-loop RGEs of gravitational couplings

Finally, we consider the RGEs for  $\xi$ ,  $\alpha$  and  $\beta$ . The RGE of  $\xi$  receives contribution from the SM couplings and  $\xi$  itself [26,27]:

$$(4\pi)^2 \frac{d\xi}{d \ln \bar{\mu}} = (1+6\xi) \left[ y_t^2 - \frac{3}{4} g_2^2 - \frac{3}{20} g_1^2 + (1+s)\lambda \right], \quad (35)$$

where  $y_t$  is the top Yukawa coupling and  $g_3$ ,  $g_2$  and  $g_Y = \sqrt{3/5}g_1$  are the gauge couplings of  $\text{SU}(3)_C$ ,  $\text{SU}(2)_L$  and  $\text{U}(1)_Y$  respectively.

The RGEs of  $\alpha$  and  $\beta$  receive two contributions: one from pure gravity loops (a rainbow and a seagull diagram), which we denote with  $\beta_\alpha^g$ , and one from matter loops,  $\beta_\alpha^m$ :

$$(4\pi)^2 \frac{d\alpha}{d \ln \bar{\mu}} = \beta_\alpha^g + \beta_\alpha^m, \quad (36)$$

$$(4\pi)^2 \frac{d\beta}{d \ln \bar{\mu}} = \beta_\beta^g + \beta_\beta^m. \quad (37)$$

One finds [28]  $\beta_\alpha^g = -1/4$  and  $\beta_\beta^g = 7/10$  (which can be removed by a suitable gauge fixing [29]).  $\beta_\alpha^m$  and  $\beta_\beta^m$  are instead gauge invariant and their expressions in the SM with the appropriate  $s$ -insertions are

$$\beta_\alpha^m = -\frac{(3+s)(1+6\xi)^2}{72}, \quad \beta_\beta^m = \frac{47}{10} + \frac{s}{60}. \quad (38)$$

#### 4.4. Higgs-to-Starobinsky inflation

Let us start this section by commenting on fine-tunings in the couplings, a relevant issue as inflation is motivated by cosmological fine-tuning problems. The first equation in (38) has an important implication; the Feynman diagram that leads to this contribution is given in Fig. 2. Generically Higgs inflation requires a rather large value of  $\xi$ , which implies a strong naturalness bound

$$|\alpha| \gtrsim \frac{\xi^2}{8\pi^2}. \quad (39)$$

A large value of  $\xi$  is necessary at the classical level (see Eq. (12) and the corresponding discussion). At quantum level one can obtain smaller values, but still  $\xi \gg 1$  [30,31].

A possible exception is Higgs inflation at the critical point [19,20]; however,  $\xi \gtrsim 10$  to fulfill the most recent observational bounds,  $r \lesssim 0.1$  [32]. Moreover, in previous analysis of Higgs inflation at the critical point the wave function renormalization of the Higgs field has been neglected, an approximation that is under control when  $\xi$  is large [30].

Since  $\xi \gg 1$  generically, (39) indicates that an additional  $R^2$  term with such a large coefficient may participate in inflation. Therefore, we write the following effective action:

$$\Gamma = \int d^4 x \sqrt{-g} \left[ \mathcal{L}_{\text{SM}}^{\text{eff}} - \left( \frac{\bar{M}_{\text{Pl}}^2}{2} + \xi |\mathcal{H}|^2 \right) R + \alpha R^2 \right], \quad (40)$$

where the  $\mathcal{L}_{\text{SM}}^{\text{eff}}$  part corresponds to the effective SM action, which is obtained, at the leading quantum level we are using, by substituting each parameter in the classical SM action with its renormalized quantity.  $\mathcal{L}_{\text{SM}}^{\text{eff}}$  is gauge independent as shown in Ref. [23] and our results will be therefore free of any gauge dependence. The scalar-tensor effective action for the metric and the physical Higgs mode  $\phi$  is

$$\Gamma_{\text{st}} = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} (\partial\phi)^2 - V_{\text{eff}} - \frac{1}{2} (\bar{M}_{\text{Pl}}^2 + \xi\phi^2) R + \alpha R^2 \right],$$

where  $V_{\text{eff}}$  is the SM effective potential. Here we have neglected the wave function renormalization of the Higgs because  $\xi$  is large.

As well-known, the  $R^2$  term corresponds to an additional scalar. In order to see this one can add to the action the term

$$- \int d^4 x \sqrt{-g} \alpha \left( R + \frac{\omega}{4\alpha} \right)^2,$$

where  $\omega$  is an auxiliary field: indeed by using the  $\omega$  field equation one obtains immediately that this term vanishes. On the other hand, after adding that term

$$\Gamma_{\text{st}} = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} (\partial\phi)^2 - V - \frac{f}{2} R - \frac{\omega^2}{16\alpha} \right], \quad (41)$$

where  $f \equiv \bar{M}_{\text{Pl}}^2 + \omega + \xi\phi^2$ .

<sup>5</sup>  $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$  is a linear combination of  $R^2$ ,  $R_{\mu\nu} R^{\mu\nu}$  and a total derivative.

Note that we have the non-canonical gravitational term  $-fR/2$ . Like we did in Section 2, we can go to the Einstein frame (where we have instead the canonical Einstein term  $-\bar{M}_{\text{Pl}}^2 R_E/2$ ) by performing a conformal transformation,<sup>6</sup>

$$g_{\mu\nu} \rightarrow \frac{\bar{M}_{\text{Pl}}^2}{f} g_{\mu\nu}. \quad (42)$$

One obtains [33]

$$\Gamma_{\text{st}} = \int d^4x \sqrt{-g} \left[ \mathcal{L}_{\phi z} - U_{\text{eff}} - \frac{\bar{M}_{\text{Pl}}^2}{2} R \right], \quad (43)$$

where

$$\mathcal{L}_{\phi z} \equiv \frac{6\bar{M}_{\text{Pl}}^2}{z^2} \frac{(\partial\phi)^2 + (\partial z)^2}{2},$$

$$U_{\text{eff}}(\phi, z) \equiv \frac{36\bar{M}_{\text{Pl}}^4}{z^4} \left[ V_{\text{eff}}(\phi) + \frac{1}{16\alpha} \left( \frac{z^2}{6} - \bar{M}_{\text{Pl}}^2 - \xi\phi^2 \right)^2 \right]$$

and we have introduced the new scalar  $z = \sqrt{6f}$ .

Notice that when  $\alpha \rightarrow 0$ , the potential  $U_{\text{eff}}$  forces  $z^2 = 6(\bar{M}_{\text{Pl}}^2 + \xi\phi^2)$  and we recover the Higgs inflation action. For large  $\alpha$  (as dictated by a large  $\xi$ ), this conclusion cannot be reached. The absence of runaway directions in  $U_{\text{eff}}$  requires  $\alpha > 0$  and  $\lambda > 0$ , which is possible within the pure SM (without gravity [35]), although in tension<sup>7</sup> with the measured values of some electroweak observables [23,31]. Ref. [33] studied a system that includes (43) as a particular case.<sup>8</sup> It was found that inflation is never dominated by the Higgs, because its quartic self-coupling  $\lambda$  (which we assume to be positive for the argument above) is unavoidably larger than the other scalar couplings, taking into account its RG flow. Even assuming that the Higgs field has a dominant initial value, in our two-field context inflation starts only after the field evolution has reached an attractor where  $\phi$  is subdominant. We have checked that this happens also when  $\xi$  is large.

Therefore, the predictions of the theory defined by Eq. (40) are closer to those of Starobinsky inflation, which are distinct from the Higgs inflation ones [37].

## 5. Conclusions

In conclusion, we have studied two different aspects of standard Higgs inflation – to seek how fine-tuned the initial conditions should be to fall into a slow-roll attractor solution in an approximate exponentially flat Higgs potential in the Einstein frame. We started with a large kinetic energy, and we found that for an initial kinetic energy density of order  $10^{-3}\bar{M}_{\text{Pl}}^4$  (this is the maximum allowed order of magnitude to avoid quantum gravity corrections) the inflaton VEV should be  $\sim 10\bar{M}_{\text{Pl}}$  to sustain inflation long enough to give rise to enough e-folds.

In the second half of the paper, we focused on the question of viability of Higgs inflation in presence of large  $\xi$ , typically required for explaining the observed CMB power spectrum and the right tilt. We found that one would incur quantum corrections (at the lowest order) to the Ricci scalar, i.e. quadratic in Ricci scalar,  $\alpha R^2$ , with a universality bound on  $\alpha$  given by Eq. (39), unless the initial value

of  $\alpha$  is fine-tuned. If one includes this  $R^2$  term in the effective action, both the Higgs and a new scalar degree of freedom are present. By taking  $\xi \sim 10^2 - 10^4$  and using the bound in Eq. (39), the potential would be effectively determined by the Starobinsky scalar component  $z$ , and the CMB predictions would be different from that of Higgs inflation.

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<sup>6</sup> Notice that one goes to the Einstein frame with the same  $f$  even in the presence of all components of the Higgs doublet.

<sup>7</sup> Such tension, however, can be eliminated by adding to the SM well-motivated new physics, which solve its observational problems [34].

<sup>8</sup> Ref. [33] has an additional scalar which, however, can be consistently decoupled by taking its mass large enough. For another treatment of the dynamical system in (43) see Ref. [36].

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