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Abstract

This paper develops a model of international trade based on the division of labour under perfect competition. International trade, by eliminating the duplication of coordination costs, leads to a greater variety of intermediate goods, each produced at a larger scale than in autarky. The greater variety of intermediate inputs implies greater division of labour and hence gains from trade. Similarly to models of international trade under imperfect competition, the volume of trade depends on the relative sizes of the trading partners. Extending the model to two factors of production yields the additional result that if the two countries are sufficiently similar in their relative endowments, then both factors of production can experience gains from trade.

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1 Introduction

"I always say there are two and a half theories of trade".

Paul Krugman to Peter Neary, as quoted in Neary (2010).

There are three main approaches to theoretical modelling of international trade: the approach based on comparative advantage and perfect competition from Ricardo to Heckscher and Ohlin, the approach based on monopolistic competition as in Krugman (1979, 1980), and the approach based on oligopoly as initially developed by Brander (1981)[1]. This paper develops a new model of international trade which takes a different approach to the preceding literature, by focussing on the division of labour as the reason for international trade. The role of the division of labour in raising per capita incomes has been recognised since at least Adam Smith (1776)[2]. The model we develop is based on trade in intermediate inputs, which constitutes over half of total goods trade, as documented by Miroudot et al (2009) and Sturgeon and Memedovic (2010), and shares features of both the comparative advantage and monopolistic competition approaches. From the comparative advantage literature, it uses a perfectly competitive market structure; from the monopolistic competition literature, countries are ex ante identical to each other, so there is no comparative advantage reason for international trade.

In our model, the division of labour is limited by both the extent of the market, and by coordination costs. International trade eliminates the duplication of coordination costs across countries, which encourages greater division of labour, and hence higher levels of output and welfare. Thus, similarly to models of trade based on monopolistic competition, we endogenise the number of varieties of intermediate goods produced; however, this is done under perfectly competitive markets. Because countries are assumed to have identical technologies in producing intermediate goods and there is no way of identifying individual intermediate goods, the direction of trade is indeterminate; however, the volume of trade is determinate, and depends on the relative sizes of the trading partners.

Having established the main features of the model with one final good and one factor of production, we then proceed to extend the model to two final goods and two factors of production, similarly to Krugman's (1981) extension of his earlier

[1] The oligopolistic approach is what Neary (2010) refers to as the half theory of trade, since it is not as widely used as the other two approaches, despite the efforts of Neary (2009).

[2] In fact, the literature on international trade under monopolistic competition may be viewed as one approach to analysing the implications of the international division of labour. The model presented in this paper presents a different approach to this issue.

(Krugman 1980) model. This enables us to consider the distributional implications of the model. With more than one factor of production, there are now two sources of the gains from trade: the division of labour as in the one factor model, and relative endowment differences as in the Heckscher-Ohlin model. As a result, it is possible, if the two countries are not too dissimilar from each other in their relative endowments, that both factors of production gain from trade. This is similar to the result in Krugman (1981) and in contrast to the traditional Heckscher-Ohlin result, where the scarce factor always loses from trade.

The role of the division of labour in international trade has been developed especially by Ethier (1979, 1982a). In the earlier paper, the distinction is not made between external and internal scale economies, while the later paper is explicit in its use of both internal and external scale economies. Francois (1990a, 1990b) makes use of the production function developed by Edwards and Starr (1987) to develop a model of international trade in which scale economies arise from producer services in a monopolistic competition model. More recently, Chaney and Ossa (2013) open up the black box of the production function in the Krugman (1979) model of monopolistic competition, modelling the production process as a series of stages produced by teams. Becker and Murphy (1992) develop a closed economy model in which the extent of the division of labour is limited by the cost of coordinating inputs. This is similar to that used by Francois (1990a, 1990b), and is the approach adopted in the present paper.

Because the present paper makes use of a perfectly competitive framework, it is different from the literature above (apart from Becker and Murphy (1992), who do not consider international trade). Nevertheless, we obtain some results which are qualitatively similar to those of models based on monopolistic competition such as Krugman (1979). Swanson (1999) develops a different model of the division of labour under perfect competition, in which a larger market enables greater specialisation and hence higher skill levels and output per worker via the endogenous development of comparative advantage. More closely related is Soo (2013), who develops a model of international trade based on the division of labour and comparative advantage in a perfectly competitive framework. Unlike Soo (2013), who makes use of comparative advantage to pin down the structure of production, in the present paper we focus on the cost of coordinating inputs that limits the extent of the division of labour.

In order to close the model, we assume that the production of intermediate goods takes place under what Ethier (1979) refers to as national scale economies which are external to the firm. This is the same assumption as in most related work in this area, for instance Markusen and Melvin (1981) and Panagariya (1981). Helpman (1984) provides an insightful survey of this literature, while Grossman and Rossi-Hansberg (2010) offer a recent treatment. Throughout the paper we focus on efficient allocations, which are those that enable the replication of the integrated equilibrium (see Krugman, 1987). This enables us to sidestep the fact that models with external scale economies exhibit multiple, inefficient and possibly unstable equilibria.

The next section outlines the main building blocks of the model. Section 3 discusses the autarkic equilibrium, while Section 4 discusses the implications and patterns of international trade. Section 5 discusses the implications of alternative assumptions in the production of intermediate goods, while Section 6 provides some concluding comments.

2 The model

The model is set up with two countries, $i \in H, F$ for Home and Foreign, although the solution method allows for easy extension to many countries. There is a single final good which is used in consumption. Let the representative consumer's utility function be:

$$U_i = C_i^{\theta}, \qquad 0 < \theta < 1. \tag{1}$$

All markets are perfectly competitive. There are many possible intermediate goods, $j = 1, \ldots, N$. The final good is assembled from intermediate goods using the following production function:

$$Q_i = (n_H + n_F)^{\beta} \min\{x_{ij}\}, \qquad 1 < \beta < 2. \tag{2}$$

Where n_i is the number of intermediate goods actually produced in each country, and x_{ij} is the quantity of each intermediate good j used in country i. The production function is such that the intermediate inputs are perfect complements, so that in equilibrium x_i is always the same across intermediate inputs. This is quite a strong assumption, but simplifies the analysis considerably. That $\beta > 1$ indicates the gains from the division of labour; the more the production process is divided into different stages, the larger the output of the final good[3]. Thus, firms will, in the absence of coordination costs, want to divide the production process into as many steps as possible; it is the coordination cost that constrains the division of labour.

[3] The model is isomorphic to one in which consumers consume the intermediate goods directly. However, while it may be reasonable to assume division of labour in the production process, it is more difficult to justify on the consumption side.

In assembling the final good from the intermediate goods, there is a coordination cost that depends on the number of intermediate goods used in the production process:

$$c_i = \psi n_i^{\rho}, \qquad \rho > \beta > 1. \tag{3}$$

The assembly process uses real resources in the sense that final output is reduced by the assembly cost (analogously to the "iceberg" trade costs in other papers). This cost is assumed to be shared by all firms producing the final good, and may be thought of as the cost of maintaining a production network; the more intermediate inputs there are, the more difficult and expensive it becomes to coordinate all the inputs. As we will see below, the restriction that $\rho > \beta$ implies that the coordination cost rises more quickly in n than the gain from the division of labour, and ensures that a larger country not only has a larger number of intermediate goods, but also that each intermediate good is produced at a larger scale.

Labour is the only factor of production, and each country has an endowment of L_i units of labour. Intermediate goods are produced using labour with a production function that exhibits external scale economies which are national in nature (Ethier, 1979). That is, output of an intermediate good j in country i depends on employment in that intermediate input in country i:

$$q_{ij} = \left(\alpha l_{ij}\right)^{\gamma}, \qquad \alpha < 1, \quad 1 < \gamma < \beta. \tag{4}$$

Where $\alpha < 1$ is labour productivity, and $\gamma > 1$ indicates external scale economies; output increases more than proportionally to labour inputs. There are two reasons for assuming external scale economies. The first, technical reason, is that it enables us to pin down the number of intermediate goods actually produced; if constant returns to scale were assumed, each final good firm could in principle demand a different set of intermediate goods.

A second reason for assuming external scale economies which are national in nature is that with international trade, the efficient, integrated equilibrium implies that production of each intermediate good will occur in only one country. As a result, international trade leads to a saving in the coordination cost of assembling the final good from the intermediate goods; these savings would not materialise if each intermediate good is produced in more than one country. This is discussed in Section 5 below. External scale economies which are national (as opposed to international) in nature may be justified by appeal to Marshall's localised external economies (see Krugman, 1991). Such localised economies may lead to the formation of industrial clusters (Porter, 1990). In the context of the present model, it is helpful to think of each intermediate input as being produced by many firms in the same location, because of the external scale economies. Different intermediate inputs may be produced in the same or in different locations.

Under perfect competition, normalising the wage rate to unity, and assuming average cost pricing (see Ethier, 1979), the zero profit condition implies that the price of each intermediate good is given by:

$$p_{ij} = \left(\alpha^{\gamma} l_{ij}^{\gamma-1}\right)^{-1}. \tag{5}$$

Since $\gamma > 1$, the larger the employment in sector j in country i, the lower the price; therefore it is more efficient for each intermediate input to be produced in only one country, as this maximises the scale of employment in that input in that country. Also, the higher is labour productivity α, the lower the price. Appendix A shows how equations (4) and (5) can be obtained from the production function for each perfectly competitive firm and the firm's profit-maximising condition, respectively.

3 Autarkic equilibrium

In autarky, all domestically produced intermediate goods are used in producing the domestic final good, and since all intermediate goods are produced and consumed in equal quantities, we have:

$$x_{ij} = q_{ij} = q_i = \left(\alpha l_{ij}\right)^{\gamma} = \left(\frac{\alpha L_i}{n_i}\right)^{\gamma}. \tag{6}$$

Substituting this into the production function (2) and subtracting the assembly cost (3) gives the production function for final goods net of assembly cost:

$$\widetilde{Q_i} = n_i^{\beta-\gamma}(\alpha L_i)^{\gamma} - \psi n_i^{\rho}. \tag{7}$$

Each firm in the final good sector chooses the number of intermediate inputs to maximise profits. All firms are identical to each other, so total industry profits are:

$$\pi_i = P_i\widetilde{Q_i} - p_i n_i x_i, \tag{8}$$

where P_i is the price of the final good, and is taken as given by the perfectly competitive firms. Substituting from equations (5), (6) and (7), we can rewrite the profit function (8) as:

$$\pi_i = P_i\left[n_i^{\beta-\gamma}(\alpha L_i)^{\gamma} - \psi n_i^{\rho}\right] - L_i. \tag{9}$$

Differentiating equation (9) with respect to n_i allows us to solve for the number of intermediate goods produced in each economy (ignoring integer constraints)[4]:

[4] It can be verified that $d^2\pi_i/dn_i^2 < 0$, so that equation (10) is indeed the profit-maximising expression for n_i.

$$n_i = \left[\frac{(\alpha L_i)^{\gamma}(\beta-\gamma)}{\psi\rho}\right]^{\frac{1}{\rho+\gamma-\beta}}. \tag{10}$$

Equation (10) shows that the assumption made above in equation (3) that $\gamma < \beta$ is required to generate positive values of n_i. In principle, each final good producing firm could demand different intermediate inputs. However, because production of intermediate inputs occurs under external scale economies, the total number of intermediate goods produced will be the minimum number that will satisfy equation (10). That is, all final good producers will use the same intermediate goods. This is one of the roles played by the assumption of external scale economies.

Since from equation (6) $q_i = (\alpha L_i/n_i)^{\gamma}$, we also have:

$$q_i = (\alpha L_i)^{\frac{\gamma(\rho-\beta)}{\rho+\gamma-\beta}}\left(\frac{\psi\rho}{\beta-\gamma}\right)^{\frac{\gamma}{\rho+\gamma-\beta}}. \tag{11}$$

Equation (11) shows that the assumption that $\rho > \beta$ implies that $dq_i/dL_i > 0$. Similarly, from equation (10), as long as $\rho + \gamma > \beta$ (which always holds since we assume that $\rho > \beta$), we have $dn_i/dL_i > 0$. That is, a larger country produces a larger number of distinct intermediate goods, and produces each of these intermediate goods at a larger scale. Following the terminology of the literature, a larger country expands both in terms of the intensive margin (more output of each intermediate is produced) and in terms of the extensive margin (more types of intermediates are produced). This gives similar results to Krugman (1979), and contrasts with the monopolistic competition literature based on the CES utility function (e.g. Krugman 1980), in which a larger country has a larger variety of goods, but not larger sectors. The extent of the division of labour depends on the size of the market as in Smith (1776), but also on the coordination cost as in Becker and Murphy (1992).

We can also obtain the price of the final good. Setting the profit function (9) equal to zero and solving gives:

$$P_i = \frac{L_i}{n_i^{\beta-\gamma}(\alpha L_i)^{\gamma} - \psi n_i^{\rho}}. \tag{12}$$

From equation (10) above, a country with a larger labour force will produce a larger number of distinct intermediate goods. This reduces the cost of production of the final good because of the division of labour, and hence reduces the price of the final good relative to intermediate goods in equilibrium.

Substituting from the number of intermediate goods (10) into the net production function for final goods (7) and then into the consumer's utility function (1), making use of $C_i = Q_i/L_i$ gives autarkic utility as a function of the model's parameters:

$$U_i^A = \left\{\frac{1}{L_i}\left[n_i^{\beta-\gamma}(\alpha L_i)^\gamma - \psi n_i^\rho\right]\right\}^\theta \tag{13}$$

Larger countries have a higher level of utility under autarky, since a larger economy enables greater division of labour; $dU_i^A/dL_i > 0$ (note that this is the case since n_i is also a function of L_i). It can also be shown that an innovation which reduces the cost of coordination (for instance, information technology) would raise utility, by encouraging greater division of labour; $dU_i^A/d\rho < 0$.

Note that the market equilibrium as described above is efficient, since it yields the same outcome as would be obtained by a benevolent central planner, whose objective is to maximise the country's utility by choosing the optimal number of intermediate inputs to maximise net output. The reason for this is that the assumptions we have made above mean that firms internalise the effects of increasing numbers of intermediate inputs on their profits, as shown in equation (9). More intermediate inputs imply greater division of labour, but also higher coordination costs, and final goods firms take both effects into account when choosing the number of intermediate inputs.

4 International trade

In this section we allow for international trade in both intermediate and final goods between the two countries. We start by considering free international trade between the two countries, then extend the model to include trade frictions, and finally consider the pattern of trade.

4.1 Free trade

Similarly to Krugman (1979, 1980), international trade is equivalent to an increase in the size of the economy, since countries have identical technologies and there is only one factor of production. The crucial assumption here is that of national scale economies in the production of the intermediate goods. This means that, when international trade is allowed, the efficient allocation of resources (the integrated equilibrium) implies that each intermediate input is produced in only one country. As a result, since the two countries effectively become one market, the coordination cost is shared between the two countries. Following the same steps as for the autarkic equilibrium, the number of intermediate goods that is consistent with profit maximisation by all final goods firms is:

$$n^T = \left\{\frac{[\alpha(L_H+L_F)]^{\gamma}(\beta-\gamma)}{\psi\rho}\right\}^{\frac{1}{\rho+\gamma-\beta}}. \tag{14}$$

And the output of each intermediate good is:

$$q^T = [\alpha(L_H + L_F)]^{\frac{\gamma(\rho-\beta)}{\rho+\gamma-\beta}}\left(\frac{\psi\rho}{\beta-\gamma}\right)^{\frac{\gamma}{\rho+\gamma-\beta}}. \tag{15}$$

These expressions also indicate how the model can be extended to allow for many countries, and the implications of such an extension. We can establish that:

$$n_H^A, n_F^A < n^T < n_H^A + n_F^A \tag{16}$$

and

$$q_H^A, q_F^A < q^T < q_H^A + q_F^A. \tag{17}$$

That is, the number of intermediate goods and the output of each intermediate good both increase compared to the autarkic number and output of each intermediate good. However, the increase is less than proportional to the expansion in market size resulting from trade liberalisation. International trade leads to an expansion on both the intensive and extensive margins.

The representative consumer's utility with free international trade is given by:

$$U_i^T = \left\{\left(\frac{1}{L_H+L_F}\right)\left[(n^T)^{\beta-\gamma}\big(\alpha(L_H + L_F)\big)^{\gamma} - (\psi(n^T)^{\rho})\right]\right\}^{\theta} \tag{18}$$

It can be shown that $U_i^T > U_i^A$; that is, there are always gains from free international trade. These gains arise from the fact that international trade enables countries to avoid duplicating the coordination cost. Whereas in autarky the coordination cost is shared only by domestic firms, in international trade it is shared by both domestic and foreign firms. This cost saving enables firms to increase the division of labour, thus yielding a productivity gain in the output of the final consumption good. Note also that since the free trade welfare is the same for all consumers in both countries whereas autarkic utility is higher in the larger country, we get the usual result that a smaller country gains more from trade than does a larger country.

4.2 Trade frictions

Now suppose that there are trade frictions that increase the cost of coordination in the presence of international trade, so that the coordination cost becomes:

$$c^{TF} = \tau\psi(n^{TF})^{\rho} \tag{19}$$

where the superscript TF denotes the outcome with trade frictions, and $\tau \geq 1$ is the additional coordination cost due to the frictions that arise from international trade (for instance, different languages or legal systems). Unlike trade costs, which affect only imported goods but not domestically produced goods, we assume that the trade friction affects both imported and domestically-produced intermediates, so has no impact on relative prices or demands. The additional trade friction incurred because of international trade must be less than the gain from spreading the coordination cost across countries; otherwise international trade would result in a welfare loss. Following the same steps as above, the equilibrium number of intermediate goods is:

$$n^{TF} = \left\{\frac{[\alpha(L_H+L_F)]^{\gamma}(\beta-\gamma)}{\tau\psi\rho}\right\}^{\frac{1}{\rho+\gamma-\beta}} \tag{20}$$

and the output of each intermediate good is:

$$q^{TF} = [\alpha(L_H+L_F)]^{\frac{\gamma(\rho-\beta)}{\rho+\gamma-\beta}}\left(\frac{\tau\psi\rho}{\beta-\gamma}\right)^{\frac{\gamma}{\rho+\gamma-\beta}}. \tag{21}$$

The representative consumer's utility with trade frictions is given by:

$$U_i^{TF} = \left\{\left(\frac{1}{L_H+L_F}\right)\left[(n^{TF})^{\beta-\gamma}\left(\alpha(L_H+L_F)\right)^{\gamma} - (\tau\psi(n^{TF})^{\rho})\right]\right\}^{\theta}. \tag{22}$$

Comparing the trade-friction outcome with the free trade outcome, the free trade outcome has a larger number of intermediate goods, and each intermediate good is produced on a smaller scale. Intuitively, the trade friction increases the cost of coordinating inputs, so reduces the incentive for firms to divide the production process into more intermediate components. As a result, utility in the presence of trade frictions is lower than utility in free trade.

Comparing the trade-friction outcome with the autarkic outcome, the number of intermediate goods is larger with trade frictions than in autarky if:

$$\tau < \left(\frac{L_H+L_F}{L_i}\right)^{\gamma}. \tag{23}$$

The output of each intermediate good is greater with trade frictions than in autarky if:

$$\tau > \left(\frac{L_i}{L_H+L_F}\right)^{\rho-\beta}. \tag{24}$$

and utility is greater than in autarky (there are gains from trade) if:

$$\tau < \left(\frac{L_H+L_F}{L_i}\right)^{\frac{\rho+\gamma-\beta-\rho\gamma}{\gamma-\beta}}. \tag{25}$$

Condition (24) is always satisfied provided $\tau \geq 1$ as we have assumed, while conditions (23) and (25) are satisfied provided τ is not too large. If conditions (23) and (24) hold, then we can also establish that, similarly to the case for free trade, that the number of intermediate goods and the output of each intermediate good both increase less than proportionally to the expansion in market size resulting from international trade.

Note also the role of the trade friction term τ in the analysis above. It can be shown that $dn^{TF}/d\tau < 0$, $dq^{TF}/d\tau > 0$, and $dU^{TF}/d\tau < 0$. That is, the higher the trade friction, the smaller the number of intermediate goods, the larger the output of each intermediate good, and the lower the utility from international trade. Equivalently, trade liberalisation which reduces τ would increase the number of intermediate goods, reduce the output of each intermediate good, and increase consumer welfare. The increase in welfare may be attributed to greater division of labour resulting from the increased number of intermediate goods, which in turn is caused by the falling coordination cost.

4.3 Trade patterns

The pattern of trade may be described as follows. There is no trade in the final good, since each country can assemble the final good using the same technology. All trade will be in intermediate goods. Since production of each intermediate good exhibits national external scale economies and technologies are identical across countries, in the integrated equilibrium, each intermediate good will be produced in only one country. The number of intermediate goods produced by each country will be proportional to its share of world labour supply: $L_i/(L_H + L_F)$. And since prices are the same across countries and preferences are homothetic, each country's demand for each intermediate good is proportional to its national income. Hence the volume of trade is equal to (since we normalise $w = 1$):

$$VT = \frac{L_H L_F}{L_H + L_F}. \tag{25}$$

This expression is identical to the expression for the volume of trade in Krugman (1979, 1980), and for the same reason: there are gains from increased variety (intermediates in the present paper, final goods in Krugman 1979, 1980). The volume of trade is maximised for a given total size of the world economy when the two countries have the same size. The larger is a country's trading partner, the more varieties of intermediate goods it will import from this trading partner. However, and again similarly to Krugman (1979, 1980), the direction of trade is indeterminate, since we do not know which country produces which intermediate input.

5 Alternative assumptions for the production of intermediate goods

In developing the model, we have made use of the assumption that production of intermediate goods takes place under conditions of national scale economies which are external to the firm. Because of the external scale economies, the fewest possible varieties of intermediate goods are produced which is consistent with the equilibrium. Because the scale economies are national in nature, each intermediate good is produced in only one country in the integrated equilibrium, and this pins down the volume of trade. In this section we discuss the implications of making alternative assumptions for the production of intermediate goods.

Perhaps the most natural alternative assumption to make on the production of intermediate goods is to assume constant returns to scale. That is, let the output of each intermediate good be linearly related to the amount of labour used in its production:

$$q_{ij} = \alpha l_{ij}. \tag{26}$$

This of course is equivalent to setting $\gamma = 1$ in equation (4). Careful examination of the results in the previous sections will show that, apart from simplifying the expressions somewhat, all the main results remain valid.

However, the mechanism by which the model operates – that international trade allows for the production of each intermediate good to be concentrated and hence leads to gains from reduced coordination costs – does not operate in this case. Under constant returns to scale, with identical technologies across countries, the location of production of intermediate inputs does not matter. But if each intermediate input is produced in both countries, the duplication of coordination costs cannot be avoided. As a result, there would be no gains from trade! To generate gains from trade, what would be required is an additional assumption, that with international trade, each intermediate input is produced in only one country. This is satisfied by the assumption made in previous sections that intermediate goods are exhibit external scale economies, provided we focus on the case of the efficient, integrated equilibrium.

The other possible assumption to make about intermediate goods production is that it takes place under *international* as opposed to *national* scale economies. Thus, instead of equation (3), the output of an intermediate good j is now a function of the total labour used anywhere in the world in that intermediate input (note the omission of the country subscript i):

$$q_j = \left(\alpha l_j\right)^{\gamma}. \tag{27}$$

Under this assumption, once again the location of production of each intermediate input does not matter; a firm can produce an intermediate input anywhere in the world and still benefit from the international scale economies. Therefore, this leads to the same problem as faced by assuming constant returns to scale – that because production of intermediates may be dispersed, the cost saving of removing the duplication of the production network does not materialise. Thus we can conclude that, for the fundamental mechanism of the model to work, external scale economies which are national in nature are essential.

6 Two factors of production and the distribution of income

In the previous sections the model was set up to have only one factor of production. This made the mechanism underlying the model more transparent, but at the same time limits the scope of the model. In this section we extend the model to introduce two different final goods and two factors of production, and explore the implications for trade and the gains from trade. In the interests of simplicity, we focus on a special case of the model in which each final good is produced using a different set of intermediate goods, which in turn are produced using only one of the two factors of production. The basic structure of the model bears close similarity to the analysis of the model of monopolistic competition with different factors of production in Krugman (1981).

Now there are two homogeneous final goods, 1 and 2. Suppose that utility takes the following Cobb-Douglas form[5]:

$$U = C_1^{\theta} C_2^{1-\theta}, \qquad 0 < \theta < 1. \tag{28}$$

There are two types of labour, 1 and 2. Final good 1 is produced using intermediate inputs which are produced using type 1 labour, while final good 2 is produced using intermediate inputs which are produced using type 2 labour. The two sectors are assumed to share the same production technologies in both intermediate and final goods stages. As a result of this structure, the production side of both final goods are decoupled from each other, and in each sector the autarkic and trading equilibria remain as in Sections 3 and 4 above.

[5] This is slightly more general than the utility function used in Krugman (1981), where he assumes that $\theta = 0.5$.

To focus attention on the implications of relative factor endowments, we follow Krugman (1981) and let each country be endowed with the following amounts of the two types of labour (with an asterisk denoting Foreign values):

$$L_1 = L_2^* = 2 - z, \qquad L_2 = L_1^* = z, \qquad 0 < z \leq 1. \tag{29}$$

Hence each country has a total of 2 units of labour, the two countries are symmetric in terms of their relative endowments, and the world has equal amounts of the two types of labour.

The interaction between the two final goods occurs on the demand side. From the consumer's maximisation problem, we have:

$$\frac{P_1}{P_2} = \frac{\theta}{1-\theta}\frac{C_2}{C_1}. \tag{30}$$

Making use of this and the market clearing condition shows that expenditure on each good is a constant fraction of total expenditure in the economy:

$$P_1C_1 = \theta(P_1C_1 + P_2C_2), \qquad P_2C_2 = (1-\theta)(P_1C_1 + P_2C_2). \tag{31}$$

Since each final good is produced using only one type of labour, the expenditure share of each final good is also the income share of the labour used in producing that good. In autarky, the representative consumer's utility in the Home country is:

$$U_H^A = \left(\tfrac{1}{2}\right)\left[n_{H1}^{\beta-\gamma}\left((2-z)\alpha\right)^{\gamma} - \psi n_{H1}^{\rho}\right]^{\theta}\left[n_{H2}^{\beta-\gamma}(z\alpha)^{\gamma} - \psi n_{H2}^{\rho}\right]^{1-\theta}. \tag{32}$$

And therefore the utility of each type of worker is:

$$U_{H1}^A = \left(\tfrac{\theta}{2-z}\right)\left[n_{H1}^{\beta-\gamma}\left((2-z)\alpha\right)^{\gamma} - \psi n_{H1}^{\rho}\right]^{\theta}\left[n_{H2}^{\beta-\gamma}(z\alpha)^{\gamma} - \psi n_{H2}^{\rho}\right]^{1-\theta}. \tag{33}$$

$$U_{H2}^A = \left(\tfrac{1-\theta}{z}\right)\left[n_{H1}^{\beta-\gamma}\left((2-z)\alpha\right)^{\gamma} - \psi n_{H1}^{\rho}\right]^{\theta}\left[n_{H2}^{\beta-\gamma}(z\alpha)^{\gamma} - \psi n_{H2}^{\rho}\right]^{1-\theta}. \tag{34}$$

In free trade, since the world has identical endowments of the two types of labour, world market clearing implies:

$$\frac{P_1}{P_2} = \frac{\theta}{1-\theta}\frac{L_2+L_2^*}{L_1+L_1^*} = \frac{\theta}{1-\theta}. \tag{35}$$

Since each final good uses only one type of labour, factor price equalisation (FPE) is always achieved, since the FPE set spans the entire endowment space. Hence, $w_1 = w_1^*$ and $w_2 = w_2^*$. Given identical technologies for producing the final goods from the intermediate goods, and for producing the intermediate goods from each type of labour, we have, from equation (35):

$$\frac{w_1}{w_2} = \frac{p_{ij1}}{p_{ij2}} = \frac{P_1}{P_2} = \frac{\theta}{1-\theta}. \tag{36}$$

Hence relative wages depend on the share of each final good in consumer expenditure. National income in the two countries is the sum of labour income, which, substituting from (36) and setting $w_2 = 1$ gives:

$$Y = w_1 L_1 + w_2 L_2 = \frac{\theta(2-z)+(1-\theta)z}{1-\theta}. \tag{37}$$

$$Y^* = w_1 L_1^* + w_2 L_2^* = \frac{\theta z+(1-\theta)(2-z)}{1-\theta}. \tag{38}$$

Since preferences are homothetic and prices are equalised under free trade, each country consumes the same proportion of the two goods, and the proportion is determined by the share of national income in world income. Hence, Home's consumer's utility under free trade is:

$$U_H^{FT} = \left[\frac{Y}{2(Y+Y^*)}\right]\left[n_{T1}^{\beta-\gamma}(2\alpha)^{\gamma} - \psi n_{T1}^{\rho}\right]^{\theta}\left[n_{T2}^{\beta-\gamma}(2\alpha)^{\gamma} - \psi n_{T2}^{\rho}\right]^{1-\theta}. \tag{38}$$

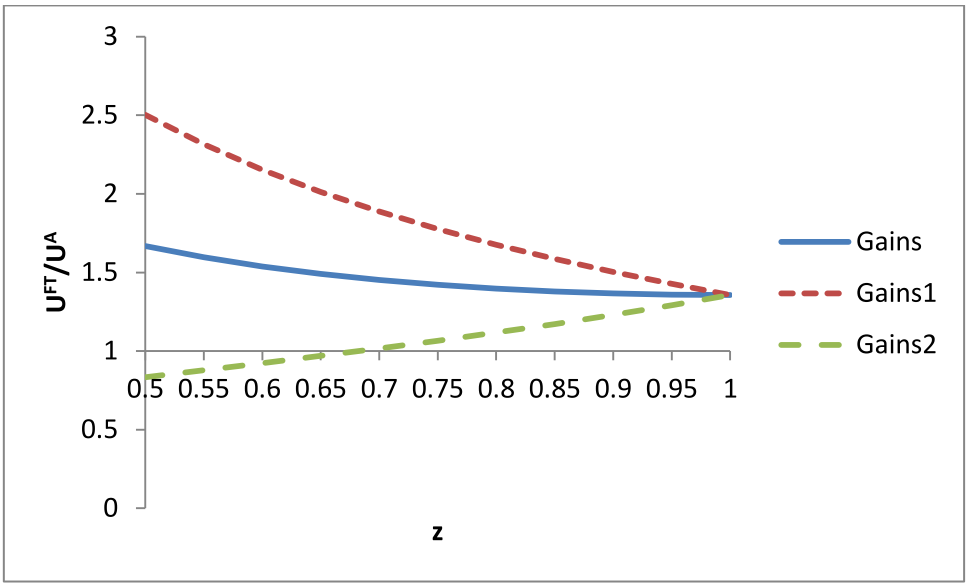
Each type of labour still gets a fraction of world income proportional to θ, but each country's share of this income is now proportional to the country's share of that type of labour, so utility of both types of labour in Home under free trade is:

$$U_{H1}^{FT} = \frac{\theta(2-z)}{2}\left[n_{T1}^{\beta-\gamma}(2\alpha)^{\gamma} - \psi n_{T1}^{\rho}\right]^{\theta}\left[n_{T2}^{\beta-\gamma}(2\alpha)^{\gamma} - \psi n_{T2}^{\rho}\right]^{1-\theta}. \tag{39}$$

$$U_{H2}^{FT} = \frac{(1-\theta)z}{2}\left[n_{T1}^{\beta-\gamma}(2\alpha)^{\gamma} - \psi n_{T1}^{\rho}\right]^{\theta}\left[n_{T2}^{\beta-\gamma}(2\alpha)^{\gamma} - \psi n_{T2}^{\rho}\right]^{1-\theta}. \tag{40}$$

The ratio of the autarkic utility (32), (33) and (34) to the free trade utility (38), (39) and (40) shows whether the country and each type of labour experiences gains from trade. Unfortunately, these ratios do not simplify. Figure 1 shows how the gains from trade vary with the relative endowment parameter z.

Figure 1: The gains from trade for the Home country.



Notes: Assumed parameter values: $\alpha = 0.8, \beta = 1.5, \gamma = 1.2, \rho = 1.8, \psi = 0.5, \theta = 0.5$.

When $z = 1$, the two countries have identical relative endowments. In this case, the only source of the gains from trade is the division of labour. The Home country experiences overall gains from trade, and both types of labour experience the same gain. As z decreases, the Home country becomes relatively more type-1 labour abundant. Type 1 labour experiences greater gains from trade, while type 2 labour's gains decrease, until after a certain point, it starts to experience losses from trade. Hence, similarly to Krugman (1981), when the two countries' relative endowments are sufficiently similar to each other, both factors of production can gain from trade.

Also, from Figure 1, the Home country's overall gains from trade increase as relative endowments become more different between the two countries. Now there are two sources of the gains from trade: the division of labour, and comparative advantage in the form of differences in relative factor endowments. It is possible to decompose the total gains from trade into the component derived from the division of labour (where $z = 1$), and the component derived from relative endowment differences. For example, in Figure 1, when $z = 0.5$, the gains from the division of labour are equal to 1.36, while the gains from relative endowment differences are equal to 1.23, for an overall gain of 1.67.

Hence, returning to the effect of international trade on the relatively scarce labour (type 2 in the Home country), when relative endowments are sufficiently similar, the gain from the division of labour more than offsets the loss from being the relatively scarce factor of production (this loss arising from the fact that the scarce factor is relatively less scarce in the free trade equilibrium, and thus experiences a fall in its real return). However, when relative endowments are sufficiently different, the decrease in the real return to the scarce factor as a result of international trade more than offsets the gain from the division of labour, leading to an overall loss for the scarce factor.

6.1 Trade patterns

As in section 4.3 above, there is no trade in final goods, since assembly of each final good does not depend on its location, hence may be assumed to be assembled locally to consumption. In each sector, each country produces a number of intermediate inputs which is proportional to its endowment of the labour used in that sector. Hence Home will produce a fraction $(2 - z)/2$ of the total number of type 1 intermediate inputs, and a fraction $z/2$ of the total number of type 2 intermediate inputs. Since preferences are homothetic, each country demands a fraction of each intermediate input which is proportional to its share of world income. Hence the value of the Home country's exports and imports of the two types of intermediate inputs are:

$$Export_{1H} = \left(\frac{2-z}{2}\right)\theta Y^* \qquad Export_{2H} = \left(\frac{z}{2}\right)(1-\theta)Y^* \tag{41}$$

$$Import_{1H} = \left(\frac{z}{2}\right)\theta Y \qquad Import_{2H} = \left(\frac{2-z}{2}\right)(1-\theta)Y \tag{42}$$

Trade is balanced; total exports equal total imports. The total value of exports depends on relative endowments and consumer preferences:

$$TE = \left(\frac{1}{2}\right)[\theta(2-z) + z(1-\theta)]Y^* = \frac{[\theta(2-z)+z(1-\theta)][\theta z+(1-\theta)(2-z)]}{2(1-\theta)} \tag{43}$$

Note that, provided $\theta \neq 0.5$, $dTE/dz > 0$; the more similar are the two countries in their relative endowments, the larger will be the total volume of trade between them. Trade may be divided into the component which is inter-industry in nature (exporting type 1 intermediates in exchange for type 2 intermediates), and the component which is intra-industry in nature (simultaneously exporting and importing the same type of intermediate). An index of intra-industry trade is given by the Grubel-Lloyd (1975) index, defined for each sector as:

$$GL = 1 - \frac{|Exports - Imports|}{Exports + Imports} \tag{44}$$

Larger values of this index imply greater intra-industry trade as a fraction of total trade. Substituting from equations (41) and (42), we get:

$$GL = \frac{2zY}{(2-z)Y^* + zY} = \frac{2z[\theta(2-z)+(1-\theta)z]}{(2-z)[\theta z+(1-\theta)(2-z)]+z[\theta(2-z)+(1-\theta)z]} \tag{45}$$

That is, the Grubel-Lloyd index of intra-industry trade depends on consumer preferences and relative endowments. It can be shown that $dGL/dz > 0$; the more similar are the two countries in their relative endowments, the greater the share of trade which is intra-industry in nature. Note that if $\theta = 0.5$ as in Krugman (1981), then we get exactly the same results as Krugman does: total exports will be equal to $0.5 \times Y^*$, and the Grubel-Lloyd index of intra-industry trade will be equal to z, the measure of similarity in relative endowments.

7 Conclusions

This paper develops a simple model of international trade based only on the division of labour; there is no comparative advantage or imperfect competition. Firms assemble final goods from intermediate inputs, and there are gains to having a larger variety of intermediate inputs. The extent of the division of labour is limited by the cost of coordinating intermediate inputs and the size of the market. International trade eliminates the duplication of coordination costs, resulting in an increased variety of intermediate inputs, greater division of labour, and hence to gains from trade. Extending the basic one-factor model to two factors of production, we obtain the result that, if relative endowments are sufficiently similar between the two countries, then both factors of production will benefit from trade. This is in contrast with the traditional factor endowments model, in which the scarce factor of production loses from trade, and arises because, when the basic model is combined with the factor endowments model, there are now two sources of the gains from trade: from the division of labour, and from comparative advantage.

The model represents an alternative treatment to the issue of scale economies and the division of labour in international trade to the now-conventional monopolistic competition approach pioneered by Krugman (1979, 1980) among others. In the conventional approach, there are scale economies which are internal to the firm; as a result, there are only a limited number of firms in the market, and each firm is associated with a different variety of the good. There, the division of labour occurs *across* firms. In the current model, the division of labour occurs *within* firms, but is constrained by the coordination cost. Some of the results we obtain are similar to those in Krugman (1979), and contrast with those in Krugman (1980): international trade leads to an increase in both the number of intermediate goods, and the scale of production of each intermediate good. Similarly, when extending the basic model to two factors of production, we obtain results which are similar to those obtained in Krugman's (1981) extension of the monopolistic competition model to more than one factor of production.

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Appendix A: Further details of the production function for intermediate goods

First we establish that the production function for an intermediate good given in equation (4) can be derived from the production function of each firm producing that intermediate good (see Panagariya, 1981). The production function for a firm k producing intermediate good j in country i depends on the total output of that intermediate good:

$$q_{ijk} = \alpha q_{ij}^{\delta} l_{ijk} \tag{A1}$$

Total output of intermediate good j is:

$$q_{ij} = \sum_k q_{ijk} = \alpha q_{ij}^{\delta} l_{ij} = \left(\alpha l_{ij}\right)^{\frac{1}{1-\delta}} = \left(\alpha l_{ij}\right)^{\gamma} \tag{A2}$$

Where $\gamma = 1/(1-\delta)$.

Next, we solve for the prices of the intermediate goods. Under perfect competition, each firm employs labour so that the value marginal product of labour is equal to the wage rate. Differentiating equation (A1) with respect to l_{ijk} gives:

$$MPL_{ijk} = \frac{dq_{ijk}}{dl_{ijk}} = \alpha q_{ij}^{\delta} \tag{A3}$$

Hence, we have:

$$w = p_{ij} \alpha q_{ij}^{\delta} \tag{A4}$$

Setting the wage rate equal to unity, we can solve for the price of each intermediate good as:

$$p_{ij} = \left(\alpha q_{ij}^{\delta}\right)^{-1} = \left[\alpha^{\gamma} l_{ij}^{\gamma-1}\right]^{-1} \tag{A5}$$

Which is equation (5) in the text. These results hold in both autarky and international trade, with the only difference being that the labour used in each intermediate good, l_{ij}, differs between autarky and international trade. Note that equation (A5) also shows that, although each firm practices marginal cost pricing, at the industry level, average cost pricing is being practiced; average cost for the industry is (substituting from equation (A2)) $wl_{ij}/q_{ij} = \left[\alpha^{\gamma} l_{ij}^{\gamma-1}\right]^{-1}$.