Lancaster University Management School

Economics Working Paper Series

2015/025

When are Capital Structure Decisions Nonseparable from Production Planning? The Case of Generalized Royalty-Based Hybrid Finance

Kim Kaivanto and Alan S.I. Zinober

The Department of Economics
Lancaster University Management School
Lancaster LA1 4YX
UK

When are Capital Structure Decisions Nonseparable from Production Planning? The Case of Generalized Royalty-Based Hybrid Finance

Kim Kaivanto*
Department of Economics, Lancaster University, Lancaster LA1 4YX, UK
Alan S.I. Zinober
Department of Applied Mathematics, University of Sheffield, Sheffield S10 2TN, UK

this version: October 28, 2015

Abstract

The well-known result that capital structure is irrelevant for firm value follows from a set of assumptions conducive to theoretical analysis. In this note we explore the implications of relaxing one of these assumptions: the independence of cash flows from capital structure. Unlike debt and equity, funding that is accompanied by a royalty payment obligation has the effect of increasing marginal cost, to which a profit-maximizing firm responds by reducing output, violating the independence assumption. We study the effect on optimal production plans of generalized royalty payment obligations in which the royalty rate need not be constant across partitions of cumulative output, resulting in piece-wise linear cumulative royalty schedules that are not everywhere differentiable. The associated optimization problem for intertemporal production planning is nonstandard as it is not time separable. Here we solve this nonstandard problem by formulating an equivalent problem that in turn can be solved by the Pontryagin Maximum Principle using numerical techniques. When generalized royalty-based finance is included in the financing mix, the optimal production plan is non-trivially related to capital structure and capital structure *is* relevant to firm value. Unless the financing mix is restricted to debt and equity, financing decisions and production planning decisions cannot be undertaken independently in general.

Keywords: production planning; capital structure; separability; Pontryagin Maximum Principle; numerical methods; royalty-based finance; hybrid instruments
JEL classification: G32

*Corresponding author: tel +44(0)1524594030; fax +44(0)1524594244; e-mail k.kaivanto@lancaster.ac.uk

1 Introduction

The compartmentalization and separation of finance functions from production planning functions is the norm in many large firms. Indeed the centrality of the Modigliani-Miller theorem in modern finance – a result which establishes the irrelevance of capital structure to firm value – appears to justify such separation (Modigliani and Miller, 1958). Nevertheless with the advent of 'hybrid' funding instruments, whose payment obligations are linked to the firm's output or sales, one of the key assumptions of the Modigliani-Miller theorem becomes untenable: the independence of capital structure from cash flows. Royalty-based finance, in which up-front finance is obtained in exchange for a royalty on sales, is an example of a funding instrument that renders financing decisions nonseparable from production planning decisions.

It is well known that royalty payments augment marginal cost, causing profit maximizing output to occur at a lower level than under the alternatives of fixed (debt repayment) or residual (profit share) payment obligations that do not affect marginal cost. This property is well established for flat-rate royalties in stylized atemporal pedagogical models of the firm. However the effect of more general forms of royalties – where the per-unit royalty is a function of cumulative output – on the intertemporal production plan has not received attention in the literature to date.

Some perspective on the effect of generalized royalties may be gleaned from the literature on dynamic economies of scale, i.e. learning economies (Dada and Srikanth, 1990; Spence, 1981; Yelle, 1979; Wright, 1936). Under learning-by-doing, a component of variable cost decreases as a function of cumulative output, in a relationship captured by the learning curve. Spence (1981) and others have shown that if production is subject to dynamic economies of scale, the profit maximizing policy must take account of the future cost reduction associated with current production. The solution to this dynamic problem has a simple characterization: the firm should set current output as if the learning had already occurred, equalizing current marginal revenue with end-of-horizon marginal cost.[1]

Similarly, the per-unit payment obligation in *generalized royalties* may be specified as a function of cumulative output. For continuous and differentiable non-linear royalty payment

[1]Discounting complicates this result somewhat. The further cost reductions occur in the future, the smaller the discounted present value of these gains, and so current marginal revenue is equalized with the sum of end-of-horizon marginal cost and a term reflecting the effect of discounting.

schedules that decrease monotonically down to some small value or vanish altogether at high levels of cumulative output, the analytics of the deterministic learning curve analysis apply: the firm should set current output as if the reduction in royalties had already occurred, equalizing current marginal revenue with end-of-horizon marginal cost adjusted for discounting. However if the royalty rate were to increase monotonically with cumulative output, current-period output would be diminished to an even greater degree than under a flat-rate royalty.

Here we study the effect of piecewise continuous linear cumulative royalty schedules on the optimal intertemporal production plan. The level of the royalty changes at prespecified cumulative output trigger points. However the time at which a particular trigger point is reached depends on the path of the production rate up to that time. Hence the times at which trigger points are reached are themselves determined by the intertemporal program, and therefore the problem is not neatly time separable. Moreover, the structure of the problem poses an interesting control-theoretic challenge, insofar as the transition points (defined by trigger points in cumulative output) between different royalty rates induce non-differentiability at those transition points in the integrand of the optimal control problem. This particular type of control-theoretic problem has no existing standard solution method.

The performance index to be maximized is a function of the unknown state variable and is also piecewise continuous. This is not a standard problem solvable using the Pontryagin Maximum Principle (PMP). An equivalent problem is solved by the PMP using numerical techniques including Runge-Kutta ODE solution, shooting method for the two point boundary value problem, and Golden Section minimization.

For the purposes of this study we specify two royalty schedules for incorporation into Spence's (1981) dynamic model of the firm. This dynamic model also incorporates a learning curve. The first royalty schedule we specify is a simple 3-regime variant, while the second royalty schedule we specify has 7 different regimes (intervals of cumulative output where the royalty takes on a specific value) over which the royalty rate first successively increases, then decreases.

We solve three variants of the model: (i) learning curve with no royalty; (ii) learning curve with a 3-regime royalty; (iii) learning curve with a 7-regime royalty. The results confirm intuition gained from the atemporal model and the Spence (1981) model. Both simple (3-regime) and complex (7-regime) generalized piece-wise continuous cumulative royalty schedules have the effect of reducing the firm's profit-maximizing output and increasing the firm's profit maximizing price.

More importantly, the optimal output and price schedules show non-trivial dependence on the generalized royalty schedule. When royalty-based finance is included in the financing mix, capital structure is not irrelevant to firm value insofar as changes in the financing mix change the optimal production and pricing schedules, which in turn affect firm value. Furthermore, as analyzed elsewhere (Damodaran, 1999; Kaivanto and Stoneman, 2007), hybrid securities such as generalized royalty-based finance improve the temporal matching of payment liabilities with revenue inflows. This creates additional debt capacity, which in turn can be exploited to take on additional leverage, thereby increasing firm value. Thus *in general* it is not safe to assume that financing decisions may be made independently of production planning decisions, unless the financing mix is limited to debt and equity.

These results show that the incorporation of hybrid securities into the financing mix should not be considered without consultation with the production planning function. Involvement of production planning is necessary not only to make the most out of a particular hybrid security, but extends to the choice of what type of hybrid security to incorporate, and in what measure. In the analysis undertaken here the emphasis is theoretical and methodological, and hence the model abstracts from operational concerns such as rigidities, adjustment costs, capacity constraints and supply chain parameters. In operations applications these factors are of utmost importance, and indeed the choice of hybrid security design – and its pricing – are heavily constrained and conditioned by these considerations.

2 Generalized royalties

2.1 Prevalence

Royalties of varying degrees of structural complexity appear in numerous settings throughout the economy, extending far beyond external licensing and intra-group transfer pricing of intellectual property rights. They appear, for instance, in royalty schemes of the publishing, media, music and film businesses, in commercial property lease contracts, in franchising contracts, in retail contracts, in natural resource development leases, in radio spectrum licenses, in the remuneration contracts between fishing boat owners and fishermen, as well as in agricultural share cropping.

In many of these arrangements, the owner of a tangible or intangible asset grants a second agent the right to use the asset in return for a royalty written on the output or sales resulting from use of the asset.

In a second category of arrangements, a generalized royalty on production or sales is exchanged for a contribution of financial resources (royalty-based financing). Technology support schemes in various countries fall into this category, including both programs that are not industry- or technology-specific – such as, for instance, the schemes in Canada, Sweden, Finland, The Netherlands, France and Israel[2] – as well as programs that are industry-specific, such as the Launch Aid schemes for civil aerospace product development programs operated in the UK, France, Germany, Spain, Canada and Japan, as well as the comparable schemes formerly operated in The Netherlands and Sweden (Kaivanto and Stoneman, 2007). But this exchange of financial resources for a generalized royalty claim is not unique to government support schemes. Royalty bonds (e.g. Intellectual Property royalty bonds) and revenue bonds share this characteristic, as do many hybrid securities, structured finance securities and project finance instruments.

2.2 Properties of royalty-based finance

Royalty-based finance is distinguished by a number of useful and valuable characteristics that are absent from conventional debt and equity finance. Raising royalty-based finance entails no loss of control nor any dilution of existing shareholders as is the case when finance is raised by issuing equity. Raising finance by issuing debt claims increases the firm's 'financial risk', whereas raising finance by issuing royalty claims reduces financial risk. This may be understood simply, in terms of improved matching between the timing of payments to financiers and the timing of cash availability: royalty payments are due precisely when cash is on hand from having produced and sold a unit of output. This may also be understood through more elaborate and explicit modeling of the variability in profits resulting from cost and revenue risk (Kaivanto and Stoneman, 2007). The effect is accentuated in firms with rated publicly traded debt, as these firms systematically employ 'dividend smoothing' payout policies (Lintner, 1956) rather than 'pass-through' dividend payout policies (Aivazian et al., 2006). Dividend smoothing payout

[2] These programs have been administered by Technology Partnerships Canada (TPC), Industrifonden [The Swedish Industrial Development Fund], TEKES [National Technology Agency of Finland], Senter [an agency of the Dutch Ministry of Economic Affairs], ANVAR [The French Agency for Innovation], and The Office of the Chief Scientist (OCS), respectively. Not all of these programs continue to operate.

policies absorb cost-revenue variability internally, rather than passing this variability through to shareholders. As a result, the financial risk of firms paying a smoothed dividend is increased relative to those that employ a pass-through payout policy – or those that raise royalty-based finance instead. Damodaran (1999) shows that the additional debt capacity created with hybrid securities must be exploited – by raising additional debt finance – in order have a positive impact on the value of the firm. Mere substitution of debt with hybrid securities such as royalty-based finance does not enhance firm value. But the total non-equity financing a firm can raise is greater when debt is combined with hybrid securities such as royalty-based finance.

2.3 Formalization

For present purposes it suffices to work with the payout function of royalty-based finance, eliding the additional structure required for valuing the stochastic royalty stream.[3] Here a simplified exposition and notation is used, tailored to the problem at hand.

In principle, a generalized royalty may be written on the output of a single product, a collection of products, the associated money-denominated turnover, or on the turnover of an entire division or corporation. To fix ideas, consider a royalty written on the output of a single good, which is produced in quantity $x(t)$ at time $t \in [0, T]$. Thus cumulative output at time t may be written as $y(t) = \int_0^t x(\tau)\mathrm{d}\tau$. It will also be convenient to denote by $i \in \mathcal{I} = \{1, 2, ..., I\}$ a particular output unit's place in the production sequence.[4] The payout function may be written either to specify the payment on each unit i or the payment at the end of each period t. In unit form, a generalized royalty admitting piecewise continuity of the cumulative royalty schedule may be written as

$$\rho_i = f(i, \boldsymbol{H}, \boldsymbol{o}) \tag{1}$$

where $\boldsymbol{H}$ is a matrix of threshold terms and associated parameters, and $\boldsymbol{o}$ is a vector of other parameters. Depending on the functional form of the royalty schedule, either $\boldsymbol{H}$ or $\boldsymbol{o}$ may be suppressed. For the piecewise continuous cumulative royalty form, the $\boldsymbol{o}$ is suppressed and $\boldsymbol{H}$ is specified as an $n \times 2$ matrix where n is the number of distinct linear royalty regimes within $\mathcal{I}$. One column of n entries records the thresholds defining the endpoints of regimes in $\mathcal{I}$, while

[3] The definition of a Sales Contingent Claim requires a precise specification of the relevant sales and the stochastic process driving these sales (Kaivanto and Stoneman, 2004).

[4] i.e. production number, manufacturer's serial number

the second column records the associated per-unit royalty of each regime.

Various piecewise linear cumulative schedules may then be specified, with the flat-rate royalty being a special case where $n = 1$ and $\boldsymbol{H} = [I \ \rho]$, giving a constant royalty of ρ over the entire production sequence: $\rho_i = \rho \ \forall\, i \in \mathcal{I}$.

At an intermediate level of complexity, which nevertheless displays non-differentiability, consider the following three-regime cumulative royalty schedule. Over an initial production run, no royalty is charged. Thereafter, a fixed-rate royalty is levied over a defined-length production run. After this, the firm is absolved of any further royalty payment obligation. Expressed in units of I and $\sum_{\mathcal{I}} \rho_i$ the following $\boldsymbol{H}'$-matrix is representative of this three-regime not-everywhere-differentiable royalty:

$$\boldsymbol{H}' = \begin{bmatrix} 0 & \frac{5}{3} & 0 \\ .2 & .8 & 1 \end{bmatrix} \tag{2}$$

Here the non-zero royalty production interval $(.2I, .8I]$ is symmetric about $.5I$, and consequently its cumulative royalty schedule lies below that of the linear royalty over the interval $(0, .5I)$ and above the linear royalty over the interval $(.5I, I)$.

Consider also a complex example with seven distinct regimes. Over an initial production run, say the estimated initial annual production, no royalty at all is required (regime 1). Thereafter, over successive intervals of cumulative output, the payable royalty is ramped up (regimes 2-5). After a particular target output, the royalty decreases first sharply, and then more moderately, over the remaining intervals (regimes 6-7). Expressed in units of I and $\sum_{\mathcal{I}} \rho_i$, the following $\boldsymbol{H}'$-matrix of is representative of this seven-regime not-everywhere-differentiable royalty:

$$\boldsymbol{H}' = \begin{bmatrix} 0 & 1.2 & 1.6 & 2 & 2.4 & .24 & .12 \\ .08 & .16 & .24 & .4 & .56 & .72 & 1 \end{bmatrix} \tag{3}$$

Here the cumulative repayment schedule crosses the linear (flat-rate) royalty's cumulative repayment schedule at $.253I$.

3 Extension of the Spence model

Demand is specified, in indirect form, as $p(x) = b_0^{\alpha} e^{\alpha\delta t} x^{-\alpha}$ or, in direct form, $x(p) = b_0 e^{\delta t} p^{-\frac{1}{\alpha}}$. Therefore gross revenue is $p(x)x = (b_0^{\alpha} e^{\alpha\delta t} x^{-\alpha})x = a(t)x^{1-\alpha}$ where $a(t) = b_0^{\alpha} e^{\alpha\delta t}$. Hence the

NPV to be maximized is

$$\int_0^T \left[a(t)x^{1-\alpha} - (\rho + m_0 + c_0 e^{-\lambda y})x(t)\right] e^{-rt}\,\mathrm{d}t \tag{4}$$

where the learning curve is captured by $\theta(y) = m_0 + c_0 e^{-\lambda y}$.

Solving this problem for generalized royalties of the form (2) or (3) presents several difficulties. Firstly, the discontinuous changes in the level of the royalty entails that the cumulative royalty schedule is not differentiable everywhere but this can be handled by the 'almost everywhere' relaxation in the PMP solution. Secondly, it is not possible to decompose the problem into separate time intervals because the times at which the non-differentiable royalty level transition points are reached depends on the overall optimization of of $x(t)$ within the entire interval $[0, T]$.

The royalty in the integrand is a function of total cumulative output $y(T)$, and the problem therefore has a *free* endpoint $y(T)$ that is not known a priori before solution using the PMP (Sethi and Thompson, 2008). An equivalent problem is to solve for the maximum of (4) using the PMP for a fixed value of $y(T)$. Call this *Problem A*. We then use the numerical optimization techniques of golden section search and Brent's method[5] – call this *Problem B* – to determine iteratively the optimal value of $y(T)$ to maximize (4) (Press et al., 2007). A shooting method using the highly accurate eighth-order Dormand-Prince Runge-Kutta ODE solver (using Newton iterations to solve for the unknown value of the costate at $t = 0$) has been used for solving the Two Point Boundary Value Problem (*Problem A*).[6]

We abstract from adjustment costs and place no limit on the maximum permissible adjustment speed of output,[7] neither do we impose an overall variation restriction on price.[8] Doing so would smooth out variation in the main parameter paths of interest: output $x(t)$ and price $p(t)$. The specification adopted reflects the dual aims of this work: to demonstrate the nonseparability of capital structure decisions from production planning decisions, and to demonstrate a solution method to the non-standard dynamic optimization problem involving not-everywhere differentiability and a non-time-separable objective function.

[5]which switches between inverse parabolic interpolation and golden section search
[6]The computer programming was undertaken in C++.
[7]which would reflect time-compression diseconomies
[8]which would reflect the fact that list prices are typically kept stable over time

4 Results

The results are reported most concisely via three graphs. Cumulative output $y(t)$ paths are reported in Figure 1a. The 'no royalty' condition is the benchmark firm with a capital structure composed of debt and equity alone, in which capital structure does not alter the firm's marginal cost. In comparison, cumulative output under the 3-regime royalty falls short of the no royalty benchmark, and in turn cumulative output under the 7-regime royalty falls short of cumulative output under the 3-regime royalty.

Figure 1b shows the time paths of the output level $x(t)$ under each of the three conditions. For the royalty conditions, there are as many output level regimes as there are royalty level regimes. Transitions between output level regimes are instantaneous due to the model's abstraction from rigidities and adjustment costs. These output paths may be understood as the result of interplay between changes in marginal costs caused by the royalty schedule and forgone learning economies caused by reduced output driven by the royalty schedule. Note, for instance, that the R3 output path does not rise to coincide with the no royalty curve in the final interval where the R3 royalty is zero. This is due to forgone learning economies in the R3 condition relative to the no royalty condition. In the R7 condition, the forgone learning economies over the entire production run are of a sufficient magnitude that even over the initial production run of $(0, .08I)$ where the royalty payable is zero, the output level falls below that of the R3 and no royalty conditions.

Finally, Figure 1c shows the time paths of the price level $p(t)$ under each of the three conditions. These paths mirror those of the output paths.

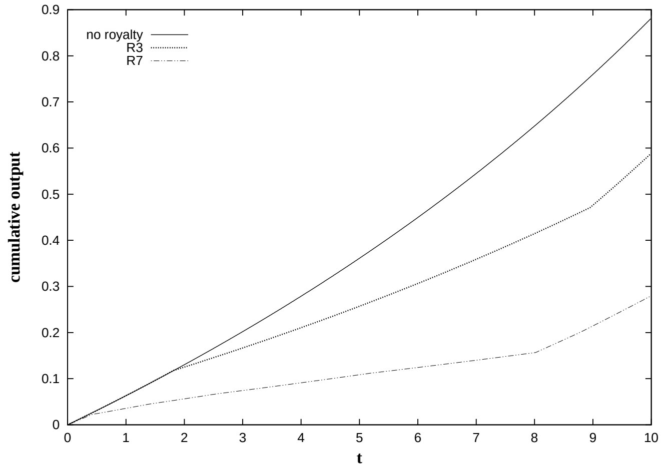
Overall these numerical results confirm prior intuition: both simple (3-regime) and complex (7-regime) generalized royalty schedules have the effect of reducing the firm's profit-maximizing output and increasing the corresponding price. In reducing output, royalties slow the firm's progress down its learning curve, which also impacts overall profitability negatively.

5 Conclusion

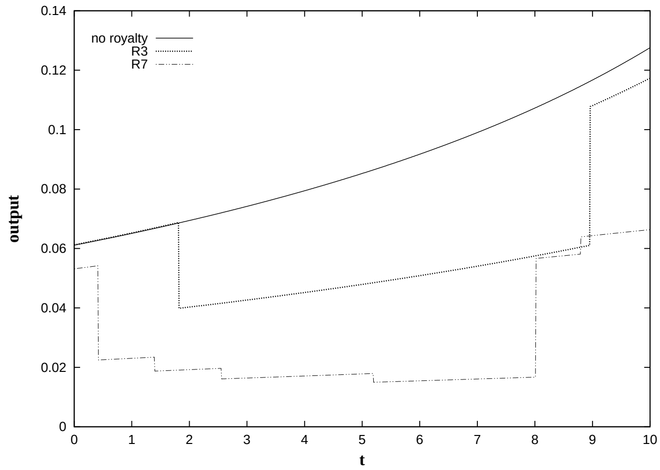
This technical note makes a two-fold contribution. Firstly, as a primary contribution, this note shows that financing decisions and production planning are not separable in general unless capital structure is restricted to equity and debt. Nevertheless large firms, in which the finance function may be sufficiently sophisticated to experiment with hybrid finance instruments, will also

Figure 1: Optimal production-parameter schedules under three conditions: (1) learning curve with no royalty; (2) learning curve with a 3-regime royalty (R3); (3) learning curve with a 7-regime royalty (R7).

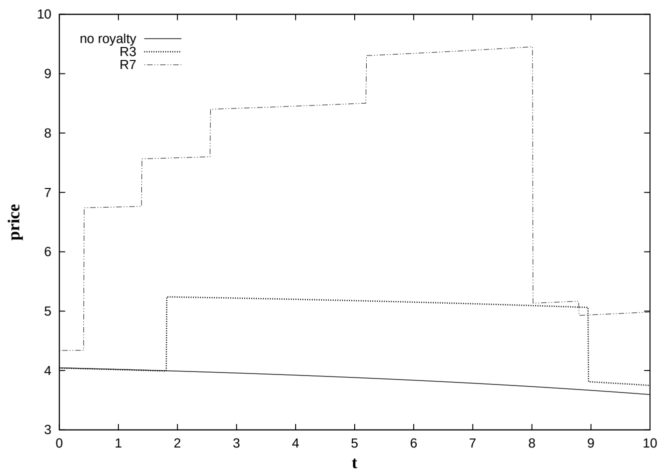
(a) Cumulative output schedule.



(b) Profit-maximizing output schedule.



(c) Profit-maximizing price schedule.



typically have more pronounced specialization and compartmentalization of finance functions and production planning functions. The present results motivate the need for cross-functional consultation when hybrid securities written on sales or output are being contemplated. Not only does the production planning function need to know the planned royalty schedule in order to optimize production, but the finance function needs to know what effect the planned royalty schedule will have on the production plan and ultimate profitability. Only when armed with this information will the finance function be in a position to choose between different hybrid security types and to calculate whether the price (or 'premium') on the hybrid finance is acceptable or too high. These considerations warrant consultation between the finance function and the production planning function. At the same time, these considerations also warrant separate attention in the design of Organizational Decision Support Systems.

Secondly, this note introduces a solution method for a technically non-trivial, non-standard problem involving non-differentiability and lacking time separability. This solution method is of independent value and interest.

Together, these contributions also fill in gaps impeding wider adoption of hybrid finance.

References

Aivazian, V.A., Booth, L., S. Cleary. 2006. Dividend smoothing and debt ratings. *Journal of Financial and Quantitative Analysis* **41**(2) 439–453.

Cabral, L.M., M.H. Riordan. 1994. The learning curve, market dominance, and predatory pricing. *Econometrica* **62**(5) 1115–1140.

Dada, M., K.N. Srikanth. 1990. Monopolistic pricing and the learning curve: An algorithmic approach. *Operations Research* **38**(4) 656–666.

Damodaran, A. 1999. Financing innovations and capital structure choices. *Journal of Applied Corporate Finance* **12**(1) 28–39.

Kaivanto, K., P. Stoneman. 2004. Risk shifting, technology policy and sales contingent claims: when is Launch Aid to the aerospace industry a subsidy? CEPR Discussion Paper no 4798, Centre for Economic Policy Research, London.

Kaivanto, K., P. Stoneman. 2007. Public provision of sales contingent claims backed finance to SMEs: A policy alternative. *Research Policy* **36**(5) 637–651.

Lintner, J. 1956. Distribution of incomes of corporations among dividends, retained earnings, and taxes. *American Economic Review* **46**(2) 97–113.

Mazzola, J.B., K.F. McCarle. 1997. The stochastic learning curve: Optimal production in the presence of learning-curve uncertainty. *Operations Research* **45**(3) 440–450.

Modigliani, F., M. Miller. The cost of capital, corporation finance and the theory of investment. *American Economic Review* **48**(3) 261-297.

Press, W.H., Teukolsky, S.A., Vetterling, W.T., B.P. Flannery. 2007. *Numerical Recipes: The Art of Scientific Computing*, 3rd edition. Cambridge University Press, Cambridge.

Sethi, S.P, G.L. Thompson. 2008. *Optimal Control Theory: Applications to Management Science and Economics*. Springer, Heidelberg.

Spence, M. 1981. The learning curve and competition *Bell Journal of Economics* **12**(1) 49–70.

Wright, T.P. 1936. Factors affecting the cost of airplanes. *Journal of the Aeronautical Society* **3**(4) 122–128.

Yelle, L.E. 1979. The learning Curve: Historical review and comprehensive survey. *Decision Sciences* **10**(2) 302–328.