Fiscal Policy Effects in a Heterogeneous-Agent
OLG Economy with an Aging Population[*]

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Abstract

This paper incorporates the aging population projected by the U.S. Social Security Administration to a heterogeneous-agent OLG model with idiosyncratic wage shocks and analyzes its effects on individual households, the government budget, and the overall economy. The fiscal gap caused by the demographic change is 2.92% of GDP under the SSA's intermediate projection. The effect of the aging population is large by itself and depends significantly on how the government finances the cost of the demographic change. There is a strong trade-off between efficiency and equity, and this paper quantitatively assesses the pros and cons of stylized fiscal reform plans.

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1   Introduction

The current fiscal policy in the United States is not sustainable. This is primarily because of the ongoing demographic changes, such as retiring baby boom generations, declining mortality rates, and stable but relatively low fertility rates. This aging population will reduce the share of the working-age population and increase the share of the retired population in the future; thus it will likely decrease labor supply per capita and output per capita relative to the economy without the aging population. The aging population will also reduce the government's tax revenue per capita, as the government relies more on taxes on labor income, and will increase the government's transfer spending per capita, as elderly households are the main recipients of Social Security benefits. Therefore, the aging population will create a sizable fiscal gap in the government's fiscal policy, and the government will eventually have to cut its transfer spending or raise tax rates or both. These additional fiscal policy changes aimed to close the fiscal gap will likely exacerbate the negative effect of the aging population in the economy.

This paper extends a heterogeneous-agent overlapping-generations (OLG) model with uninsurable wage and mortality shocks by incorporating an aging population projected by the U.S. Social Security Administration (SSA) and analyzes the general-equilibrium effect of the aging population on the behavior and welfare of individual households, the government's budget, and the aggregate economy. The households are heterogeneous with respect to their age, working ability (hourly wage), wealth, and the average historical earnings that will determine their future Social Security benefits. Each year, households receive idiosyncratic wage shocks and determine their consumption, working hours, and saving to maximize their lifetime utility, taking factor prices, the government policy schedule, and the population projection as given. The government fiscal policies, such as the income taxes and the Social Security programs, involve both inter-generational and intra-generational redistribution and risk sharing. Therefore, this setting is critically important to analyze the effect of the aging population as well as the future fiscal policy changes.

This paper then calibrates the model to the 2014 U.S. economy. One of the difficulties in incorporating the aging population to a heterogeneous-agent OLG model is, however, the population distribution in the current year (2014 in this paper) is not stationary, thus, the model cannot describe the baseline economy as a stationary equilibrium. For that reason, this paper calibrates the model to the 2014 U.S. economy by solving it for an equilibrium transition path between 1975 and 2200. As discussed in the above, the aging population creates the government's budget deficits in the future and surpluses in the past under the current-law policy. In the aging-population baseline economy, this paper assumes that the government decreases or increases its consumption spending so that the debt per capita stays at the same level after productivity-

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growth adjustment.[1]

This paper finally conducts a set of policy experiments that replace the government's financing assumption to close the fiscal gap generated by the aging population. Although most of the fiscal imbalances are generated by the Social Security program, especially its old age and survivors insurance (OASI) program and hospital insurance (HI) program, there is no reason for the government to close the fiscal gap within the Social Security system. Therefore, this paper introduces the proportional change in the progressive income tax rates in addition to the changes in Social Security benefit schedules and payroll tax rates.[2] The paper addresses how the behavior and well-being of individual households and the overall economy will differ under these financing assumptions of the cost of the aging population. Not surprisingly, there are a trade-off between efficiency (e.g., GDP growth) and equity (e.g., social welfare) and a trade-off between the well-beings of the current households and the future households; and this paper quantitatively assesses the pros and cons of each alternative fiscal policy.

The main findings of this paper are as follows: First, the macroeconomic and budgetary effects of the aging population are large by itself. In the aging-population baseline economy, where the feedback effect of future fiscal policy changes are turned off, under SSA's population projection Alternative 2 (intermediate scenario), the projected demographic changes will decrease GDP per capita by 5.4% in 2039 and 7.0% in the long run, primarily because the share of the working-age households in the economy declines over time. To balance the budget, the government will have to cut its primary deficits as a share of GDP by 3.6 percentage points in 2039 and by 5.4 percentage points in the long run. The fiscal gap in the current-law fiscal policy, measured by the present discount sum of the current and future primary deficits, is 2.92% as a percentage of the present discount sum of the current and future GDP. In other words, the government will have to reduce its primary deficits by 2.92% of GDP each year starting in 2014 in order to make its current fiscal policy (programs) sustainable.

Second, if the government tries to close this fiscal gap, more realistically, by increasing taxes or cutting transfer spending, the effect of the demographic changes with the additional fiscal policy change will differ significantly, depending on how the government finances the budgetary cost of the aging population. For example, GDP per capita will decrease by 10.0% in 2039 and 24.7% in the long run, relative to the economy without the aging population, if the government increases marginal income tax rates proportionally; but GDP

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[1]In the model economy of this paper, the government's consumption is not in the household's utility function. Thus, by assumption, its change does not affect the household's behavior and wellbeing. The current-law fiscal policy is not sustainable, but it is uncertain when and how the government will change its policy to make it sustainable. To construct an aging-population baseline economy, therefore, this paper adjusts the government consumption to turn off any behavioral feedback from future policy changes.

[2]In its earlier version, this paper also introduced the changes in lump-sum taxes (transfers), the flat consumption tax rate, and both the Social Security payroll tax rates and benefit rates on 50-50 basis. These numerical results are available upon request.

per capita will increase by 2.3% in 2039 and 3.3% in the long run if the government instead decreases Social Security benefits proportionally. Cutting Social Security benefits looks better than raising marginal income tax rates in terms of its effects on output. However, the welfare results indicate that households in the current generation—those aged 21 and older at the time of the policy change—strongly prefer increasing marginal income tax rates to cutting Social Security benefits, at the expense of households in the future. As the fiscal gap generated by the demographic changes increases over time, delaying the policy reform (by increasing the government debt) is not desirable in terms of both the economic output and the intergenerational equity.

This is not the first paper that incorporates the U.S. aging population into a general-equilibrium OLG model and analyzes its macroeconomic and welfare effects. Relative to the previous literature, however, the present paper has a variety of strengths in the model economy and calibration procedure.

To the best of my knowledge, Auerbach and Kotlikoff (1987) are the first to incorporate the historical and predicted changes in the fertility rate to a large-scale deterministic general-equilibrium OLG model. Kotlikoff, Smetters, and Walliser (2007) incorporate the U.S. demographic changes to their 12-income-level deterministic general-equilibrium OLG model. Ludwig, Schelkle, and Vogel (2012) also incorporate the U.S. demographic changes to their deterministic general-equilibrium OLG model with human capital accumulation. The present paper conducts the similar exercises in an incomplete-market OLG economy with idiosyncratic wage shocks, so that the model can capture not only the inter-generational redistribution effect of the government fiscal policy but also its intra-generational redistribution and risk-sharing effects.[3] De Nardi, İmrohoroğlu, and Sargent (1999) incorporate the SSA's population projection to their stochastic general-equilibrium OLG model with idiosyncratic preference and endowment shocks. Yet, they use linear-quadratic approximation around the certainty-equivalent equilibrium to deal with idiosyncratic shocks. Therefore, their paper does not fully consider precautionary motives for saving and labor supply.[4]

Most recently, Kitao (2014) incorporates the demographic change based on Census Bureau's population projection into her heterogeneous-agent OLG model with idiosyncratic wage and mortality shocks, which is similar to the one in the present paper. Kitao first calibrates the model to the 2010 stationary economy with the stationary population distribution, then she changes the conditional survival rate by age and the

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[3]The previous literature that uses this class of models, without the aging population, includes İmrohoroğlu, İmrohoroğlu, and Joines (1995), Huggett (1996), Nishiyama and Smetters (2007), and Conesa, Kitao, and Krueger (2009).

[4] There are several other papers that analyze the effects of the demographic changes of countries or regions outside the United States. For example, Rios-Rull (2001) incorporates the Spanish population distribution to his stochastic general-equilibrium OLG model with aggregate fertility shocks. Attanasio, Kitao, and Violante (2006, 2007) construct a two-region large-open economy general-equilibrium OLG model and incorporate the United Nations global population projections to it. Krueger and Ludwig (2007) construct a three-region large-open economy general-equilibrium OLG model and incorporate the United Nations population projections, too. Chen, İmrohoroğlu, and İmrohoroğlu (2007) incorporate the Japanese population distribution to their deterministic general-equilibrium OLG model. Braun, Ikeda, and Joines (2009) also incorporate the Japanese population projection to their deterministic general-equilibrium OLG model.

population growth rate of newborn households. However, her 2010 stationary benchmark economy does not fully capture the current population distribution with a large group of baby boomers.[5] The present paper instead incorporates SSA's population projection as well as historical population directly and calibrates the model to the 2014 benchmark economy by solving the model for an equilibrium transition path between 1975 and 2200.

The rest of the paper is organized as follows: Section 2 describes the heterogeneous-agent OLG model with the projected aging population; Section 3 explains the calibration of the model to the 2014 benchmark economy; Section 4 shows the and the aging-population baseline economy; Section 5 explains the policy experiments that replace the future government policies to close the fiscal gap; and Section 6 concludes the paper.

2 The Model Economy

The model economy consists of a large number of overlapping-generations households, a perfectly competitive representative firm with constant-returns-to-scale technology, and a government that can credibly commit to a fiscal policy. The time is discrete, and one model period is a year, which is denoted by t. In a long-run stationary equilibrium, the economy is assumed to be on a balanced-growth path with a constant labor-augmenting productivity growth rate, μ, and a constant (long-run) population growth rate, ν. In the following model description, individual variables other than working hours are growth-adjusted by $(1+\mu)^{-t}$, and aggregate variables are adjusted by $[(1+\mu)(1+\nu)]^{-t}$.

2.1 The Households

Households in the model economy are heterogeneous with respect to their age, $i = 21, \ldots, I$, beginning-of-year wealth (net worth), $a \in A = [a_{\min}, \infty)$, average historical earnings used to calculate Social Security Old-Age and Survivors Insurance (OASI) benefits, $b \in B = [0, b_{\max}]$, and labor productivity, $e \in E = [0, \infty)$. The households enter the economy and start working at age $i = 21$, and they can live at most to age $I = 100$. In each year, t, the households receive idiosyncratic and age-dependent working ability shocks, e, and they each choose their consumption, c, working hours, h, and thus wealth at the beginning of the next year, a', to maximize their expected (remaining) lifetime utility.

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[5]See, for example, Figure 13 in Kitao (2014) and Figure 2 in the present paper. Labor supply is higher with the demographic shift during the first 30 years (2010–2040), relative to the benchmark economy, in Kitao (2014), whereas labor supply is 8.1% lower by 2039 in the present paper. The relatively quick decline in labor supply in the latter is mainly predicted by the retirement of the baby-boom generation.

The households are, for simplicity, assumed to start receiving OASI benefits at their own full retirement age (FRA), $i = I_{\tilde{R}}$, which is between ages 65 and 67, depending on their birth year, although they can continue working after their FRA if they find it optimal.[6] The average historical earnings, b, are used to approximate the average indexed monthly earnings (AIME) and determine OASI benefits. The individual working ability, e, follows a first-order Markov process.

2.1.1 The State Variables

Let $\mathbf{s} = (i, a, b, e)$ and $\mathbf{S}_t = (X_t(\mathbf{s}), w_{G,t})$ denote the individual state of the household and the aggregate state of the economy in year t, respectively, where $X_t(\mathbf{s})$ is the cumulative distribution function of households, and $w_{G,t}$ is the government's net worth per household, both at the beginning of year t. Let $p_{i,t}$ be the (long-run) growth-adjusted population of age-i households in year t. The distribution function, $X_t(\mathbf{s})$, satisfies $\int_{A \times B \times E} dX_t(i, a, b, e) = p_{i,t}$.

Let $\boldsymbol{\Psi}_t = \left\{ c_{G,s}, tr_{LS,s}, \tau_{I,s}(\cdot), \tau_{P,s}(\cdot), tr_{SS,s}(\cdot), \tau_{C,s}, w_{G,s+1} \right\}_{s=t}^{\infty}$ be the government's policy schedule as of the beginning of year t, where $c_{G,t}$ is the government's consumption per household, $tr_{LS,t}$ is a lump-sum transfer per household, $\tau_{I,t}(\cdot)$ is a progressive income tax function, $\tau_{P,t}(\cdot)$ is a Social Security Old-Age, Survivors, Disability, and Hospital Insurance (OASDHI) payroll tax function, $tr_{SS,t}(\cdot)$ is an OASDHI benefit function, and $\tau_{C,t}$ is a flat consumption tax rate.[7] The government's consumption is not assumed to be in the household's utility function and thus does not affect the household's decision on private consumption and labor supply, while lump-sum transfers directly affect the household's decision through the intertemporal budget constraint. The flat consumption tax, $\tau_{C,t}$, is assumed to approximate other federal revenues, such as those from excise taxes and customs duties.

Finally, let $\boldsymbol{\Phi}_t = \left\{ (p_{i,s})_{i=0}^{I}, (\phi_{i,s})_{i=0}^{I} \right\}_{s=t}^{\infty}$ be the population projection as of the beginning of year t, where $\phi_{i,t}$ is the conditional survival rate at the end of age i in year t, given that the household is alive at the beginning of age i in year t. For simplicity, the population distribution and the corresponding mortality rates are assumed to be deterministic, and households have perfect foresight about the future population distribution.

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[6]Social Security OASI benefits are partially taxable when the sum of adjusted gross income and 50 percent of benefits is above $25,000 (single) or $32,000 (married filing jointly).

[7]In this paper, the population growth rate changes over time, and the growth rates also depend on the population projection. For a better comparison of alternative fiscal policies over time as well as under different population assumptions, the *current-law* fiscal policy in this paper is assumed to keep the government's net worth per household, $w_{G,t}$, and consumption per household, $c_{G,t}$, at the same levels after the productivity growth adjustment, rather than to keep the government's aggregate net worth and aggregate consumption unchanged.

2.1.2 The Optimization Problem

Let $v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)$ be the value function of a household at the beginning of year t. Then, the household's optimization problem is

$$(1) \quad v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) = \max_{c,h,a'} \left\{ u(c,h) + \tilde{\beta}\phi_{i,t} E\left[v(\mathbf{s}', \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1}, \mathbf{\Phi}_{t+1}) \,|\, \mathbf{s} \right] \right\}$$

subject to the constraints for the decision variables and the law of motion of the individual state,

$$(2) \quad c > 0, \qquad 0 \le h < 1, \qquad a' \ge a'_{\min}(\mathbf{s}),$$

$$(3) \quad a' = \frac{1}{1+\mu}\Big[(1+r_t)a + w_t eh + tr_{SS,t}(i,b) + tr_{LS,t} + q_t(i) - \tau_{I,t}(w_t eh, r_t a, tr_{SS,t}(i,b)) $$
$$ - \tau_{P,t}(w_t eh) - (1+\tau_{C,t})c \Big],$$

$$(4) \quad b' = \mathbf{1}_{\{i<I_R\}} \frac{1}{i-20}\Big[(i-21)\, b \,\frac{w_t}{w_{t-1}} + \min(\eta w_t eh, \vartheta_{\max}) \Big] + \mathbf{1}_{\{i \ge I_R\}} b,$$

where $u(\cdot)$ is a period utility function, a combination of Cobb–Douglas and constant relative risk aversion (CRRA), $\tilde{\beta}$ is a growth-adjusted discount factor explained below, and $E[\,\cdot\,|\,\mathbf{s}\,]$ denotes the conditional expected value given the household's current state. The household's decision variables are constrained: consumption, c, is strictly positive; working hours, h, are nonnegative and are less than a time endowment (unity); and wealth at the beginning of the next year, a', satisfies a borrowing constraint, $a' \ge a'_{\min}(\mathbf{s})$.[8] In the law of motion, r_t is the interest rate, w_t is the wage rate per efficiency unit of labor, q_t is the distribution of accidental bequests explained below, $\mathbf{1}_{\{\cdot\}}$ is an indicator function that returns 1 if the condition in { } holds and 0 otherwise, I_R is set at 65 so that the household's OASI benefits are calculated based on its growth-adjusted earnings between ages 21 and 64, η is the ratio of taxable labor income to total labor income, and $\vartheta_{\max}$ is the maximum taxable earnings for the OASI program.[9] The household's wealth at the beginning of the next year, a', is adjusted by the productivity growth rate, $1 + \mu$. The average historical earnings for a household at the beginning of the following year, b', are calculated as wage-indexed, and the price indexation of AIME after age 60 is reflected in the Social Security benefit function.[10]

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[8] For simplicity, the borrowing limit (the lowest-possible wealth level) is assumed to depend only on the household's age i. See Section 3.4 for the assumption of $a'_{\min}(\mathbf{s})$.

[9] In the current Social Security system, AIME is calculated based on the highest 35 years of growth-adjusted earnings. In the model economy, AIME is approximated by the growth-adjusted earnings of all ages before $I_R = 65$, for computational simplicity.

[10] See Appendix D in SSA (2014) for a detailed explanation on wage indexing.

2.1.3 The Distribution of Households

The household's problem is solved for c, h, and a' for all possible states. The household's decision rules are obtained as $c(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t)$, $h(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t)$, and $a'(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t)$, and the average historical earnings in the next year are also obtained as $b'(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t)$ from equation (4).

Households are assumed to enter the economy at age 21 without any assets and working histories, thus,

$$\int_{A \times B \times E} dX_t(21, a, b, e) = \int_E dX_t(21, 0, 0, e) = p_{21,t},$$

where $p_{21,t}$ is the population of age-21 households in year t. In the benchmark year, which is 2014 in this paper, the growth-adjusted population of age-21 households is normalized to unity, that is, $p_{21,2014} = 1$. Let $x_t(\mathbf{s}')$ be the density function of households corresponding to $X_t(\mathbf{s}')$. Then, the law of motion of the growth-adjusted population distribution is, for $i = 21, \ldots, I$,

$$x_{t+1}(i+1, a', b', e') = \frac{\phi_{i,t}}{1+\nu} \int_{A \times B \times E} \mathbf{1}_{\{a'=a'(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t), \, b'=b'(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t)\}} \pi_i(e' \,|\, e) \, dX_t(\mathbf{s}),$$

where ν is the long-run population growth rate, and $\pi_i(e' \,|\, e)$ is a probability density function of working ability e' at age $i+1$ given that the working ability is e at age i.

2.1.4 The Aggregation

Total private wealth, $W_{P,t}$, the government's net worth, $W_{G,t}$, capital stock, K_t, and labor supply in efficiency units, L_t, are calculated as

$$(5) \quad K_t = \sum_{i=21}^{I} \int_{A \times B \times E} a \, dX_t(\mathbf{s}) + w_{G,t} \sum_{i=21}^{I} p_{i,t},$$

$$(6) \quad L_t = \sum_{i=21}^{I} \int_{A \times B \times E} eh(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t) \, dX_t(\mathbf{s}).$$

2.2 The Firm

In each year, the representative firm chooses the capital input, $\tilde{K}_t$, and efficiency labor input, $\tilde{L}_t$, to maximize its profit, taking factor prices, r_t and w_t, as given. Thus, the firm's optimization problem is

$$(7) \quad \max_{\tilde{K}_t, \tilde{L}_t} F(\tilde{K}_t, \tilde{L}_t) - (r_t + \delta)\tilde{K}_t - w_t \tilde{L}_t,$$

where $F(\cdot)$ is a constant-returns-to-scale production function, $F(\tilde{K}_t, \tilde{L}_t) = A\tilde{K}_t^\theta \tilde{L}_t^{1-\theta}$, with total factor productivity A, and δ is the depreciation rate of capital. The firm's profit-maximizing conditions are

$$(8) \quad F_K(\tilde{K}_t, \tilde{L}_t) = r_t + \delta, \qquad F_L(\tilde{K}_t, \tilde{L}_t) = w_t,$$

and the factor markets are cleared when $K_t = \tilde{K}_t$ and $L_t = \tilde{L}_t$.

2.3 The Government

The government policy schedule, $\boldsymbol{\Psi}_t$, as of year t is assumed to be credible. The government collects taxes and makes its consumption and transfer spending as scheduled in $\boldsymbol{\Psi}_t$. In addition, the government collects wealth left by deceased households (accidental bequests) and distributes it uniformly to all working-age households.

2.3.1 The Government's Revenue and Expenditure

The government's income tax revenue, $T_{I,t}$, payroll tax revenue for Social Security, $T_{P,t}$, and consumption (or other) tax revenue, $T_{C,t}$, are calculated as

$$(9) \quad T_{I,t}(\varphi_t) = \sum_{i=21}^{I} \int_{A \times B \times E} \tau_{I,t}(w_t eh(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t), r_t a, tr_{SS,t}(i, b); \varphi_t) \, dX_t(\mathbf{s}),$$

$$(10) \quad T_{P,t}(\tau_{O,t}, \tau_{D,t}, \tau_{H,t}) = \sum_{i=21}^{I} \int_{A \times B \times E} \tau_{P,t}(w_t eh(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t); \tau_{O,t}, \tau_{D,t}, \tau_{H,t}) \, dX_t(\mathbf{s}),$$

$$(11) \quad T_{C,t}(\tau_{C,t}) = \sum_{i=21}^{I} \int_{A \times B \times E} \tau_{C,t} \, c(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t) \, dX_t(\mathbf{s}),$$

where φ_t is one of the parameters of the income tax function, and $(\tau_{O,t}, \tau_{D,t}, \tau_{H,t})$ is a vector of the OASI, DI, and HI payroll tax rates. The government's consumption spending, $C_{G,t}$, non-Social Security transfer spending, $TR_{LS,t}$, and Social Security transfer spending, $TR_{SS,t}$, are

$$(12) \quad C_{G,t}(c_{G,t}) = c_{G,t} \sum_{i=21}^{I} p_{i,t}, \qquad TR_{LS,t}(tr_{LS,t}) = tr_{LS,t} \sum_{i=21}^{I} p_{i,t},$$

$$(13) \quad TR_{SS,t}(\psi_{O,t}, \psi_{D,t}, \psi_{H,t}) = \sum_{i=21}^{I} \int_{A \times B \times E} tr_{SS,t}(i, b; \psi_{O,t}, \psi_{D,t}, \psi_{H,t}) \, dX_t(\mathbf{s}),$$

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where $\psi_{O,t}$ is a parameter of the OASI benefit function, and $\psi_{D,t}$ and $\psi_{H,t}$ are adjustment factors of DI and HI benefits, respectively.

For simplicity, the government is assumed to collect remaining wealth held by deceased households at the end of year t and distributes it in a lump-sum manner in the same year. Because there are no aggregate shocks in the model economy, the government can perfectly predict the sum of accidental bequests during the year. Suppose that the government redistributes the accidental bequests uniformly to all working-age households.[11] Then, for $i = 21, \ldots, I_R - 1$, the bequest received by each working-age household is

$$(14) \quad q_t = \left(\sum_{i=21}^{I_R-1} \int_{A \times B \times E} dX_t(\mathbf{s}) \right)^{-1} \sum_{i=21}^{I} \int_{A \times B \times E} (1 - \phi_{i,t})(1 + \mu) a'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) \, dX_t(\mathbf{s}).$$

Finally, the government budget satisfies the following constraint,

$$(15) \quad (1 + \mu)(1 + \nu) W_{G,t+1} = (1 + r_t) W_{G,t} + T_{I,t}(\varphi_t) + T_{P,t}(\tau_{O,t}, \tau_{D,t}, \tau_{H,t}) + T_{C,t}(\tau_{C,t})$$
$$- C_{G,t}(c_{G,t}) - TR_{LS,t}(tr_{LS,t}) - TR_{SS,t}(\psi_{O,t}, \psi_{D,t}, \psi_{H,t}).$$

2.4 Social Welfare Measure

Suppose that the economy is in the initial equilibrium in year $t = 0$ and that the government introduces a new policy at the beginning of year 1. Then, the rest-of-the-lifetime value of a household of state $\mathbf{s} = (i, a, b, e)$ is denoted by $v(\mathbf{s}, \mathbf{S}_0; \mathbf{\Psi}_0, \mathbf{\Phi}_0)$ before the policy change and $v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)$ for $t = 1, \ldots, \infty$ after the policy change.

Under the consumption equivalence welfare measure, welfare gains or losses of newborn households (age $i = 21$) at the beginning of $t = 1, \ldots, \infty$ are calculated by uniform percent changes, $\lambda_{21,t}$, in the baseline consumption path that would make their expected lifetime utility equivalent to the expected utility after the policy change, that is,[12]

$$\lambda_{21,t} = \left[\left(\frac{E \, v(\mathbf{s}_{21}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)}{E \, v(\mathbf{s}_{21}, \mathbf{S}_0; \mathbf{\Psi}_0, \mathbf{\Phi}_0)} \right)^{\frac{1}{\alpha(1-\gamma)}} - 1 \right] \times 100.$$

Similarly, the average welfare changes of households of age $i = 22, \ldots, I$ at the time of the policy

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[11]Uniformly distributing the accidental bequests is a common simplification in the literature. Conesa, Kitao, and Krueger (2009) assume that all surviving households receive accidental bequests uniformly. Nishiyama and Smetters (2007) assume that working-age surviving households receive uniform accidental bequests with probability η, where η is the ratio of deceased households to the working-age surviving households.

[12]The calculation of $\lambda_{i,t}$ depends on the period utility function. With the Cobb–Douglas and CRRA utility function, equation (??), when consumption proportionally increases by $\lambda_{i,t}/100$, the household value increases by $(1 + \lambda_{i,t}/100)^{\alpha(1-\gamma)} - 1$.

change ($t = 1$) are calculated by the uniform percent changes, $\lambda_{i,1}$, required in the baseline consumption path so that the rest of the lifetime value would be equal to the rest of the lifetime value after the policy change, that is,

$$\lambda_{i,1} = \left[\left(\frac{E\, v(\mathbf{s}_i, \mathbf{S}_1; \boldsymbol{\Psi}_1, \boldsymbol{\Phi}_1)}{E\, v(\mathbf{s}_i, \mathbf{S}_0; \boldsymbol{\Psi}_0, \boldsymbol{\Phi}_0)} \right)^{\frac{1}{\alpha(1-\gamma)}} - 1 \right] \times 100.$$

Note that $\lambda_{i,1}$ for $i = I, \ldots, 21$ shows the cohort-average welfare changes of all current households alive at the time of the policy change, and $\lambda_{21,t}$ for $t = 2, \ldots, \infty$ shows the cohort-average welfare changes of all future households.

3 Calibration

The current U.S. population is aging, that is, the population distribution is not stationary. So, a stationary equilibrium in an overlapping-generations economy cannot, in general, describe an economy with the aging population. This section, therefore, constructs the aging-population baseline economy as an equilibrium transition path between 1975 and 2200 and calibrates the model to the 2014 U.S. economy such that the model economy in 2014 (on the equilibrium transition path) is consistent with the 2014 U.S. economy in terms of macroeconomic statistics and population distribution. Because the government's fiscal policy is not sustainable under the aging-population assumption, the baseline economy assumes that the government cuts (or increases) its consumption so that the government's debt-per capita stays at the same level after the growth adjustment.[13] Because the government's consumption does not affect the household's decision in this model economy, the aging baseline economy incorporates the effect of the demographic changes without any feedback effects from the government's fiscal policy changes. This paper calls the 2014 economy in the aging baseline economy the benchmark economy and uses it as the benchmark for comparison in the policy experiments.
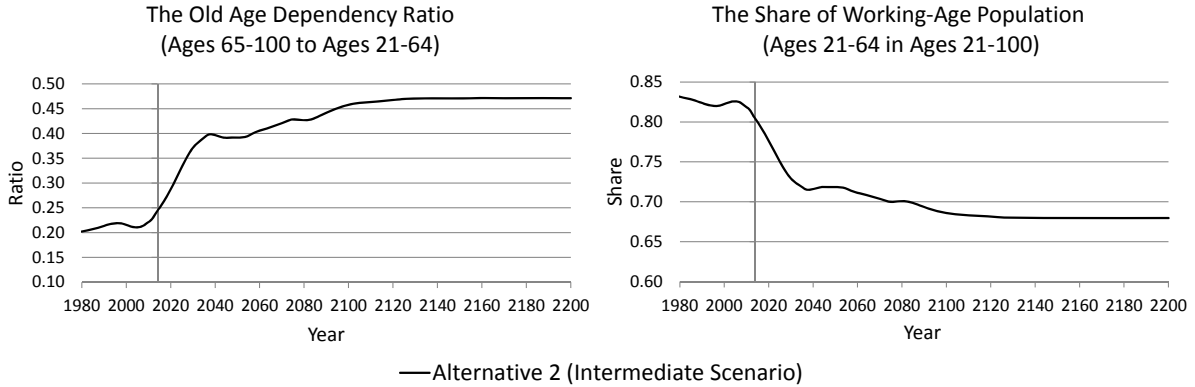
3.1 Population Projection

This paper uses Social Security Administration's population projection Alternative 2 (intermediate scenario) used in the 2014 trustees report. This population projection consist of the historical population by age and gender over 1941–2012 and the projected population by age and gender over 2013–2100.

This paper extrapolates the population and the survival rate by age beyond 2100. In this extrapolation,

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[13]In other words, the government's debt per capita is assumed to grow at the same rate of the labor augmenting productivity growth rate, μ. This assumption assures that the debt-to-GDP ratio stays constant in the long run.

Figure 1: The Old-Age Dependency Ratio and the Share of Working-Age Population

Source: Author's calculations from the population projection used in Social Security Administration (2014).

for simplicity, this paper keeps the survival rate by age at the same level beyond 2099, that is, $\phi_{i,t} = \phi_{i,t-1}$ for $i = 0, \ldots, 100$ and $t = 2100, \ldots, 2200$. The paper also assumes the age-specific fertility rates to be constant beyond 2100. More specifically, this paper estimates the population of age-0 (newborn) babies, $\hat{p}_{0,t}$, for $t = 2101, \ldots, 2200$ recursively as

$$\hat{p}_{0,t} = \zeta_{2100} \left(0.041 \sum_{i=15}^{19} \hat{p}_{i,t}^{f} + 0.103 \sum_{i=20}^{24} \hat{p}_{i,t}^{f} + 0.115 \sum_{i=25}^{29} \hat{p}_{i,t}^{f} + 0.099 \sum_{i=30}^{34} \hat{p}_{i,t}^{f} \right.$$
$$\left. +0.047 \sum_{i=35}^{39} \hat{p}_{i,t}^{f} + 0.010 \sum_{i=40}^{44} \hat{p}_{i,t}^{f} + 0.001 \sum_{i=45}^{49} \hat{p}_{i,t}^{f} \right),$$

where ζ_{2100} is the fertility rate adjustment factor for 2100 such that the above equation holds in 2100, and $\hat{p}_{i,t}^{f}$ is the population of women of age i in year t. The age-specific fertility rates used in the above equation are those for 2008 in United Nations (2013).

Then this paper obtains the long-run population growth rate as $\nu = \sum_{i=0}^{100} \hat{p}_{i,2200} / \sum_{i=0}^{100} \hat{p}_{i,2199} - 1 = 0.003217$, and calculates the growth-adjusted population by age as $p_{i,t} = (1 + \nu)^{-(t-2014)} \hat{p}_{i,t} / \hat{p}_{21,2014}$ for $i = 21, \ldots, 100$ and $t = 1975, \ldots, 2200$.

Figure 1 shows the old-age dependency ratio (the ratio of households ages 65 and older to those ages between 21 and 64) and the share of working age population (the share of households ages between 21 and 64 in those ages 21 and older) between 1980 and 2200. The old-age dependency ratio goes up from 0.244 in 2014 to 0.398 in 2039 and 0.471 in the long run. The increase in this old-age dependency ratio explains the increase in the cost of the Social Security program in the future. The share of working-age population in total adult population goes down from 0.804 in 2014 to 0.715 in 2039 and 0.680 in the long run. The

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decline in the share of working-age population explains the decreases in the labor supply per capita and gross domestic output per capita in the future.

3.2 Calibration Strategy

Because the current U.S. population distribution is aging and thus not stationary, it is impossible to construct the 2014 benchmark economy as a stationary equilibrium. So, this paper constructs the aging-population baseline economy as an equilibrium transition path between 1975 and 2200 by the following procedure.

1. This paper first solves the model for a stationary equilibrium with the population distribution in 1975, where the households in the model economy falsely believe that the population distribution does not change and the economy is stationary, although the model calculates aggregate variables by using the non-stationary 1975 population distribution.

2. This paper next solves the model for a stationary equilibrium with the population distribution in 2200. The population projection in 2200 is constructed to be stationary.

3. This paper then solves the model for an equilibrium transition path between 1975 and 2200 using one of population projections as well as historical population. The households in the model economy realize, at the beginning of 1975, that the population distribution is not stationary, and they precisely project the changes in fertility rates and mortality rates through 2200.

This paper repeats Step 1 through Step 3 to find a set of preference and policy parameters that make the 2014 model economy on the equilibrium transition path consistent with the 2014 U.S. economy in terms macroeconomic and fiscal policy statistics. The current-law fiscal policy is not sustainable in the presence of the aging population. Therefore, as explained above, this paper assumes that the government adjusts its consumption to keep its government's net worth per capita at the 2014 level after the productivity growth adjustment.

Table 1 shows the values of main preference and technology parameters in the aging-population baseline economy over 1975–2200 under SSA's population projection Alternative 2 (intermediate scenario); and Table 2 (on page 17) shows the values of the government policy parameters in the same economy.

3.3 Demographics

The maximum possible age of households in the model economy, I, is 100. This paper calls households of ages 21 to 64 working-age households, and households of ages $I_R = 65$ and older elderly households,

Table 1: Main Parameter Values of the Aging-Population Baseline Economy

Parameter		Value	Comment
Demographics			
Maximum age	I	100	
Maximum age households can work		80	
Minimum age of elderly households	I_R	65	Medicare eligibility age
Productivity growth rate	μ	0.0150	Real GDP per capita growth in 1989–2014
Long-run population growth rate	ν	0.0032	Population growth projected in 2200
Total population (ages 21–120) in 2014		51.297	When $p_{21,t} = 1.0$ in 2014
Working-age population (ages 21–64) in 2014		41.241	
Preferences			
Discount factor	β	1.0084	Target: $K_t/Y_t = 2.92$ in 2014
Growth-adjusted discount factor	$\tilde{\beta}$	0.9872	$\tilde{\beta} = \beta(1+\mu)^{\alpha(1-\gamma)}$
Coefficient of relative risk aversion	γ	3.0000	
Share parameter of consumption	α	0.7149	Target: Frisch elasticity 0.5 in 2014
Production technology and wage process			
Share parameter of capital stock	θ	0.4100	Labor income share 0.59 in 2010–2014
Depreciation rate of capital stock	δ	0.0804	Target: $r_t = 0.06$ in 2014
Total factor productivity	A	0.8798	Target: $w_t = 1.0$ in 2014
Auto correlation parameter of log wage	ρ	0.9500	
Standard deviation of log wage shocks	σ	0.2600	Var. of log earnings in 2013 SCF
Average hourly wage by age	$\bar{e}_i$		Estimated by OLS with 2013 SCF

Note: This baseline calibration uses the population projection Alternative 2 (intermediate scenario) for Social Security Administration (2014).

even though they can possibly work until age 80. When this paper normalizes the population of age-21 households in 2014 to unity, the total population and the working-age population are 51.30 and 41.24, respectively. For simplicity, the total population is that of ages 21 and older in this paper, and it excludes the population of ages between 0 and 20. Households in the model economy are assumed to be a mixture of married (50%) and single (50%) households.[14] The labor-augmenting productivity growth rate, μ, is set at 1.5%, which is close to the average growth rate, 1.48%, of real GDP per capita over the past 25 years, 1989–2014. The long-run population growth rate, ν, is set at 0.3137% as explained above, although the population grows faster in the short and medium run.

3.4 Preferences

The household's period utility function is a combination of Cobb–Douglas and constant relative risk aversion, $u(c, h) = \left[c^\alpha (1-h)^{1-\alpha} \right]^{1-\gamma} / (1-\gamma)$, which is consistent with a growth economy. The coefficient of relative risk aversion, γ, for the combination of consumption and leisure is set at 3.0, which

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[14] According to Table 1 in Vespa, Lewis, and Kreider (2013), which is constructed from the U.S. Census Bureau's 2011 American Community Survey, 48.3% of all households and 51.4% of households ages 25 to 64 are married households in 2011.

is roughly in the middle of the range assumed in macroeconomic public finance literature.[15] The share parameter of consumption, α, is set at 0.7149 so that the Frisch elasticity of working hours of the average household (a household whose working hours are the average of those of all working-age households, $\bar{h} = 0.6183$) is 0.5 in the 2014 benchmark economy, where the Frisch elasticity is calculated as $(1 - \bar{h})\bar{h}^{-1}(1 - \alpha(1 - \gamma))\gamma^{-1} = 0.5$. The discount factor, β, of households is set at 1.0084 so that the capital–output ratio in the 2014 benchmark economy is equal to the ratio, 2.92, in 2014 in the national income and product account (NIPA) data. Then, this paper obtains the growth-adjusted discount factor as $\tilde{\beta} = \beta(1 + \mu)^{\alpha(1-\gamma)} = 0.9872$.

This paper assumes that the household's minimum wealth level depends only on its age, as $a_{\min}(i) = \big[(1+\mu)a_{\min}(i+1) - 0.15\bar{w}e_{\min}(i)h_{\max}\big]/(1+\bar{r})$ if $i = 22, \ldots, I$, and $a_{\min}(i) = 0$ if $i = 21$ or $I+1$, where $e_{\min}(i)$ is the lowest working ability for age i, and where $\bar{r}$, $\bar{w}$, and $\bar{h}$ are the interest rate, the wage rate, and the average working hours, respectively, in the 2014 benchmark economy. One might expect minimum assets to be set at what a household could repay under the worst possible realizations for working ability. However, this paper multiplies labor income with the lowest working ability and average working hours (repaying ability) by 0.15 to align household debt to the U.S. economy. According to the 2013 Survey of Consumer Finances (SCF) (Federal Reserve Board, 2014), 11.3% of households (ages 21-99) have negative net worth with average net debt $32,292, and average debt per household is $3,659 . In the 2014 benchmark economy, 20.2% of households have negative wealth with average net debt $18,257, and average debt per household is $3,688.

3.5 The Production Technology and the Wage Process

The production function of the representative firm is one of Cobb–Douglas, $Y = F(K, L) = AK^\theta L^{1-\theta}$. The share parameter of capital stock, θ, is set equal to 0.41, which reflects 1 minus the average labor income share, $w_t L_t / Y_t = 0.59$, for 2009–2013 in the NIPA data. When the capital–output ratio, K_t/Y_t, is 2.92, the gross rate of return to capital is $r_t + \delta = \theta Y_t/K_t = 0.41/2.92 = 0.1404$ or 14.04%. The depreciation rate of capita, δ, is set at 0.0804 so that the real rate of return to capital, r_t, is 0.06 or 6.0% in the 2014 benchmark economy. Total factor productivity, A, is fixed at 0.8798 so that the average wage rate, w_t, is normalized to unity in the benchmark economy.

The working ability, e_i, of an age i household in the model economy is assumed to satisfy $\ln e_i = \ln \bar{e}_i + \ln z_i$ for $i = 21, \ldots, 80$, where $w_t \bar{e}_i$ is the average wage rate of age i households in year t, and z_i is

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[15]For example, Domeij and Heathcote (2004) use 1.0, İmrohoroğlu, İmrohoroğlu, and Joines (1995) use 2.0, and Auerbach and Kotlikoff (1987) and Conesa, Kitao, and Krueger (2009) use 4.0.

the persistent shock that follows an AR(1) process, $\ln z_i = \rho \ln z_{i-1} + \epsilon_i$ for $i = 21, \ldots, 80$. The temporary shock, ϵ_i, is normally distributed, $\epsilon_i \sim N(-\frac{1}{2}\sigma^2, \sigma^2)$, and the initial distribution of the log-persistent shock satisfies $\ln z_{20} \sim N(-\frac{1}{2}\sigma^2_{\ln z_{20}}, \sigma^2_{\ln z_{20}})$.

The hump-shaped average working ability, $\bar{e}_i$, for ages 21 to 80 is constructed from the 2013 SCF (Federal Reserve Board, 2014). This paper first calculates the unconditional average hourly wage and the average hourly wage given earnings are strictly positive for ages 21 to 80 in the 2013 SCF.[16] Then, this paper estimates the average hourly wage (working ability) by age by using ordinary least squares with the weighted average of the unconditional and conditional average hourly wages. Thus, this paper implicitly assumes that some households do not work because their working ability is very low, but other households, especially elderly households, decide not to work even though their working ability is high. The average of $\bar{e}_i$ for ages 21 to 64 is also normalized to unity in the model economy.

The autocorrelation parameter, ρ, of the log-persistent shock is set at 0.95, which is approximately in the middle of the range in the previous literature.[17] Given the initial variance, $\sigma^2_{\ln z_{20}}$, the variance of log-persistent shock, $\ln z_i$, is calculated recursively as $\sigma^2_{\ln z_i} = \rho^2 \sigma^2_{\ln z_{i-1}} + \sigma^2$ for $i = 21, \ldots, 80$. To align the wage process to the U.S. data, the standard deviation, σ, of the transitory shock, ϵ_i, is set at 0.26, and the initial variance of the log-persistent shock is set at a fraction of its limiting variance, $\sigma^2_{\ln z_{20}} = 0.4 \lim_{i \to \infty} \sigma^2_{\ln z_i} = 0.2773$. The number $0.4 < 1$ is chosen so that the variance of log earnings increases with the age of the household.

This paper uses the Rouwenhorst method (Rouwenhorst, 1995) to discretize the log-persistent shock for each age and construct the Markov transition matrix by age. The Markov transition matrix of an age i household, $\Pi_i = [\pi(e^{j'}_{i+1} \mid e^j_i)]$, that corresponds to $\rho = 0.95$ is obtained as, for $i = 21, \ldots, 79$,

$$\Pi_i = \begin{pmatrix} 0.8591 & 0.1322 & 0.0085 & 0.0003 & 0.0000 & 0.0000 & 0.0000 \\ 0.0220 & 0.8619 & 0.1103 & 0.0057 & 0.0001 & 0.0000 & 0.0000 \\ 0.0006 & 0.0441 & 0.8636 & 0.0883 & 0.0034 & 0.0001 & 0.0000 \\ 0.0000 & 0.0017 & 0.0662 & 0.8642 & 0.0662 & 0.0017 & 0.0000 \\ 0.0000 & 0.0001 & 0.0034 & 0.0883 & 0.8636 & 0.0441 & 0.0006 \\ 0.0000 & 0.0000 & 0.0001 & 0.0057 & 0.1103 & 0.8619 & 0.0220 \\ 0.0000 & 0.0000 & 0.0000 & 0.0003 & 0.0085 & 0.1322 & 0.8591 \end{pmatrix},$$

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[16]To calculate the unconditional average hourly wage, this paper assumes that the hourly wage is zero when the household is not working. So, the unconditional average is the lower bound of the average potential wage, whereas the conditional average is the upper bound of the average potential wage.

[17]For example, Domeij and Heathcote (2004) use 0.90, Huggett (1996) uses 0.96, and Conesa, Kitao, and Krueger (2009) use 0.98.

Table 2: Fiscal Policy Parameter Values of the Aging-Population Baseline Economy

Parameter		Value	Comment
Model units			
Taxable labor income ratio	η	0.7220	SS covered earnings / NIPA labor income in 2013
Scale adjustment[a]		139.193	Average earnings $67,936 in 2014
Progressive income tax			
Income tax: tax rate limit	φ_t	0.3745	Target: $T_{I,t}/Y_t = 0.10$ in 2014
: curvature	φ_1	0.7525	} Estimated by OLS
: scale	φ_2	0.7368	
: deduction/exemptions[b]	d	0.1094	$0.5 \times \$20,300 + 0.5 \times \$10,150$ in 2014
Social Security system			
S.S. payroll tax rate: OASI	$\tau_{O,t}$	0.1007	Target: revenue 3.70% of GDP in 2014
: DI	$\tau_{D,t}$	0.0172	Target: revenue 0.63% of GDP in 2014
: HI	$\tau_{H,t}$	0.0305	Target: revenue 1.32% of GDP in 2014
: HI surtax	$\tau_{HS,t}$	0.0090	Current-law rate
Maximum taxable earnings[c]	$\vartheta_{\max}$	1.0507	$(0.5 \times 1.5 + 0.5)\$117,000$ in 2014
Repl. rate threshold: 0.90 & 0.32[c]	ϑ_1	0.0879	$(0.5 \times 1.5 + 0.5)\$816 \times 12$ in 2014
: 0.32 & 0.15[c]	ϑ_2	0.5299	$(0.5 \times 1.5 + 0.5)\$4,917 \times 12$ in 2014
HI surtax threshold[b]	ϑ_3	1.6165	$0.5 \times \$250,000 + 0.5 \times \$200,000$ in 2014
Benefit adjustment factor: OASI	$\psi_{O,t}$	1.5786	Target: benefits 4.01% of GDP in 2014
: DI	$\psi_{D,t}$	0.0100	Target: benefits 0.84% of GDP in 2014
: HI	$\psi_{H,t}$	0.0779	Target: benefits 1.56% of GDP in 2014
Other policy variables			
Government's consumption per capita	$c_{G,t}$	0.0342	Calculated as a residual
Lump-sum transfers per capita	$tr_{LS,t}$	0.0727	Target: non-SS transfers 7.44% of GDP in 2014
Consumption tax rate	$\tau_{C,t}$	0.0246	Target: $T_{C,t}/Y_t = 0.016$ in 2014
Government's net worth per capita	$w_{G,t}$	0.0293	Target: $W_{G,t}/Y_t = 0.03$ in 2014

Note: This baseline calibration uses the population projection Alternative 2 (intermediate scenario) for Social Security Administration (2014). [a] A unit in the model economy corresponds to $139,193 in 2014 growth-adjusted dollars. [b] 50% of all households are assumed to be married, and the other 50% of households are single. [c] Each married household is assumed to have 1.5 full-time equivalent workers.

and the unconditional probability distribution of the 7 nodes is $\pi_i = (0.0156, 0.0938, 0.2344, 0.3125, 0.2344, 0.0937, 0.0156)$ for $i = 21, \ldots, 80$.

3.6 Taxable Earnings and the Progressive Income Tax Function

According to Table 4.B1 in Social Security Administration (2015), total covered earnings—the tax base of the HI payroll tax—are $7,147 billion in 2013, which are 42.6% of GDP and 72.2% of total labor income in the same year. In this paper, therefore, the ratio of taxable labor income to total labor income, η, is set at 0.722. The average household labor income for ages 21 to 64 is $65,766 in the 2013 SCF (Federal Reserve Board, 2014). Because labor income per capita increased by 3.3% in NIPA between 2013 and 2014, the average labor income is estimated at $67,936 in 2014. In the units employed in the model economy, the

average labor income of households ages 21 to 64 is calculated as 0.6759 in the 2014 benchmark economy. Under this assumption, one model unit for an individual household corresponds to $67,936/(0.6759 \times 0.722) \approx \$139,193$ in 2014 growth-adjusted U.S. dollars. This scale factor converts some of the policy variables under the current law into the model parameters.

The individual income tax function is one of Gouveia and Strauss (1994),

$$\tau_{I,t}(w_t eh, r_t a, tr_{SS,t}(i,b)) = \tilde{\tau}_{I,t}(y) = \varphi_t \left[y - \left(y^{-\varphi_1} + \varphi_2 \right)^{-1/\varphi_1} \right],$$

where y is the household's taxable income after deductions and exemptions. The three parameters, φ_t, φ_1, and φ_2, are set at 0.3745, 0.7525, and 0.7368, respectively, so that the function approximates the progressive income tax schedule in 2014. See Appendix for detailed explanation.

3.7 The Social Security System

The Social Security OASDHI payroll tax function is

$$\tau_{P,t}(w_t eh) = (\tau_{O,t} + \tau_{D,t}) \min(\eta \cdot w_t eh, \vartheta_{\max}) + \tau_{H,t}\, \eta \cdot w_t eh + \tau_{HS,t} \max(\eta \cdot w_t eh - \vartheta_3, 0),$$

where $\tau_{O,t}$ is the flat Old-Age and Survivors Insurance (OASI) tax rate, $\tau_{D,t}$ is the flat Disability Insurance (DI) tax rate, $\tau_{H,t}$ is a flat Hospital Insurance (HI, Medicare Part A) tax rate, and $\tau_{HS,t}$ is a flat HI surtax rate. These payroll tax rates include the portion of taxes paid by employers. The payroll tax revenues for the OASI, DI, and HI programs are 3.70%, 0.63%, and 1.32%, respectively, as a share of GDP in 2013, according to Tables 4.A1, 4.A2, and 8.A1 in Social Security Administration (2014). This paper sets $\tau_{HS,t}$ at 0.009, which is the statutory HI surtax rate. Then, this paper sets $\tau_{O,t}$, $\tau_{D,t}$, and $\tau_{H,t}$ at 0.1007, 0.0172, and 0.0305, respectively, so that the payroll tax revenues for the OASI, DI, and HI programs as a share of GDP in the 2014 benchmark economy are consistent with the data in 2013.[18]

The maximum taxable earnings per worker for the OASDI payroll tax were $117,000 in 2014 (SSA, 2015). This paper assumes that 50% of households are married households, and each of those household has 1.5 full-time equivalent workers. Thus, this paper sets the maximum taxable earnings, $\vartheta_{\max}$, as the weighted average of 1.5 (married households) and 1.0 (single households). In the 2014 benchmark economy, the maximum taxable earnings for a household are at 1.0507 in model units, which correspond to $(0.5 \times$

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[18]When labor income is below the maximum taxable earnings, the statutory OASI tax rate is 10.6%, including 5.3% paid by employers. The effective OASI tax rate is, therefore, $0.106/1.053 = 0.1007$ or 10.07%, which is almost equal to the effective OASI tax rate in the 2014 benchmark economy. The statutory DI and HI tax rates are 1.8% and 2.9%, respectively. The effective HI tax rate in the benchmark economy is higher than the statutory rate because the model cannot capture the revenue from the HI surtax for high-income workers perfectly.

$1.5 + 0.5 \times 1.0)\$117{,}000 = \$146{,}250$ per household in 2014. The threshold for the HI surtax is currently $\$250{,}000$ for a married household and $\$200{,}000$ for a single household. In the benchmark economy, the surtax threshold, ϑ_3, is at 1.6164 in model units, which corresponds to $0.5 \times \$250{,}000 + 0.5 \times \$200{,}000 = \$225{,}000$ per household in 2014.

The Social Security OASDHI benefit function is

$$
\begin{aligned}
tr_{SS,t}(i,b) = {} & \mathbf{1}_{\{i \geq I_{\tilde{R}}\}} \psi_{O,t} \frac{1}{(1+\mu)^{i-60}} \big\{ 0.90 \min(b, \vartheta_1) + 0.32 \max\left[\min(b, \vartheta_2) - \vartheta_1, 0\right] \\
& + 0.15 \max(b - \vartheta_2, 0) \big\} + \mathbf{1}_{\{i < I_{\tilde{R}}\}} \psi_{D,t} \cdot \kappa_{D,i} + \mathbf{1}_{\{i \geq I_R\}} \psi_{H,t} \cdot \kappa_{H,i},
\end{aligned}
$$

where $I_{\tilde{R}}$ is the full retirement age and set between ages 65 and 67 in two-month increments, depending on the household's birth year; I_R is the Medicare eligibility age (age 65); ϑ_1 and ϑ_2 are the thresholds for the three replacement rate brackets, 90%, 32%, and 15%, that calculate the OASI benefit from the average historical earnings; $\psi_{O,t}$ is the adjustment factor of the OASI benefits; $\kappa_{D,i}$ is the ratio of the DI eligibility rate for age i to the average DI eligibility rate; $\psi_{D,t}$ is the average DI benefit in year t; $\kappa_{H,i}$ is the ratio of the average HI benefit (cost) for age i to the average HI benefit for all beneficiaries; and $\psi_{H,t}$ is the average HI benefit in year t.

The current U.S. Social Security system determines the thresholds to calculate primary insurance amounts for each age cohort when a worker reaches age 62. This paper fixes the growth-adjusted thresholds at the same level for all age cohorts and adjusts the PIA later by using the long-term productivity growth rate and years after age 60. Thus, the model simply uses the thresholds for the age 62 cohort in 2014 after scale adjustment.[19] This paper estimates the ratio of the DI eligibility rate by age to the overall average DI eligibility rate (the likelihood of receiving DI benefits by age), $\kappa_{D,i}$, by OLS to the distribution of DI benefits by age. Numan, Cubanski, and Damico (2015) estimate the average cost of HI benefits by age in 2011. Using their estimates, this paper also calculates $\kappa_{H,i}$ as the ratio of the average HI benefit by age to the overall average HI benefit.

The benefit expenditures for the OASI, DI, and HI programs are 4.01%, 0.84%, and 1.56%, respectively, as a share of GDP in 2013, according to Tables 4.A1, 4.A2, and 8.A1 in Social Security Administration (2014). This paper, therfore, sets $\psi_{O,t}$, $\psi_{D,t}$, and $\psi_{H,t}$ at 1.5786, 0.0100, and 0.0779, respectively, so that the benefit expenditures for the OASI, DI, and HI programs as a share of GDP in the 2014 benchmark

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[19]Under the current system, AIMEs for those ages 60 and older are not wage-indexed but price-indexed, and the bracket thresholds are also price-indexed for each age cohort. To simplify the computation in the growth economy, the model first assumes all of the above variables are wage-indexed. Then, the model converts the Social Security benefits to be price-indexed by dividing the benefits by $(1+\mu)^{i-60}$.

economy are consistent with the data in 2013.[20]

3.8 The Other Policy Variables

The government's (beginning-of-year) debt held by the public is 70.9% as a share of GDP in 2014, according to the Treasury Department and NIPA. The government's (beginning-of-year) capital stock is 74.0% as a share of GDP in the same year, according to NIPA. This paper, for simplicity, assumes that the government's capital stock and debt held by the public cancel out to each other, and the model uses the government's net worth per household, $w_{G,t}$, as one of its policy variables. In the 2014 benchmark economy, the government's (beginning-of-year) net worth is assumed to be 3.0% of GDP.

The government's non-Social Security (OASDHI) transfer spending is 7.44% as a share of GDP in 2013, according to NIPA and Social Security Administration (2015). In the 2014 benchmark economy, lump-sum transfers per household, $tr_{LS,t}$, is assumed to be 7.44% of GDP. The government's consumption spending is 5.7% as a share of GDP in 2013, according to NIPA. In the 2014 benchmark economy, however, it is calculated as a residual for each year so that the government's net worth (capital stock net of debt) per household stays at the same level after the productivity-growth adjustment. So, it is assumed to be 3.5% of GDP in the 2014 benchmark economy, which is smaller than 5.7% in NIPA.[21] The rate of the national consumption tax, which approximates the excise tax and other taxes, is set at $\tau_{C,t} = 0.0246$ or 2.46% so that the consumption tax revenue is 1.6% of GDP in the 2014 benchmark economy.

4 The Aging Baseline Economies

This section shows the aging-population baseline economy with SSA's population projection Alternative 2 (intermediate scenario). This baseline economy assumes that the government cuts its consumption spending to keep its growth-adjusted net worth per household at the same level. In this paper, therefore, this baseline economy does not consider any feedbacks effects of the government's policy changes on individual behaviors and the overall economy. This section also explains the fiscal gap in the aging-population baseline economy. The fiscal gap shows how much the government will have to reduce its primary deficits in the future, relative to the deficits under the current-law fiscal policies, in order to make the government budget

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[20]There is no disability risk or health risk in the model economy, for simplicity. DI benefits and HI benefits are age dependent but uniformly distributed for each age. So, the model implicitly assumes that the government transfers the actuarially-fair insurance premiums for DI and HI, rather than realized insurance benefits, to those households.

[21]In other words, there is a fiscal gap as large as 2.2% of GDP in 2014 before assuming any demographic changes in the future. For simplicity, however, this paper only considers the fiscal gap caused by the projected aging population, which is around 3.0% of GDP under the population projection alternative 2. This existing fiscal gap, 2.2%, is relatively large partly because the model assumes that the government's capital stock and its debt change in parallel so that its net worth stays at the same level.

sustainable.

4.1 The Aging-Population Baseline Relative to the 2014 Benchmark Economy

Figure 2 shows the aging-population baseline economy for the years from 1980 to 2200. Each panel of the figure shows the changes from the 2014 benchmark economy, which year is indicated by the vertical line in each panel. In other words, the horizontal line at 0.0 indicates the balanced growth path that goes through the 2014 economy in the absence of the aging population; and each panel shows the effect of the aging population (demographic changes) under the current-law (2014 benchmark) fiscal policies.

According to the model, if fiscal policies had been unchanged at the 2014 benchmark levels, labor supply per capita would have increased since the mid-1980s and peaked around 2006–2007. The years from 1980 to 2006 coincide with the period when the baby boomers had a central role in the U.S. labor market. The baby boomers started retiring in 2008, and the proportion of working-age households and labor supply per capita would have declined since then and will continue declining over time.[22] This is the direct effect of the aging population on the U.S. economy. Labor supply per capita will be 8.1% lower in 2039 and 11.9% lower in the long run relative to the economy without the aging population (the 2014 benchmark economy).

It is, in general, difficult to predict the economy's capital stock, because it depends not only on the households' saving decision but also on the government's debt in the future. In this paper, however, the government's net worth per capita is kept constant, after the productivity-growth adjustment, at the 2014 benchmark level. If the government cuts its consumption spending—in other words, if no additional budgetary feedbacks to the household's budget are considered—capital stock per capita will be 1.5% lower in 2039 and 0.1% higher in the long run relative to the 2014 benchmark economy. The capital stock will increase slightly in the long run, because the households will have to accumulate larger wealth for their retirement if life expectancy is higher and the government's fiscal policies are not changed.
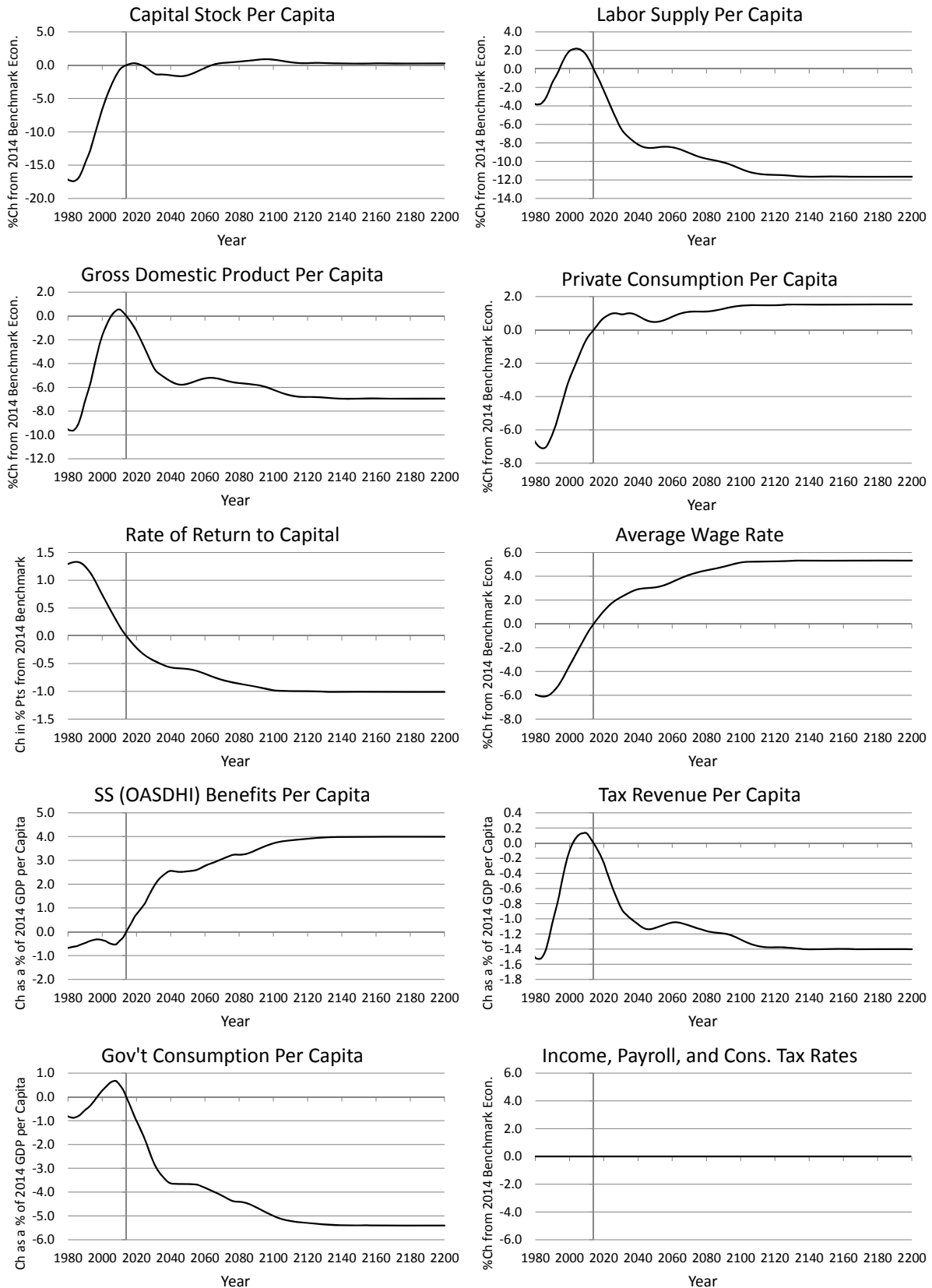
In the aging baseline economy, other things being equal, GDP per capita would have increased since the early-1980s and peaked around 2010; then, it will likely decrease over time. The potential GDP per capita growth rate would have been higher than the long-run productivity growth rate until 2010 and will be lower than the long-run rate,at least, until the mid-2040s, because of the change in the share of the working-age households. GDP per capita will be 5.4% lower in 2039 and 7.2% lower in the long run with the aging population if the budgetary costs of aging are financed by cuts in government consumption.

Because of the decrease in labor supply per capita and GDP per capita, the disposable income of house-

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[22]In this paper, percent changes in labor supply per capita are equal to percent changes in labor supply per household but not equal to percent changes in labor supply per worker or working-age household. Labor supply per working-age household will increase as households expect to live longer in the aging baseline economy.

Figure 2: The Aging-Population Baseline Economy Relative to the 2014 Benchmark Economy

holds will also decrease, on average, by 2.0% in 2039 and 2.3% in the long run. However, the private saving rate will also decrease, on average, by 2.6 percentage points in 2039 and 3.4 percentage points in the long run. In the aging baseline economy, capital stock per capita will increase slightly in the long run, although the household's average saving rate falls. This is because the the private saving rate is above its steady-state level in the 2014 benchmark economy, and it gradually decreases to the steady-state level in the long run. Private consumption per capita, therefore, will increase modestly by 0.9% in 2039 and 1.4% in the long run.

Though capital stock per capita will stay around the 2014 benchmark level after productivity-growth adjustment, labor supply per capita will decrease significantly. Thus, the rate of return to capital will fall gradually by 0.56% in 2039 and 1.02% in the long run, and the average wage rate will increase by 2.9% in 2039 and 5.4% in the long run.

The aging population will significantly affect the government budget in the future. If the government's fiscal policy is unchanged, except for the size of the government consumption that, by assumption, does not affect the households' behavior, the government's Social Security (OASDHI) benefit expenditure will grow rapidly by 2.5% as a share of GDP in 2039 and by 4.1% as a share of GDP in the long run. To make things worse, the government's revenue from taxes (the income tax, the payroll tax, and the consumption tax) will decrease by 1.1% as a share of GDP in 2039 and 1.4% as a share of GDP in the long run.

In the aging-population baseline economy, accordingly, the government will have to decrease its consumption spending as a share of GDP by 3.6 percentage points in 2039 and by 5.5 percentage points in the long run. It is obviously unrealistic to assume that the government will be able to cut its consumption by these percentage points, as the government's consumption is 5.7% of GDP in 2013 and 3.5% of GDP in the 2014 benchmark economy. In Section 5, therefore, this paper introduces alternative fiscal policy changes—raising taxes or cutting Social Security benefits—to the aging-population baseline economy and analyzes the overall effect of the aging population on the economy.

4.2   The Fiscal Gaps of the Current Government Policies

The fiscal gap of the government's current-law fiscal policies is the present discount sum of the government's primary deficits from the present year through the infinite future. Economists often show the fiscal gap as a percentage of the present discount sum of GDP over the same period. Table 3 shows the fiscal gaps—total and by the government's program— in the aging-population baseline economy. When the model uses the realized real interest rate, r_t, over the transition path, the total fiscal gap is 2.92% as a share of GDP. This number 2.92% indicates that the government will have to reduce its primary deficits by 2.92% of GDP

Table 3: The Fiscal Gap in the Aging-Population Baseline Economy

	GDP	Total	Social Security				Rest of the
			Total	OASI	DI	HI	government
Discount rate $= r_t$							
Present discount value	1456.7	42.53	45.54	26.98	2.99	15.57	-3.01
Fiscal gap (%GDP)		2.92	3.13	1.85	0.20	1.07	-0.21
Discount rate $= 6.0\%$							
Present discount value	1278.7	34.37	37.48	22.14	2.63	12.71	-3.11
Fiscal gap (%GDP)		2.69	2.93	1.73	0.21	0.99	-0.24

Note: The present discount values are in model units as of the beginning of 2014.

every year from the present year (2014) through the infinite future in order to make the policy sustainable.[23] The fiscal gap of the Social Security (OASDHI) programs is 3.13% as a share of GDP, and the fiscal gap of the rest of the government budget is –0.21% . In this paper, the fiscal gap of the rest of the government is negative because the model constructs the government's fiscal policies in the 2014 benchmark economy to be sustainable in the absence of the aging population.

In the aging-population baseline economy, when the government's growth-adjusted consumption per household is fixed at the 2014 level, the primary deficits as a share of GDP will increase over time. Therefore, the fiscal gap measure is sensitive to the choice of the discount rate to calculate the present discount values. If the discount rate is fixed at the real interest rate, 6.0%, in the 2014 benchmark economy, the fiscal gap will be smaller. The total fiscal gap in the aging-population baseline economy is 2.69% as a share of GDP.

5 Financing the Aging Population

Section 4 calibrates the model to the U.S. economy with the aging population. The projected aging population will affect individual households through the changes in their mortality rates and factor prices in the economy; the overall economy (per household) through the decrease in the share of the working-age households as well as the population growth rate; and the government budget through the decrease in tax revenue and the increase in Social Security benefit spending. In the aging-population baseline economy, however, the government is assumed to balance its budget, thus stabilize the debt-to-GDP ratio, by cutting government consumption without affecting the rest of the economy. In this model economy, the government consumption is not in the household's utility function or its budget constraint. Therefore, the aging baseline

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[23]In 2014, the government's income tax revenue is about 10.0% of GDP, and the Social Security payroll tax revenue is 5.65% of GDP. Cutting primary deficits every year by 2.92% of GDP is not an easy policy change for the government, and it will generate significant effects on individual households and the overall economy, which will be discussed in Section 5.

Figure 3: The Assumption for the Policy Experiments: Recreating the Fiscal Gap

Government Consumption
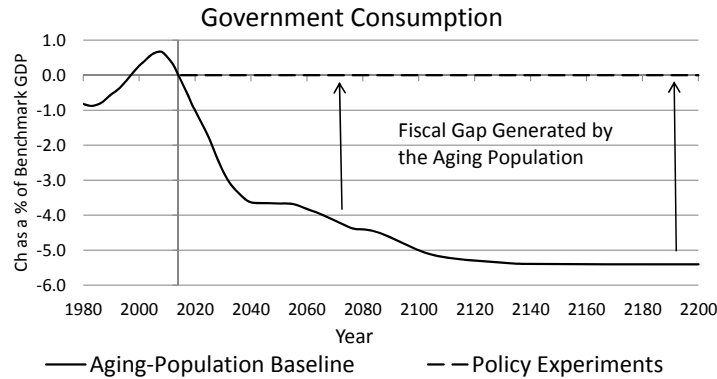
economy shows the effect of the demographic changes projected in the United States without considering its budgetary feedback (the effect from the government's future policy changes).

This section, more realistically, assumes that the government keeps its consumption spending per household between 2014 and 2200 at the 2014 benchmark level instead of cutting it to balance the budget (See Figure 3). This assumption creates a fiscal imbalance each year in 2014–2200 or, equivalently, creates a fiscal gap in the current fiscal policy. The government then introduces an alternative fiscal policy to close this fiscal gap and make the fiscal policy sustainable. The alternative fiscal policies introduced in this section are

- raising progressive income tax rates proportionally,

- raising Social Security (OASI, DI, HI) payroll tax rates independently,

- cutting Social Security (OASI, DI, HI) benefits independently, and

to keep the government's net worth per household at the 2014 level after the growth adjustment. Under each of the above policy assumptions, this paper solves the model with the aging population for an equilibrium transition path in 2014–2200. Thus, each policy experiment shows the effect of the aging population on the economy with the additional budgetary feedback through a specific policy change to close the fiscal gap.

Figure 4 shows the macroeconomic and budgetary effects of these policy changes, and Figure 5 shows the welfare effect of the same policy changes. For easier comparison, the figures also include the results in the aging-population baseline economy, where the government cuts its consumption each year to close the fiscal gap caused by the demographic changes.

Figure 4: The Macroeconomic and Budgetary Effects of the Aging Population (Changes from the 2014 Benchmark Economy)
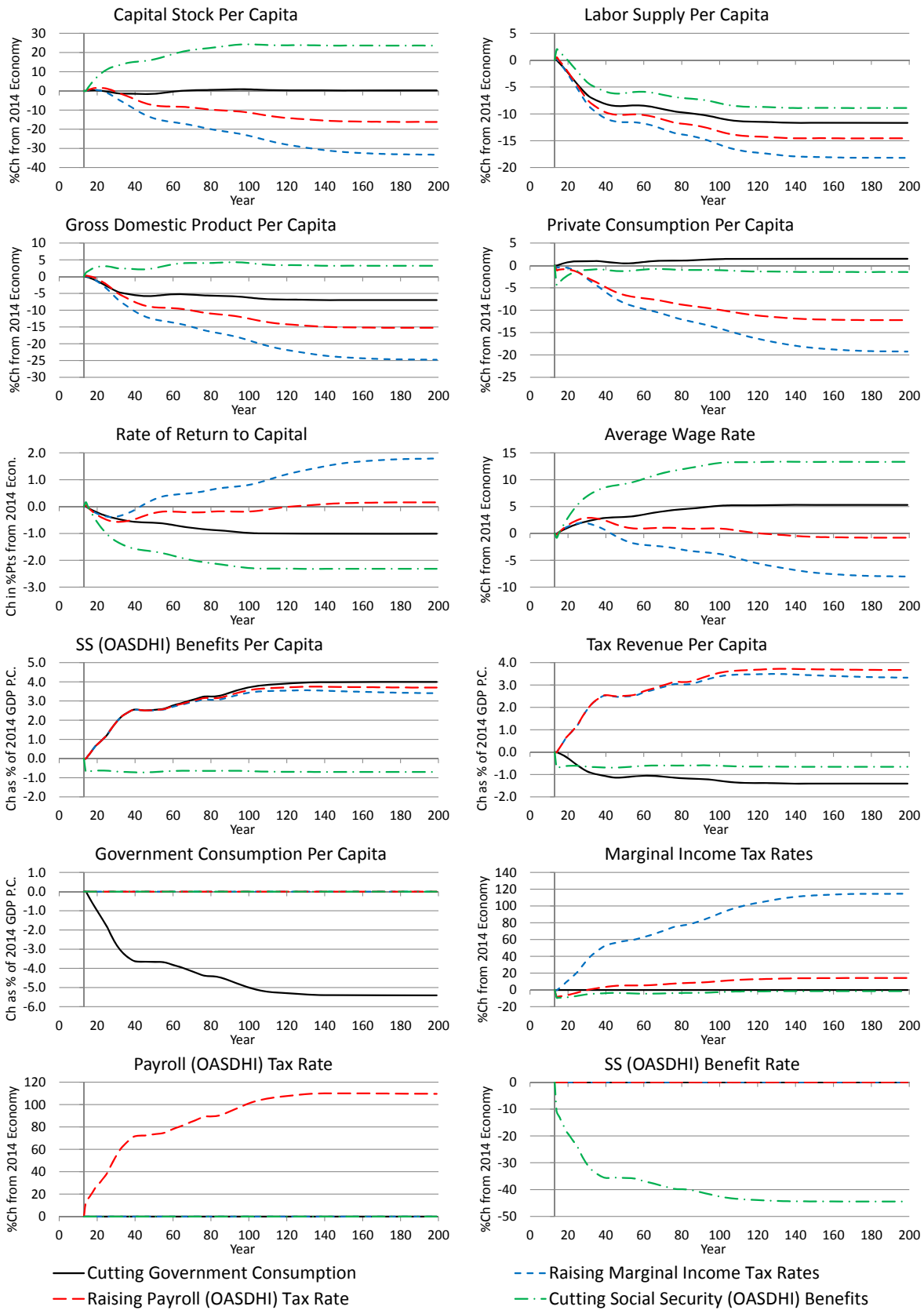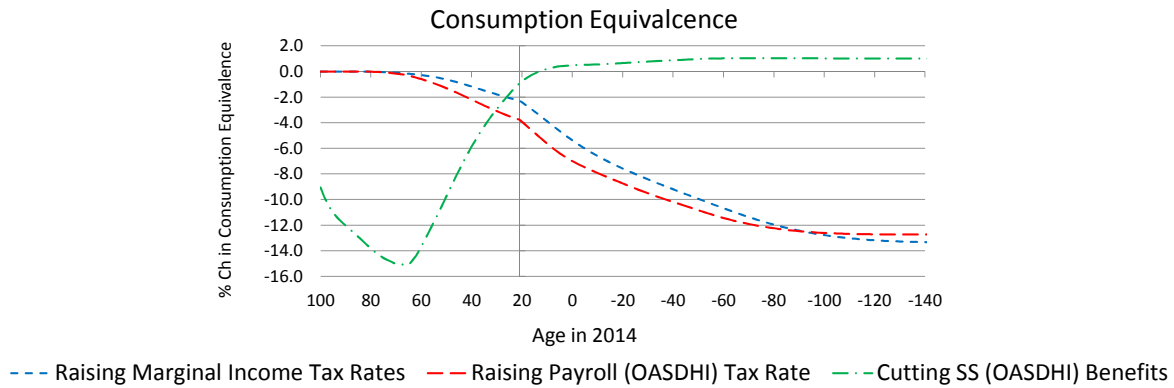

Cutting Government Consumption
Raising Payroll (OASDHI) Tax Rate
Raising Marginal Income Tax Rates
Cutting Social Security (OASDHI) Benefits

Figure 5: The Welfare Effects of the Aging Population (Changes from the Aging-Population Baseline Economy)

5.1 Raising Marginal Income Tax Rates Proportionally

Blue dashed lines in Figure 4 show the macroeconomic effects when the government closes the fiscal gap by raising marginal income tax rates proportionally instead of cutting its consumption as in the aging-population baseline economy. Raising marginal income tax rates proportionally will reduce the household's labor supply, relative to the aging baseline economy, as the substitution effect of the tax rate change is larger than the income effect of the tax payment increase. The policy change will also decrease the household's saving. Relative to the 2014 benchmark economy, labor supply per capita will decrease by 10.7% in 2039 and 18.2% in the long run. Capital stock per capita will decrease by 9.0% in 2039 and 33.3% in the long run. The negative effect of the policy change on capital stock is especially large because the policy change hits high-income households, who have the highest saving rate, most severely. Accordingly, GDP per capita will decrease by 10.0% in 2039 and 24.7% in the long run. Thus, closing the entire fiscal gap by increasing marginal income tax rates proportionally intensifies the negative effect of the aging population on the overall economy by forcing households work less and save less.

To close the fiscal gap, under this assumption, the government will have to raise marginal income tax rates proportionally by 52.0% in 2039 and 114.7% in the long run.[24] This policy change will decrease the household's disposable income and consumption significantly. Relative to the 2014 benchmark economy, private consumption per capita will decrease by 5.7% in 2039 and 19.2% in the long run. Blue dashed lines in Figure 5 show the welfare changes by age cohort. Not surprisingly, raising marginal income tax rates will decrease the household's average consumption significantly, thus the policy change will make most age cohorts, excluding very old households at the time of the policy change, on average worse off. What might

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[24]This means that, the marginal tax rate of 15% in 2014 will have to increase to 22.8% in 2039 and 32.2% in the long run, and the tax rate of 35% will be 53.2% in 2039 and 75.1% in the long run.

be surprising is that, for the current and near-future households, their welfare loss is the smallest of the three policy changes discussed in this section, even though its effect on the overall economy is by far the worst of the three. This is because raising marginal income tax rates proportionally does not hurt low-income and retired households very much and keeps the risk-sharing effect of the current government policy.

5.2 Raising Social Security Payroll Taxes Proportionally

Red long-dashed lines in Figures 4 show the macroeconomic effects when the government closes the fiscal gap by increasing Social Security (OASI, DI, HI) payroll tax rates independently to balance each of the three budgets and adjusting the marginal income tax rates to balance the rest of the government budget. Raising Social Security payroll tax rates will generate a similar effect on the household's labor supply to that of increasing marginal income tax rates. Since payroll tax rates are flat rather than progressive, however, raising these tax rates will not decrease the overall labor supply as much as increasing marginal income tax rates proportionally. As there is no change in the tax rate on capital income, raising payroll tax rates will not also discourage the household's saving by itself. Relative to the 2014 benchmark economy, labor supply per capita will decrease by 9.6% in 2039 and 14.5% in the long run. Capital stock per capita will decrease by 4.0% in 2039 and 16.2% in the long run. Accordingly, GDP per capita will decrease by 7.3% in 2039 and 15.2% in the long run. Thus, closing the fiscal gap primarily by raising Social Security payroll tax rates exacerbates the negative effect of the aging population on the overall economy, but this policy is less harmful than closing the fiscal gap solely by increasing marginal income tax rates.

To balance the Social Security budget, under this assumption, the government will have to raise the OASI payroll tax rate (below the maximum taxable earnings) by 66.9% in 2039 and100.4% in the long run, the DI payroll tax rate by 32.8% in 2039 and 42.6% in the long run, and the HI payroll tax rate by 106.8% in 2039 and 177.6% in the long run. The OASDHI payroll tax rate, in total, will have to increase by 71.2% in 2039 and 109.7% in the long run. To balance the rest of the government budget, the government will also have to raise the marginal income tax rates by 3.3% in 2039 and 14.3% in the long run. Relative to the 2014 benchmark economy, private consumption per capita will decrease, on average, by 4.6% in 2039 and 12.2% in the long run. Red long-dashed lines in Figure 5 show the welfare changes by age cohort under this assumption. Raising Social Security payroll tax rates will hurt all age cohorts except for the elderly households at the time of the policy change. The welfare loss by age cohort is similar to that when the government increases marginal income tax rates. Therefore, most current households prefer raising Social Security payroll tax rates to other stylized policy changes discussed in this section. However, this policy change will affect the macro economy significantly and hit future households most severely among all policy

changes.

5.3 Cutting Social Security Benefits Proportionally

Green dot-dash lines in Figure 4 show the macroeconomic effects when the government closes the fiscal gap by cutting Social Security (OASI, DI, HI) benefits independently to balance each of the three budgets and adjusting the marginal income tax rates to balance the rest of the government budget. Cutting Social Security benefits will generate a very large effect on the household's life cycle saving for their retirement and a modest effect on their precautionary saving. To accumulate higher wealth before retirement, relative to the aging baseline economy, the household will work longer and increase their life cycle and precautionary savings. Relative to the 2014 benchmark economy, labor supply per capita will decrease by 5.7% in 2039 and 8.9% in the long run. Capital stock per capita will increase by 15.1% in 2039 and 23.6% in the long run. Accordingly, GDP per capita will increase by 2.3% in 2039 and 3.3% in the long run. Thus, closing the fiscal gap by cutting Social Security benefits actually increases the output and capital stock per capita in the economy despite that labor supply per capita decreases because of the demographic change.

To balance the Social Security budget, under this assumption, the government will have to cut the OASI replacement rates proportionally by 35.2% in 2039 and 45.9% in the long run, the DI benefits by 17.8% in 2039 and 16.9% in the long run, and the HI benefits by 46.0% in 2039 and 55.4% in the long run. On weighted average, the OASDHI benefits will have to decrease by 35.5% in 2039 and 44.4% in the long run. To balance the rest of the government budget, however, the government will be able to reduce the marginal income tax rates by 3.9% in 2039 and 1.4% in the long run. Cutting Social Security benefits will decrease the household's disposable income and consumption as well. Relative to the 2014 benchmark economy, private consumption per capita will decrease, on average, by 0.9% in 2039 and 1.4% in the long run. Green dot-dashed lines in Figure 5 show the welfare changes by age cohort under this assumption. Cutting Social Security benefits will hurt the current elderly and working-age households, especially those near the retirement age, seriously. In the consumption equivalence measure, households age 66 at the time of the policy change will be worse off, on average, by 15.1% of their (rest of the) lifetime consumption. Because of that, cutting Social Security benefits will be politically difficult to introduce, although this policy change will generate strong positive effects on the economy and will be least harmful for most future age cohorts.

6 Concluding Remarks

This paper incorporates historical and projected demographic changes, provided by the U.S. Social Security Administration, to a heterogeneous-agent OLG model with uninsurable wage and mortality shocks. The paper analyzes the general-equilibrium effects of the aging population on the behavior and well-being of heterogeneous households, the government's budget, and the aggregate economy over 2014–2200. The projected improvement in longevity will make working-age households work longer and save more to prepare for their retirement. However, the decreasing share of the working-age households will significantly decrease the aggregate labor supply and GDP relative to the benchmark economy without the aging population. These changes in macroeconomic variables will also decrease the government's tax revenue and increase its transfer spending. Thus, the aging population will make the current-law fiscal policy unsustainable and force the government to change its policy eventually.

This paper has two main contributions to the literature. First, this paper incorporates the SSA's population projection directly to a general-equilibrium OLG model so that the model can capture the effects of retiring the current baby-boom generation, declining mortality rates, and stable but relatively low fertility rates at the same time. Second, this paper uses an incomplete-market OLG model with idiosyncratic wage shocks so that the model can appropriately replicate the government's fiscal policy tools—the progressive income tax and the progressive Social Security system—and analyze the policy effects through inter-generational and intra-generational income redistribution and risk sharing. One of the difficulties to incorporate historical and projected population distributions to an incomplete-maket model is that the current population distribution is not stationary and, thus, a stationary equilibrium cannot describe the current economy. To deal with this problem, this paper creates the 2014 benchmark economy by solving the model for an equilibrium transition path between 1975 and 2200.

This paper then introduces three stylized fiscal policy changes to finance the budgetary cost of the aging population in the future. This paper intentionally chooses simple stylized policy changes to emphasize to what extent the effect of the aging population will differ depending on how the government closes the fiscal gap. Updating the model used in this paper, we can easily analyze the effects of more complex policy changes, such as a deficit-financing policy change, a phased-in policy change, or more specifically, an increase in the full retirement age, an increase in the maximum taxable earnings for the OASDI payroll tax, and a means-tested OASI benefits. The model and the procedure demonstrated in this paper will work as the platform of the future fiscal policy analyses on the aging population and help policymakers by providing both macroeconomic and distributional effects of their reform plans.

A Recursive Competitive Equilibrium

A recursive competitive equilibrium for the model economy is defined as follows:

Definition of Recursive Competitive Equilibrium: Let $\mathbf{s} = (i, a, b, e)$ be the individual state of households, let $\mathbf{S}_t = (X_t(\mathbf{s}), w_{G,t})$ be the state of the economy in year t, let $\boldsymbol{\Psi}_t$ be the government policy schedule committed at the beginning of year t,

$$\boldsymbol{\Psi}_t = \left\{ c_{G,s}, tr_{LS,s}, \tau_{I,s}(\cdot), \tau_{P,s}(\cdot), tr_{SS,s}(\cdot), \tau_{C,s}, w_{G,s+1} \right\}_{s=t}^{\infty},$$

and let $\boldsymbol{\Phi}_t = \left\{ (p_{i,s})_{i=0}^I, (\phi_{i,s})_{i=0}^I \right\}_{s=t}^{\infty}$ be the population projection and corresponding survival rates as of the beginning of year t. A time series of factor prices and the government policy variables,

$$\boldsymbol{\Omega}_t = \left\{ r_s, w_s, q_s, c_{G,s}, tr_{LS,s}, \varphi_s, (\tau_{O,s}, \tau_{D,s}, \tau_{H,s}), (\psi_{O,s}, \psi_{D,s}, \psi_{H,s}), \tau_{C,s}, w_{G,s+1} \right\}_{s=t}^{\infty},$$

the value functions of households, $\{v(\mathbf{s}, \mathbf{S}_s; \boldsymbol{\Psi}_s, \boldsymbol{\Phi}_s)\}_{s=t}^{\infty}$, the decision rules of households,

$$\mathbf{d}(\mathbf{s}, \mathbf{S}_s; \boldsymbol{\Psi}_s, \boldsymbol{\Phi}_s) = \left\{ c(\mathbf{s}, \mathbf{S}_s; \boldsymbol{\Psi}_s, \boldsymbol{\Phi}_s), h(\mathbf{s}, \mathbf{S}_s; \boldsymbol{\Psi}_s, \boldsymbol{\Phi}_s), a'(\mathbf{s}, \mathbf{S}_s; \boldsymbol{\Psi}_s, \boldsymbol{\Phi}_s) \right\}_{s=t}^{\infty},$$

and the distribution of households, $\{X_s(\mathbf{s})\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, for all $s = t, \ldots, \infty$, each household solves the optimization problem of equations (1) through (4), taking $\mathbf{S}_s$, $\boldsymbol{\Psi}_s$, and $\boldsymbol{\Phi}_s$ as given; the firm solves its profit maximization problem of equations (7) and (8); the government policy schedule satisfies equations (9) through (15); and the goods and factor markets clear. The economy is in a stationary (steady-state) equilibrium—thus, on the balanced-growth path—if in addition, $\mathbf{S}_s = \mathbf{S}_{s+1}$, $\boldsymbol{\Psi}_{s+1} = \boldsymbol{\Psi}_s$, and $\boldsymbol{\Phi}_{s+1} = \boldsymbol{\Phi}_s$ for all $s = t, \ldots, \infty$.[25]

B The Progressive Income Tax Function

The individual income tax function is one of Gouveia and Strauss (1994),

$$\tau_{I,t}(w_t eh, r_t a, tr_{SS,t}(i,b)) = \varphi_t \left[y - \left(y^{-\varphi_1} + \varphi_2 \right)^{-1/\varphi_1} \right],$$

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[25]To solve the model for a stationary equilibrium, the population distribution needs to be constant, after growth adjustment, in the long run. Even if this paper assumes the fertility rates to be constant after 2100, it takes several hundred years before the population distribution completely stabilizes because the number of newborn babies depends on the population distribution of women ages 15–50.

where $y = \max\{r_t a + \eta\, w_t eh + r_t a + \zeta(\eta\, w_t eh + r_t a, tr_{SS,t}(i, b)) - d, 0\}$ is the household's taxable income with constant deductions and exemptions d, and $\zeta(\cdot)$ calculates the taxable portion of Social Security benefits.[26]

The first parameter, φ_t, of the Gouveia–Strauss function shows the limit of the effective marginal income tax rate as taxable income goes to infinity. This paper initially sets this first parameter at 0.396 or 39.6%, which is the marginal income tax rate for the highest tax bracket. Then, this paper estimates the other two parameters, φ_1 and φ_2, by OLS with the 2014 statutory marginal tax rate schedule, assuming 50% of households are married filing jointly and 50% of households are single. The second parameter, $\varphi_1 = 0.7525$, shows the curvature of the tax function; and the third parameter, $\varphi_2 = 0.7368$, is used to adjust the scale of the tax function.[27] The federal income tax revenue, the sum of the individual income tax revenue and the corporate income tax revenue, is 10.0% as a share of GDP in fiscal year 2014. So, this paper finally adjusts the first parameter, φ_t, to 0.3745 so that the income tax revenue as a share of GDP is 10.0% in the 2014 benchmark economy.

In 2014, the standard deduction is \$6,200 for a single household and \$12,400 for a married household filing return jointly. In the same year, the exemption is \$3,950 per person. When 50% of households are assumed to be married and 50% are assumed to be single, the sum of the deduction and exemptions is, on average, $0.5(12{,}400 + 7{,}900) + 0.5(6{,}200 + 3{,}950) = 15{,}225$ in 2014 growth-adjusted dollars. The parameter, d, is set at $15{,}225/139{,}193 = 0.1094$ by converting the above number into model units.

References

[1] Auerbach, Alan J., and Laurence J. Kotlikoff. 1987. *Dynamic Fiscal Policy*. Cambridge, UK: Cambridge University Press.

[2] Attanasio, Orazio P., Sagiri Kitao, and Giovanni L. Violante. 2006. "Quantifying the Effects of the Demographic Transition in Developing Economics." *The B.E. Journal of Macroeconomics*, 6 (1): 1–44.

[3] Attanasio, Orazio P., Sagiri Kitao, and Giovanni L. Violante. 2007. "Global Demographic Trends and Social Security Reform." *Journal of Monetary Economics*, 54: 144–198.

[4] Board of Governors of the Federal Reserve System. 2014. "2013 Survey of Consumer Finances." http://www.federalreserve.gov/econresdata/scf/scfindex.htm.

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[26]For the taxation on Social Security benefits, see Tables 2.A.31–32 in Social Security Administration (2014).

[27]This paper initially estimates the third parameter, φ_2, as 0.017958 by using labor income in thousands of dollars, then the paper rescales the Gouveia–Strauss tax function to model units as $\varphi_2 = 0.017958 \times 139.193^{\varphi_1} = 0.7368$, where 139.193 is a scale parameter explained above.

[5] Braun, R. Anton, Daisuke Ikeda, and Douglas H. Joines. 2009. "The Saving Rate in Japan: Why It Has Fallen and Why It Will Remain Low." *International Economic Review*, 50 (1): 292–321.

[6] Chen, Kaiji, Ayşe İmrohoroğlu, and Selahattin İmrohoroğlu. 2007. "The Japanese Saving Rate Between 1960 and 2000: Productivity, Policy Changes, and Demographics." *Economic Theory*, 32: 87–104.

[7] Conesa, Juan C., Sagiri Kitao, and Dirk Krueger. 2009. "Taxing Capital? Not a Bad Idea after All!" *American Economic Review*, 99 (1): 25–48.

[8] De Nardi, Mariacristina, Selahattin İmrohoroğlu, and Thomas J. Sargent. 1999. "Projected U.S. Demographics and Social Security." *Review of Economic Dynamics*, 2: 575–615.

[9] Domeij, David, and Jonathan Heathcote. 2004. "On the Distributional Effects of Reducing Capital Taxes." *International Economic Review*, 45 (2): 523–554.

[10] Gouveia, Miguel, and Robert P. Strauss. 1994. "Effective Federal Individual Income Tax Functions: An Exploratory Empirical Analysis." *National Tax Journal*, 47 (2): 317–339.

[11] Huggett, Mark. 1996. "Wealth Distribution in Life-Cycle Economies." *Journal of Monetary Economics*, 38 (3): 469–494.

[12] İmrohoroğlu, Ayşe, Selahattin İmrohoroğlu, and Douglas H. Joines. 1995. "A Life-Cycle Analysis of Social Security." *Economic Theory*, 6 (1): 83–114.

[13] Internal Revenue Service. 2015. "Form 1040 Instructions 2014." Department of the Treasury, `http://http://www.irs.gov/pub/irs-pdf/i1040gi.pdf`.

[14] Kitao, Sagiri. 2014. "Sustainable Social Security: Four Options." *Review of Economic Dynamics*, 17: 756–79.

[15] Kotlikoff, Laurence J., Kent Smetters, and Jan Walliser. 2007. "Mitigating America's demographic Diremma by Pre-funding Social Security ." *Journal of Monetary Economics*, 54: 246–66.

[16] Krueger, Dirk, and Alexander Ludwig. 2007. "On the Consequences of Demographic Change for Rates of Returns to Capital, and the Distribution of Wealth and Welfare." *Journal of Monetary Economics*, 54: 49–87.

[17] Ludwig, Alexander, Thomas Schelkle, and Edgar Vogel. 2012. "Demographic Change, Human Capital and Welfare." *Review of Economic Dynamics*, 15: 94–107.

[18] Neuman, Patricia, Juliette Cubanski, and Anthony Damico. 2015. "Medicare Per Capita Spending By Age And Service: New Data Highlights Oldest Beneficiaries." *Health Affairs*, 34, no.2 (2015):335–39 (published online January 14, 2015; 10.1377/hlthaff.2014.1371)

[19] Nishiyama, Shinichi, and Kent Smetters. 2007. "Does Social Security Privatization Produce Efficiency Gains?" *Quarterly Journal of Economics*, 122(4): 1677-719.

[20] Nishiyama, Shinichi, and Kent Smetters. 2014. "Analyzing Fiscal Policies in a Heterogeneous-Agent Overlapping-Generations Economy." Kenneth L. Judd and Karl Schmedders, eds., *Handbook of Computational Economics, Volume 3*, Chapter 3.

[21] Ríos-Rull, José-Víctor. 2001. "Population Changes and Capital Accumulation: The Aging of the Baby Boom." *The B.E. Journal of Macroeconomics*, 1 (1): 1–48.

[22] Rouwenhorst, K. Geert. 1995. "Asset Pricing Implications of Equilibrium Business cycle Models." Thomas F. Cooley, ed., *Frontiers of Business Cycle Research*, Chapter 10.

[23] Social Security Administration. 2014. *The 2014 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds.* The U.S. House of Representatives, Document 113–139.

[24] Social Security Administration. 2015. *Annual Statistical Supplement to the Social Security Bulletin, 2014.* Social Security Administration, Publication 13-11700.

[25] United Nations. 2008. "World Fertility Data 2008: Period Fertility Indicators." United Nations, `http://www.un.org/esa/population/publications/WFD%202008/Main.html`.

[26] Vespa, Jonathan, Jamie M. Lewis, and Rose M. Kreider. 2013. "America's Families and Living Arrangements: 2012—Population Characteristics." U.S. Census Bureau, P20–570.