

# Making, Buying and Concurrent Sourcing: Implications for Operating Leverage and Stock Beta\*

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## Abstract

We present a real options model of a firm's make-or-buy decision under demand uncertainty. "Making" is subject to decreasing returns to scale, fixed costs and capital investment. "Buying" happens at a fixed price and requires no investment. Three distinct procurement regimes endogenously arise: buying, making or concurrent sourcing for, respectively, low, intermediate and high demand. Capital constraints encourage buying or concurrent sourcing. Operating leverage peaks when the firm switches between buying and making, and it is lowest (and negative) at the switch between making and concurrent sourcing. This non-monotonic pattern mirrors and drives the behavior of the firm's beta.

*Keywords:* sourcing, operating flexibility, operating leverage, stock beta

*JEL codes:* D24, D81, G31, L23

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# 1 Introduction

Whether to make or to buy a product is for many firms an important decision. Empirical studies (see Harrigan (1986), Dutta et al. (1995) and Parmigiani (2007), among others) have shown that some firms rely exclusively on internal production, others procure the good exclusively through external suppliers, while yet others simultaneously rely on internal production and outsourcing. The last category is particularly puzzling as a firm that both makes and buys the same good incurs the costs that are associated with both outsourcing and internal production. Furthermore, concurrent sourcing appears to be widespread.<sup>1</sup>

There is also empirical evidence that aggregate outsourcing activity within an economy is not constant over time, but occurs in waves and that these waves are linked to the state of the economy.<sup>2</sup> Furthermore, the extent to which an individual firm relies on outsourcing can vary dramatically over time, with firms switching back and forth between procurement modes in response to economic shocks. Yet, existing models of the make-or-buy decision are essentially static and lack time series implications. Joskow (2005) highlights the need to understand better why firms' modes of procurement change over time and how firms adapt to changing demand and supply conditions. We neither understand how changes in the mode of procurement affect a firm's beta and stock returns.

This paper develops a real options model that explains some of the observed behavior and addresses some unanswered questions. Using analytical solutions, we examine how operating flexibility affects a firm's operating leverage, its beta and expected returns. The five novel

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<sup>1</sup>Heriot and Kulkarni (2001), Heide (2003) and Parmigiani (2007) find that, respectively, 57%, 31% and 28% of the firms in their sample rely on concurrent sourcing. Empirical observations have also shown that many local governments both make and buy the same service (Warner and Hefetz, 2008).

<sup>2</sup>Although outsourcing has been used for more than a century, the first wave of outsourcing started in the 1970s and 1980s in the manufacturing sector. The reader is referred to Domberger (1999) for further details on past outsourcing activity.

features of our paper are as follows. First, we model internal production as a more capital intensive procurement mode than buying. The existing literature, which is largely static in nature, has ignored the role of capital investment because there is no essential difference between fixed operating costs and sunk investment costs in a static model without a time dimension or without meaningful variation in the production policy. However, in a dynamic, uncertain environment where firms may switch production mode or face excess capacity, both types of costs are very different in nature. For example, by switching from making to buying, a firm may be able to cut its fixed production costs but not its sunk investments.

Second, in our setting firms have the flexibility to engage in concurrent sourcing. This generates a number of new insights compared to the existing literature which traditionally assumes that firms can either outsource or produce in-house, but not both at the same time.

Third, our paper characterizes analytically how demand shocks affect the firm's optimal operating mode. We show how firms, in response to economic shocks, switch back and forth between outsourcing, internal production and concurrent sourcing, and analyze how this affects operating leverage. We show that concurrent sourcing is not a weighted average of the make and buy strategies. Instead it is a distinctly different strategy that is situated to the right of the buy and make strategies along the firm's output spectrum. Our theoretical model supports recent empirical work by Parmigiani (2007) who explores whether concurrent sourcing should be interpreted as lying somewhere on a continuous make-buy spectrum, reflecting a partial degree of integration as implicit in the transaction cost theories, or whether it should be considered as a third, distinct strategy. She finds strong support for the latter hypothesis. It appears that firms first decide whether or not to produce internally, and only subsequently determine the percentage of good that is internally produced.

Fourth, we analyze the effect of capital constraints and irreversible investment. Often firms may not be able to invest the optimal amount due to capital constraints. We therefore examine how the optimal output level and operating regime vary with the severity of the

capital constraint.

Finally, we derive testable implications regarding the effect of the firm's procurement flexibility on its expected returns. We show that the firm's beta is closely linked to the degree of operating leverage (see also Carlson, Fisher, and Giammarino (2004), who introduced the operating leverage hypothesis). Beta is also determined by a complex interplay between the returns to scale associated with each operating regime and the economies of scale associated with internal production.

We now briefly sketch our real options model and its main results. We consider a firm that produces output for which demand is stochastic and determined by an iso-elastic inverse demand function. The output can be made internally, or it can be bought at an exogenously given fixed unit price.<sup>3</sup> Internal production is governed by a Cobb-Douglas production function that has capital and labor as inputs and is subject to decreasing returns to scale. We consider both the polar case where capital is irreversibly fixed once and for all, and the other extreme scenario where capital can continuously and costlessly be altered. Buying output does not require labor nor capital.

Three distinct procurement regimes endogenously arise. For low demand levels, the firm purchases *all* required quantity of the good (i.e., "*buying*"). For intermediate demand, the firm produces *all* units in-house (i.e., "*making*"). Finally, for high demand, the firm produces a fixed threshold quantity internally and outsources any quantity in excess of the threshold (i.e., "*concurrent sourcing*"). In such a case, outsourcing is used as an "overflow" facility to deal with excess demand. Importantly, with decreasing returns to scale concurrent sourcing survives even if the firm can make internal capital (dis)investments in a frictionless manner.

We show that a tighter capital constraint encourages buying or concurrent sourcing. The availability of cheap outsourcing imposes a minimum capital requirement on firms. Firms that cannot operate at a sufficiently large scale stop producing internally, irrespective of

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<sup>3</sup>In appendix C we consider the case where the outsourcing price depends on the quantity ordered.

the state of the economy. The cheaper it is to buy, the larger the scale required for internal production. This may explain why the arrival of cheap overseas outsourcing led to the closure of many small labor intensive production units.

Our analysis provides new insights into the behavior of operating leverage when firms can switch between different operating modes.<sup>4</sup> We show that operating leverage equals zero under buying and jumps up significantly at the point where the firm switches from buying to making. At this switch the firm experiences a big increase in its output. For the lowest levels of internal production the marginal cost is significantly lower than the average cost, as there are initially economies of scale. As internal production rises, the marginal cost increases while the average cost decreases causing operating leverage to fall. When the marginal cost equals the outsourcing price it is optimal for the firm to switch from making to concurrent sourcing. Crucially, at this switching point the average cost is lower than the marginal cost (because the firm switches to concurrent sourcing *after* diseconomies of scale have kicked in), causing operating leverage to be negative. Operating leverage remains negative under concurrent sourcing but asymptotes to zero as output goes to infinity. On average, firms engaging in concurrent sourcing have a lower level of operating leverage than firms that outsource or firms that produce internally at output levels that are not too high. In summary, operating leverage peaks (reaches a minimum) when the firm switches from buying to making (from making to concurrent sourcing).

The behavior of the stock beta mirrors our measure of operating leverage. However, (unlike operating leverage) beta is a smooth function of the state variable as beta is based on the sensitivity of the firm's value (which is based on future expectations). Beta is low under buying, and gradually reaches a peak as the firm moves towards making. Under

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<sup>4</sup>The measure used for operating leverage in this paper is  $OL = 1 - \frac{d\text{Total Cost}}{d\text{Output}} \frac{\text{Output}}{\text{Total Cost}} = 1 - \frac{\text{Marginal Cost}}{\text{Average Cost}}$ . Other measures commonly used for operating leverage are  $OL = \frac{d\text{Profit}}{d\text{Output}} \frac{\text{Output}}{\text{Profit}}$  and  $OL = \frac{d\text{Profit}}{d\text{Sales}} \frac{\text{Sales}}{\text{Profit}}$  (see section 4.4 for further details).

making beta gradually declines as operating leverage decreases. As diseconomies of scale set in (i.e., the marginal cost exceeds the average cost) beta moves towards a minimum, but then gradually increases again as the firm starts engaging in concurrent sourcing. Since beta peaks around the buying-to-making switching point, and reaches a minimum in the neighborhood of the making-to-concurrent sourcing switching point, it follows that the beta exhibits a characteristic non-monotonic pattern.

Our work and research methods are related to the real options literature that studies a firm's optimal capacity choice under uncertainty and irreversibility. Key papers on this topic are Abel (1983, 1984), Pindyck (1988), Triantis and Hodder (1990), He and Pindyck (1992) and Kandel and Pearson (2002). Triantis and Hodder (1990) value flexible production systems with multiple products, whereas we consider a firm with multiple operating modes all generating the same product. Kandel and Pearson (2002) analyze the optimal investment and production decision of a firm facing stochastic demand with access to two alternative production technologies. Our paper differs in two important respects. First, by including a more general cost function (based on the Cobb-Douglas production technology), we are able to rationalize the choice of the reversible technology (buying) and a combination of both technologies (concurrent sourcing). Second, by focusing on either completely irreversible investment or perfectly reversible investment, we are able to generate insightful closed form solutions and a rich set of comparative statics results regarding the timing and size of investment, the value of the option to invest, the firm's operating leverage and its systematic risk (beta).

Several papers on investment in costly capacity (Baldursson (1998), Grenadier (2002), Aguerrevere (2003) and Aguerrevere (2009)) have focused on capacity as a strategic variable when firms are competing with other firms. None of these papers captures the make-or-buy decision because they allow for one production method only.

Our paper is also related to a growing literature pioneered by Berk, Green, and Naik

(1999) that links corporate decisions to asset returns. In Berk et al. (1999) firms that perform well tend to be those that discovered valuable investment opportunities. Good news is associated on average with lower systematic risk. Carlson et al. (2004) introduce operating leverage in the real options framework and demonstrate that a book-to-market effect can be directly related to fixed operating costs. Zhang (2005) presents an industry equilibrium model of asymmetric convex adjustment costs of investment in which value firms have more difficulty severing their capital stocks than growth firms. Cooper (2006) analyzes the effects of nonconvex adjustment costs, irreversibility of investment, and operating leverage on the behavior of financial asset prices. Novy-Marx (2011) derives and tests implications of the operating leverage hypothesis and documents that operating leverage predicts returns in the cross-section.<sup>5</sup> The above papers all find that positive productivity shocks lead to lower expected returns.<sup>6</sup> Most notably, Carlson et al. (2004) show that a decrease in demand for output increases the riskiness of returns because high fixed costs lead to higher operating leverage in downturns. We confirm this result for firms that produce internally, but show that operating leverage, and the resulting relation between the state of the economy and firm risk, can be highly non-monotonic for firms that switch between different procurement options.<sup>7</sup> Hackbarth and Johnson (2014) show theoretically and empirically that combining real options with operating leverage can also lead to a non-monotonic relation between profitability and risk.

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<sup>5</sup>More recently, the focus has shifted to examining the relation between stock returns and financing decisions (see Carlson, Fisher, and Giammarino (2006, 2010), Whited and Wu (2006), Li, Livdan, Lyandres, Sun, and Zhang (2008), Zhang (2009), Livdan, Saprizza, and Zhang (2009), Gomes and Schmid (2010) and Ozdagli (2012), among others).

<sup>6</sup>Studies focusing on financial leverage tend to come to the same conclusion (see Li, Livdan, and Zhang (2009) and Ozdagli (2012) among others).

<sup>7</sup>Our paper is also related to the literature on outsourcing and vertical integration (see Joskow (2005) for a comprehensive review). Only a few theoretical papers consider the possibility of concurrent sourcing (see, e.g., Carlton (1979), Rob and Vettas (2003), Alvarez and Stenbacka (2007) and Allon and Van Mieghem (2010)) and typically focus on the trade-off between cost and responsiveness.

The remainder of the paper is organized as follows. Section 2 presents the basic assumptions of the model and analyzes the optimal procurement decision for a fixed level of capital. Section 3 studies the optimal procurement decision with fully reversible capital. Section 4 examines how output price uncertainty and the outsourcing price affect optimal investment, production and the company beta. Section 5 discusses the model’s empirical implications. Section 6 concludes.

## 2 Optimal Production with a Fixed Level of Capital

A firm sells a product that can either be produced in-house (“making”) or can be bought from an external supplier (“buying”). The firm can also produce some units internally and buy the remainder (“concurrent sourcing”). The firm faces an isoelastic inverse demand function which is given by  $H(q) = \alpha q^{-\theta}$ , with  $\alpha > 0$ ,  $\theta \in (0, 1)$  and where  $q$  is the firm’s level of output.

Making requires capital  $K$  and labor  $L$  whereas buying does not. In this section, the amount of installed capital  $K$  is assumed to be given, whereas labor is variable. The firm’s (instantaneous) profit function under *buying*,  $\pi_1$ , (i.e., when *all* units are bought) is given by:

$$\pi_1(q_B) = q_B H(q_B) - pq_B = \alpha q_B^{1-\theta} - pq_B \quad (1)$$

where  $p$  is the unit price at which the product can be bought from a supplier.<sup>8</sup>

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<sup>8</sup>Goods that are bought (or produced internally) cannot be sold back to the upstream suppliers. The market structure adopted is similar to monopolistic competition. Each firm has monopoly power (by virtue of its brand differentiation or location, for example) while at the same time there are a sufficient amount of suppliers of the product in its generic form for the purchase price  $p$  to be constant. The market structure may particularly apply to industries such as cereals, toothpaste, clothing or catering and services in large cities. In appendix C we consider an extension in which a larger order attracts a bigger discount and  $p(q) = p_0 q^{-\nu}$ , where  $p_0 > 0$  and  $\nu \in [0, \theta)$  are constants.



Consider now a situation where the firm makes all its output. The firm's production technology is associated with a Cobb-Douglas production function (cf., e.g., Bertola (1998) and Gomes (2001))

$$F(K, L) = AK^\zeta L^\eta \quad (2)$$

with  $A$ ,  $\zeta$  and  $\eta$  being positive constants and  $\zeta + \eta < 1$  (i.e., the production function is subject to decreasing returns to scale).<sup>9</sup> For the moment we assume that the capital  $K$  is fixed and a sunk investment. Labor  $L$  is variable, and the wage rate is  $w$ . The firm also incurs a fixed cost of production  $f$ . (The necessary condition for concurrent sourcing ever to occur is that  $\eta < 1$ , which we assume to be satisfied.<sup>10</sup>) The production function implies that the cost of making  $q_M$  units of output equals

$$C(q_M; K) = wA^{-1/\eta}K^{-\zeta/\eta}q_M^{1/\eta} + f \equiv c(K)q_M^\xi + f \quad (3)$$

The firm's profit function under *making*,  $\pi_2$ , (i.e., when *all* output is produced internally) is therefore given by:

$$\pi_2(q_M; K) = q_M H(q_M) - c(K)q_M^\xi - f = \alpha q_M^{1-\theta} - c(K)q_M^\xi - f \quad (4)$$

where  $q_M$  is the quantity that is produced internally and  $c(K)q_M^\xi$  is the variable cost associated with producing  $q_M$  units in-house. The average cost function under internal production is:

$$AC(q_M; K) = c(K)q_M^{\xi-1} + \frac{f}{q_M} \quad (5)$$

Given that  $\xi (\equiv 1/\eta) > 1$ , the average cost is a standard U-shaped function of quantity. The presence of the fixed production cost  $f$  means that there are initially economies of scale,

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<sup>9</sup>Our results are robust to alternative specifications for the demand and cost functions. A previous version of the paper that combined a linear demand function with a quadratic cost function generated results that are qualitatively very similar.

<sup>10</sup>If outsourcing price were to decrease with quantity bought (cf. footnote 8), the necessary condition for concurrent sourcing to occur becomes  $\eta < 1/(1 - \nu)$ .

because the fixed cost can be spread over a larger amount of units produced. However, since the marginal cost of production is increasing, at some point diseconomies of scale kick in. If the firm is able to increase its capital stock  $K$  (see section 3), it is able to mitigate the diseconomies of scale as  $\partial c(K)/\partial K < 0$ .

We can immediately derive the profit function when the firm combines outsourcing with internal production. If the firm produces  $q_M$  units internally and buys  $q_B$  units then the operating profit function under the *concurrent sourcing* strategy,  $\pi_3$ , is given by:

$$\begin{aligned}\pi_3(q_B, q_M) &= (q_B + q_M) H(q_M) - c(K) q_M^\xi - pq_B - f \\ &= \alpha (q_B + q_M)^{1-\theta} - c(K) q_M^\xi - pq_B - f \quad \text{with } q_B, q_M \geq 0\end{aligned}\tag{6}$$

The problem can now be formulated as follows: what level of internal ( $q_M$ ) and external ( $q_B$ ) production will be adopted in order to maximize the profit flow?

The firm maximizes its instantaneous profit flow by solving the following problem:

$$\max_{i, q_B, q_M} \pi_i \tag{7}$$

Problem (7) is solved in two stages. We first derive the maximum profit flow  $\pi_i^o$  that can be achieved in a given regime  $i$  ( $i = 1, 2, 3$ ). Subsequently we determine which of these three regimes is optimal and under what conditions. The following proposition characterizes the solution (proofs are given in appendix A):

**Proposition 1** *If  $p \leq [(\frac{\xi f}{\xi-1})^{\xi-1} \xi c(K)]^{1/\xi} \equiv p_{\min}(K)$ , then buying is always optimal. If  $p_{\min}(K) < p$  then:*

- buying (regime 1) is optimal for  $\alpha \in (0, \alpha^*(K)]$ ,*
- making (regime 2) is optimal for  $\alpha \in (\alpha^*(K), \alpha^{**}(K)]$ ,*
- concurrent sourcing (regime 3) is optimal for  $\alpha \in (\alpha^{**}(K), +\infty)$ ,*

*The demand parameter  $\alpha^*(K)$  at which it is optimal to switch from buying to making is the smaller root of equation  $\pi_1^o(\alpha) = \pi_2^o(\alpha)$ . The cutoff  $\alpha^{**}(K)$  above which concurrent sourcing*

commences is given by

$$\alpha^{**}(K) = \frac{1}{1-\theta} p^{\frac{\theta+\xi-1}{\xi-1}} (\xi c(K))^{-\frac{\theta}{\xi-1}}$$

The optimal profit flow ( $\pi_i^o$ ), the amount bought ( $q_{iB}^o$ ) and the amount made ( $q_{iM}^o$ ) under regime  $i$  is given by:

$$\begin{aligned} \pi_1^o(\alpha) &= \Pi_1 \alpha^{\frac{1}{\theta}} && \text{with } q_{1B}^o = \left( \frac{(1-\theta)\alpha}{p} \right)^{\frac{1}{\theta}} \text{ and } q_{1M}^o = 0 \\ \pi_2^o(\alpha) &= \Pi_2 \alpha^{\frac{\xi}{\theta+\xi-1}} - f && \text{with } q_{2B}^o = 0 \text{ and } q_{2M}^o = \left( \frac{(1-\theta)\alpha}{\xi c} \right)^{\frac{1}{\theta+\xi-1}} \\ \pi_3^o(\alpha) &= \Pi_1 \alpha^{\frac{1}{\theta}} + C_1 - f && \text{with } q_{3B}^o = q_{1B}^o - q_{3M}^o \text{ and } q_{3M}^o = \left( \frac{p}{\xi c} \right)^{\frac{1}{\xi-1}} \end{aligned}$$

where constants  $\Pi_1$ ,  $\Pi_2$ ,  $\Pi_3$ , and  $C_1$  are given in appendix A. Furthermore,  $q_{1B}^o(\alpha^*) < q_{2M}^o(\alpha^*)$  and  $q_{2M}^o(\alpha^{**}) = q_{3B}^o(\alpha^{**}) + q_{3M}^o(\alpha^{**}) = q_{3M}^o(\alpha^{**})$ .

Proposition 1 conveys a number of important insights about the economics of outsourcing and internal production. A key determinant in the make-or-buy decision is the price  $p$  at which the product can be bought versus the cost at which it can be produced. This comparison is reflected in the condition  $p \geq p_{\min}(K)$ , where  $p_{\min}(K)$  is determined by the parameters of the cost function. The proposition states that if the purchasing price  $p$  is sufficiently low then outsourcing always dominates internal production because it is cheaper to buy than to make for all possible output levels  $q$ .<sup>11</sup>

The condition for internal production to be optimal for some demand levels (i.e.,  $p > p_{\min}(K)$ ) is equivalent to  $C_1 > f$ . This latter condition implies that the cost saving from making some input internally becomes positive for a sufficiently high level of demand. Furthermore, the condition is also equivalent to  $K > \left( \frac{(\xi w)^{1/\xi}}{pA} \left( \frac{\xi-1}{\xi f} \right)^{-(\xi-1)/\xi} \right)^{1/\zeta} \equiv K_{13}^*$  (see Figure 1 and its discussion below). Therefore  $K_{13}^*$  is the minimum scale required for making.

Another important determinant in the make-or-buy decision is the output price captured by the demand parameter  $\alpha$ . The price at which the firm can sell the finished production

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<sup>11</sup>This may explain why large firms, such as Nike or Apple, do none of their manufacturing despite having huge volumes.

determines the marginal revenue and therefore the optimal output level. The optimal output level is (globally) increasing in  $\alpha$ , creating a link between  $\alpha$  and the optimal production regime: buying, making and concurrent sourcing are optimal for low, intermediate and high demand levels, respectively.

The results lend support to the notion of an optimal production range. The firm can only efficiently produce internally for output levels in the range  $[q_{2M}^o(\alpha^*), q_{2M}^o(\alpha^{**})]$ . It is more efficient to buy quantities below  $q_{2M}^o(\alpha^*)$ , and to outsource any production in excess of  $q_{2M}^o(\alpha^{**})$ . Note that concurrent sourcing occurs even though the firm does not face a binding capacity constraint (the production function allows the firm to install more labor to increase output).

[Please insert Figure 1 about here.]

Proposition 1 is illustrated in Figure 1 for the base set of parameter values  $A = 0.2$ ,  $\zeta = 0.3$ ,  $\eta = 0.65$ ,  $\theta = 0.5$ ,  $f = 0.1$ ,  $w = 0.2$  and  $p = 1.5$ . It plots the optimal procurement regime for given levels of capital  $K$  and the demand parameter  $\alpha$  in the  $(\alpha, K)$  space. Buying prevails for low levels of capital or demand. Internal production occurs when  $\alpha$  exceeds  $\alpha^*(K)$  and the firm has a minimum level of capital. The absolute minimum level of capital required is  $K_{13}^*$ , as indicated by the vertical solid line in the figure. A looser capital constraint (i.e.,  $K > K_{13}^*$ ) reduces the threshold  $\alpha^*(K)$  at which the firm starts making, and increases the threshold  $\alpha^{**}(K)$  at which the firm switches to concurrent sourcing. Conversely, a tighter capital constraint widens the demand range over which concurrent sourcing and, particularly, buying prevail.

While internal production always becomes optimal for a sufficiently high demand level if the firm cannot outsource (the firm will hire more labor to make up for the lack of capital), this is no longer the case when firms have access to cheap outsourcing. The availability of outsourcing imposes a minimum capital requirement on firms: firms that cannot operate at a

sufficiently large scale stop producing internally, irrespective of the state of the economy. The cheaper it is to buy, the larger the scale required for internal production (i.e.,  $\partial K_{13}^*/\partial p < 0$ ). This may explain why the arrival of cheap overseas outsourcing led to the closure of many small, labor intensive production units.

### 3 Optimal Production with Fully Reversible Capital

In the previous section we focused on the case where the level of installed capital  $K$  is fixed and cannot be changed. The reader might wonder whether relaxing this assumption alters our results. We now consider the opposite polar case, i.e., when investment in capital can be incremental and is fully reversible. This is equivalent to the firm renting a desired amount of capital at each instant. Without loss of generality, we normalize the purchase cost of one unit of capital to one. Therefore, the (flow) cost of renting one unit of capital is  $r$ . The quantity of hired capital solves the firm's instantaneous optimization problem

$$\min_{K,L} rK + wL$$

subject to

$$AK^\zeta L^\eta = q_{iM}^o \text{ with } i \in \{1, 2, 3\}$$

where  $q_{iM}^o$  is the desired level of output from making (in itself being a solution to the profit maximization problem). The necessary and sufficient for concurrent sourcing to occur is now  $\eta + \zeta < 1$ .<sup>12</sup>

The solution to the problem gives the optimal amount of factors  $K(q_M)$  and  $L(q_M)$ ,

$$K(q_M) = \left(\frac{q_M}{A}\right)^{\frac{1}{\zeta+\eta}} \left(\frac{\zeta w}{\eta r}\right)^{\frac{\eta}{\zeta+\eta}} \quad (8)$$

$$L(q_M) = \left(\frac{q_M}{A}\right)^{\frac{1}{\zeta+\eta}} \left(\frac{\eta r}{\zeta w}\right)^{\frac{\zeta}{\zeta+\eta}} \quad (9)$$

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<sup>12</sup>If a larger order attracted a bigger discount, so  $p(q) = p_0 q^{-\nu}$ , where  $p_0 > 0$  and  $\nu \in [0, \theta)$ , the condition for concurrent sourcing to occur would be  $\eta + \zeta < 1/(1 - \nu)$ .

which can be translated into the cost function

$$\hat{C}(q_M) = \Phi(r, w; A)\Theta q_M^{\frac{1}{\zeta+\eta}} + f \equiv \hat{c}q_M^{\hat{\xi}} + f \quad (10)$$

where

$$\Phi(r, w; A) = \left( \frac{r^\zeta w^\eta}{A} \right)^{\frac{1}{\zeta+\eta}} \quad (11)$$

$$\Theta = \left( \frac{\zeta}{\eta} \right)^{\frac{\eta}{\zeta+\eta}} + \left( \frac{\zeta}{\eta} \right)^{-\frac{\zeta}{\zeta+\eta}} \quad (12)$$

We now compare the cost of making under the irreversible and reversible investment scenarios, as given by the cost functions (3) and (10), respectively. Since the irreversible investment scenario entails a sunk investment  $K$ , we have to add a capital charge  $rK$  to (3) to make both cost functions comparable. One can then verify that

$$\hat{c}q^{\hat{\xi}} \leq c(K)q^\xi + rK \quad \text{for all } q \text{ and } K \quad (13)$$

In other words, the cost of internal production when investment is fully reversible is always lower than the cost of internal production when investment is irreversible. This is due to the fact that the optimal mix of capital and labor is used in the former case. More interestingly, the benefit from flexibility increases with the output elasticity of capital,  $\zeta$ . In the limiting case where  $\zeta = 0$ , we obtain that  $\hat{c} = c(K)$  and  $\hat{\xi} = \xi$ , indicating that flexibility of changing the capital stock has no value as it is not optimal for the firm to use capital (see equation 8). Now we are able to formulate the following proposition.

**Proposition 2** *The optimal procurement regimes, profit and output levels with fully reversible capital are as given in Proposition 1 but with  $\xi$  and  $c$  replaced by  $\hat{\xi}$  and  $\hat{c}$ , defined in (10), respectively.*

It follows that the firm's production and procurement decisions are qualitatively the same under the irreversible and reversible investment scenarios. One merely has to replace the

parameters  $\xi$  and  $c$  by  $\hat{\xi}$  and  $\hat{c}$ . In the remainder of the paper, all parameters that refer to the reversible case are labeled with the caret (hat) symbol.<sup>13</sup> Note that investment in capacity occurs for some  $\alpha$  as long as  $p > \hat{p}_{\min}$ . As shown in Figure 1:  $\alpha^*(K) < \hat{\alpha}^*$  and  $\alpha^{**}(K) < \hat{\alpha}^{**}$  (the latter for  $K < K(q_{3M}^o)$ , i.e., for the level that the firm may optimally invest in). Therefore, outsourcing (concurrent sourcing) is less (more) likely to be adopted by firms for which investment in productive capacity is highly irreversible. The intuition behind  $\alpha^*(K)$  being smaller than  $\hat{\alpha}^*$  is as follows. Once the capital is in place, its cost is sunk and cannot be recuperated even if the firm decides not to use it and buys the good. Therefore, the relative benefit of making versus buying does not take into account the cost of capital and making becomes more attractive. Therefore, even with suboptimal amount of capital  $K$ , the firm will find it optimal to make for lower  $\alpha$  than under the reversible capital scenario. The reason why threshold  $\hat{\alpha}^{**}$  is greater than  $\alpha^{**}(K)$  is as follows. If capacity is fully reversible, the firm increases the level of capital installed when demand increases. By doing so, it keeps the marginal cost low enough for concurrent sourcing only to be adopted when  $\alpha$  becomes sufficiently high. In the decision whether to make or source concurrently, the sunk cost argument does not play a role as capital is employed in both regimes.

## 4 Optimal Production and Capital Investment with Demand Uncertainty

So far we have assumed that the amount of capital  $K$  is either exogenously fixed or fully reversible, and we analyzed in both scenarios the optimal output and procurement policies for different levels of demand. In this section, we determine the optimal investment in irreversible capital under uncertain demand. To do so, we allow  $\alpha$ , the constant in the inverse demand function, to vary with the state of nature. Parameter  $\alpha$  reflects consumers'

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<sup>13</sup>They are no longer a function of capital  $K$ , which is chosen endogenously in this case.

willingness to pay for the product, which could vary over time because of taste or income shocks. The effect of economic uncertainty on demand is captured by letting  $\alpha_t$  follow a geometric Brownian motion (cf. Pindyck (1988))

$$d\alpha_t = \mu\alpha_t dt + \sigma\alpha_t dw_t, \quad (14)$$

where  $\mu$  and  $\sigma$  correspond to the instantaneous growth rate and the volatility of the constant in the demand function, respectively, and  $dw_t$  denotes a Wiener increment. The firm is risk-neutral and we assume that  $r > \mu/\theta + 0.5(1 - \theta)(\sigma/\theta)^2$  to obtain finite valuations.<sup>14</sup> To simplify future notation, we define  $\delta(n) \equiv r - n\mu - 0.5n(n - 1)\sigma^2$ .

We assume that capital does not depreciate. Investment in capital is irreversible and, once chosen, the level of capital cannot be changed. The firm can, however, change the procurement regime in response to economic shocks. Changes in the procurement regime are fully reversible (there are no switching costs).

Introducing a dynamic framework separates the investment decision and production decision and requires that the problem be solved in two stages: 1) installing the optimal capital level,  $K$ , at the optimal investment threshold  $\bar{\alpha}$ ; 2) given the capital put in place, selecting the optimal production level and procurement regime, and continuously updating these decisions as uncertainty unfolds.

As is standard with dynamic optimization, we solve the problem backwards. We first solve for the optimal production level and procurement regime taking the firm's investment  $K$  as given. This problem corresponds to the one described in section 2 (for a given level of  $K$ ) with the parameter  $\alpha$  changing randomly over time according to process (14).<sup>15</sup> Next, we solve for the optimal investment size,  $K$ , and investment threshold,  $\bar{\alpha}$ .

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<sup>14</sup>This restriction is due to the fact that variable cash flows are proportional to powers  $1/\theta$  and  $\xi/(\theta + \xi - 1)$  of  $\alpha_t$ . Since the profit elasticity  $1/\theta$  is always greater than  $\xi/(\theta + \xi - 1) > 1$  (as  $\theta < 1$  and  $\xi > 1$ ), this condition ensures that the firm values for all 3 procurement regimes are finite.

<sup>15</sup>The dynamics therefore corresponds to vertical fluctuations in the  $(K, \alpha)$  space in Figure 1.



We are primarily interested in the effect of demand uncertainty ( $\sigma$ ) and the outsourcing price ( $p$ ) on the optimal investment level  $K$ , the optimal investment threshold  $\bar{\alpha}$ , the value of the option to invest, and the procurement regime boundaries.

We start off by deriving the value of a *mature* firm, that is, after the capital investment  $K$  is made. Subsequently, we calculate the value of a *young* firm that currently purchases the entire required quantity of the good and has a single option to invest.

#### 4.1 THE VALUE OF A MATURE FIRM

Once the investment  $K$  has been made, the value of a mature firm can be obtained by noting that all the flexibility arises from the reversibility regarding the procurement decision. As there is no cost associated with regime changes, the procurement regime and the optimal output level are selected to maximize the instantaneous profit flow and, therefore, are determined by the cutoff values and the output levels established earlier in the paper. In particular, critical level of  $\alpha$  that separates the make and buy regimes is given by  $\alpha^*(K)$  as defined in Proposition 1. The cutoff level of demand associated with a switch between making and concurrent sourcing is given by  $\alpha^{**}(K)$ , the explicit expression for which is given in the same proposition. Quantities bought and made across different regimes correspond to the output levels derived in Proposition 1. The value of the firm in each regime equals the sum of the present value of the associated profit flow and the value of the options to switch procurement regime in response to upward or downward moves in the stochastic demand parameter  $\alpha_t$ . The option values are determined by solving the system of equations in which there is value matching and smooth pasting of firm values at the regime boundaries.

The value of the firm is considered for the general case of  $K \in [K_{13}^*, \infty)$ . As shown in section 2, the optimal investment in capacity is at least  $K_{13}^*$  as otherwise it is always optimal

to buy the entire input. The value of the mature firm,  $V_M$ , is:

$$V_M(\alpha; K) = \begin{cases} PV_1(\alpha) + A_1\alpha^{\lambda_1} & \alpha \leq \alpha^*(K), \\ PV_2(\alpha; K) + A_2\alpha^{\lambda_1} + B_2\alpha^{\lambda_2} & \alpha \in (\alpha^*(K), \alpha^{**}(K)], \\ PV_3(\alpha; K) + B_3\alpha^{\lambda_2} & \alpha > \alpha^{**}(K). \end{cases} \quad (15)$$

where  $\lambda_1$  and  $\lambda_2$  are the positive and negative root, respectively, of the characteristic equation  $0.5\sigma^2\lambda(\lambda - 1) + \mu\lambda - r = 0$ . Present value components  $PV_i$  as well as constants  $A_i$  and  $B_i$  for  $i \in \{1, 2, 3\}$  are defined in appendix B.<sup>16</sup> In general, three regimes of capacity utilization are present. If the demand is low, the capital in place is not utilized at all and the firm outsources production. If the state variable  $\alpha$  is between  $\alpha^*$  and  $\alpha^{**}$ , capital is (fully) utilized and the firm makes the entire required quantity of the good. Finally, when demand is sufficiently high (i.e.  $\alpha > \alpha^{**}$ ) the firm switches to concurrent sourcing because the marginal cost of internal production exceeds the outsourcing price  $p$ .<sup>17</sup>

Having obtained the expression for  $V_M(\alpha)$  allows us to analyze the relation between the state of the economy and the firm's Tobin's  $q$  (cf., e.g., Bertola (1998)). The firm's (average)  $q$  is defined as the ratio of the value of the firm and the replacement cost of its capital stock, that is,  $q_A \equiv V_M(\alpha)/K$ .

It is meaningful to analyze the relation between  $q_A$  and the state of the economy, measured by the level of  $\alpha$ , for the reversible case (as otherwise capital stock is fixed). Optimal capital stock is zero under buying, constant under concurrent sourcing and an increasing function of  $\alpha$  under making. Upon substituting  $\hat{q}_{2M}^o$  into (8), the optimal capital stock can be written

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<sup>16</sup>The dependence of constants  $A_i$  and  $B_i$  on  $K$  has been omitted for the brevity of notation. Moreover,  $B_1$  ( $A_3$ ) equals zero as there is no flexibility associated with a downward (an upward) movement of the state variable  $\alpha$  under buying (concurrent sourcing).

<sup>17</sup>Obviously, the value of only those components that correspond to procurement regimes in which capacity is utilized ( $PV_2$  and  $PV_3$ ) depends on the amount of installed capacity  $K$ . In addition, the present value of the incremental profit flow resulting from the output made under concurrent sourcing, which we denote by  $\Delta PV_{31}$  ( $\equiv PV_3 - PV_1$ ), is determined by  $\alpha^{**}$  and does *not* depend on the actual realization of the demand shock  $\alpha$ .

as  $f(A, r, w, \zeta, \eta)\alpha^{\hat{\xi}/(\theta+\hat{\xi}-1)}$  under making and is constant under concurrent sourcing. Apart from buying, in which no capital stock is employed and the ratio is not properly defined, average  $q$  can be shown to increase with  $\alpha$ .<sup>18</sup>

Note that an increase in  $q_A$  associated with an improving state of the economy is not always associated with a corresponding increase in investment in capital. In fact,  $\partial K/\partial\alpha$  becomes zero once the firm adopts concurrent sourcing.<sup>19</sup> This is due to the fact that a better state of the economy is associated with a procurement mode that is relatively less capital intensive. This observation leads to a conclusion that the average  $q$  may be a particularly poor predictor of corporate investment (cf. Erickson and Whited (2000)) if less capital intensive production technologies are adopted during market booms.

## 4.2 THE VALUE OF A YOUNG FIRM AND THE OPTIMAL CAPITAL INVESTMENT

The value of the “young” (that is, before the investment option is exercised) firm,  $V_Y(\alpha)$ , can be obtained by maximizing the value of the investment opportunity with respect to the investment threshold  $\bar{\alpha}$  and the investment size  $K$ . Therefore, it can be represented as

$$V_Y(\alpha) = PV_1(\alpha) + OV(\alpha), \quad (16)$$

where  $PV_1(\alpha)$  denotes the present value of cash flows of a firm that can only buy the good with no option to invest in capital (cf. (15)).  $OV(\alpha)$  denotes the option value of the optimal capital investment:

$$OV(\alpha) = \max_{\bar{\alpha}, K} (V_M(\bar{\alpha}; K) - PV_1(\bar{\alpha}) - K) \left(\frac{\alpha}{\bar{\alpha}}\right)^{\lambda_1} \quad \text{for } \alpha \leq \bar{\alpha} \quad (17)$$

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<sup>18</sup>If options to switch regimes and fixed costs were to be ignored, average  $q$  would be constant under making.

<sup>19</sup>If making was the only option,  $\partial K/\partial\alpha$  would still be positive and proportional to  $\alpha^{(1-\theta)/(\theta+\hat{\xi}-1)}$ .

The maximization in (17), which is essentially the optimal lumpy capital investment problem, is performed in two stages. First, for a given  $K$  the optimal investment threshold  $\bar{\alpha}(K)$  is calculated by (numerically) solving the familiar system of value-matching and smooth-pasting conditions. Second, the value of the investment option is maximized with respect to  $K$ . The upper bound on the optimal capital investment can be shown to equal  $K(q_{2M}^o(\alpha^*); \hat{p}_{\min})$ , which is a solution to  $\Delta PV_{31}(K; \hat{p}_{\min}) = K$ .

The value of the option to invest ( $OV$ ), the value of the installed investment net of the investment cost, and the optimal investment threshold ( $\bar{\alpha}$ ) are depicted in Figure 2 for the base set of parameter values (cf. section 2) combined with  $\mu = 0.01$ ,  $r = 0.05$ , and  $\sigma = 0.1$ . At the optimal investment threshold,  $\bar{\alpha}$ , the value of the option to invest smooth-pastes to the NPV of the installed investment. The latter NPV is bounded from below by the investment cost  $-K$ , and from above by the asymptote  $\Delta PV_{31}(K) - K$  which denotes the NPV of the *incremental* profit flow associated with the output that is internally produced under concurrent sourcing. This incremental profit flow is determined by the cost side only and is therefore independent of  $\alpha$ .

[Please insert Figure 2 about here.]

### 4.3 THE EFFECT OF UNCERTAINTY AND OUTSOURCING PRICE

In this section we explore how uncertainty ( $\sigma$ ) and outsourcing price ( $p$ ) affect the timing of irreversible investment, the size of the investment, and the value of the option to invest. Since the procurement regime is fully reversible, irreversibility only applies to capital investment. Analytical expressions for the comparative statics with respect to  $\sigma$  and  $p$  are not available. Therefore, we rely on a numerical analysis with its results depicted in Figure 3 (parameter values correspond to the base set, unless stated otherwise).

[Please insert Figure 3 about here.]

Starting off with the investment timing, panel A of Figure 3 confirms the standard result from the real options literature (see Dixit and Pindyck (1994)) that uncertainty unambiguously delays investment, i.e.:

$$\frac{d\bar{\alpha}}{d\sigma} > 0$$

It is also true that higher outsourcing price accelerates investment (see panel B):

$$\frac{d\bar{\alpha}}{dp} < 0$$

For  $p \downarrow \hat{p}_{\min}$ , the firm becomes indifferent between buying the entire amount of good and making it even if it was to optimally utilize installed capital at all times. There is no investment for  $p \leq \hat{p}_{\min}$ , no matter how high the output demand. This result differs from standard real option models in which investment ultimately takes place provided the level of the demand state variable is high enough. The reason for this difference is that in standard models there is no alternative procurement mode to in-house production, an assumption which is equivalent in our model to setting  $p$  infinitely high. At  $p = \hat{p}_{\min}$  the investment threshold is infinitely high. Once outsourcing price increases, making becomes more attractive and  $\bar{\alpha}$  asymptotically decreases to the standard real option threshold, for which making is the only available procurement mode.<sup>20</sup>

Consider next the effect of uncertainty on the option value to invest. The payoff from investing is given by:

$$V_M(\alpha; K) - PV_1(\alpha) - K \tag{18}$$

This payoff is an S-shaped function of  $\alpha$ , as is illustrated in Figure 2. The payoff from investing in capital is bounded above (by the horizontal dashed line  $\Delta PV_{31}(K) - K$ ) because the

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<sup>20</sup>  $\lim_{p \rightarrow \infty} \bar{\alpha} = \left( \kappa \frac{f/r + \tilde{K}}{\Pi_2(\tilde{K})} \right)^{(\xi + \theta - 1)/\xi}$ , where  $\kappa \equiv \frac{\lambda_1}{\lambda_1 - \xi / (\xi + \theta - 1)}$  and  $\tilde{K} = \underset{K}{\operatorname{argmax}} \frac{K^{\kappa \xi} \frac{1 - \theta}{\xi + \theta - 1}}{f/r + K}$ . For  $\sigma = 0.1$  (0.15, 0.05 and 0.025), the limiting value of the threshold equals 0.945 (1.367, 0.765, and 0.711, respectively).

firm switches from making to concurrent sourcing once demand exceeds a critical threshold. Standard real options models assume the firm sticks to in-house production (no matter how high the output price) allowing the upside from investment in capital to be unbounded. The (net) payoff for low demand levels is similarly capped from below (by the dashed line  $-K$ ) because the firm switches from making to buying once demand drops below a particular threshold. The caps on the payoff from investing in capital have important implications for how uncertainty affects the option value to invest. Panel C of Figure 3 shows that uncertainty increases (reduces) the option value to invest for low (high) levels of  $\alpha$ , i.e.:

$$\frac{dOV(\alpha; \sigma)}{d\sigma} > (<) 0 \text{ for low (high) } \alpha$$

When  $\alpha$  is high there is little or no upside in value left because the firm switches to concurrent sourcing when demand is high. Hence volatility reduces the option value to invest. Conversely, when demand is low there is little downside (because the firm can always buy the product) but a lot of upside. As a result, volatility increases the option value to invest for low  $\alpha$ .

The effect of outsourcing price on the value of the option to invest is straightforward: higher  $p$  unambiguously increases the payoff from investing for all values of  $\alpha$ , and therefore also increases the value of the option to invest (cf. panel D), i.e.:

$$\frac{dOV(\alpha; p)}{dp} > 0$$

with  $OV(\alpha; \hat{p}_{\min}) = 0$ .<sup>21</sup>

Panel E of Figure 3 shows that volatility increases the optimal capital invested, a result we now turn to in more detail (the optimal level of capital for the benchmark case of  $\sigma = 0.1$  and  $p = 1.5$  is  $K = 9.338$ ). The effect of uncertainty on the capital installed upon investment

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<sup>21</sup>The option value asymptotes to a bounded value as  $p \rightarrow \infty$ . E.g., for  $\sigma = 0.1$ ,  $\lim_{p \rightarrow \infty} OV(0.75; p) = 2.562$

can be expressed as:

$$\frac{dK(\bar{\alpha}; \sigma)}{d\sigma} = \underbrace{\frac{\partial K(\bar{\alpha}; \sigma)}{\partial \sigma}}_{\text{Direct effect}}^{(+/-)} + \underbrace{\frac{\partial K(\bar{\alpha}; \sigma)}{\partial \bar{\alpha}} \frac{d\bar{\alpha}}{d\sigma}}_{\text{Waiting effect}}^{(+)(+)} > 0 \quad (19)$$

The first term describes the *direct* effect of volatility on the capital level when holding the entry threshold  $\bar{\alpha}$  constant. Numerical analysis reveals that the direct effect can have either sign. The direction of the indirect *waiting* effect is unambiguous. Higher volatility delays investment ( $\frac{d\bar{\alpha}}{d\sigma} > 0$ ), which in turn increases the capital installed at the time of investment ( $\frac{\partial K}{\partial \bar{\alpha}} \geq 0$ ). The results in Panel E show that for feasible parameter values the combined direct and indirect effect is positive. Consequently, higher volatility increases the capital installed upon investment. This result is in line with previous studies that showed a positive relation between uncertainty and the optimal capital level (Capozza and Li (1994) and Bar-Ilan and Strange (1999)).

The effect of outsourcing price ( $p$ ) on the optimal capacity level can also be decomposed into a direct and an indirect (waiting) effect:

$$\frac{dK(\bar{\alpha}; p)}{dp} = \underbrace{\frac{\partial K(\bar{\alpha}; p)}{\partial p}}_{\text{Direct effect}}^{(+)} + \underbrace{\frac{\partial K(\bar{\alpha}; p)}{\partial \bar{\alpha}} \frac{d\bar{\alpha}}{dp}}_{\text{Waiting effect}}^{(+)(-)} < 0 \quad (20)$$

The direct effect is positive: a higher outsourcing price,  $p$ , *ceteris paribus* leads to more capital installed. But, a higher  $p$  accelerates investment, which in turn lowers the capital installed, creating a negative indirect waiting effect of  $p$  on the optimal capital investment.

The results in panels E-F show that higher outsourcing price *reduces* the amount of capital investment when investment is irreversible as the indirect effect of waiting dominates. Indeed, we know that  $\frac{d\bar{\alpha}}{dp} \rightarrow -\infty$  as  $p \downarrow \hat{p}_{\min}$ , implying that  $\frac{dK(\bar{\alpha}; p)}{dp}$  is negative there. Numerical simulations indicate that the result extends for the entire domain of  $p$ . The intuition for it is simple: if the outsourcing price is high, investment occurs at a lower level of demand

so capital investment is relatively modest.<sup>22</sup> The upper bound for capital invested equals  $K(q_{2M}^o(\alpha^*); \hat{p}_{\min})$ , which is 12 for the base set of parameter values.<sup>23</sup>

Finally, consider the effect of uncertainty and the outsourcing price on the optimal regime switching thresholds  $\alpha^*(K)$  and  $\alpha^{**}(K)$ . It is helpful at this point to circle back to Figure 1 which plots both cut-off levels as a function of the installed capital level. Cutoff  $\alpha^*(K)$  decreases and  $\alpha^{**}(K)$  increases with  $K$ .

While  $\sigma$  does not affect  $\alpha^*(K)$  and  $\alpha^{**}(K)$  directly because the switch at both cutoffs is perfectly reversible, there is an *indirect* effect through the capital level installed. From our earlier results regarding  $\frac{dK}{d\sigma}$ , it follows that:

$$\frac{d\alpha^*(K)}{d\sigma} < 0 \quad \text{and} \quad \frac{d\alpha^{**}(K)}{d\sigma} > 0$$

Hence, higher volatility requires greater changes of demand to trigger the switch from making to both outsourcing and concurrent sourcing.

The effect of outsourcing price on the regime boundaries is both direct and indirect. For a given  $K$ , higher  $p$  results in an increased gap between  $\alpha^*(K)$  and  $\alpha^{**}(K)$ . On the other hand,  $p$  negatively affects the amount of installed capital, which has the opposite effect on the distance between the two cutoffs. Therefore, the combined effect is generally ambiguous.<sup>24</sup>

We conclude that uncertainty affects investment in costly capital in non-standard ways if the firm has access to alternative procurement options. The reason for this is that alternative (less capital intensive) modes of production cap the upside and downside risk associated with capital investment, which in turn alters the effect of uncertainty compared to standard real

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<sup>22</sup>In the limiting case of  $p \rightarrow \infty$ ,  $K = 1.937$  for  $\sigma = 0.1$  (with  $p \rightarrow \infty$ ,  $K$  equals 4.762, 1.168, and 0.990 for  $\sigma = 0.15, 0.05$ , and  $0.025$ , respectively.)

<sup>23</sup>Note that the upper bound for installed capital in the fully *reversible* case is higher and equals  $K(q_{3M}^o) = 44.052$ , which is the maximum amount of capital that will be utilized for any future  $\alpha$ .

<sup>24</sup>Numerical simulations indicate that  $\alpha^*(K)$  ( $\alpha^{**}(K)$ ) either decreases or increases (increases) with  $p$ , with the sensitivity of  $\alpha^{**}(K)$  with respect to  $p$  being substantially higher. Consequently, the width of the region in which (pure) making is optimally adopted increases with  $p$ .



option models. In addition, access to cheaper outsourcing actually increases the amount of capital invested, given that the firm optimally delays investment until a higher level of demand  $\alpha$  is reached.<sup>25</sup>

#### 4.4 OPERATING LEVERAGE

The model's analytical solution allows us to analyze how operating leverage is affected by the firm's operating policy. We define operating leverage in regime  $i$  as (cf., e.g., Lev (1974) and Reilly and Bent (1975))

$$OL_i(q_i^o(\alpha)) = 1 - \left. \frac{\partial TC_i(q)}{\partial q} \right|_{q=q_i^o(\alpha)} \frac{q_i^o(\alpha)}{TC_i(q_i^o(\alpha))} = 1 - \frac{MC_i(q_i^o(\alpha))}{AC_i(q_i^o(\alpha))} \quad (21)$$

where  $q_i^o \equiv q_{Bi}^o + q_{Mi}^o$ , and  $AC_i(q)$ ,  $MC_i(q)$ , and  $TC_i(q)$ , is the average, marginal and total cost, respectively, associated with generating output  $q$  in regime  $i$ .

The advantage of this measure of operating leverage is that it is not confounded by product market effects such as the firm's market power, mark-up as well as the shocks to the demand itself. A quick inspection of (21) reveals that the notion of operating leverage is intrinsically linked to the concept of the economies of scale. Positive (negative) economies of scale are equivalent to the marginal cost being lower (higher) than the average cost, which – in turn – is equivalent to a positive (negative) operating leverage.<sup>26</sup>

The operating leverage ( $OL_i$ ) is depicted in Figure 4 as a function of the demand function parameter  $\alpha$  for the set of parameter values as in section 4.3 and  $K = 9.338$ .<sup>27</sup> Kinks in  $OL_i$

<sup>25</sup>With fully reversible capital, the amount of invested capital  $K$  under making does not depend on the outsourcing price and *increases* with  $p$  under concurrent sourcing, which is adopted for higher  $\alpha$ .

<sup>26</sup>Recall that positive (negative) economies of scale are defined as the presence of a decreasing (increasing) average cost. A decreasing (increasing) average cost is equivalent to the marginal cost being lower (higher) than the average cost.

<sup>27</sup>Recall that  $K = 9.338$  is the optimal amount of capital for the base set of parameter values:  $\mu = 0.01$ ,  $r = 0.05$ ,  $\sigma = 0.1$ ,  $A = 0.2$ ,  $\zeta = 0.3$ ,  $\eta = 0.65$ ,  $\theta = 0.5$ ,  $f = 0.1$ ,  $w = 0.2$ , and  $p = 1.5$ .

correspond to cutoffs  $\alpha^*$  and  $\alpha^{**}$ , which delineate regime boundaries.

[Please insert Figure 4 about here.]

Under buying (regime 1),  $OL_1(q_1^o(\alpha))$  equals zero as there is no fixed cost and  $MC_1(q) = p = AC_1(q)$  for all  $q$ .

When making is optimally adopted (regime 2), a fixed cost  $f$  is present, and the resulting operating leverage equals

$$OL_2 = \frac{f - (\xi - 1)c(q_2^o(\alpha))^\xi}{c(q_2^o(\alpha))^\xi + f} \geq 0 \Leftrightarrow \alpha \leq \alpha_0 \equiv \frac{\xi}{1 - \theta} c^{\frac{1-\theta}{\xi}} \left( \frac{f}{\xi - 1} \right)^{\frac{\theta + \xi - 1}{\xi}}$$

In regime 2, there are two opposing effects. First, the presence of the fixed cost contributes to a higher operating leverage. This creates a positive spike in operating leverage at  $\alpha^*$  (the point where the firm switches back and forth between buying and making). Second, an increasing marginal cost of production affects operating leverage negatively. For low  $\alpha$  the former effect dominates and the outcome is consistent with the textbook notion that a technology associated with a positive fixed cost component is generally more risky.<sup>28</sup> However, for sufficiently high  $\alpha$  ( $\alpha > \alpha_0$ ), operating leverage becomes *negative* as the firm enters the region of negative scale economies (i.e.,  $MC_2(q) > AC_2(q)$  for  $\alpha > \alpha_0$ ).<sup>29</sup> Importantly, operating leverage is negative and reaches a minimum at  $\alpha^{**}$ , the point where the firm switches between making and concurrent sourcing. At  $\alpha^{**}$ , the marginal cost equals the outsourcing price  $p$  and exceeds the average cost.

Once  $\alpha$  exceeds  $\alpha^{**}$  and concurrent sourcing is optimally adopted (regime 3), operating leverage can be expressed as

$$OL_3 = \frac{-C_1 + f}{pq_3^o(\alpha) - C_1 + f} < 0$$

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<sup>28</sup>Note that  $q_i^o$  is globally increasing with  $\alpha$ .

<sup>29</sup>We can show that  $\alpha_0$ , for which  $OL_2 = 0$ , indeed falls into the making regime.

where  $C_1$  is defined by (A.10). Under concurrent sourcing the firm’s cost structure is the same as under buying ( $MC_3(q) = p$ ) except that the firm also realizes a constant cost saving of  $C_1 - f$ , which results from making  $q_{3M}^o$  units of output. Therefore, its operating leverage is still negative.<sup>30</sup> As output increases through concurrent sourcing the saving is spread over an increasing output quantity. Therefore,  $OL_3$  increases asymptotically to zero as a consequence of  $AC_3(q_3^o(\alpha))$  increasing asymptotically to  $p$ . While the negative sign of operating leverage under concurrent sourcing is a feature specific to our measure of operating leverage, the finding that operating leverage under concurrent sourcing is lower than under making or buying is quite robust.<sup>31</sup>

In summary, operating leverage peaks (reaches a minimum) when the firm switches from buying to making (from making to concurrent sourcing). Firms engaging in concurrent sourcing have negative operating leverage, and their operating leverage is lower than for firms that outsource or firms that produce internally at output levels that are not too high.

## 4.5 COMPANY BETAS AND EQUITY RETURNS

Our model has direct implications for a firm’s systematic risk, measured as the elasticity of the firm value with respect to the demand variable  $\alpha$  (cf. Carlson et al. (2004)). For the

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<sup>30</sup>A firm that engages in concurrent sourcing is, in a way, similar to a firm that has a “negative” fixed operating cost equal to the difference between the cost of buying the total internal production ( $q_{3M}^o$ ) and making it. This difference generates a constant, fixed cost savings flow (similar to a negative fixed cost) for as long as concurrent sourcing takes place.

<sup>31</sup>To check robustness, we analyzed different measures of operating leverage. For example,  $OL = \frac{d \text{Profit}}{\text{Profit}} \frac{\text{Output}}{d \text{Output}}$  (see Mandelker and Rhee (1984) and Guthrie (2011)) and  $OL = \frac{d \text{Profit}}{\text{Profit}} \frac{\text{Sales}}{d \text{Sales}}$  (see O’Brien and Vanderheiden (1997)) both equal 1 under buying, shoot up at  $\alpha^*$ , and monotonically decline up to  $\alpha^{**}$ . Both measures drop below 1 at  $\alpha^{**}$  and asymptotically increase towards 1 as  $\alpha \rightarrow \infty$ . Both measures behave in a similar way as ours, except that operating leverage does not become negative. Importantly, for all three measures operating leverage is lower under concurrent sourcing than under making or buying.

mature firm, beta ( $\frac{\partial V_M}{\partial \alpha} \frac{\alpha}{V_M}$ ) equals

$$\beta_M(\alpha) = \begin{cases} \frac{1}{\theta} + \frac{(\lambda_1 - \frac{1}{\theta})A_1\alpha^{\lambda_1}}{V_M(\alpha)} & (> \frac{1}{\theta}) & \alpha \in (0, \alpha^*], \\ \frac{\xi}{\theta + \xi - 1} + \frac{(\lambda_1 - \frac{\xi}{\theta + \xi - 1})A_2\alpha^{\lambda_1} + (\lambda_2 - \frac{\xi}{\theta + \xi - 1})B_2\alpha^{\lambda_2} + \frac{\xi f}{(\theta + \xi - 1)r}}{V_M(\alpha)} & (\geq \frac{1}{\theta}) & \alpha \in (\alpha^*, \alpha^{**}], \\ \frac{1}{\theta} + \frac{(\lambda_2 - \frac{1}{\theta})B_3\alpha^{\lambda_2} - \frac{C_1 - f}{\theta r}}{V_M(\alpha)} & (< \frac{1}{\theta}) & \alpha > \alpha^{**}. \end{cases} \quad (22)$$

The beta of a mature firm,  $\beta_M$ , is  $1/\theta$  for  $\alpha \rightarrow 0$  (see (22)), which is the absolute value of the price elasticity of demand (recall that the present value of the firm's cash flow under buying is proportional to  $\alpha^{\frac{1}{\theta}}$ ). For  $\alpha \rightarrow \infty$ ,  $\beta_M$  tends to  $1/\theta$  as well. For the intermediate levels of  $\alpha$ ,  $\beta_M$  can be either higher or lower than  $1/\theta$  and  $\beta_M$  is affected by the firm's operating leverage as well as the options to change procurement regime.<sup>32</sup>

Compare first a firm that can only buy (hereafter "buy-only") with a firm that can only make (hereafter "make-only"). The beta of a buy-only firm is constant and equal to the absolute value of the price elasticity of demand (i.e.  $1/\theta$ ). The beta of a make-only firm is high for low states of the economy (and above the beta of a buy-only firm) because of fixed production costs and the resulting high operating leverage; the make-only beta, monotonically decreases with the state of the economy, and asymptotes to a level (equal to  $\xi/(\theta + \xi - 1)$ ) that is below the beta of a buy-only firm. The beta of a make-only firm is therefore counter-cyclical.<sup>33</sup>

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<sup>32</sup>In appendix C, we consider an extension in which a larger order attracts a bigger discount and  $p(q) = p_0 q^{-\nu}$ , where  $p_0 > 0$  and  $\nu \in [0, \theta)$ . A decreasing outsourcing cost has a first-order effect on the firm's beta. For very low demand levels, outsourcing prevails and beta of the firm approaches  $\frac{1-\nu}{\theta-\nu}$ , which is greater than buy-only beta with a fixed outsourcing cost. The same limit occurs for  $\alpha \rightarrow \infty$  when firm adopts concurrent sourcing. A higher beta is in both cases due to the fact that the procurement cost is negatively related to the level of demand, through changes in output, which causes the firm's profit to respond to demand fluctuations more strongly.

<sup>33</sup>This result echoes that of Cooper (2006), who demonstrates that the presence of idle capacity and high book-to-market ratios associated with bad states of the economy lead to higher expected returns. Other explanations for counter-cyclical expected returns include limited operating flexibility due to unionization

Why are make-only firms safer than buy-only firms in good states of the economy? The reason is that the internal production function is subject to decreasing returns to scale<sup>34</sup>, which dampens the sensitivity of output to economic shocks (compared to outsourcing, which is a constant return to scale technology).<sup>35</sup>

Does operating flexibility lower or raise a firm's beta? The answer again depends on the state of the economy. In states with high product demand a firm that can engage in concurrent sourcing will have a higher beta than its corresponding counterpart that can only produce the good internally. This result follows from concurrent sourcing being a higher return to scale technology, and therefore also a riskier operating mode, than making. In states with low product demand the beta of a buy-only firm (the natural benchmark for low states) is lower than the beta of a firm that has the flexibility to switch from buying to making, because the latter has the option to switch to a regime with high fixed costs and therefore high operating leverage.

We can now also understand the non-monotonic behavior of beta. Beta is low under buying, and gradually reaches a peak as the firm moves towards making. Under making, beta gradually declines as operating leverage lowers. As diseconomies of scale set in (i.e., the marginal cost exceeds the average cost) beta moves towards a minimum, but then gradually increases again as the firm starts engaging in concurrent sourcing and therefore adopts a higher return to scale technology. Thereafter beta rises and asymptotes towards the absolute value of the price elasticity of demand. Since beta peaks around the buy-to-make switching point, and reaches a minimum in the neighborhood of the make-to-concurrent

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<sup>34</sup>With increasing returns to scale we would get the opposite result, i.e. make-only firms are riskier than buy-only firms.

<sup>35</sup>This relative dampening effect is even stronger if the outsourcing price decreases with the quantity that is bought.

sourcing switching point, it follows a characteristic non-monotonic pattern. The main result is summarized in the following corollary:

**Corollary 1** *The beta and expected equity returns of a mature firm are the highest (lowest) when the firm is on the cusp of switching between buying and making (making and concurrent sourcing).*

The behavior of beta is visualized in Figure 5 which plots beta as a function of demand for various levels of volatility. The company beta is generally larger in bad states of the economy than in good states. This effect is more pronounced for low volatility levels as it is more likely then that the firm will stay in a given regime for longer so the regime type will have a stronger influence on beta.<sup>36</sup> As a result of the presence of regime switching options, volatility has a more pronounced effect on the beta of a firm with procurement flexibility ( $\beta_M$ ) than on the one of a make-only firm (denoted with  $\beta_{M(2)}$ ).

[Please insert Figure 5 about here.]

For sufficiently good states of the economy, the beta of the mature firm that can be flexible regarding the procurement regime it adopts is *lower* than the beta of an otherwise identical firm that can only rely on outsourcing. This result may be somewhat unexpected as it seemingly contradicts the standard finance textbook argument that a higher fixed to variable cost ratio, which is associated with concurrent sourcing and making, leads to a higher firm risk (see, e.g., Brealey, Myers, and Allen (2010), p. 250). The standard argument, however, is based on the implicit assumption of a constant marginal cost, which – in combination with a presence of a fixed cost – results in a positive scale economies and positive operating leverage. Since in our model negative scale economies kick in for a sufficiently high level

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<sup>36</sup>When volatility is lower (higher), the instantaneous cost structure influences beta more (less) heavily whereas the effect of the options to switch the production regime is smaller (greater).

of output, operating leverage becomes negative, which eventually contributes to beta being lower than under a constant average cost technology (buying).

## 5 Empirical Implications

Our model has several novel testable implications. Here are the main ones:

(1) Our model predicts that concurrent sourcing is not a “mixed” production strategy that lies somewhere on a continuous make-buy spectrum. Instead, it should be considered a third, distinct strategy used in response to high demand (as it is optimal for  $\alpha > \alpha^{**}$ ). Our paper is therefore capable of explaining the empirical findings of Parmigiani (2007).

(2) Outsourcing activity is likely to display a U-shaped pattern as a function of the state of the economy (as no outsourcing occurs for  $\alpha \in (\alpha^*, \alpha^{**})$ ). Economic recessions generate outsourcing motivated by cost-cutting, whereas in economic booms firms engage in concurrent sourcing to meet “peak” demand. Domberger (1999) gives an overview of outsourcing activity in a historical context and illustrates that outsourcing occurs in waves.

(3) The availability of outsourcing imposes a minimum capital requirement on firms: firms that cannot operate at a sufficiently large scale stop producing internally, irrespective of the state of the economy. Our model therefore predicts that the advent of cheap outsourcing leads to the closure of many small, labor intensive production units. The cheaper it is to buy, the larger the scale required for internal production and the fewer production units survive.

(4) Market-to-book ratio (a proxy for Tobin’s  $q$ ) is a poorer predictor of corporate investment (cf. Erickson and Whited (2000)) for those sectors or firms that are likely to adopt a less capital intensive technology in response to positive economic shocks.

(5) García-Feijóo and Jorgensen (2010) find a positive association between the degree of operating leverage and stock returns. Novy-Marx (2011) documents that operating lever-

age predicts returns in the cross-section, and that strategies formed by sorting on operating leverage earn significant excess returns. Our model provides complementary empirical predictions regarding the link between stock returns and operating leverage for firms that can adopt multiple operating strategies:

(i) All else equal, production technologies with higher return to scale parameters generate higher expected stock returns than technologies with lower returns to scale.

(ii) All else equal, (dis)economies of scale coincide with (lower) higher expected stock returns.

(iii) Betas are counter-cyclical for firms that produce all output internally. Counter-cyclicity of stock betas has been empirically confirmed by Lettau and Ludvigson (2001), among others.

(iv) In economic recessions (booms) betas of firms that can only make output are higher (lower) than betas of firms that can only buy output.

(v) Operating leverage is non-monotonic for firms that can switch to other operating regimes (such as outsourcing and concurrent sourcing). The beta of a firm with different procurement options may therefore display a more complex, non-monotonic behavior. In particular, our model predicts a peak in beta when a firm switches between making and buying. By focussing on a sample of firms that switch from buying to making (or vice versa), one could empirically verify whether there is a statistically significant increase in beta in the run-up (or run-down) to this switch. Similarly, one could test whether firms that engage in concurrent sourcing have, on average, lower expected stock returns than their counterparts that only make or buy.<sup>37</sup>

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<sup>37</sup>Although the current paper does not focus on the financing side of the firm's activities, the presented model yields a prediction about the expected capital structure following the leverage trade-off option hypothesis (cf. Van Horne (1977), see also Mandelker and Rhee (1984) and Kahl, Lunn, and Nilsson (2012)). According to this hypothesis, a change in the production technology associated with an increase in operating



(vi) Due to the presence of regime switching options, procurement flexibility increases the sensitivity of stock beta to market volatility.

While some of our predictions have already been tested and confirmed empirically, many predictions have yet to be explored and could provide the basis for future empirical research.

## 6 Conclusions

This paper examines a firm's make-or-buy decision assuming that making is subject to fixed production costs and decreasing returns to scale, while buying happens at a constant unit price. We find that three different procurement regimes arise: buying, making, and concurrent sourcing. Firms optimally switch back and forth between regimes in response to economic shocks. Our model shows that concurrent sourcing is an operating strategy that is distinctly separate from buying and making. Capital investment plays an important role in the choice of the optimal procurement mode. A lower level of capital installed makes internal production more expensive compared to buying and this compresses the output demand range over which making is optimal. Therefore, it encourages buying or concurrent sourcing because these operating modes are less capital intensive.

We show that a firm's procurement strategy affects its operating leverage. Operating leverage is zero if the firm buys all its output from an external supplier. Operating leverage under making declines in the output level: operating leverage is initially strongly positive but becomes negative for high output levels. Operating leverage is negative under concurrent sourcing. Hence, operating leverage peaks (reaches a minimum) when the firm switches from buying to making (from making to concurrent sourcing). This non-monotonic pattern of leverage is likely to be associated with a reduction of financial leverage in order for the risk of the firm's equity stock to remain unchanged. As concurrent sourcing results in a lower operating leverage than buying and (largely) making, it is expected to be associated with a higher level of debt than the remaining two regimes.

operating leverage determines the behavior of the stock beta. Beta is low under buying, and gradually reaches a peak as the firm moves towards making. Under making, beta gradually declines as operating leverage lowers. As diseconomies of scale set in, beta moves towards a minimum, but then gradually increases again as the firm starts engaging in concurrent sourcing and therefore adopts a higher return to scale technology. Thereafter beta rises and asymptotes towards the (absolute value of the) price elasticity of demand. Since beta peaks around the buy-to-make switching point, and reaches a minimum in the neighborhood of the make-to-concurrent sourcing switching point, it follows that the beta follows a characteristic non-monotonic pattern. Low product demand volatility amplifies the stock beta's peak and through. Positive (negative) scale economies generate higher (lower) beta and higher (lower) expected stock returns.

Our results open up avenues for future research. For instance, our model could be applied to related problems such as a firm's decision to export or to create productive capacity via Foreign Direct Investment (FDI). The latter usually allows for a lower marginal cost but involves a higher investment in overseas capacity.<sup>38</sup> Our paper also has implications for merger and takeover activity. Our results show that value can be created when a firm engaging in concurrent sourcing merges with a firm that outsources its production, if the latter firm has idle production capacity. The value creation arises from two potential sources. First, both firms can decrease their average cost of production by transferring production from the former to the latter firm. Second, the firms' capital in place can be used more efficiently if the latter firm has excess capacity. This type of mergers reduces outsourcing activity and causes a shift towards large vertically integrated firms. Finally, our model could be extended to examine the effect of financial leverage and adjustment costs.

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<sup>38</sup>An important paper that discusses FDI in a dynamic context is Kogut and Kulatilaka (1994) who model a multinational firm's operating flexibility to shift production between two manufacturing plants located in different countries.

## A Proofs of Propositions

**Proof of Proposition 1.** We start off by determining  $\pi_i^o$ , the maximum profit flow that can be achieved in a given regime  $i$  ( $i = 1, 2, 3$ ), and subsequently determine which of the 3 regimes is optimal.

No capital is needed for regime 1. Optimizing  $\pi_1(q_{1B})$  with respect to  $q_{1B}$ , the first and second order conditions are given by:

$$\frac{\partial \pi_1(q_{1B})}{\partial q_{1B}} = \alpha(1 - \theta)q_{1B}^{-\theta} - p = 0 \quad (\text{A.1})$$

$$\frac{\partial^2 \pi_1(q_{1B})}{\partial q_{1B}^2} = -\alpha\theta(1 - \theta)q_{1B}^{-\theta-1} < 0 \quad (\text{A.2})$$

Substituting  $q_{1B}^o$  into  $\pi(q_{1B})$  we find that the maximum profit flow in regime 1 is:

$$\pi_1 = \Pi_1 \alpha^{\frac{1}{\theta}} \text{ for } q_{1M}^o = 0 \text{ and } q_{1B}^o = \left( \frac{(1 - \theta)\alpha}{p} \right)^{\frac{1}{\theta}} \quad (\text{A.3})$$

where

$$\Pi_1 \equiv \left( \frac{1 - \theta}{p^{1-\theta}} \right)^{\frac{1}{\theta}} [(1 - \theta)^{-1} - 1]$$

It follows that  $\pi_1^o(\alpha) > 0$  for all  $\alpha > 0$ , so buying is always viable.

In regime 2, optimizing  $\pi_2(q_{2M})$  with respect to  $q_{2M}$  gives the following optimality conditions:

$$\frac{\partial \pi_2(q_{2M})}{\partial q_{2M}} = \alpha(1 - \theta)q_{2M}^{-\theta} - \xi c(K)q_{2M}^{\xi-1} = 0 \quad (\text{A.4})$$

$$\frac{\partial^2 \pi_2(q_{2M})}{\partial q_{2M}^2} = -\alpha\theta(1 - \theta)q_{2M}^{-\theta-1} - \xi(\xi - 1)c(K)q_{2M}^{\xi-2} < 0 \quad (\text{A.5})$$

Substituting  $q_{2M}^o$  into  $\pi(q_{2M})$  we find that the maximum profit flow in regime 2 is:

$$\pi_2^o = \Pi_2 \alpha^{\frac{\xi}{\theta+\xi-1}} - f \text{ for } q_{2B}^o = 0 \text{ and } q_{2M}^o = \left( \frac{(1 - \theta)\alpha}{\xi c} \right)^{\frac{1}{\theta+\xi-1}} \quad (\text{A.6})$$

where

$$\Pi_2 \equiv \left( \frac{1 - \theta}{\xi c} \right)^{\frac{1-\theta}{\theta+\xi-1}} \left( 1 - \frac{1 - \theta}{\xi} \right)$$

Finally, optimizing the profit flow for regime 3,  $\pi_3(q_{3B}, q_{3M})$ , gives as first order conditions:

$$\frac{\partial \pi_3(q_{3B}, q_{3M})}{\partial q_{3B}} = \alpha(1 - \theta)(q_{3B} + q_{3M})^{-\theta} - p \quad (\text{A.7})$$

$$\frac{\partial \pi_3(q_{3B}, q_{3M})}{\partial q_{3M}} = \alpha(1 - \theta)(q_{3B} + q_{3M})^{-\theta} - \xi c(K)q_{2M}^{\xi-1} = 0 \quad (\text{A.8})$$

Solving gives the expressions for  $q_{3B}^o$  and  $q_{3M}^o$  given in the proposition. One can show that the first and second order leading principal minors of the Hessian are, respectively  $-\theta(1 - \theta)\alpha(q_{3B} + q_{3M})^{-\theta-1}$  and  $\theta(1 - \theta)\alpha\xi(\xi - 1)cq_{3M}^{\xi-2}(q_{3B} + q_{3M})^{-\theta-1}$ . Since all constants are positive,  $\theta < 1$  and  $\xi > 1$ , it follows that the Hessian is negative definite and that the critical point  $(q_{3B}^o, q_{3M}^o)$  is a maximum. Substituting the solution into  $\pi(q_{3B}^o, q_{3M}^o)$  gives:

$$\pi_3^o(q_{3B}^o, q_{3M}^o) = \Pi_1 \alpha^{\frac{1}{\theta}} + C_1 - f \quad (\text{A.9})$$

where

$$C_1 \equiv \left( \frac{p^\xi}{\xi c} \right)^{\frac{1}{\xi-1}} (1 - \xi^{-1}) \quad (\text{A.10})$$

To decide whether to opt for buying or making, firms compare the profit flow under either regime by calculating and analyzing the difference  $\Delta_{12}(\alpha) \equiv \pi_1^o - \pi_2^o$ :

$$\Delta_{12}(\alpha) = \Pi_1 \alpha^{\frac{1}{\theta}} - \Pi_2 \alpha^{\frac{\xi}{\xi+\theta-1}} + f \quad (\text{A.11})$$

$$\frac{\partial \Delta_{12}(\alpha)}{\partial \alpha} = \frac{1}{\theta} \Pi_1 \alpha^{\frac{1-\theta}{\theta}} - \frac{\xi}{\xi + \theta - 1} \Pi_2 \alpha^{\frac{1-\theta}{\xi+\theta-1}} \quad (\text{A.12})$$

$$= \left( \frac{1}{\theta} \Pi_1 \alpha^{\frac{(\xi-1)(1-\theta)}{\theta(\xi+\theta-1)}} - \frac{\xi}{\xi + \theta - 1} \Pi_2 \right) \alpha^{\frac{1-\theta}{\xi+\theta-1}} \quad (\text{A.13})$$

It follows that  $\Delta_{12}(\alpha)$  is a function defined on the non-negative domain with  $\Delta_{12}(0) = f$ . The first-order derivative of  $\Delta_{12}(\alpha)$  is positive for sufficiently large  $\alpha$  and negative for sufficiently small  $\alpha$  (the sign of the expression in the bracket changes only once).

$\Delta_{12}(\alpha)$  is negative for some range of  $\alpha$  if and only if  $p > \hat{p}$ . For  $p = \hat{p}$  the firm is indifferent between buying the entire output and making a fraction of it internally, i.e.:

$$c(q_{3M}^o(\hat{p}))^\xi + f = \hat{p}q_{3M}^o(\hat{p})$$

Cutoff  $\hat{p}$  is therefore given by  $[(\frac{\xi f}{\xi-1})^{\xi-1} \xi c]^{1/\xi}$ .

As for sufficiently small  $\alpha$  buying dominates making, the lower bound of the internal production region,  $\alpha^*$ , is characterized by the smaller root of  $\Delta_{12}(\alpha) = 0$ . The switching point  $\alpha^*$  satisfies the value-matching condition:  $\pi_1^o(\alpha^*) = \pi_2^o(\alpha^*)$ .

As  $\alpha$  rises above  $\alpha^*$ , it may become optimal for the firm to start outsourcing any production in excess of some critical level. Let us denote the demand level  $\alpha$  at which switching from regime 2 to regime 3 is optimal by  $\alpha^{**}$ , where  $\alpha^{**}$  is the solution to  $\pi_2^o(\alpha^{**}) = \pi_3^o(\alpha^{**})$  and given by  $\alpha^{**} = \frac{1}{1-\theta} p^{\frac{\theta+\xi-1}{\xi-1}} (\xi c)^{-\frac{\theta}{\xi-1}}$ . It is straightforward to show that  $\alpha^{**}$  also satisfies the following smooth-pasting condition:

$$\left. \frac{d\pi_2(q_{2M}^o(\alpha); \alpha)}{d\alpha} \right|_{\alpha=\alpha^{**}} = \left. \frac{d\pi_3(q_{3M}^o(\alpha); \alpha)}{d\alpha} \right|_{\alpha=\alpha^{**}} \quad (\text{A.14})$$

The condition states that the optimal value function needs to be differentiable (“smooth”) at  $\alpha^{**}$ . Calculating both sides of the smooth-pasting condition reveals that the smooth-pasting condition is equivalent to  $q_{2M}^o(\alpha^{**}) = q_{3B}^o(\alpha^{**}) + q_{3M}^o$ . Substituting  $\alpha^{**}$  into the optimal capacity and output levels gives:  $q_{2M}^o(\alpha^{**}) = q_{3M}^o = \left(\frac{p}{\xi c}\right)^{\frac{1}{\xi-1}}$  and  $q_{3B}^o(\alpha^{**}) = 0$ . Since  $\frac{\partial q_{3B}^o(\alpha)}{\partial \alpha} = \frac{q_{1B}^o}{\alpha \theta} > 0$ , it follows immediately that  $q_{3B}^o(\alpha) > 0$  for all  $\alpha > \alpha^{**}$ . ■

**Proof of Proposition 2.** The optimal procurement regime is a function of demand, the cost of buying, and the cost of making. The two former are the same under fully reversible and fixed capital, whereas the latter has an identical functional form (cf. (3) and (10)). The problem under fully reversible capital is therefore equivalent to the problem under fixed level of capital. Finally, note that under fixed capital, its cost is sunk and not taken into account when making the procurement decision. Under fully reversible capital, its unit cost  $r$  is reflected in the cost parameter  $\hat{c}$ . ■

## B Derivation of Value Functions under Uncertainty

An arbitrary claim,  $F(\alpha)$ , that is contingent on  $\alpha$  and yields instantaneous cash flow  $M\alpha^n + N$ , where  $M, N \in \mathbb{R}$ , satisfies the ordinary differential equation (ODE)

$$rF(\alpha) = \mu\alpha \frac{\partial F(\alpha)}{\partial \alpha} + \frac{1}{2}\sigma^2\alpha^2 \frac{\partial^2 F(\alpha)}{\partial \alpha^2} + M\alpha^n + N \quad (\text{B.1})$$

Consequently, the value of the mature firm,  $V_M(\alpha; K)$ , young firm,  $V_Y(\alpha)$ , and the value of the firm with no option to invest in capacity,  $V_0(\alpha)$ , all satisfy ODE (B.1). The general solution to (B.1) is

$$F(\alpha) = \frac{M\alpha^n}{\delta(n)} + \frac{N}{r} + A\alpha^{\lambda_1} + B\alpha^{\lambda_2} \quad (\text{B.2})$$

where the first two components equal the present value of profit flow if the firm was to remain in the same procurement regime forever and the last two components represent options to move to one of the two adjacent regimes.<sup>39</sup> Given the profit flows for each of the procurement regimes as defined in section 2, the present values of relevant cash flows for  $V_M(\alpha; K)$  are given by:

$$PV_1(\alpha) = \frac{\Pi_1\alpha^{\frac{1}{\theta}}}{\delta(1/\theta)} \quad (\text{B.3})$$

$$PV_2(\alpha; K) = \frac{\Pi_2\alpha^{\frac{\xi}{\xi+\theta-1}}}{\delta(\xi/(\xi+\theta-1))} - \frac{f}{r} \quad (\text{B.4})$$

$$PV_3(\alpha; K) = PV_1(\alpha) + \frac{C_1 - f}{r} \quad (\text{B.5})$$

$V_M(\alpha; K|\alpha \in \text{regime } i)$  can be therefore written as

$$V_M(\alpha; K|\alpha \in \text{regime } i) = PV_i(\alpha; K) + A_i\alpha^{\lambda_1} + B_i\alpha^{\lambda_2} \quad (\text{B.6})$$

with  $i \in \{1, 2, 3\}$  corresponding to buying, making and concurrent sourcing, respectively. Given that the boundaries between the pairs of adjacent regimes are given by  $\alpha^*$  and  $\alpha^{**}$ , respectively, constants  $A_i$  and  $B_i$  are found derived by solving the system of two following pairs

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<sup>39</sup>The solution reflects the fact that the present value of an  $n$ -th power of the stochastic variable  $\alpha$  is associated with an effective discount rate equal to  $r - n\mu - 0.5n(n-1)\sigma^2 \equiv \delta(n)$ , cf. Dixit (1993), p. 13.

of value-matching and smooth-pasting conditions (recall that in this case smooth-pasting is not associated with optimality but merely with differentiability of the value function at a reversible threshold):

$$V_M(\alpha; K)|_{\alpha \uparrow \alpha_j} = V_M(\alpha; K)|_{\alpha \downarrow \alpha_j} \quad (\text{B.7})$$

$$\left. \frac{\partial V_M(\alpha; K)}{\partial \alpha} \right|_{\alpha \uparrow \alpha_j} = \left. \frac{\partial V_M(\alpha; K)}{\partial \alpha} \right|_{\alpha \downarrow \alpha_j} \quad (\text{B.8})$$

where  $\alpha_j \in \{\alpha^*, \alpha^{**}\}$ . Given that there are no adjacent regions associated with a decrease (an increase) in  $\alpha$  under buying (concurrent sourcing), two additional conditions

$$V_M(0; K) = 0 \quad (\text{B.9})$$

$$V_M(\alpha; K)|_{\alpha \uparrow \infty} = PV_3(\alpha; K) \quad (\text{B.10})$$

imply that constants  $B_1$  and  $A_3$  are zero.

Having derived  $V_M(\alpha; K)$  we can proceed to determining the value of the young firm,  $V_Y(\alpha)$ .  $V_Y(\alpha)$  consists of two components,  $PV_1(\alpha)$  and  $OV(\alpha)$ .  $PV_1(\alpha)$  is given by (B.3), whereas the option value of incremental capacity investment,  $OV(\alpha)$ , and the optimal investment threshold,  $\bar{\alpha}$ , are found by solving

$$OV(0; K) = 0 \quad (\text{B.11})$$

$$OV(\bar{\alpha}; K) = V_M(\bar{\alpha}; K) - PV_1(\bar{\alpha}) - kK \quad (\text{B.12})$$

$$\left. \frac{\partial OV(\alpha; K)}{\partial \alpha} \right|_{\alpha=\bar{\alpha}} = \left. \frac{\partial [V_M(\alpha; K) - PV_1(\alpha)]}{\partial \alpha} \right|_{\alpha=\bar{\alpha}} \quad (\text{B.13})$$

(Note that  $OV(\bar{\alpha}(K); K)$  satisfies (B.1) with  $M = N = 0$ .)  $OV(\bar{\alpha})$  is simply defined as  $\max_K OV(\bar{\alpha}(K); K)$ .

## C Declining Cost of Outsourcing

Below, we demonstrate the effects of relaxing the assumption of a constant outsourcing price  $p$ . More specifically, we now assume that larger orders attract a bigger discount, so

$p(q) = p_0 q^{-\nu}$ , where  $p_0 > 0$  and  $\nu \in [0, \theta)$  are constants.<sup>40</sup> The maximization of the profit function if the entire output is bought yields

$$\tilde{\pi}_1^o(\alpha) = \tilde{\Pi}_1 \alpha^{\frac{1-\nu}{\theta-\nu}} \text{ where } \tilde{q}_{1B}^o = \left( \frac{(1-\theta)\alpha}{p_0(1-\nu)} \right)^{\frac{1}{\theta-\nu}} \text{ and } \tilde{q}_{1M}^o = 0 \quad (\text{C.1})$$

with

$$\tilde{\Pi}_1 \equiv p_0^{-\frac{1-\theta}{\theta-\nu}} \left[ \left( \frac{1-\theta}{1-\nu} \right)^{\frac{1-\theta}{\theta-\nu}} - \left( \frac{1-\theta}{1-\nu} \right)^{\frac{1-\nu}{\theta-\nu}} \right] \quad (\text{C.2})$$

Equation C.1 implies that the elasticity of the firm's profit with respect to demand parameter  $\alpha$  is  $\frac{1-\nu}{\theta-\nu} > \frac{1}{\theta}$ . A decreasing outsourcing cost has therefore a first-order effect on the firm's beta. When buying is the only procurement mode or dominates, that is, for small and for very large  $\alpha$ , respectively, beta of the firm approaches  $\frac{1-\nu}{\theta-\nu}$ , which is greater than buy-only beta with a fixed outsourcing cost. This is due to the fact that the reduction in the costs partially offsets the dumping effect of demand elasticity on the output price and the firm's profit can respond to demand fluctuations more strongly.

Allowing for the decreasing cost of outsourcing results in the following system of equations that jointly determine the quantity bought,  $\tilde{q}_{3M}^o$ , and made,  $\tilde{q}_{3B}^o$ :

$$\frac{\partial \tilde{\pi}_3(q_{3B}, q_{3M})}{\partial q_{3B}} = \alpha(1-\theta)(q_{3B} + q_{3M})^{-\theta} - (1-\nu)p_0 q_{3B}^{-\nu} \quad (\text{C.3})$$

$$\frac{\partial \tilde{\pi}_3(q_{3B}, q_{3M})}{\partial q_{3M}} = \alpha(1-\theta)(q_{3B} + q_{3M})^{-\theta} - \xi c(K) q_{2M}^{\xi-1} = 0 \quad (\text{C.4})$$

Solving the above system numerically yields  $\tilde{q}_{3B}^o$  and  $\tilde{q}_{3M}^o$ . The resulting Hessian is negative definite and that the critical point  $(q_{3B}^o, q_{3M}^o)$  is a maximum if  $\xi(\xi-1)c(K)(\tilde{q}_{3M}^o)^{\xi-2} > \nu(1-\nu)p_0(\tilde{q}_{3B}^o)^{-\nu-1}$ .

Since the cost of outsourcing is not constant, the amount of utilized capital and the quantity made under the fully reversible case are going to fluctuate with  $\alpha$  even when concurrent sourcing is adopted. This results from the fact that the marginal cost of buying is no longer constant and the marginal cost of making has to follow changes in the former.

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<sup>40</sup>So for  $\nu = 0$ , we are back to the base case of a constant outsourcing cost.



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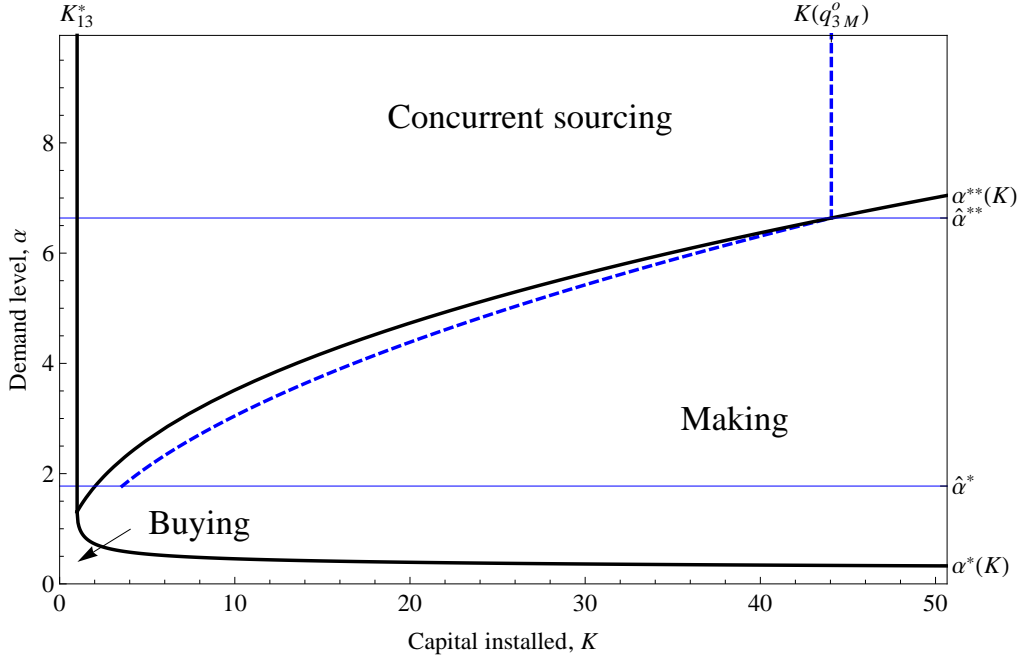


Figure 1: Optimal procurement regime for a given level of installed capital,  $K$ , and the demand parameter  $\alpha$  for the following set of parameter values:  $A = 0.2$ ,  $\zeta = 0.3$ ,  $\eta = 0.65$ ,  $\theta = 0.5$ ,  $f = 0.1$ ,  $w = 0.2$  and  $p = 1.5$ . Thick solid lines represent boundaries between three regions in which either making, buying or concurrent sourcing are optimally adopted if capital level  $K$  is exogenously given. Thin solid lines represent the level of demand consistent with boundary regimes under fully reversible case (section 3). The dashed line represents the optimal capital expansion path  $K(q_M(\alpha))$  in the fully reversible case.

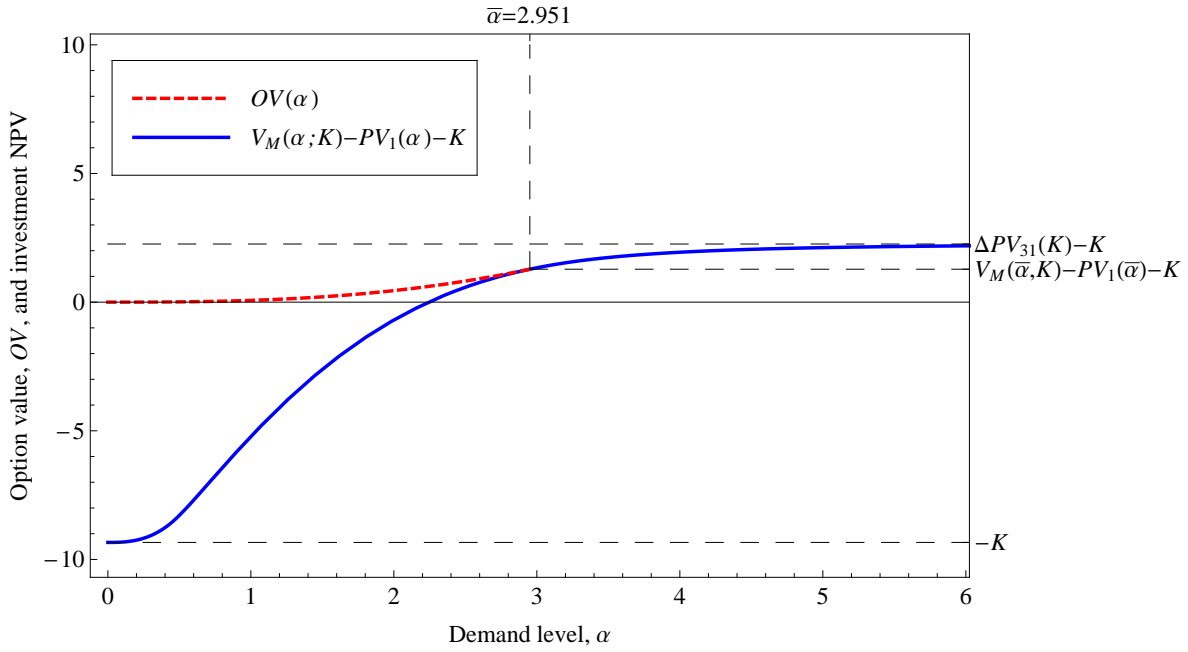


Figure 2: The option value,  $OV(\alpha)$ , and the net present value (NPV) of capacity investment,  $V_M(\alpha; K) - PV_1(\alpha) - K$ , as a function of the demand parameter  $\alpha$  for the following set of parameter values:  $\mu = 0.01$ ,  $r = 0.05$ ,  $\sigma = 0.1$ ,  $A = 0.2$ ,  $\zeta = 0.3$ ,  $\eta = 0.65$ ,  $\theta = 0.5$ ,  $f = 0.1$ ,  $w = 0.2$ ,  $p = 1.5$  and  $K = 9.338$  (optimal level).

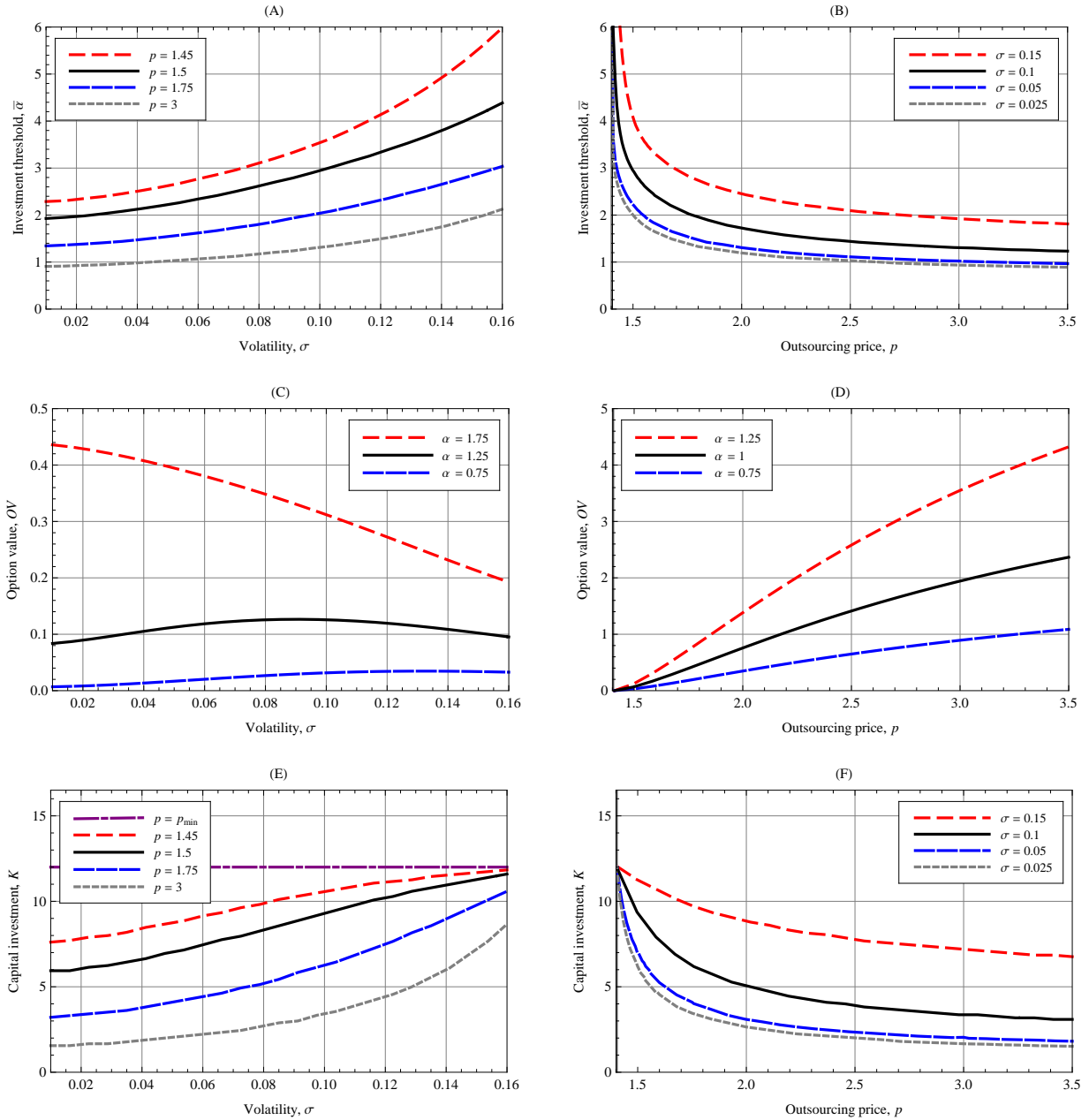


Figure 3: Investment threshold (panels A-B), option value (panels C-D) and capacity level (panels E-F) for the following set of parameter values:  $\mu = 0.01$ ,  $r = 0.05$ ,  $A = 0.2$ ,  $\zeta = 0.3$ ,  $\eta = 0.65$ ,  $\theta = 0.5$ ,  $f = 0.1$ , and  $w = 0.2$ . Unless indicated otherwise,  $\sigma = 0.1$  and  $p = 1.5$ . The minimum outsourcing price for which making is viable is  $\hat{p}_{\min} = 1.406$ .



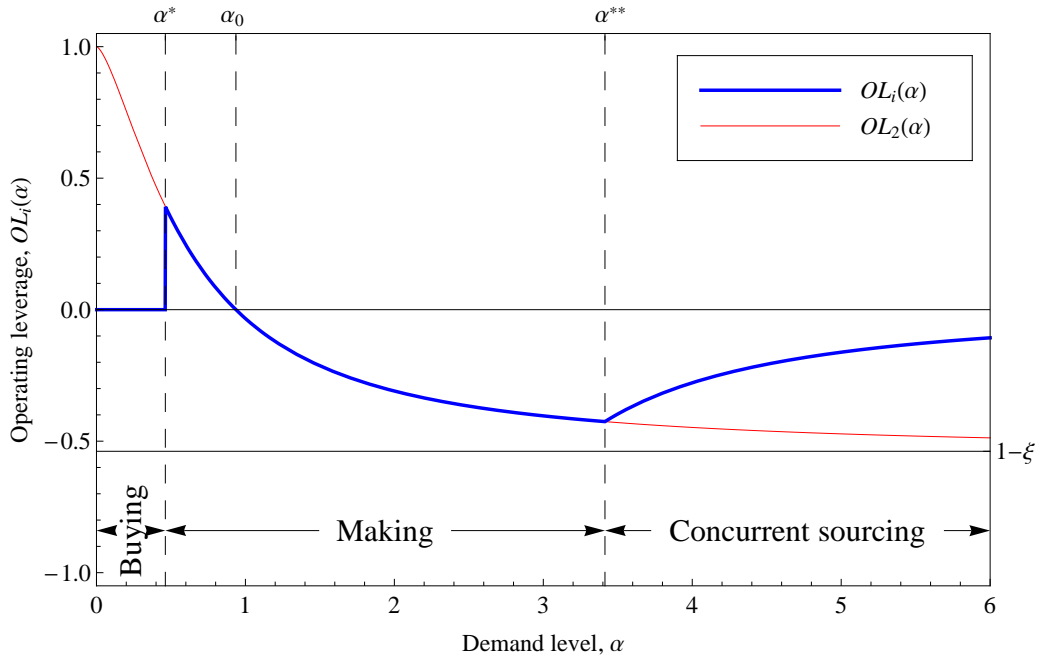


Figure 4: Operating leverage ( $OL_i$ ) is depicted as a function of the demand parameter  $\alpha$  for the following set of parameter values:  $A = 0.2$ ,  $\zeta = 0.3$ ,  $\eta = 0.65$ ,  $\theta = 0.5$ ,  $f = 0.1$ ,  $w = 0.2$ ,  $p = 1.5$  and  $K = 9.338$ . Operating leverage of a buy-only firm ( $OL_1$ ) is zero for all  $\alpha$ . Operating leverage of a make-only firm ( $OL_2$ ), depicted with the thin line, is 1 for  $\alpha = 0$  and asymptotes to  $1 - \xi$  for  $\alpha \rightarrow \infty$ .

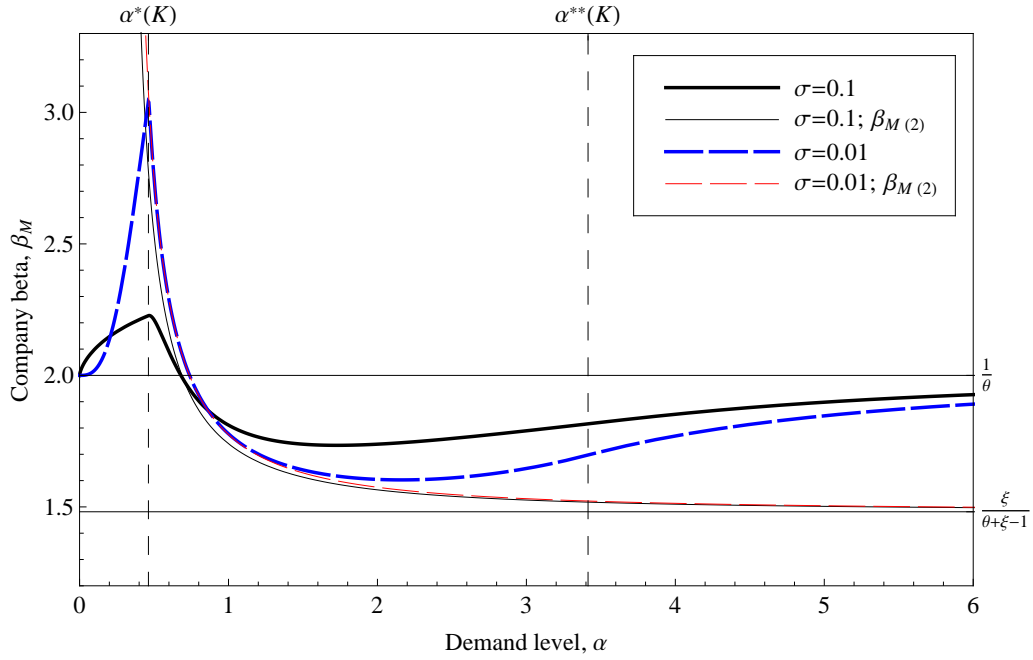


Figure 5: The beta of the mature firm as a function of the demand parameter  $\alpha$  for following set of parameter values:  $\mu = 0.01$ ,  $r = 0.05$ ,  $A = 0.2$ ,  $\zeta = 0.3$ ,  $\eta = 0.65$ ,  $\theta = 0.5$ ,  $f = 0.1$ ,  $w = 0.2$ ,  $p = 1.5$  and  $K = 9.338$ .  $\beta_{M(2)}$  denotes the beta of an otherwise identical, make-only firm (i.e., that adopts regime 2 for all  $\alpha$ ).