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Episodes of Exuberance in Housing Markets: In Search of the Smoking Gun*

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Abstract

After a prolonged period characterized by rapid real appreciation in house prices, there is now broad recognition of the severe correction in housing markets that followed as one of the causes of the 2008-09 global recession. We investigate the time series characteristics of three relevant price indicators of the housing market—real house prices, price-to-income, and price-to-rent ratios—for the U.S. and 21 other countries during the period 1975Q1-2013Q2 (see Mack and Martínez-García (2011)) for evidence of explosive behavior as a plausible explanation for the boom and bust. The empirical detection of explosive behavior in house prices provides a precise timeline as well as empirical content to the narrative connecting the evolution of housing markets to the global recession; our rich cross-country dataset offers a novel international perspective. For testing and detection, we adopt a pair of novel techniques based on a right-tail variation of the standard Augmented Dickey-Fuller (ADF) test—the supremum ADF ($SADF$) (Phillips et al. (2011)) and the generalized SADF ($GSADF$) (Phillips et al. (2012) and Phillips et al. (2013))—where the alternative hypothesis is of a mildly explosive process (even periodically collapsing with the $GSADF$ test) behavior within sample. Statistically significant periods of exuberance are found in most countries, with our empirical estimates suggesting an unprecendented synchronization across countries preceeding the global recession. The boom in housing begins during the late 90s in the U.S. spreading to most countries by the early 2000s, until it bursts for most during $2007-08$ as the impact on economic activity was being felt. In this regard, our findings corroborate the narrative of the 2008-09 global recession. In this paper, we also discuss more generally the use of these procedures to monitor international housing markets and as a warning signal.

JEL Classification: C22, G12, R30, R31

KEY WORDS: House Prices; 2008-09 Global Recession; Timeline; Unit-root Tests; Mildly Explosive Time Series; Sup ADF test; Generalized sup ADF test; Periodically Collapsing Bubbles.

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1 Introduction

The latest boom and bust in international housing markets has generated an increased interest in the dynamics of house prices. The policy concerns on housing are related to the effectiveness of fiscal, monetary and regulatory policies to restore financial stability after the collapse of house prices. While the consequences of the housing correction are still being felt, shedding light on the emergence and evolution of the period of exuberance that preceded the 2008-09 global recession can improve our understanding of the housing market and is of great importance to policy-making. Explosive behavior in house prices can give rise to such boom periods which, in turn, can lead to a misallocation of resources, distort investment patterns, and have serious repercussions in real economic activity. The depth and complexity of the 2008-09 global recession is such, that exploring its root causes in the housing market, and even mapping its timeline, has been anything but straightforward. Our paper provides a unique look at the evolution of housing markets in the U.S. and internationally.

We examine empirically the performance of international housing markets—since the housing market correction is viewed as one of the factors leading to the 2008-09 global recession—to elicit and document the facts, and explore their interconnectedness. To achieve this, we use data from the International House Price Database of the Federal Reserve Bank of Dallas (Mack and Martínez-García (2011)) as it offers a comprehensive and comparable panel from the first quarter of 1975 to the second quarter of 2013 for the U.S. and 21 other countries. We focus on periods of housing exuberance in the testing and detection of explosive autoregressive behavior in real house prices and house price to income ratios found in this dataset.

The evidence of exuberance in housing markets—understood as mildly explosive behavior—that we detect and identify is compatible with several different explanations. Different propagating mechanisms can lead to explosive characteristics in the data, the most prominent of which are perhaps models with rational bubbles (see, e.g., Blanchard (1979) and Blanchard and Watson (1982)). In that sense, our motivation and work is related to the rational bubbles literature and the large body of empirical studies on rational bubbles that followed—but we refrain from using the term bubbles in connection with our findings because bubble-detection requires the specification and estimation of an economic model for the housing market which is something that we do not attempt here (see, e.g., Himmelberg et al. (2005)).

Time series methods that can detect and date periods of explosive behavior are particularly relevant for our empirical analysis, but standard unit root and cointegration tests have difficulties detecting such behavior. Flood and Hodrick (1990), Gürkaynak (2008) and Homm and Breitung (2012) survey existing econometric methodologies and their findings. For testing and date-stamping the origination and termination of periods of exuberance, we implement a novel econometric procedure and mildly explosive regression asymptotics to data on real house prices as well as house price to economic fundamental ratios—namely the house price to income ratio.

In a recent series of papers, Phillips et al. (2011) propose a new recursive flexible-window testing procedure for identifying periods of mild explosiveness (supremum ADF, $SADF$) and Phillips et al. (2012) and Phillips et al. (2013) extend it for time series that display periodically collapsing behavior (Generalized SADF, $GSADF$). The detection strategy employed is based on a right-tail variation of the standard Augmented Dickey-Fuller (ADF) test where the alternative hypothesis is of a mildly explosive process.[1]

[1] Mildly explosive behavior is modeled by an autoregressive process with a root that exceeds unity, but remains within the vicinity of one. This represents a small departure from martingale behavior, but is consistent with the submartingale property

Rejection of the null with the $SADF$ and $GSADF$ tests provides empirical evidence of explosive behavior within sample, but also enables us in a subsequent step to date-stamp its occurrence.[2] These tests better detect explosive behavior in time series data than standard methods such as unit root/cointegration tests (e.g., Diba and Grossman (1988)), but also variance bound tests (e.g., LeRoy and Porter (1981), Shiller (1981)), specification tests (e.g., West (1987)), and Chow and CUSUM-type tests (e.g., Homm and Breitung (2012)).

A number of recent studies have implemented similar time series techniques in the context of housing markets. For example, Phillips and Yu (2011) use the $SADF$ test to date stamp bubbles in the U.S. housing market, corporate bond spreads and oil prices, during the crisis. Yiu et al. (2013) document multiple episodes of exuberance in the Hong Kong housing market using the $GSADF$ test. Our work complements theirs, but contributes to the existing literature by exposing the characteristics of the last boom and bust episode in international housing markets.

Our findings indicate that the period of exuberance that started in the U.S.—and Ireland—in the late 1990s, became rapidly widespread across countries in the early 2000s, and continued until sometime in 2008 preceding the 2008-09 global recession. This pattern of near-simultaneous explosive behavior across multiple countries, whose emergence cuts across significant differences in their domestic housing markets and the non-tradability of housing, has no precedent in our sample since the 1970s. The complex financial linkages and greater economic integration during this period may have facilitated the propagation, and perhaps amplified the potential consequences of the housing collapse that followed. While the precise mechanism still remains the source of much academic and professional debate, failing to recognize the emergence of exuberance episodes can have devastating implications. In that regard, it is worth noting that the empirical identification of explosive behavior in real time for surveillance and monitoring, and even the documentation of a timeline, can be greatly aided by these techniques.

The remainder of the paper proceeds as follows: Section 2 outlines the standard asset pricing equation on housing and describes how explosive behavior in house prices may arise. In section 3 we provide extensive discussion and further details on the $SADF$ (Phillips et al. (2011)) and $GSADF$ test procedures (Phillips et al. (2012) and Phillips et al. (2013)) that we implement with the data from the International House Price Database of the Federal Reserve Bank of Dallas. Then we present novel quantitative findings in section 4 based on the implementation of these tests and date-stamping procedures across multiple international housing markets. We also note the strong synchronization of periods of explosive behavior in the last boom and bust episode across all countries covered in the International House Price Database. Section 5 provides some additional discussion and concludes.

often used in the rational bubbles literature (see section 2 for further details). Phillips and Magdalinos (2007) and **?**) provide a large sample asymptotic theory for this class of processes that enables econometric inference in this case, unlike for purely explosive processes.

[2] Since the second quarter of 2013, the Federal Reserve Bank of Dallas' International House Price Database in partnership with the Department of Economics at Lancaster University Management School publishes indicators of exuberance in real house prices and house-prices-to-income ratios based on the $SADF$ and $GSADF$ methodologies.

2 House Price: Models and Specification

One conventional framework for the study of exuberance in asset prices is provided by the rational bubbles literature, as it is well-known that the presence of bubbles can result in explosive behavior. The main take-away from this strand of the literature is that the existence of a bubble in an asset price—in house prices in our case—should manifest in the dynamic and stochastic properties of the observed price of housing, therefore allowing statistical inference to detect the bubble. More generally, episodes of mildly explosive behavior in house prices—whether derived from the formation and collapse of a rational bubble or due to other factors such as behavioral biases, pricing errors, etc.—should be similarly amenable to statistical testing with the observed data on house prices. Our empirical strategy aims to detect mildly explosive behavior in the data from its time series properties.

Based on asset pricing theory, the price of housing in equilibrium can be derived from the following no-arbitrage condition under the assumption of risk neutrality,

$$\underbrace{r}_{\text{constant risk-free rate}} = \underbrace{\mathbb{E}_t\left(R_t\right)}_{\text{expected return on housing}}, \tag{1}$$

where $r > 0$ is the discount rate, the expectations operator $\mathbb{E}_t$ is based on all information available up to time t, and the return on housing at time $t+1$ is defined as,

$$R_t \equiv \frac{P_{t+1} + F_{t+1}}{P_t} - 1. \tag{2}$$

P_t denotes the house price at time t, and F_t is the stream of payoffs (pecuniary or otherwise) derived from housing at time t.

We equate the discount rate $r > 0$—that is, the expected (net) return on an alternative investment opportunity—with the constant risk-free rate r_t such that $\mathbb{E}_t\left(r_t\right) = r$ for all t. We will consider later on discount rates that vary over time, as those variations can also contribute to propagate explosive behavior in house prices. We refer to F_t as the economic fundamentals of the housing market, and work out our analysis with the help of two related specifications:

A general specification of F_t includes the payoff stream X_t that is given by the economic rents of housing, including housing services, but recognizes the possibility of unobserved fundamentals U_t driving the price of housing, i.e.,

$$F_t = X_t + U_t. \tag{3}$$

where $\{U_t\}_{t=1}^{\infty}$ represents a stream of fundamental factors driving the price of housing that are otherwise unobservable.

An alternative specification of F_t relates the payoff stream of housing rents to macroeconomic fundamentals through the demand equation for rental housing. Under additional constraints on preferences, we can derive a linear expenditure system where the demand for rental housing linearly relates housing rents X_t to macroeconomic fundamentals such as disposable income Y_t, i.e.,

$$F_t = \theta + \delta Y_t + U_t. \tag{4}$$

Appendix A provides details on the derivation of this relationship that is meant to capture the affordability determinants of housing.[3]

Replacing the definition of the return on housing R_t in (2) and re-arranging the no-arbitrage condition in (1), house prices can be expressed as follows,[4]

$$P_t = \frac{1}{1+r}\mathbb{E}_t\left[P_{t+1} + F_{t+1}\right], \tag{5}$$

which indicates that the price today must be equal to the discounted present-value of the expected fundamentals plus the re-sale price of housing tomorrow. Recursive substitution of this asset pricing equation yields the standard present value model for the price of housing (see, e.g., Clayton (1996)).

Solving equation (5) recursively T periods forward, we obtain an expression for the house price as a function of the expected discounted flow of all future payoffs up to time T plus a terminal condition that determines the present discounted of the time T re-sale value of the house, i.e.,

$$P_t = \mathbb{E}_t\left[\sum_{i=1}^{T}\left(\frac{1}{1+r}\right)^{i} F_{t+i}\right] + \mathbb{E}_t\left[\left(\frac{1}{1+r}\right)^{T} P_{t+T}\right]. \tag{6}$$

Letting T go to infinity and imposing the transversality condition,

$$\lim_{T\to\infty} \mathbb{E}_t\left[\left(\frac{1}{1+r}\right)^{T} P_{t+T}\right] < \infty, \tag{7}$$

the (unique) no-bubbles solution to the expectational difference equation that characterizes house prices in (5) yields,

$$P_t^* = \mathbb{E}_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^{i} F_{t+i}\right], \tag{8}$$

where P_t^* is referred to as the fundamental value of housing.

Equation (8) indicates that, when housing is treated as an investment (or asset), the value of housing should be equal to the present discounted value of all future economic rents that it generates as captured by the economic fundamentals. In other words, the factors that determine the price are the expected fundamentals $\mathbb{E}_t\left[F_{t+i}\right]$ for all $i \geq 1$ and the discount rate r. For further details on the present value model, see Gordon and Shapiro (1956) for the standard dividend discount model—assuming the payoff stream $\{X_t\}_{t=1}^{\infty}$ grows at a constant rate. Blanchard and Watson (1982) and Campbell et al. (1997) discuss the present value model in the context of more general processes for $\{X_t\}_{t=1}^{\infty}$.

Imposing the transversality condition in (7) rules out non-fundamental behavior (bubbles), and this implies that the housing price corresponds to its fundamental value (i.e., $P_t = P_t^*$). Explosive behavior can still be inherited by the house price through its fundamental price P_t^*, if the observable fundamentals such as

[3] As housing rents are difficult to measure in the data, we often use the affine transformation implied by the demand equation for rental housing to relate house prices to personal income. In doing so, however, the definition of fundamentals has to be augmented with a particular specification of the rental housing demand which may affect the inferences we draw from the analysis.

[4] Log-linear approximations of this equation—see, e.g., Campbell and Shiller (1989) and Chapter 7 in Campbell et al. (1997)— are commonly used, but may be less relevant with nonstationary data where sample means do not converge to population constants. Further discussion on these approximations can be found in Lee and Phillips (2011). In this paper, we discuss the theory and report our applied work in levels, but we find very similar results in log levels.

housing rents $\{X_t\}_{t=1}^{\infty}$ or disposable income $\{Y_t\}_{t=1}^{\infty}$ display such behavior. Unobserved fundamentals—not directly reflected in the measures of housing rents—$\{U_t\}_{t=1}^{\infty}$ can also be driving the behavior of house prices. Explosive behavior can also arise from time-variation in the discount rate r, as indicated before.

Without imposing the transversality condition in (7), the forward solution to the expectational difference equation for the price of housing P_t given in (5) is no longer unique (see, e.g., Sargent (1987), Diba and Grossman (1988) and LeRoy (2004)). It includes the fundamental price determined in (8)—that is, the no-bubbles solution P_t^* to the asset pricing model—plus a non-stationary component in the following form,

$$P_t = P_t^* + (1+r)^t c_t, \tag{9}$$

where $\{c_t\}_{t=1}^{\infty}$ is a martingale—that is, a stochastic process that satisfies $\mathbb{E}_t c_{t+1} = c_t$. If the non-stationary (or bubble) component cannot be ruled out, it introduces explosive behavior that affects the time series of house prices even when economic fundamentals are not explosive themselves. Moreover, there are infinitely many solutions of the form presented in (9) that solve equation (5).

We can re-define the non-stationary component of the solution in (9) as $B_t = (1+r)^t c_t$. With this characterization, the rational bubble, B_t, can simply be expressed as the difference between the housing price, P_t, and its fundamental-based value, P_t^*, i.e.,

$$B_t = P_t - P_t^*. \tag{10}$$

The bubble component B_t shows mildly explosive behavior since it satisfies the submartingale property,

$$\mathbb{E}_t (B_{t+1}) = (1+r)B_t, \tag{11}$$

given that the underlying component c_t follows a martingale process and the discount factor satisfies that $r > 0$. Making the discount factor $r_t > 0$ either stationary or integrated of order 1 is not going to alter the implications of the submartingale in (11), implying an explosive process for B_t even if F_t is not explosive (see, e.g., Phillips and Yu (2011) on this point).

With this framework, we define rational bubbles in house prices, B_t, as departures from the fundamental value of housing. If $B_t = 0$, there is no rational bubble and prices are determined only by the expected future discounted fundamentals of housing. In turn, if $B_t \neq 0$ there is a bubble that induces explosiveness into the time series of house prices P_t. If house prices include a non-stationary (bubble) component B_t that satisfies condition (11), then it is because investors operating in the housing market are expecting the non-fundamental component of the price of housing (the bubble) to keep growing at a rate that equals the discount rate $r > 0$. The theory of rational bubbles under the expectational difference equation in (5) can be understood from that logic.

For simplicity, let us assume that B_t is strictly positive in order to illustrate the rational bubbles' argument. An investor is willing to pay today $B_t > 0$ units more than its fundamental value P_t^* for a house only if he expects to be sufficiently compensated through future price increases (rather than through future housing rent increases) for the higher payment he is making today. If enough investors share the same belief about house price appreciation, then they will buy the houses driving their price up. In turn, this confirms the expectation of future price increases that had been anticipated by the investors sustaining the bubble for a period in what is referred to as a self-fulfilling prophecy.

2.1 Non-Fundamental Rational Bubbles

Adopting the representation proposed by Campbell and Shiller (1987) for the fundamental value of housing P_t^* and replacing it into the asset pricing model solution—including non-fundamental behavior (bubbles)—in (10), house prices can be expressed as,

$$P_t - \frac{1}{r}F_t = \left(\frac{1+r}{r}\right)\mathbb{E}_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i \Delta F_{t+i}\right] + B_t, \tag{12}$$

where Δ is the difference operator (i.e., $\Delta F_t = F_t - F_{t-1}$). If a (non-fundamental) rational bubble is a large part of the total price of housing, the price itself would become disconnected from the fundamentals of the housing market in ways that can be exploited for statistical inference. Therefore, a natural path to follow is to test for the possibility of such a disconnect by seeking evidence of explosive behavior in the observed house price series directly.

To illustrate the disconnect that arises between housing prices and fundamentals in the presence of a (non-fundamental) rational bubble we must specify the stochastic process for the fundamentals F_t. A plausible assumption would be that the economic rents on housing, F_t, follows either a stationary or integrated of order 1 (i.e., $I(1)$) process, i.e.,

$$F_t = \mu + \rho F_{t-1} + \epsilon_t,\ \epsilon_t \sim WN\left(0, \sigma_\epsilon^2\right), \tag{13}$$

where ϵ_t is white noise. The stochastic process in (13) is stationary if $|\rho| < 1$, and becomes $I(1)$ with a unit root whenever $\rho = 1$. The $I(1)$ process is said to be a random walk with drift. In the absence of bubbles (i.e., if $B_t = 0$ for all t), equation (12) implies that the house price equates its fundamental value and is given by,[5]

$$P_t = P_t^* = \left(\frac{r}{\rho(1+r-\rho)}\right)\left(\frac{1+r}{r^2}\right)\mu + \left(1 + (1-\rho)\left(\frac{1+r}{1+r-\rho}\right)\right)\frac{1}{r}F_t. \tag{14}$$

The house price P_t is either stationary or has a unit root depending on the specification of the fundamentals F_t. The house price P_t is also cointegrated with F_t such that $P_t - \left(1 + (1-\rho)\left(\frac{1+r}{1+r-\rho}\right)\right)\frac{1}{r}F_t$ is stationary. In other words, house prices and fundamentals—housing rents—should be driven by either the same stationary autorregressive process of order 1 or by the same integrated process of order 1 (i.e., $I(1)$) if $B_t = 0$ for all t.

Diba and Grossman (1988) observe that fundamental asset prices—the fundamental house price P_t^* in our context—are integrated of the same order as the fundamental process F_t in the absence of bubbles (as illustrated for the $I(1)$ case here). In the presence of a bubble (i.e., $B_t \neq 0$), the house price in (10) contains the explosive root from B_t and so does the linear combination $P_t - \frac{1}{r}F_t$ in (12). This differentiates the fundamental value P_t^* from the bubble process B_t underlying the house price P_t, as the stochastic process for the fundamental value is inherited from the stochastic process for the fundamentals while the (non-fundamental) rational bubble component is characterized by an explosive autoregressive process implied by the submartingale condition (11) instead.

Hence, if house prices P_t and fundamentals F_t are integrated of the same order (i.e., $I(1)$) or stationary with the same autorregressive order, then we could exclude the presence of non-fundamental rational bubbles in the data. More generally, if house prices exhibit explosive autoregressive behavior or shifts from $I(1)$ to

[5] For a discussion of a more general solution with log-linear approximation methods, see the work of Engsted et al. (2012).

mildly explosive behavior such as the one characterized by the submartingale property in (11), then we could argue that house prices have become disconnected from $I(1)$ −fundamentals. When the process for house prices loses its explosiveness, then we expect house prices to be again aligned with that characteristics of the process driving the fundamentals—then we could claim that the episode of exuberance has ended.

2.2 Explosive Behavior and Fundamentals

In this paper we use a recursive procedure based on the augmented Dickey-Fuller (ADF) test which allows for the testing, *ex post* identification and date stamping of mildly explosive behavior in economic time series. This econometric method has been developed in a series of papers by Phillips et al. (2011), Phillips et al. (2012) and Phillips et al. (2013) with tests that deal with the structural change from a random walk ($I(1)$ process) to mildly explosive behavior.

Our empirical strategy applies this procedure to data on real house prices to detect empirical evidence of mildly explosive behavior, but exploits the possible disconnect between house prices and fundamentals discussed in the context of (non-fundamental) rational bubbles for identification purposes. The house-price-to-rent ratio is often used as an indicator of over or undervaluation of housing relative to the expenses of renting. If this ratio deviates from its long-run average, it can be an indication that house prices have become misaligned from fundamentals. Working with housing rents is not without its problems, as housing rents are often measured with great error or not available at all.

Since housing rents are not readily available, we extend the present-value model in (5) with other economic relationships that relate housing rents to a set of macroeconomic variables (fundamentals). We focus our attention primarily on the demand-side of the housing market using—in particular—data on real disposable income and on the house price to income ratio to investigate the dynamics and stochastic properties of fundamentals in connection with our investigation of explosiveness in real house prices. The price-to-income ratio is commonly used in the literature as well as the house price to rent ratio to assess whether house prices are sustainable in the sense of being consistent with the economic fundamentals of the housing market.[6]

In exploring the dynamics of fundamentals we aim to shed light on the question of whether mildly explosive behavior can be attributed to some extent to the behavior of fundamentals themselves or whether it could be due to other factors such as unobserved fundamentals, the behavior of the discount rate, (non-fundamental) rational bubbles, etc. In this regard, three observation qualifications are in order to understand the specification of the tests we perform and the identification implied by our results:

1. Non-fundamental explosive behavior can lead to a disconnect between the stochastic process for fundamentals F_t and for house prices P_t, as illustrated by the (non-fundamental) rational bubbles literature. However, explosive behavior in house prices can also be inherited from the fundamental process itself. Assume that the fundamentals $F_t = F_t^* + B_t^F$ can be described by a random walk with drift term F_t^* and a bubble-component B_t^F, i.e.,

$$F_t = F_t^* + B_t^F, \tag{15}$$

where F_t^* is given by,

$$F_t^* = \mu^F + F_{t-1}^* + \epsilon_t^F,\ \epsilon_t^F \sim WN\left(0, \sigma_{\epsilon^F}^2\right), \tag{16}$$

[6] The house price to income ratio provides a metric of house prices relative to the ability of households to pay (see, e.g., Himmelberg et al. (2005) and Girouard et al. (2006)). In that way, it incorporates one the key determinants of the demand for housing.

and B_t^F satisfies that submartingale property, i.e.,

$$\mathbb{E}_t\left(B_{t+1}^F\right) = (1+c)\, B_t^F,\ 0 < c < r. \tag{17}$$

The unique solution to house prices P_t corresponds to the fundamental value of housing P_t^* in equation (8) which can be re-written à la Campbell and Shiller (1987) as follows,

$$P_t = P_t^* = \frac{1}{r} F_t + \left(\frac{1+r}{r}\right) \mathbb{E}_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i \Delta F_{t+i}\right]. \tag{18}$$

Under the specification of the fundamentals given by $(15)-(17)$, equation (18) implies that house prices are given by,

$$P_t - \frac{1}{r} F_t = \left(\frac{1+r}{r^2}\right)\mu^F + \left(\frac{c}{r-c}\right)\left(\frac{1+r}{r}\right) B_t^F.$$

The house price P_t in this example has a unit root from F_t^*, but also inherits the explosiveness from the bubble component B_t^F that has been added to the fundamentals F_t. The linear combination between the house price P_t and fundamentals F_t given by $P_t - \frac{1}{r}F_t$ is not stationary, but displays explosive behavior as well.

In other words, if a bubble drives the process for the fundamentals F_t, then we would not expect a similar disconnect between house prices and fundamentals to emerge as discussed in the presence of non-fundamental rational bubbles. This illustration indicates that whenever data on fundamentals is observable, then exploring the properties of the fundamental process can shed some light on the question of the source of explosiveness. If we specialize the present-value model described to the case where housing rents X_t and disposable income Y_t are related, then looking at both the disposable income as well as the house price to income ratio can help us in the identification—and naturally we make that part of our empirical strategy. However, the bubble component may arise also from through the unobserved fundamentals U_t. Even if we find a disconnect between housing rents or disposable income on one hand and house prices on the other, we cannot truly rule out the possibility that the explosiveness may have been inherited from the unobserved fundamentals.

2. Fundamentals F_t may generate observed run-up trajectories on house prices P_t that are non-explosive, but need to be taken into account for identification. As an example, a random walk with drift as specified for the fundamentals F_t in equation (13) whenever $\rho = 1$ can generate run-up periods in the trajectory of house prices and fundamentals, if the variance of the innovation is small and the drift itself is positive and strong enough. In this case, there is no disconnect between the dynamics of fundamentals and house prices. The observation of a large appreciation and a subsequent fall in the time series, however, does not necessarily reveal explosivenes as it can be attributed to the drift.

The run-up rate of growth in house prices coming from the drift is linear, while it is exponential in the case of an explosive process satisfying the submartingale property specified in equation (11). This distinction enables statistical inference to identify periods of explosive behavior whenever a non-negligible drift is present in the data. We may consider more general specifications as well in which the drift itself is not simply a constant, but a function of time. In our empirical analysis, we include a constant in the specification of the regression equations underlying our tests—but also evaluate alternative specifications of the drift with a linear-time trend—to account for the effect of the drift on the dynamics of house prices and fundamentals to avoid incorrectly attributing some of it to explosive behavior.

3. Time variation in the discount rate r can also lead to explosive behavior in the time series of house prices P_t, even when fundamentals F_t remain non-explosive (either stationary or integrated). As an illustration, let us allow a time-varying discount rate in the asset pricing equation in (5), i.e.,

$$P_t = \frac{1}{1+r_t}\mathbb{E}_t\left[P_{t+1}+F_{t+1}\right], \tag{19}$$

and assume a random walk without drift for the fundamental process in (13), i.e.,

$$F_t = F_{t-1}+\epsilon_t,\ \epsilon_t \sim WN\left(0,\sigma_\epsilon^2\right). \tag{20}$$

The trajectory of the discount rate can have an important effect on the characteristics of the fundamental price of housing implied by (19) and (20)—even inducing explosive behavior and a disconnect with the fundamentals. We illustrate this with a simple (deterministic) time-varying discount rate that captures a gradual and anticipated decline in interest rates over a period of time, i.e.,[7]

$$1+r_{t+s} = \begin{cases} 1+r', \text{ for } 0\leq s\leq k, \\ (1+r_{t+s+1})\,g, \text{ for } k+1\leq s<k', \\ 1+r, \text{ for } s\geq k', \end{cases} \tag{21}$$

where $0<k<k'<\infty$ defines the time window of decline for the discount rate, and $g\geq 1$ determines the gross rate of decline in (21). This interest rate specification collapses to the constant discount rate whenever $g=1$ and implies $r'>r$ whenever $g>1$. In a stylized manner, the time-varying discount rate in (21) captures the idea that declining rates were an important factor in the run-up of house prices leading to the 2008-09 global recession.

Imposing the transversality condition $\lim_{T\to\infty}\mathbb{E}_t\left[\left(\frac{1}{\Pi_{s=1}^{T}(1+r_{t-1+s})}\right)P_{t+T}\right]<\infty$ to rule out non-fundamental bubbles, the present value model for the price of housing derived in equation (19) under (20) − (21) has a unique solution of the form,

$$P_t = P_t^* = \theta_{t-1}F_t, \tag{22}$$

where θ_{t-1} is time-varying and obeys the following (deterministic) difference equation,

$$(1+r_t)\,\theta_{t-1} = (1+\theta_t). \tag{23}$$

If we combine the solution in (22) with the specification of the fundamentals in (20), we derive the following process for the price of housing,

$$P_t = \frac{\theta_{t-1}}{\theta_{t-2}}P_{t-1}+\varepsilon_t,\ \varepsilon_t\sim WN\left(0,\theta_{t-1}^2\sigma_\epsilon^2\right), \tag{24}$$

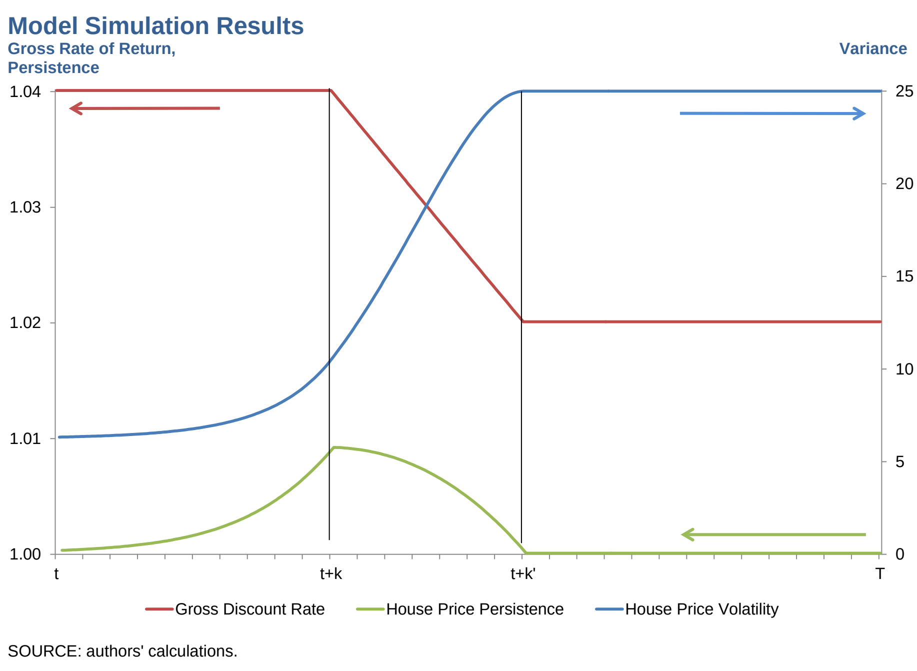
[7] The recursive representation of the discount rate is equivalent to the following alternative characterization,

$$1+r_{t+s} = \begin{cases} 1+r' = (1+r)\,g^{k'-k}, \text{ for } 0\leq s\leq k, \\ (1+r)\,g^{k'-s}, \text{ for } k+1\leq s<k', \\ 1+r, \text{ for } s\geq k'. \end{cases}$$

which indicates the potential impact of time variation in the discount rate on the persistence and volatility of house prices.[8]

In the constant discount rate case where $g = 1$, the unique solution implies $\theta_t = \frac{1}{r}$. In that case, house prices inherit the unit root of the fundamentals and its volatility is that of the fundamentals scaled by the constant discount rate. The solution in the general case where $g > 1$ can be characterized by backward induction as follows: for $s \geq k'$, the unique solution corresponds to the case where the low discount rate remains constant with $\theta_{t+s} = \frac{1}{r}$; taking $\theta_{t+k'} = \frac{1}{r}$ as given and using the specification of the discount rate given in (21) and the difference equation in (23), we recover $\theta_{t+k'-1}$, and this can be used to recover $\theta_{t+k'-2}$ similarly and so on until we recover the entire trajectory back to time t.[9] However, there is time variation in θ_t during the period of declining interest rates and also during the preceding period as the decline is anticipated. We show that such variation alone induces mildly explosive behavior and higher volatility in house prices as well as a disconnect with the fundamentals F_t from t to $t+k'$. Figure 1 illustrates the impact of the discount rate r_t in (21) on the properties of the house price series P_t in (24) with a simple numerical example setting $r = 0.02$, $g = 1.0002774397$, $k' - k = 70$, and $\sigma_\epsilon^2 = 0.01$.

Figure 1: Real House Prices: The Dynamic Effects of An Anticipated Decline in the Discount Rate



[8] For a discussion on the characteristics of the volatility process in house prices with data from the International House Price Database, see for instance Mack and Martínez-García (2012). These authors provide some empirical evidence of an increase in house price volatility that would be consistent as well with the stylized implications of declining discount rates laid out here.

[9] We can also show that the persistence term $\frac{\theta_{t-1}}{\theta_{t-2}}$ in the house price equation in (24) is bounded below by g and above by $g^{k'-k}$ over the period from t up to $t+k'$.

We have discussed in sub-section 2.1 how rational bubbles can emerge when fundamentals are otherwise an integrated process. If there is no evidence of explosive behavior in the fundamentals, real house prices that display patterns of explosiveness could be driven by the non-fundamental (bubble) component of the solution. However, even after controlling for house price run-ups induced by the drift component, testing for explosiveness in fundamentals offers only an incomplete picture of the disconnect between prices and fundamentals—as we suspect the presence of important unobserved fundamentals. Our findings, therefore, do not preclude the possibility that the explosiveness in house prices may be inherited from other unobserved fundamentals.[10] Moreover, an alternative interpretation based on a declining discount rate can also rationalize the observed run-up in house prices, the explosiveness in the time series and the disconnect with fundamentals without having to appeal to a rational bubble. While our empirical strategy focuses on real house prices and their relationship to disposable income and housing affordability (the house price to income ratio), we take into consideration the potential impact of the discount rate also as a possibility in our discussion of the identification and in setting the timeline of events for the 2008-09 global recession.

3 Testing for Explosive Behavior

Asset pricing theory suggests that the dynamics and stochastic properties of an observed asset price may be indicative of the existence of explosive behavior. Departures from $I(1)$ to mildly explosive behavior allows to formulate statistical tests that can detect evidence of such explosiveness in the data. Diba and Grossman (1988) provided one of the first attempts to test it in the context of the stock market. The authors suggest comparing stock prices and observable market fundamentals, using reduced form stationarity tests. The rationale is that bubbles cannot be ruled out in the case that stock prices are found to be explosive when market fundamentals are not. However, the presence of periodically collapsing bubbles (i.e., those that emerge and burst at recurring times), a feature of actual stock price data, puts into question the power of stationarity-based tests as indicated by Evans (1991). Evans (1991) showed using simulation methods that standard unit root and cointegration tests cannot reject the null of no explosive behavior, when periodically collapsing bubbles are present in the data.[11]

Based on a recursive and rolling right-tail variation of the standard Augmented Dickey-Fuller (ADF) unit root test, Phillips et al. (2011)—with the supremum ADF ($SADF$)—and Phillips et al. (2012) and Phillips et al. (2013)—with the generalized SADF ($GSADF$)—have developed new detection strategies to provide a better identification. These strategies enable us to detect mildly explosive behavior in the data and to date-stamp their occurrence. These tests consider as a null hypothesis the unit root, while the alternative is a mildly explosive process. Phillips et al. (2012), Phillips et al. (2012) and Phillips et al. (2013) show how using recursive and rolling tests increases the power in the detection of explosiveness, as compared to standard unit root tests on the whole sample.

[10] We note that, apart from income and rent, there are other fundamental drivers of housing prices, such as the cost of foregone interest, the cost of property taxes and maintenance costs (see, e.g., the discussion in Himmelberg et al. (2005)). Since explosive behavior in prices may be induced by these other fundamental drivers, the presence of explosive dynamics in house-price-to-income and price-to-rent ratios cannot be considered conclusive of a disconnect or lack thereof between house prices and fundamentals.

[11] Let us assume a large price increase develops during a period of explosive behavior. When the episode bursts, the price tends to decline rapidly (but usually only for a short time period). The price increase followed by the decline makes the process look like it mean reverts—the process will not look explosive but, on the contrary, stationary. Intuitively, this is the reason why many non-recursive unit root tests wrongly suggest that processes that incorporate periodically collapsing boom-bust episodes are stationary—the main point of the Evans (1991) paper.

Phillips et al. (2012) and Phillips et al. (2013) find that their strategy based on the $GSADF$ test outperforms that of the $SADF$ test originally proposed in Phillips et al. (2011) in the presence of multiple episodes of exuberance (explosive behavior). The findings of Homm and Breitung (2012), using Monte Carlo simulation, also support the view that the $GSADF$ test strategy performs relatively well in the detection of explosiveness when compared against standard time series tests for the detection of bubbles, particularly when periodically occurring bubbles are present, and for real time monitoring.

We adopt these same procedures for testing and date-stamping, which will now be presented in brief, to investigate the dynamics and stochastic properties of real house prices and some commonly observed housing fundamentals.

3.1 The $SADF$ and $GSADF$ Procedures

The time series econometric method used for testing and detecting explosive behavior is based on the following Augmented Dickey-Fuller (ADF) regression equation,

$$\Delta y_t = a_{r_1,r_2} + \beta_{r1,r2} y_{t-1} + \sum_{i=1}^{k} \psi^i_{r_1,r_2} \Delta y_{t-i} + \epsilon_t,\ \ \epsilon_t \overset{iid}{\sim} N(0, \sigma^2_{r_1,r_2}), \tag{25}$$

where y_t denotes a time series process (in our case, real house prices, personal disposable income, or the house price to income ratio), Δy_{t-i} for $i = 1, ..., k$ are the differenced lags of the time series, and ϵ_t is the error term. Moreover, k is the maximum number of lags included in the specification, r_1 and r_2 denote fractions of the total sample size that specify the starting and ending points of a subsample period, a_{r_1,r_2} is the intercept, and $\psi^i_{r_1,r_2}$ for $i = 1, ..., k$ are the coefficients on the differenced lags of the time series, and $\beta_{r1,r2}$ is the coefficient of interest in using the ADF regression equation for testing.[12]

As noted in the previous section, the emergence and popping of a bubble process $\{B_t\}_{t=1}^{\infty}$ that satisfies condition (11) is indicated by a shift from a random walk—under the assumption that fundamentals are $I(1)$—to an explosive autoregressive process. Explosiveness in house prices can also be induced by time variation in the discount rate in the context of the present value model of housing presented in equation (5)—and it would be indicated by a shift from a random walk to a mildly explosive autorregressive process as well. Therefore, we are interested in testing with equation (25) the null hypothesis of a unit root, $H_0 : \beta_{r_1,r_2} = 0$, against the alternative of mildly explosive behavior in y_t, $H_1 : \beta_{r_1,r_2} > 0$. Let

$$ADF^{r_2}_{r_1} = \frac{\widehat{\beta}_{r1,r2}}{\text{s.e.}(\widehat{\beta}_{r1,r2})} \tag{26}$$

denote the test statistic corresponding to this null hypothesis. It is easy to see that setting $r_1 = 0$ and $r_2 = 1$ yields the standard ADF test statistic, ADF^1_0. The limit distribution of ADF^1_0 is given by,

$$\frac{\int_0^1 W dW}{\int_0^1 W^2}, \tag{27}$$

[12] While the intercept in the specification of the estimation regression, a_{r_1,r_2}, is constant we also evaluate an alternative where it is replaced with a linear trend to account for the possibility that the underlying process driving the data has a time-varying (though deterministic) trend component.

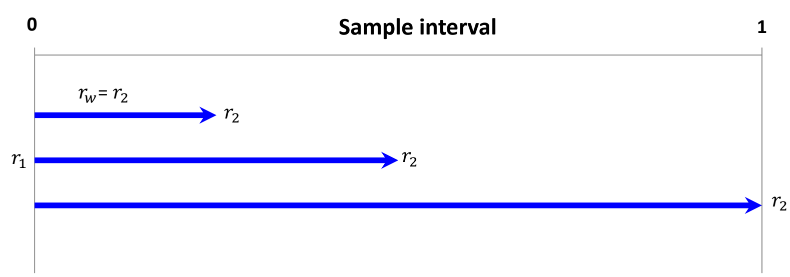
where W is a Wiener process. The ADF test compares the ADF_0^1 statistic with the right tail critical value from its limit distribution. When the test statistic exceeds the critical value, the unit root hypothesis is rejected in favor of explosive behavior.

Although widely employed, the standard ADF test has extremely low power in detecting episodes of explosive behavior when these episodes end with a large drop in prices, i.e. in the presence of boom-bust dynamics. Nonlinear dynamics, such as those displayed by periodically collapsing bubbles, frequently lead to finding spurious stationarity even though the process is inherently explosive as noted by Evans (1991).

In order to deal with the effect of a price collapse on the test's performance, Phillips et al. (2011) proposed a recursive procedure based on the estimation of the ADF regression on subsamples of the data. Detection of mildly explosive behavior is reduced to testing for a change from $I(1)$ to explosive in a univariate time series, where the change point is unknown. In particular, normalizing the end of the original sample to $T = 1$, the authors propose estimating (25) using a forward expanding sample with the end of the sample period r_2 increasing from r_0 (the minimum window size for the fixed initial window) to one (the last available observation). In this procedure, the beginning of the sample is held constant at $r_1 = 0$, and the expanding window size of the regression (over the normalized sample) is denoted by $r_w = r_2 - r_1$.

Figure 2 illustrates the nature of the estimation procedure proposed by Phillips et al. (2011): The first observation in the sample is the starting point of the estimation window (i.e., $r_1 = 0$) and the end point of the initial estimation window, r_2, is set to ensure a minimum window size of r_0 (so $r_w = r_0 = r_2$ for the initial estimation). Then, while the starting point of the estimation is kept fixed at $r_1 = 0$, the test regression is recursively estimated, while incrementing the window size, $r_2 \in [r_0, 1]$, by adding one additional observation at a time. Each estimation yields an ADF statistic denoted as $ADF_0^{r_2}$.

Figure 2: SADF: Illustration of the Rolling Window Procedure



The test statistic, called sup ADF ($SADF$), is defined as the supremum value of the $ADF_0^{r_2}$ sequence expressed as follows,

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2}. \tag{28}$$

Under the null hypothesis, the limit distribution of the $SADF$ statistic is given by,

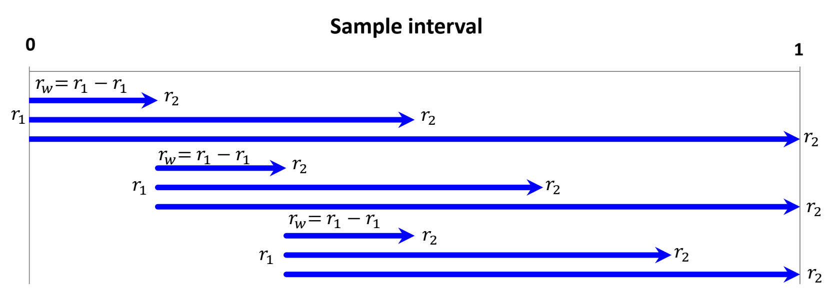
$$\sup_{r_2 \in [r_0, 1]} \frac{\int_0^{r_2} W dW}{\int_0^{r_2} W^2}. \tag{29}$$

Similarly to the standard ADF test, when the $SADF$ statistic exceeds the right tailed critical value from its limit distribution, the unit root hypothesis is rejected in favor of explosive behavior.

The $SADF$ test performs well when there is a single boom-bust episode in the time series. Simulation experiments in Homm and Breitung (2012) reveal that the $SADF$ outperforms alternative methods, such as the modified Bhargava (1986), the modified Busetti and Taylor (2004), and the modified Kim (2000) (with the corrections of Kim et al. (2002)), in terms of power. These approaches are used to test a permanent change in persistence from a random walk to an explosive process. As a consequence, they perform well only in the case that a bubble develops but never bursts. In the presence of periodically collapsing bubbles they exhibit very low power, they are inconsistent, and are outperformed by the $SADF$.[13]

More recently, Phillips et al. (2012) and Phillips et al. (2013) derived a new unit root test, the Generalized $SADF$ ($GSADF$), that covers a larger number of subsamples than the $SADF$ by allowing both the ending point, r_2, and the starting point, r_1, to change. This extra flexibility on the estimation windows illustrated in 3 results in substantial power gains in comparison to the $SADF$. Moreover, the test is consistent with multiple boom-bust episodes within a given time series.

Figure 3: GSADF: Illustration of the Rolling Window Procedure



The $GSADF$ statistic is defined by,

$$GSADF(r_0) \;=\; \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}. \tag{30}$$

[13] These test procedures were proposed to test for a change in persistence between $I(0)$ and $I(1)$. In the simulations of Homm and Breitung (2012), Chow-type break test is also considered. The Chow-type test often exhibits the highest power in their estimations and its estimators for the unknown break date tend to be most reliable in finite samples.

Under the null hypothesis, the limit distribution of the $GSADF$ statistic is

$$\sup_{r_2 \in [r_0,1], r_1 \in [0, r_2 - r_0]} \left\{ \frac{\frac{1}{2} r_w [W(r_2)^2 - W(r_1)^2 - r_w] - \int_{r_1}^{r_2} W(r) dr [W(r_2) - W(r_1)]}{r_w^{1/2} \{ r_w \int_{r_1}^{r_2} W(r)^2 dr - [\int_{r_1}^{r_2} W(r) dr]^2 \}^{1/2}} \right\}, \tag{31}$$

where, again, the window size of each estimation is $r_w = r_2 - r_1$. Rejection of the unit root hypothesis in favor of explosive behavior requires that the test statistic exceeds the right tailed critical value from its limit distribution given by (31).

3.2 The Date-Stamping Strategy

In many cases, it is of prime interest to detect the period(s) that display explosive dynamics. Moreover, it is important for policy formation and monitoring purposes to examine whether the time series is currently in an explosive regime or not. The $SADF$ and $GSADF$ test procedures also deliver, under general regularity conditions, a date-stamping strategy to consistently estimate the beginning and end of periods of mildly explosive behavior—often referred to as periods of exuberance when the underlying time series is on the upswing. If the null hypothesis of either of these tests is rejected, then one can infer the start and end times of such episodes of exuberance—the implementation of the date-stamping procedure, however, differs depending on whether we use the $SADF$ or $GSDAF$ approach.

The date-stamping strategy proposed by Phillips et al. (2011) under the $SADF$ approach compares each element of the estimated $ADF_0^{r_2}$ sequence against the corresponding right-tailed critical value of the standard ADF statistic. The beginning of the period of exuberance corresponds to the first observation, denoted T_{r_e}, for which the $ADF_0^{r_2}$ statistics crosses the corresponding critical value from below. Analogously, the end of the period of exuberance, denoted T_{r_r}, is dated from the first observation after T_{r_e} for which the $ADF_0^{r_2}$ statistics crosses the corresponding critical value from above. The estimates of the beginning and end of the period of exuberance under the $SADF$ approach are given as,

$$\widehat{r}_e = \inf_{r_2 \in [r_0,1]} \left\{ r_2 : ADF_0^{r_2} > scu_{\lfloor r_2 T \rfloor}^{\alpha} \right\}, \tag{32}$$

$$\widehat{r}_f = \inf_{r_2 \in [\widehat{r}_e,1]} \left\{ r_2 : ADF_0^{r_2} < scu_{\lfloor r_2 T \rfloor}^{\alpha} \right\}, \tag{33}$$

where $scu_{\lfloor r_2 T \rfloor}^{\alpha}$ is the $100\,(1-\alpha)\,\%$ critical value of the standard ADF statistic based on $\lfloor r_2 T \rfloor$ observations. In applied work it is standard to set α to 5%, but we also consider statistical significance at the 1% and 10% levels.

Under the $GSDAF$ approach proposed by Phillips et al. (2012) and Phillips et al. (2013), the first step of the date-stamping procedure is to test the unit root hypothesis by comparing the $GSADF(r_0)$ to the $1-\alpha$ critical value, where α is the nominal significance level as before. Statistical significance at the 1%, 5% and 10% levels is conventionally reported. Suppose that the $GSADF$ test rejects the null hypothesis of a unit root. Hence, the second step of the procedure is to identify period(s) of explosive behavior if $GSADF$ test rejects the null.

Phillips et al. (2012) and Phillips et al. (2013) recommend a dating strategy based on the backward sup

ADF statistic, i.e.,[14]

$$BSADF_{r_2}(r_0) \;=\; \sup_{r_1 \in [0, r_2 - r_0]} SADF_{r_1}^{r_2}. \tag{34}$$

The authors define the origination date of the period of exuberance as the first observation that the $BSADF$ statistic exceeds its critical value,

$$\widehat{r}_e \;=\; \inf_{r_2 \in [r_0, 1]} \{ r_2 : BSADF_{r_2}(r_0) > scu_{\lfloor r_2 T \rfloor}^{\alpha} \}, \tag{35}$$

and the termination date as the first observation after $\widehat{r}_e$ for which the $BSADF$ falls below its critical value,

$$\widehat{r}_f \;=\; \inf_{r_2 \in [\widehat{r}_e, 1]} \{ r_2 : BSADF_{r_2}(r_0) < scu_{\lfloor r_2 T \rfloor}^{\alpha} \}, \tag{36}$$

where $scu_{\lfloor r_2 T \rfloor}^{\alpha}$ is the $100\,(1-\alpha)\,\%$ critical value of the sup ADF based on $\lfloor r_2 T \rfloor$ observations and α is the chosen significance level. When the $BSADF$ statistic exceeds the finite-sample critical values of the $SADF$, we argue that the empirical evidence suggests that the time series displays explosive behavior. The consistency of the above dating strategy in the presence of one or two periodically collapsing bubbles is established in Phillips et al. (2012).

Because the distributions of the $SADF(r_0)$ in (28) and $GSADF(r_0)$ in (34) are non-standard, critical values have to be obtained through Monte Carlo simulations. The Monte Carlo procedure consists of the following steps:

1. Generate a driftless random walk series of size T.
2. Estimate equation (25) using least squares and compute the $SADF$ and $GSADF$ statistics.
3. Repeat steps 1 and 2 a large number of times, say 2000, to obtain the distribution of the $SADF$ and $GSADF$ statistics.
4. The $100\,(1-\alpha)\,\%$ critical value of each test statistic is given by the $100\,(1-\alpha)$ percentile of the corresponding distribution obtained in step 3.

The researcher may choose to neglect very short periods of exuberance by setting a minimum duration period. The researcher may also decide to combine adjacent periods of exuberance by setting a minimum duration period to elapse between any two consecutive periods of exuberance to be treated as separate. Phillips et al. (2013) recommend that we define $\log(T)/T$ as a minimal interval for date-stamping a period of mildly explosive in a time series, which we apply also to set the minimum length of periods in between two consecutive episodes. Since all our time series have 154 quarterly observations, the minimum interval that we adhere to in our empirical evaluation corresponds to 5 quarters.

3.3 Other Technical Details

The computation of the $SADF$, $GSADF$ and $BSADF$ test statistics necessitates the selection of the minimum window size r_0 and the autoregressive lag length k. Regarding the minimum window size, this has to be large enough to allow initial estimation but it should not be too large to avoid missing short

[14] The backwward sup ADF ($BSADF$) statistic relates to the $GSADF$ statistic as follows,

$$GSADF(r_0) \;=\; \sup_{r_2 \in [r_0, 1]} \{ BSADF_{r_2}(r_0) \}.$$

episodes of exuberance. We follow Phillips et al. (2012) and set the minimum size equal to 36 observations. Exploring alternative minimum window sizes can be computationally demanding since for each r_0 we have to compute new critical values, and for this paper changes in r_0 within the neighborhood of 36 seem to make little difference on our findings.

With respect to the autoregressive lag length k, we evaluate our results primarily for two cases, k equal to 1 and 4. Our results do not appear very sensitive sensitive to a fixed lag specification when we allow for fewer than 4 lags. Hence, our findings in the remainder of the paper will be reported only for the case of lag length set at $k = 4$, unless otherwise noted, to save space. The choice of a fixed lag length is appealing because it allows us to employ a recursive least squares approach which reduces substantially the computational cost of estimation.

More sophisticated lag length selection procedures in ADF-type tests based on information criteria (such as the Modified Information Criteria of Ng and Perron (2001)) and sequential hypothesis testing (see, e.g., Ng and Perron (1995)) could, in principle, be applied but with a higher computational cost. Phillips et al. (2012) show that sequential hypothesis testing for the determination of the lag length can result in severe size distortions, and a reduction in power of both the $SADF$ and $GSADF$ tests. That is, $SADF$ and $GSADF$ tests based on sequential hypothesis testing perform poorly, frequently rejecting the null hypothesis when the data follow a unit root process, and failing to reject the null when there is explosiveness in the time series. Nonetheless, we consider for the paper the lag selection criteria (using the Modified Akaike Information Criterion) with up to 4 lags as a robustness check and, once again, our results (not reported) do not appear very sensitive to the lag specification in this case.

The implementation of the unit root tests also requires the limit distributions of the $SADF$, $GSADF$ and $BSADF$ test statistics. These distributions are non-standard and depend on the minimum window size. Hence, critical values have to be obtained through Monte Carlo simulations. We obtain finite sample critical values by generating $2,000$—and up to $20,000$ robustness—replications of a random walk process with $N(0,1)$ errors. Asymptotic $SADF$ and $GSADF$ critical values are provided in Phillips et al. (2012) Table 1. Although it is preferred to use finite-sample critical values when sample size is relatively small, using asymptotic critical values doesn't change qualitatively our results.

4 Empirical Evidence on International House Prices

A novel dataset on house prices and personal disposable income per capita from the International House Price Database of the Federal Reserve Bank of Dallas is studied in the empirical work that we report here. The sources and methodology used to construct this panel of 22 countries are documented in Mack and Martínez-García (2011).[15] The data on real house prices and real personal disposable income per capita is reported quarterly, deflated with the PCE deflator, and covers the period from the first quarter of 1975 to the second quarter of 2013. We construct an affordability index for housing as the ratio (in percent) of real house prices over real personal disposable income per capita for each country.

[15] See the Appendix for a description of the data sources on house prices.

Figure 4: Real House Prices: Cross-Country Characteristics

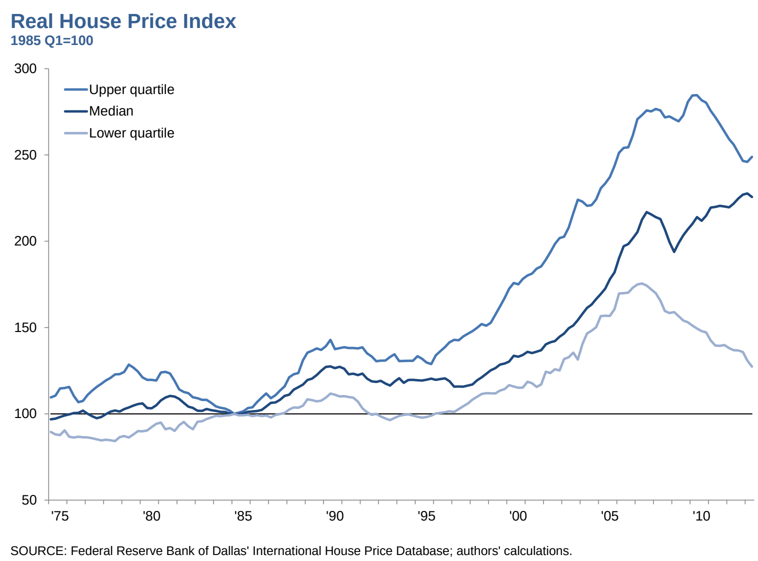
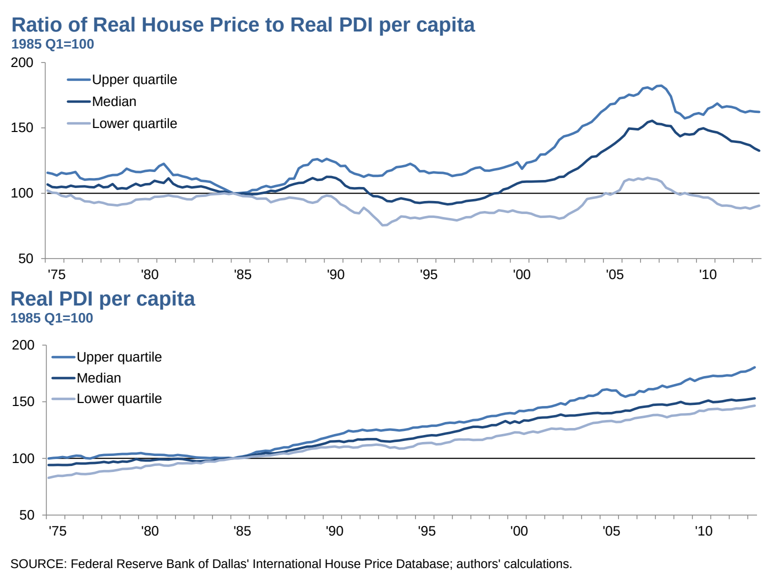


Figure 5: Price-to-Income Ratio and Real PDI: Cross-Country Characteristics



The sample period covered in this database includes several prior recessions, which makes it ideal for us to contrast the timeline of the boom and bust in international housing markets prior to the 2008-09 global recession against other periods of contraction in economic activity. Longer time series may be available for some countries, but the empirical findings reported here are generally robust. The median, lower and upper quartile of the time series of all 22 countries are displayed for illustration in Figure 4. We observe that for the median country, real house prices troughed in the mid-1990's and peaked around 2006. This run-up in real house prices appears widespread, which has fueled the view that this episode of housing exuberance had a part in the subsequent 2008-09 global recession. Our paper provides empirical evidence to partly substantiate the claim of exuberance in house prices, but also provides a time line of events and shows that there is less of a common pattern in the cross-country data that meets the eye.

For all these countries, we investigate the explosive behavior of real house prices, the price-to-income ratio and real personal disposable income per capita (PDI). The ratio of real house prices to real personal disposable income is a long-run anchor in the determination of house prices. The median, upper and lower quartiles of the price-to-income ratio, as well as the real PDI, are plotted in Figure 5. While the price-to-income ratio exhibits a timeline similar to that of real house prices—a boom period during the late 1990s and the first half of the 2000s, followed by a severe correction afterwards—the pattern of the real PDI does not suggest a clear connection with prices.

4.1 Empirical Findings: A Timeline of Periods of Exuberance in International Housing Markets

Table 2, included in Appendix B, reports results for real house prices, the price-to-income ratio, and the real PDI for each of the 22 countries covered in the International House Price Database. With the $SADF$ test statistics of Phillips et al. (2011), we cannot reject the null on real house prices for 11 out of the 22 countries at conventional significance levels. The evidence is even weaker when we look at the price-to-income ratio and the real PDI, as fewer countries show evidence of explosive behavior in the time series according to this test.

We adopt the $GSADF$ procedure of Phillips et al. (2012) and Phillips et al. (2013) as our primary tool for testing and date-stamping periods of exuberance in international housing markets. The $GSADF$ procedure is expected to have higher power than the $SADF$ test whenever there are periodically collapsing explosive dynamics within sample.

The $GSADF$ statistics, reported in Table 2, offers strong evidence of exuberance in real house prices for all countries except three (Finland, Italy, and South Korea). Hence, our results indicate that periods of explosive behavior were widespread across a large number of countries given our available time series. The evidence for mildly explosive behavior is equally robust when we look at the price-to-income ratio, except for Norway, for which we cannot reject the null. When we apply the same testing procedure to the real PDI series, we fail to find evidence of explosive behavior at conventional statistical significance levels for 4 out of the 22 countries (Germany, Denmark, France, and New Zealand), but the timeline of these episodes suggests a disconnect between periods of exuberance in housing markets and the observable fundamental factors (real PDI) driving the demand for housing.

Figure 6 and Figure 7 complete the description of the results derived with the procedure of Phillips et al. (2012) and Phillips et al. (2013) by plotting the periods of exuberance for all countries in the database and for all three series of interest (real house prices, price-to-income ratios, and real PDI). As before, we identify periods of explosiveness by comparing the time evolution of the $BSADF$ statistics for each series against the $95\% - GSDAF$ critical value sequence in finite-samples.

We include in Figure 6 two subplots for real house prices (top) and the house-price-to-income ratio (bottom). We observe that for the most part there is accordance between the periods of exuberance detected with these two variables. However, it is worth pointing out, that periods of explosiveness based on non-fundamental behavior detected by the house-price-to-income ratio tend to be somewhat shorter than those we see with the real house price data. Three phases can be identified from this graph in connection with the last boom and bust episode in international housing markets. According to our evidence, each phase involves a specific evolution of the episode.

In the first phase, between the mid-1990s and the early 2000s, explosiveness appears in the U.S. and Ireland. There were concurrent episodes detected for Norway and Switzerland at the time, but those seemed to have evolved and eventually collapsed on their own. The leading role of the U.S. during this phase completely disappears if we look at the price-to-income ratio. The following hypothesis is central for our interpretation of the timeline of events: That explosiveness in house prices may have originally been driven by fundamentals in the U.S. The 1990s was a period of strong growth in income, partly due to the impact of the new information technologies.

We include in Figure 7 two subplots for real house prices (top) and the real PDI (bottom). We observe evidence of explosiveness in the real PDI during the first phase, but not in the case of the U.S. The robust upward trend of real PDI can explain the more muted picture of exuberance in the U.S. housing market than we get from the price-to-income ratio, but as we argue in the preceding section, there are other factors that can explain explosiveness in the U.S. real house prices, other than the characteristics of the fundamentals. Our evidence, therefore, does not allow us to rule out the possibility of other unobserved fundamentals played a major role in the origination of the boom, or even to rule the possibility of bubbles.

Furthermore, there is an unusual synchronization in the episodes of explosive behavior across most of the countries in the sample since the early-to-mid 2000s. This period of near simultaneous exuberance was pervasive across very different housing markets whose fundamentals where not necessarily aligned, and it is unprecedented with the sample covered within the International House Price Database. This constitutes the second phase of the boom period, that eventually collapsed before the start of the 2008-09 global recession. While differences in the timeline for each country vary depending on whether we look at episodes of exuberance on real house prices or the price-to-income ratio, there is a broad synchronization during this second phase with both variables.

Figure 6: Date-Stamping with Real House Prices and the Price-to-Income Ratio Across Countries

NOTE: Shaded areas indicate periods of exuberance determined by the GSADF test.
SOURCE: Federal Reserve Bank of Dallas international house price database; authors' calculations.

Figure 7: Date-Stamping with Real House Prices and Real PDI Across Countries

NOTE: Shaded areas indicate periods of exuberance determined by the GSADF test. Real PDI exuberance measure includes a linear time trend.
SOURCE: Federal Reserve Bank of Dallas international house price database; authors' calculations.

The evidence of exuberance in the real PDI is significantly different with very few countries showing a pattern of explosiveness that could account for the behavior of house prices during this second phase. Spain and the U.K. stand out as two of the major economies in the narrative of the boom and bust of housing markets whose distinct experiences are illustrative of the second phase. In both cases, we document a significant disconnect between this particular fundamental and the behavior of house prices—in countries such as Spain and the U.K., a period of explosiveness in income arises only after the correction in housing markets takes place. We will look at both country's experiences compared against the U.S. in closer inspection later on.

During the second phase, the boom in house prices that originated in the U.S., propagated to other housing markets with perceived greater opportunities or lower risks. Our evidence on the real PDI does not rule out a contribution from fundamentals, but the pattern of propagation and the high synchronization observed in real house prices suggest that a common factor may have contributed to house price exuberance spreading to other countries. In this regard, the decline in world interest rates experienced during the 2000s may have been an important contributing factor, as suggested in Section 2.2 by theory. Figure 1 illustrates the potential impact of time variation in the discount rate r_t in (21) on the features of the house price series P_t in (24) indicating that explosiveness in house prices due to declines in the discount rate may be associated with higher volatility as well. Mack and Martínez-García (2012) provide some corroborating empirical evidence as they detect (with essentially the same dataset) an increase in house price volatility during this phase that would be consistent with the stylized implications of a declining discount rate.

In the third and final phase, the run-up in house prices started to be perceived as not sustainable, and risks to real economic activity in both the U.S. and around the world grew. The episode ended in a near simultaneous collapse of house prices around 2005-06, and then the economic implications for the U.S. and the world became apparent.

These three phases provide a timeline for the boom and bust in international housing markets that preceded the 2008-09 global recession. We identify periods of mildly explosive behavior in house prices for each country, and investigate the contribution of fundamentals (real PDI) as well. While our evidence does not rule out the possibility that housing fundamentals would be the main driver, pinning down the sources of the observed behavior is more complicated. The strong run-up in income during the 1990s may have contributed to the overheating in the U.S. housing market, but non-fundamental bubbles or the behavior of unobserved fundamentals could have also contributed. The pattern of propagation that we observe across housing markets during the second phase could also be consistent with a period of decline in world interest rates.

4.2 Empirical Findings: The Cases of the U.S., the U.K., and Spain

We highlight the experiences of the U.S., the U.K., and Spain—for the period between the first quarter of 1975 and the second quarter of 2013—for their economic size and significance, and because they describe the distinct patterns observed during the three main phases of the timeline of events that we describe in the paper.

The real house price appreciation has been very significant for these three countries since the mid 1980s. The time series shows that the real house price run-up seen in the U.K. and Spain has been larger over time than that experienced in the U.S. This sets Spain and the U.K. apart, but as our evidence shows, it does not mean that explosive behavior is somehow weaker for the U.S. In fact, during the period of exuberance leading to the 2008-09 global recession, the U.S. played the leading role, while the U.K. and Spain lagged.

Interestingly, the differences between the U.K. and Spain become more noticeable when we look at the price-to-income ratio and the real PDI data. The price-to-income ratio during the 1990s reverted back to its pre-1985 average for the U.K., but remained elevated for Spain. Spain's correction since 2006 has been more severe than that of the U.K., whose price-to-income ratio has remained elevated since the mid-2000s in spite of experiencing a housing bust. The time series also shows that growth in real PDI was consistently more robust for the U.K. than for Spain since the mid-1990s.

In spite of these differences, our findings show that both countries went through a simultaneous period of exuberance during the second phase of the timeline. While country specific differences play a distinctive role for both countries, there is still a pattern of propagation that we observe, where explosiveness in real house prices in the U.S. may have migrated and amplified the effect of domestic factors in these very different housing markets.

Table 1: Evidence of Explosive Behavior in the Housing Markets

Panel A: Test Statistics	**Real House Prices**		**Price-to-Income Ratio**		**Real PDI**	
Country	$SADF$	$GSADF$	$SADF$	$GSADF$	$SADF$	$GSADF$
United States	1.52^{**}	3.81^{***}	-0.78	3.47^{***}	-1.15	1.30^{**}
United Kingdom	1.83^{**}	3.34^{***}	1.50^{**}	2.65^{***}	-0.34	1.31^{**}
Spain	0.39	3.34^{***}	0.01	1.84^{**}	1.39^{***}	1.77^{***}
Panel B: Critical Values						
90%	0.98	1.54	0.98	1.54	0.12	0.71
95%	1.25	1.80	1.25	1.80	0.39	0.95
99%	1.89	2.39	1.89	2.39	0.91	1.43

Note: *, **, and ***, denote statistical significance at the 10, 5 and 1 percent significance level respectively. All results are for autoregressive lag length k=4. The estimates for Real Personal Disposable Income (PDI) are based on a estimation regression that includes a linear time trend.

The top panel of Table 1 summarizes the estimated $SADF$ and $GSADF$ test statistics for the U.S., the U.K., and Spain on the three variables of interest: Real house prices, the price-to-income ratio and real PDI. The bottom panel of Table 1 reports the 90%, 95% and 99% critical values for the $SADF$ and $GSADF$ statistics.

Focusing on real house prices, we observe that the $SADF$ test statistics are greater than the 95% critical values for the U.S. and the U.K. but not for Spain. Further, the $GSADF$ statistics for all three countries are greater than the 99% critical values in accordance with the higher power of the $GSADF$ test. Overall, there is strong evidence that real house prices have exhibited periods of explosive behavior in the given time series of these three countries.

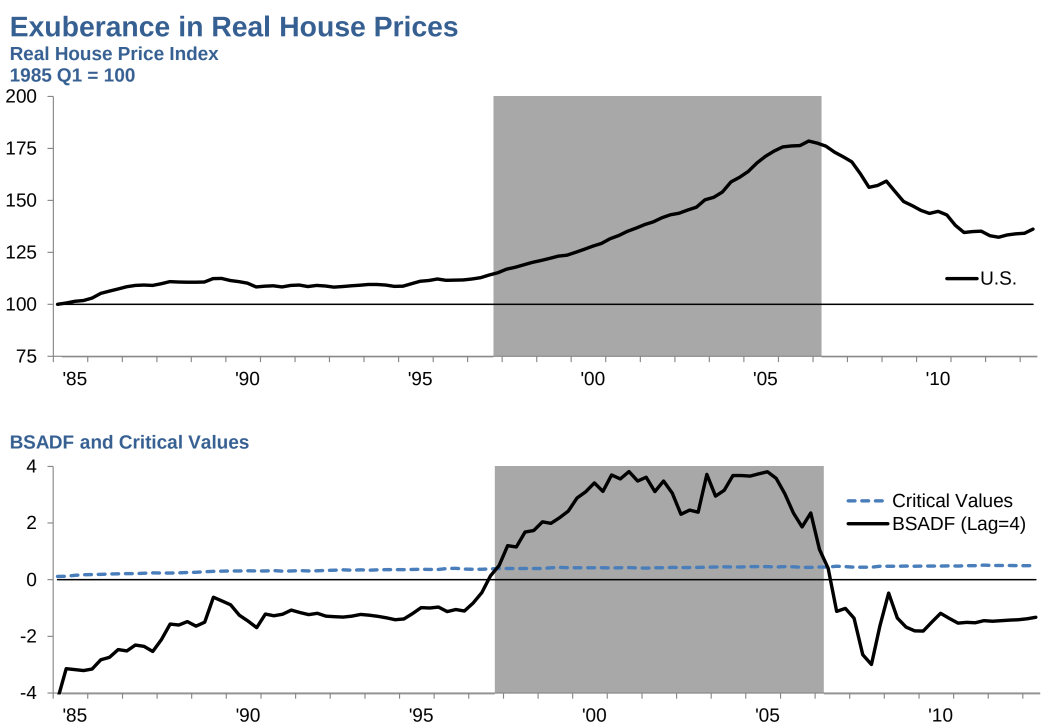
Table 1 also reports results for price-to-income ratios and real PDI from the International House Price Database for the same set of countries. What we observe with the price-to-income ratio is similar to the evidence on real house prices. The $GSADF$ statistics for house price to income ratio is above the 99% critical value for the U.S. and the U.K., and above the 95% critical value for Spain. Hence, the evidence from this ratio seems to corroborate that these three countries have all experienced periods of explosive behavior. The $GSADF$ statistics for real PDI are statistically significant at the conventional 95% level for the U.S. and the U.K. and at the 99% level for Spain. We interpret this as evidence that domestic fundamental factors may exhibit explosiveness, but cannot conclude that they have played a role in supporting this period of exuberance in housing markets.

Having established that there is strong empirical support for explosiveness in real house prices, the next step of the procedure of Phillips et al. (2012) and Phillips et al. (2013) is to identify the actual period(s) of explosive behavior in the time series—which we do next.

United States Figures 8, 9 and 10 display our main results for the U.S. In the top panel, we observe the respective time series of all three variables examined (real house prices, house-price-to-income ratio and real PDI). The bottom panel shows the time evolution of the $BSADF$ statistics for real house prices, house-price-to-income ratios and real PDI respectively—together with the 95% $GSADF$ critical value sequence (in finite-samples).

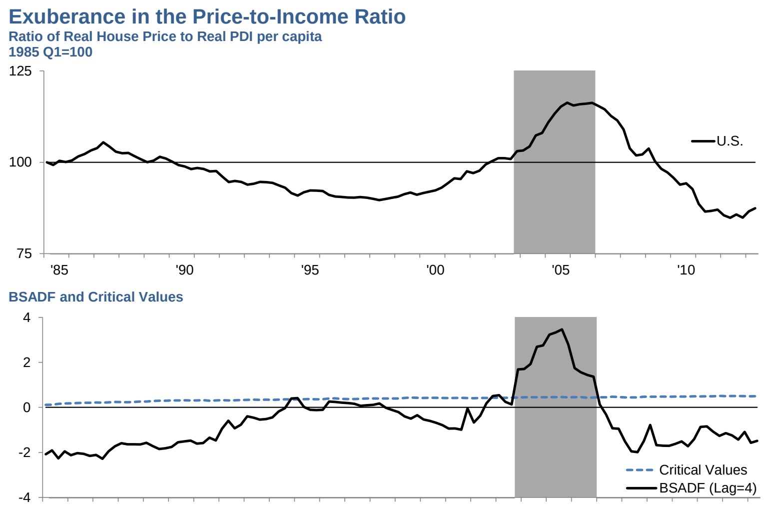
We see that real house prices for the U.S. entered an explosive regime around the second half of the mid-1990s until 2006. With the long-run anchor of the house-price-to-income ratio, we observe a period of exuberance that tends to be shorter and coincides with the second phase of the timeline described in the paper. The evidence for real PDI does not show a distinct pattern of explosiveness in the data prior to the collapse of real house prices documented in 2006, once we account for the trend component in the series. Our findings signal a period of explosive behavior occurring in the U.S. since the mid-1990s that may have multiple causes, but may have been partly supported and partly masked by the strong growth in income experienced at the same time.

Figure 8: Date-Stamping with U.S. Real House Prices



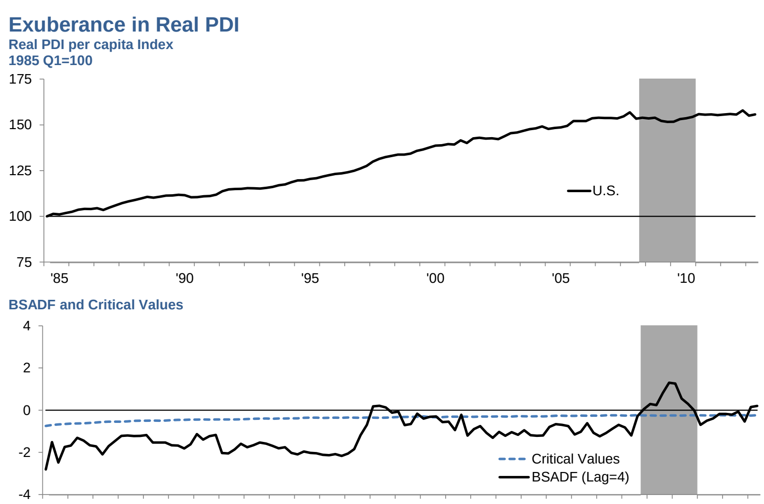
NOTE: Shaded area indicates period when real house prices surpass critical value.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Figure 9: Alternative Date-Stamping with U.S. Price-to-Income Ratio



NOTE: Shaded area indicates period when price-to-income ratio surpasses critical value.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

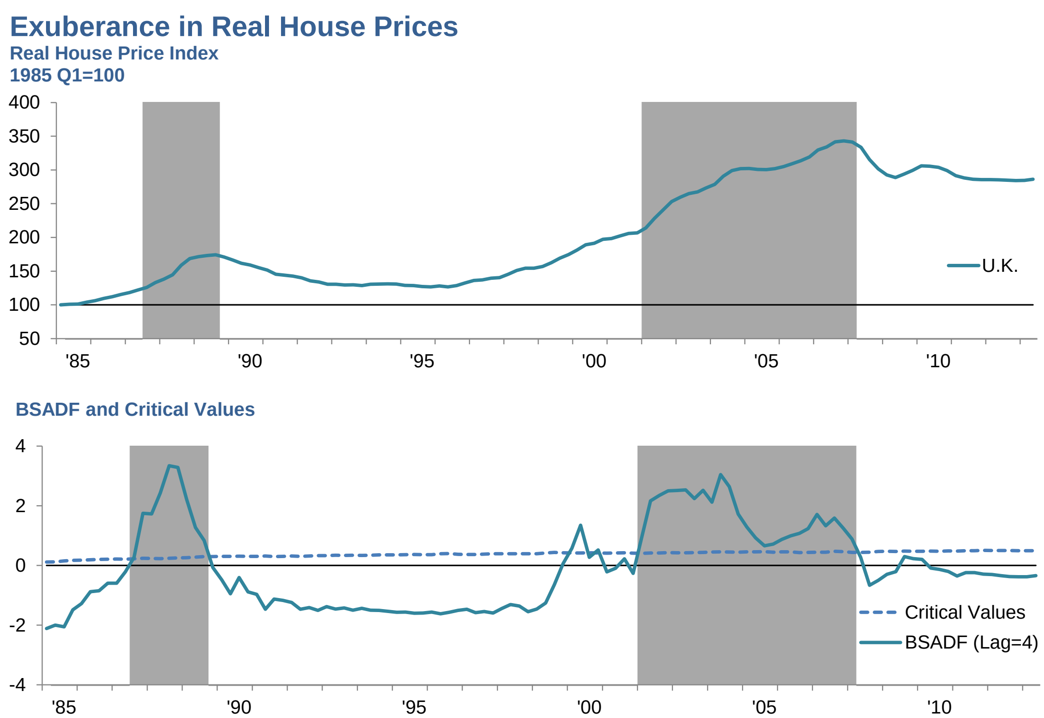
Figure 10: Alternative Date-Stamping with U.S. Real PDI



NOTE: Shaded area indicates period when real PDI index surpasses critical value. Real PDI statistics calculated with a linear time trend.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

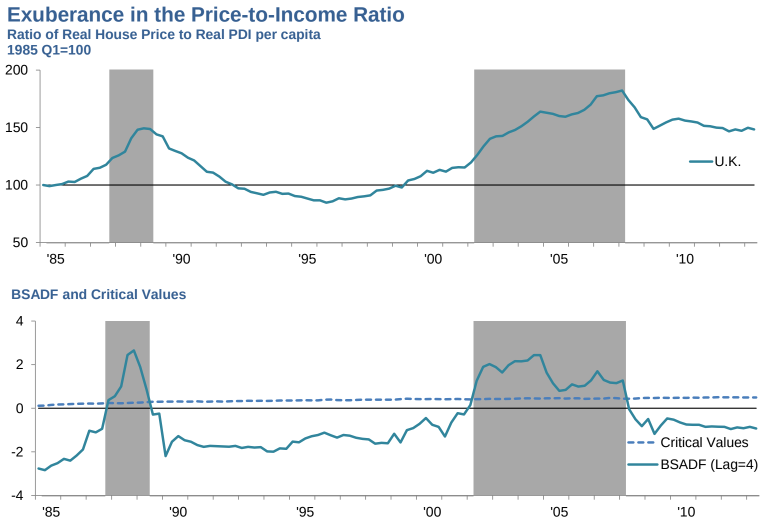
United Kingdom Figures 11, 12 and 13 display our main results for the U.K. The interpretation of these figures is analogous to those of the U.S. The U.K. displays an additional episode of explosive behavior in the late 1980s when looking at real house prices and the price-to-income ratio, although not seen in the real PDI data. We observe that the periods of exuberance are aligned whether we look at real house prices or the house-price-to-income ratio. These findings point towards a period of exuberance in house prices during the first half of the 2000s that corresponds to the second phase of our timeline. But once again, it does not overlap with explosiveness in income, as measured by the real PDI. The strength of income growth since the mid-1980s in the U.K. may have played a similar role as in the case of the U.S., but the evidence suggests that this was not enough to insulate the housing market from a period of exuberance preceding the bust in 2007-08.

Figure 11: Date-Stamping with U.K. Real House Prices



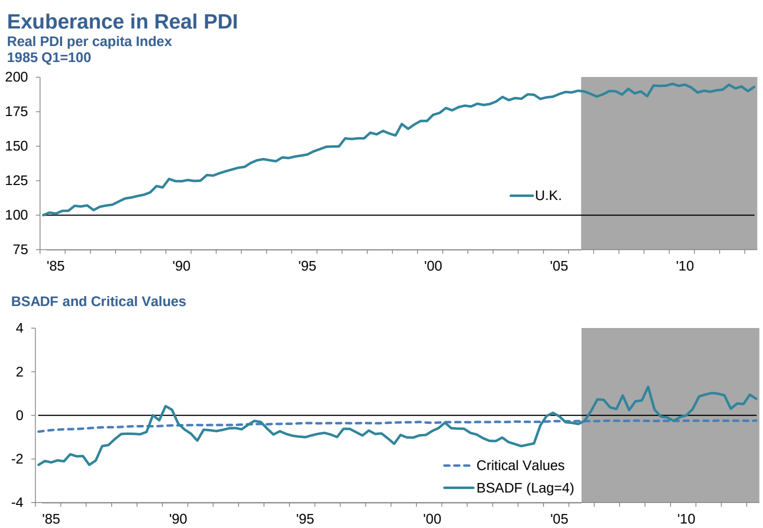
NOTE: Shaded areas indicate period when real house prices surpass critical value.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Figure 12: Alternative Date-Stamping with U.K. Price-to-Income Ratio



NOTE: Shaded area indicates period when price-to-income ratio surpasses critical value.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Figure 13: Alternative Date-Stamping with U.K. Real PDI

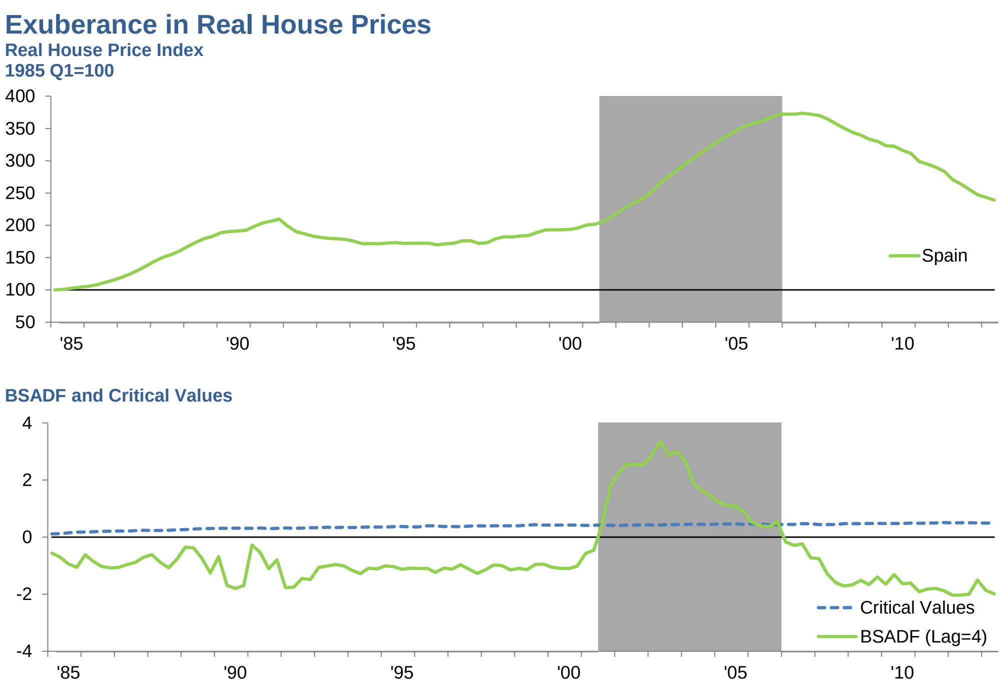


NOTE: Shaded area indicates period when real PDI index surpasses critical value. Real PDI statistics calculated with a linear time trend.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Spain Figures 14, 15 and 16 display our main results for Spain. Once again, the interpretation of these figures is the same as for the U.S. The rapid acceleration of house prices was not driven by explosive behavior except for a period of exuberance in the first half of the 2000s. The house-price-to-income ratio tends to give us a shorter period of exuberance for the latest boom-bust episode in the Spanish housing market, suggesting that part of the appreciation could be due to the behavior of fundamentals. When we look at the evidence on explosiveness in the real PDI series for Spain, it appears as statistically significant at conventional confidence levels, following the correction of housing prices in 2006-07. Once again, our results point to a disconnect between the behavior of house prices and that of income during this boom and bust cycle.

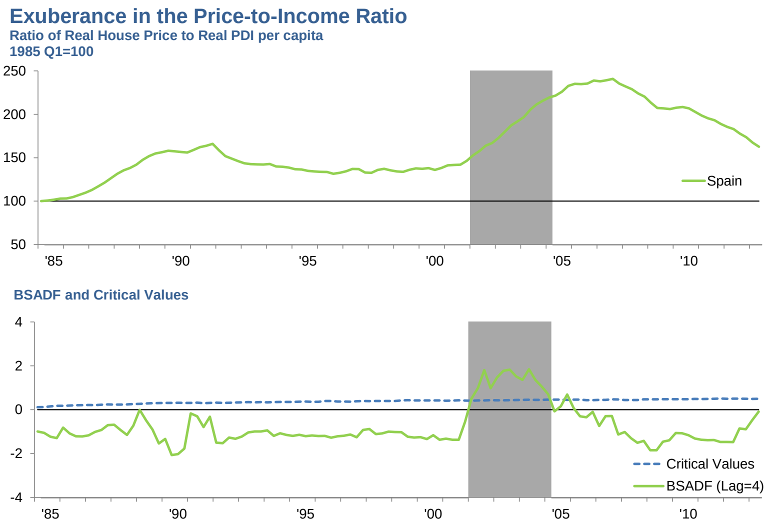
However, in the case of Spain, the collapse in the housing market may have contributed to the behavior of the real PDI series since 2008. We also observe that there is some evidence of explosive behavior in real PDI in the second half of the 1980s—a period that coincided with Spain's accession into the present-day European Union, but before the collapse of the European Monetary System in 1992-93 and the severe recession that affected the country in the early 1990s. We find during that time no evidence of explosiveness in house prices that may have either triggered or been induced by the behavior of real PDI, indicating again a disconnect between them.

Figure 14: Date-Stamping with Spain Real House Prices



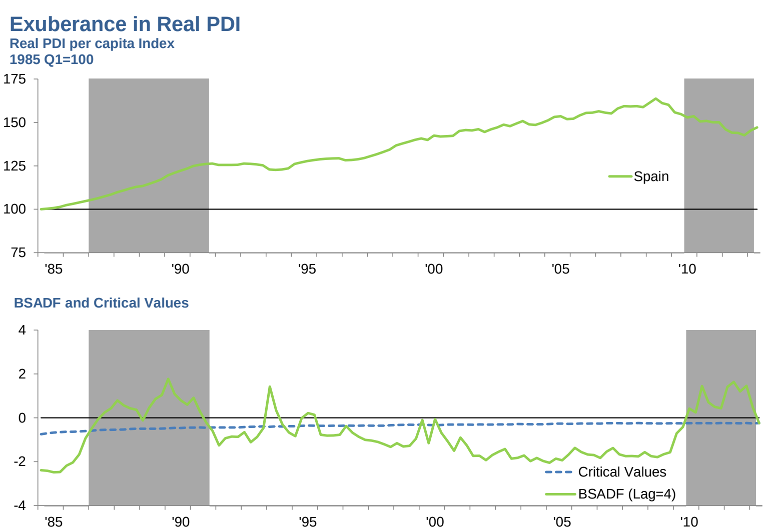
NOTE: Shaded area indicates period when real house prices surpass critical value.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Figure 15: Alternative Date-Stamping with Spain Price-to-Income Ratio



NOTE: Shaded area indicates period when price-to-income ratio surpasses critical value.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

Figure 16: Alternative Date-Stamping with Spain Real PDI



NOTE: Shaded area indicates period when real PDI index surpasses critical value. Real PDI statistics calculated with a linear time trend.
SOURCE: Federal Reserve Bank of Dallas' International House Price Database; authors' calculations.

5 Concluding Remarks

In this paper we describe a novel test ($GSADF$) proposed by Phillips et al. (2012) and Phillips et al. (2013) applied to house price data from the International House Price Database of the Federal Reserve Bank of Dallas documented in Mack and Martínez-García (2011). We show that this test is useful for detecting and date-stamping periods of explosive behavior in housing markets, and more broadly to monitor the macroeconomic developments in housing.

We cannot attribute our findings to the observable fundamentals (real personal disposable income per capita) for most countries. However, a timeline of events that emerges from the empirical evidence suggests that the latest boom-bust cycle in international housing markets evolved in three phases: One of origination that can be identified with the U.S. experience primarily during the second half of the 1990s; a second phase of propagation that is characterized by widespread and synchronized episodes of exuberance across very different housing markets during the first half of the 2000s; and a final phase where this episode of exuberance bursts within a short period of time for the affected countries preceding the severe contraction in economic activity of the 2008-09 global recession.

If the discount rate is time invariant, the identification of explosive characteristics in the house price data is observationally equivalent to the detection of a rational bubble (see, e.g., Diba and Grossman (1988)), and theory suggests this could also be a propagating factor during the second phase described in this paper that can also explain the increase in volatility documented by Mack and Martínez-García (2012). Otherwise, unobserved fundamentals or even non-fundamental rational bubbles may have played a role in the observed behavior of house prices and the apparent disconnect that we document here with the behavior of the real PDI (as a key determinant of the demand for housing).

We pay special attention to the cases of the U.S., the U.K., and Spain, which were greatly affected by the latest boom and bust in the housing markets. Our evidence suggests that even controlling for fundamental behavior, there is strong evidence that explosive behavior appeared in the U.S. in the second half of the 1990s and followed in the U.K. and Spain during the first half of the 2000s. This evidence contrasts with existing work that employs other more conventional methods of bubble-detection which produce less clear readings. Our results show that the cases of the U.S., the U.K. and Spain were far from isolated. In fact, they were fairly typical of a period of exuberance that affected most of the countries covered currently in the International House Price Database. The correction of such a widespread period of explosive behavior resulted in the most severe recession of the post-WWII period for most of them.

Our work also shows how this methodology can be implemented to detect the presence of explosive behavior that are not identifiable with other existing methodologies. As a result, we view this procedure as a valuable tool for policy analysis as well as scholarly work in helping us detect explosiveness in international housing markets.

Appendix

A Demand Equation for Rental Housing

Consider the maximization of the Stone-Geary utility function with housing units rented, H_t, and consumption of other goods, C_t, i.e.,[16]

$$U\left(H_t, C_t\right) = \left(H_t - \theta_H\right)^{\alpha} \left(C_t - \theta_C\right)^{1-\alpha}, \ 0 < \alpha < 1,$$

subject to the intratemporal budget constraint,

$$C_t + x_t H_t = Y_t,$$

where the price of the consumption good is normalized to one. $X_t \equiv x_t H_t$ is the housing rents—rental expenditures—paid and x_t the rental rate per unit rented, Y_t refers to disposable income, while $0 < \alpha < 1$, θ_H and θ_C are preference parameters.

From first-order conditions, the Stone-Geary utility function subject to the standard intratemporal budget constraint gives a linear expenditure system where the demand for rental housing takes the following form,

$$H_t = \theta_H + \frac{\alpha}{x_t}\left(Y_t - x_t \theta_H - \theta_C\right), \tag{37}$$

or in expenditure terms,

$$X_t \equiv x_t H_t = \alpha Y_t + \left(1 - \alpha\right) \theta_H x_t - \alpha \theta_C. \tag{38}$$

Under the assumption that in equilibrium the units rented are constant (i.e., $H_t = H$) and normalized to one, the demand equation that determines housing rents in (38) reduces to an affine transformation of disposable income Y_t, i.e.,

$$X_t = x_t = \theta + \delta Y_{t+1}, \tag{39}$$

where $\delta \equiv \frac{\alpha}{1-(1-\alpha)\theta_H}$ and $\theta \equiv -\frac{\alpha}{1-(1-\alpha)\theta_H}\theta_C$.

[16] While the Stone-Geary reduces to the Cobb-Douglas utility function whenever the parameters θ_H and θ_C are both set equal to zero, the specification permits both the rental rate elasticity and the income elasticity to vary with both rental rates and income—unlike the Cobb-Douglas where both elasticities are constant or the constant elasticity of substitution utility function for which the income elasticity is constant.

B $SADF$ and $GSADF$ Statistics for All Countries in the Database

Table 2: Evidence of Explosive Behavior in the Housing Markets

Panel A: Test Statistics

	Real House Prices		**Price-to-Income Ratio**		**Real PDI**	
Country	$SADF$	$GSADF$	$SADF$	$GSADF$	$SADF$	$GSADF$
Australia	2.23***	6.18***	1.08*	2.57***	1.02***	1.07**
Belgium	0.97	2.98***	−0.25	2.92***	0.93***	1.06**
Canada	0.32	3.76***	−1.13	2.16**	−0.30	1.25**
Switzerland	1.64**	2.55**	1.20*	2.30**	−0.09	0.72*
Germany	−0.59	2.10**	0.57	2.55***	−1.08	−0.07
Denmark	1.31**	3.00***	−0.03	2.15**	−0.32	0.54
Spain	0.39	3.34***	0.01	1.84**	1.39***	1.77***
Finland	0.94	1.45	−0.89	0.96	0.17*	1.81***
France	1.35**	2.21**	−0.03	2.46***	−0.46	0.50
United Kingdom	1.83**	3.34***	1.50**	2.65***	−0.34	1.31**
Ireland	2.59***	3.71***	2.01***	2.19**	2.41***	3.92***
Italy	−1.28	−0.31	−1.52	1.08	0.18*	1.54***
Japan	1.67**	3.77***	0.88	4.63***	1.27***	1.37**
South Korea	−0.60	−0.32	0.50	0.50	−0.48	1.64***
Luxembourg	1.66**	3.89***	−0.27	1.59*	0.95***	1.34**
Netherlands	−0.43	4.13***	−0.17	3.13***	−0.49	0.74*
Norway	0.85	1.75*	0.22	0.36	0.59**	1.06**
New Zealand	1.77**	2.35**	0.43	3.19***	−0.61	0.31
Sweden	0.19	3.79***	0.30	3.34***	−0.44	1.93***
United States	1.52**	3.81***	−0.78	3.47***	−1.15	1.30**
South Africa	−0.52	3.93***	−0.78	3.44***	0.43**	1.88***
Croatia	0.08	1.46*	0.87	3.52***	2.90***	3.33***
Panel B: Critical Values						
90%	0.98	1.54	0.98	1.54	0.12	0.71
95%	1.25	1.80	1.25	1.80	0.39	0.95
99%	1.89	2.39	1.89	2.39	0.91	1.43

Note: *, **, and ***, denote statistical significance at the 10, 5 and 1 percent significance level respectively. All results are for autoregressive lag length k=4. The estimates for Real Personal Disposable Income (PDI) are based on a estimation regression that includes a linear time trend.

C National Sources of House Price Data

	House Price Definition	Source and Time Coverage
Australia	Weighted average of 8 capital cities, new and existing detached house price index, per dwelling	Australia Bureau of Statistics 1986Q3-present
	Weighted average of 6 capital cities, new and existing dwelling price index, per dwelling	Australian Treasury 1960Q3-present
Belgium	Nationwide existing single-family house price index, per dwelling	Statistics Belgium 1973Q1-present
Canada	10 metropolitan areas, “fair” price of existing detached bungalows and two story executive dwellings, per dwelling	Royal Le Page 1993Q1-present
	10 metropolitan areas, “fair” price of existing detached bungalows and two story executive dwellings, per dwelling	University of British Columbia 1975Q1-2012Q1
Switzerland	Nationwide new and existing single-family house price index, per dwelling	Swiss National Bank 1970Q1-present
Germany	Nationwide existing terraced house price index, per dwelling	Deutsche Bundesbank 1995-present (annual)
	W. Germany existing terraced house price index, per dwelling	Deutsche Bundesbank 1990-2010 (annual)
	W. Germany new terraced house price index, per dwelling	Deutsche Bundesbank 1975-2010 (annual)
Denmark	Nationwide new and existing single-family house price index, per dwelling	Statistics Denmark 1992Q1-present
	Nationwide new and existing single-family house price index, per dwelling	Danmarks Nationalbank 1971Q1-present
Spain	Nationwide average price of existing dwellings, per square meter	Ministerio de Fomento 1995Q1-present
	Nationwide average price of new and existing dwellings, per square meter	Ministerio de la Vivienda 1987Q1-2004Q4
	Madrid average price of new dwellings, per square meter	Tecnigrama 1976-1986 (annual)
Finland	Nationwide existing single-family house price index, per square meter	Statistics Finland 1985Q1-present
	Nationwide existing apartment price index, per square meter	Statistics Finland 1970Q1-2009Q4
France	Nationwide existing detached house and apartment price index, per dwelling	INSEE 1996Q1-present
	Nationwide existing apartment price index, per dwelling	CEGDD - Ministère de l’Écologie 1936-2009 (annual)

	House Price Definition	Source and Time Coverage
United Kingdom	Nationwide new and existing dwelling price index, per dwelling	Department of Communities and Local Government 1968Q2-present
Croatia	Nationwide new and existing dwelling price index, pure price	Croatian National Bank 1997Q1-present
	Nationwide average price of new dwellings, per square meter	Croatian Bureau of National Statistics, 1965-2011 (annual)
Ireland	Nationwide average price of existing dwellings, per dwelling	Central Statistics Office 2005Q1-present
	Nationwide average price of existing dwellings, per dwelling	Department of Environment, Community & Local Government 1978Q1-present
	Nationwide average price of existing dwellings, per dwelling	Department of Environment, Community & Local Government 1974-2009 (annual)
Italy	13 main metropolitan area average price of new and existing dwellings, per square meter	Nomisma 1988S1-present
	13 main metropolitan area average price of new dwellings, per square meter	Il Consulente Immobiliare 1967-2001 (bi-annual)
Japan	Nationwide urban residential land price index, per square meter	Japan Real Estate Institute 1955S1-present
South Korea	Nationwide new and existing dwelling price index, per dwelling	Kookmin Bank 1986M1-present
	Kyung-Hwan Kim (1993) index:	
	- Nationwide quoted transactions and estimations of real estate agents	Korea Housing Bank 1982-1990 (annual)
	- Nationwide standard construction costs (excluding land)	Korea Housing Bank 1978-1981 (annual)
	- Nationwide weighted average of total factor costs single-family house and apartment construction	Korea Housing Bank 1974-1977 (annual)
Luxembourg	Nationwide new and existing house price index, per dwelling	L'Observatoire de l'Habitat 2005Q1-present
	Nationwide new and existing dwelling price index, per dwelling	Banque centrale du Luxembourg 1974-2009 (annual)
Netherlands	Nationwide existing single-family house price index, per dwelling	Statistics Netherlands 1995M1-present
	Nationwide average price of existing dwellings, per dwelling	Kadaster 1976M1-2010M12

Norway	Nationwide new and existing detached house price index, per dwelling	Statistics Norway 1992Q1-present
	Norges Bank forecasting model index:	
	- Nationwide sales reports of Norges Eindomsmeglerforbund real estate agents	Norges Eindomsmeglerforbund 1987Q1-2003Q4
	- Dwelling price based on national property register	GAB Register 1984Q1-1986Q4
	- Nationwide building cost index	Statistics Norway 1979Q1-1983Q4
	- Housing rent component of the Consumer Price Index	Statistics Norway 1972Q1-1978Q4
New Zealand	Nationwide new and existing detached house price index, per dwelling	Reserve Bank of New Zealand 1962Q2-present
Sweden	Nationwide new and existing one- and two-family house price index, per dwelling	Statistics Sweden 1986Q1-present
	Nationwide new and existing one- and two-family house price index, per dwelling	Statistics Sweden 1975-2010 (annual)
United States	Nationwide existing single-family house price index, per dwelling	FHFA 1975Q1-present
South Africa	Nationwide new and existing single-family house price index, per dwelling	ABSA 1966M1-present

Note: Time series backcasting is used to extend the house price indexes of Spain and the Netherlands from the first quarter of 1976 back to the first quarter of 1975. Time series nowcasting is used for Italy, Germany and Japan in order to complete the quarterly dataset and avoid long lags in its public release. Nowcasting are subsequently replaced with actual data from the national sources, as it becomes available.

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