Quantum Properties of Black Holes

[On the analogue model of Bose-Einstein Condensate of Gravitons]

Kiran Kolishetty
Lancaster University 2013-14

A thesis submitted for the degree of
Masters by Research
Lancaster 2014

Quantum Properties of Black Holes

[On the analogue model of Bose-Einstein Condensate of Gravitons]

by **Kiran Kolishetty**
M.sc by Research in Physics 2013-14
Lancaster University
October 10, 2014

Abstract

In this research report I would like to present some of the results obtained based on the published research, it is on the analogue model of Bose Einstein condensate of gravitons. In my investigation with different approach I could reach similar conclusion for Schwarzschild radius which is derived for Bose Einstein condensate of N-gravitons . I could calculate the pressure inside the condensed black hole. It is found that the Bogoliubov dispersion relation for the elementary excitations of the weakly-interacting N-gravitons is shown to hold for the case of the weakly-interacting Graviton gas or N weakly interacting gravitons. where both the cases of attraction and repulsion is taken into consideration between the weakly coupled N-gravitons. It is interesting to notice that weakly interactions among gravitons have differences in attractive and repulsive case, additionally I will be looking into the possible way to understand the entropy of the N-graviton in a condensed Black hole. Finally I could derive the critical point for Black hole for N gravitons in case of weakly attractive interacting N gravitons , In attractive nature of N-gravitons I found that it has critical point at which black holes do not form the condensate , above this critical value k Black hole seems to form a condensate of N-gravitons. In this approach Black hole entropy is understood from condensed matter physics perspective as excited states of condensed matter of N-gravitons. Furthermore, this includes the literature review in order to support the results obtained in my calculations.

Declaration of Authorship

I, Kiran Kolishetty, declare that this thesis titled, 'Quantum properties of Black Holes' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself under supervision,I have made clear exactly what I have contributed myself.
- All the images used in this document are part of my own intellectual property, I have 3D visualFX computer skills I created them, I own them.

Signed: **Kiran Kolishetty**

Date: October 10, 2014

Dedication

I would like to dedicate my work to beautiful “Nature” (Laws of Nature, The complex physical system in the universe). To all the good life force in this universe and perhaps multi-verse. To all the disabled people and to all the students with learning disability who are Dyslexic like me, who always work hard to reach their goals and dreams, “Who Never Give up in life !” even when odds are always against you. To all hard working Physicists,Inventors, Scientists, Mathematicians, Philosophers, Engineers and to all the creative minds, who shape the planet earth and humanity with their scientific wisdom. Especially to Prof. Stephen W. Hawking for all the works on Black Holes theories, you are impossible! “Please never say that black holes do not exist !” and Sir.Isaac Newton, Galileo Galilei, Albert Einstein, Michael Faraday, Nicholas Tesla, Thomas Alva Edison, Marie Skłodowska-Curie, Emmy Noether , Émilie du Châtelet, Lise Meitner, Archimedes, Euclid, Aristotle, Plato, Socrates, Pythagoras, Srinivasa Ramanujan, René Descartes, Évariste Galois and Leonardo Da Vinci for inspiring me a lot. Also to my love and respect for “Physics, Science, Mathematics”. My excitement and curiosity is the bond between me and my universe. Since the start of this academic research project I sacrificed every thing , I sacrificed most of the things only to understand this research project deeply, most of the time went in solitude space, so that I could get more creative but I could not understand papers of Prof.Gia Dvali and Prof.Cesar Gomez deep enough since they are advanced concepts mixed with string theory, CFT, Advanced topics in quantum field theory etc. I really enjoyed working for this research project I loved the every moment and enjoying beauty of mathematics and physics. However, love for physics and resistance caused in words/verbal/numerical processing by dyslexia is ironic weird combination in my mind. I needed to process all the 20,300+ words in my head with equations in the end it felt like $E = mc^2$ of my mind, I had to do it anyway for the project my neurotransmissions really heated up in my head. I believed in my will power and self motivation that helped me to finish this project to current level limited time for a year. Finally, to my “solitude” as it is my only friend with deep relationship since childhood days,thank you for all the things you taught me, as this research was done in isolation, powered by “Solitude”.

Kiran Kolishetty

Acknowledgement

I would like to acknowledge Dr.Vadim Cheianov, who is my very first supervisor for Masters by Research in Physics at Lancaster University 2013-14. During the academic year I enjoyed his lectures on Mechanics and Variations. I find it as passionate and admiring way of teaching which is exciting. I could get chance to meet my supervisor during supervision sessions and had interesting discussions on amazing topics in physics. This made me happy, I enjoyed discussing physics topics with him. I am glad to be his student, I would like to thank him for supervising me and giving me his valuable time for this intriguing research project, also like to thank him for giving me this research topic. The advice and guidance for thesis research provided under his supervision. Moreover, I like to thank Lancaster University and physics department for providing me wonderful opportunity for study and research. I would like to thank all my examiners and assessment team who allotted their time for going through my project work.

Kiran Kolishetty

Lancaster, UK
September 2014

Contents

ABSTRACT i

DECLARATION i

DEDICATION ii

ACKNOWLEDGMENT iii

LIST OF FIGURES viii

LIST OF TABLES ix

PREFACE x

I INTRODUCTION 1

1 REVIEW OF THE QUANTUM MECHANICS 2
- 1.1 Particle wave duality . . . 2
- 1.2 Schrödinger equation . . . 3
  - 1.2.1 Particle in a box . . . 4
- 1.3 Energy levels and Degeneracy . . . 5
- 1.4 Heisenberg's Uncertainty Principle . . . 6

2 REVIEW OF THE QUANTUM FIELD THEORY 7
- 2.1 Coupling constant . . . 7
- 2.2 Ultrarelativistic Particle . . . 7
- 2.3 Klein-Gordon Equation . . . 7

3 REVIEW OF THE THERMODYNAMICS 9
- 3.1 Laws of thermodynamics . . . 9
  - 3.1.1 Zeroth law of thermodynamics . . . 9
  - 3.1.2 First law of thermodynamics . . . 9
  - 3.1.3 Second law of thermodynamics . . . 9
  - 3.1.4 Third law of thermodynamics . . . 10
- 3.2 Entropy . . . 10
- 3.3 Violation of laws of thermodynamics . . . 10
  - 3.3.1 Maxwell Demon . . . 10
- 3.4 Absolute zero . . . 12
  - 3.4.1 Lowest temperature in the universe . . . 12
- 3.5 Zero point energy and Vacuum Energy . . . 13

4 REVIEW OF THE STATISTICAL MECHANICS 14
- 4.1 Macrostates and Microstates . . . 14
- 4.2 Maxwell-Boltzmann statistics . . . 15
- 4.3 Bose-Einstein statistics . . . 16
- 4.4 Fermi-Dirac statistics . . . 16

4.5 Bose-Einstein Condensate . . . 17

5 REVIEW OF CONDENSED MATTER PHYSICS **19**
5.1 Quasiparticle . . . 19
5.1.1 Bogolons . . . 20
5.1.2 Quasi ion in an electrolyte liquid . . . 20
5.1.3 Quasi particle as electron . . . 22
5.2 Collective excitations . . . 23
5.2.1 Phonons . . . 23
5.2.2 The collective excitations of plasmons . . . 23
5.3 Bogoliubov theory for weakly interacting Bose gas . . . 24
5.4 Gross Pitaevskii equation . . . 24

II BLACK HOLE LITERATURE REVIEW **26**

6 REVIEW OF BLACK HOLES **27**
6.1 Newton's Classical view of 3D Space "and" Universal Time . . . 27
6.2 Newton's Gravity . . . 27
6.3 Limitations of Newton's Universal gravity . . . 28
6.3.1 Universally Isolated mass system of M_1, one body problem . . . 28
6.3.2 Mercury's perihelion shift . . . 29
6.3.3 Incompatibility of Newton's gravity with special theory of relativity . . . 29
6.4 General Relativity . . . 30
6.5 The gravitational effects of general relativity . . . 31
6.6 Tests of general relativity and gravitational waves . . . 32
6.7 Schwarzschild metric . . . 34
6.8 Schwarzschild radius as Event Horizon . . . 34
6.9 Gravitational Singularity . . . 35
6.10 Life cycle of stars . . . 35
6.10.1 Stellar evolution-The Protostar . . . 36
6.10.2 Young star, it's Balance of Life and Death . . . 36
6.10.3 The Red Giant . . . 36
6.10.4 Supernova . . . 36
6.10.5 Chandrasekhar limit for White Dwarf . . . 36
6.10.6 Tolman Oppenheimer Volkoff limit/TOV limit for Neutron star . . . 37
6.10.7 The Black holes: The battle of Quantum mechanics Vs. Classical physics . . . 37
6.11 No-hair conjecture . . . 38
6.12 The Thermodynamic laws of black hole mechanics . . . 38
6.12.1 The Zeroth Law . . . 38
6.12.2 The First Law . . . 38
6.12.3 The Second Law . . . 39
6.12.4 The Third Law . . . 39
6.12.5 Understanding the four laws . . . 40
6.13 Entropy of Black hole . . . 40
6.14 Hawking radiation . . . 40
6.15 Trans-Planckian problem . . . 42
6.16 What is the Minimum mass required to bend space ? . . . 42

6.17 General relativity Vs. Quantum mechanics . . . 42
6.18 Quantum gravity . . . 43
6.19 Why gravity is too weak? . . . 43
6.20 N-Gravitons . . . 43

7 ANALOGUE MODELS OF BLACK HOLES **45**
7.1 Acoustic analogue model: The Dumb Hole . . . 45
7.2 Quantum Liquids . . . 47
7.3 Shallow water waves . . . 47
7.4 Analogue gravity from Bose-Einstein condensates . . . 48

8 BOSE EINSTEIN CONDENSATE OF GRAVITONS **49**
8.1 Black Hole's Quantum N picture . . . 49

III MASTERS BY RESEARCH in PHYSICS **52**

9 A GRAVITON IN A BOX **53**
9.1 Particle in a box . . . 53
9.2 Bose-Einstein Condensate energy distribution for graviton . . . 53
9.3 Particle in a square box . . . 53

10 PRESSURE INSIDE THE BLACK HOLE **58**

11 ULTRA-RELATIVISTIC CASE **60**

12 CONCLUSION **67**
12.1 Temperature and Bose Einstein condensate, Thinking in the practice perspective . . 68
12.2 No Black holes ? The current research . . . 69

REFERENCES **74**

List of Figures

1 The 3D model illustrates an electron orbiting around the atom, as the electron can be both wave and a particle according to De Broglie's pilot-wave. 3
2 The 3D model of the box with particle such as electron inside the box, as the rectangular box with lengths L_x, L_y, L_z. 5
3 The Maxwell's Demon, as illustrated in my image that two containers with A and B are placed side by side with the barrier. A and B are in thermal equilibrium with each other and have same temperature. Molecules with red are with high kinetic energy (denoted with velocity vectors) and molecules with blue are low in average velocity. 11
4 The Maxwell's Demon, as illustrated in my image that two containers with very hot Box A and B being very cold are placed side by side with the barrier. A and B are no longer in thermal equilibrium with each other and have different temperature. Molecules with red are with high kinetic energy (denoted with velocity vectors) enclosed in Box A creating the perfect order and molecules with blue are low in average velocity enclosed in Box B. This separation is caused by the ability of Maxwell's demon to move the particle through the barrier creating the perfect order, Entropy of isolated system decreases violating the second law of thermodynamics. There is relationship with The information plays key role, Demon uses information to create the order in a box. 12
5 The group of positive and negative ions forming quasi particles as blue clouds representing the formation 21
6 (i). Shows the introduction of (+) ion into the system, (ii). Shows as (+) ion gains cloud or shield of (-) ions making it as quasi particle, (iii). The quasi ion moves through the system 21
7 (i).Conduction of an electron through pure metal of (+) ions, (ii).Electron passes through impure metal 22
8 (a) Shows plasma wave in electron gas with high and low density regions, (b) Shows particle-hole picture of plasma wave as black represents holes and white as particle . 24
9 shows fundamental degrees of freedom for mass M in 3D Space and Universal Time . 27
10 shows Newton's Universal law of gravity 28
11 shows Perihelion precession of Mercury around the sun, the orbit size, planet and sun size are not to scale 29
12 Illustrates the concept of general relativity, massive object such as sun can distort space-time geometry. 31
13 Illustrates the gravitational waves from the massive state 32
14 My image illustrates the concept of General relativity replacing the concept of the Newton's view of gravity ,consider an imaginary situation where the planet Mars orbits around planet Jupiter, the curvature of space itself determines the orbit of the planet Mars around Jupiter. 33
15 image illustrates the concept of the moving fluid which will drag sound pulses along with it. This may form supersonic and subsonic regions. 46
16 I would like to imagine from "artistic" perspective of Bose Einstein Condensate of N-gravitons, as gravitons are taken as black particles to represent black hole. This is non-scientific but based on the discussion on Bose Einstein condensate of gravitons, perhaps light may escape from event horizon as shown in the image which is not reflection. 51

17 The graph illustrating the relation between $\omega(k)$ and k for positive value of large values of k in repulsive case as coupling constant $+U_o$ for the equation 11.24 62
18 The graph illustrating the relation between $\omega(k)$ and k for positive value of large values of k in repulsive case as coupling constant $+U_o$ for the equation 11.24 63
19 The graph illustrating the relation between $\omega(k)$ and k for positive value of large values of k in attractive case as coupling constant $-U_o$ for the equation 11.25 The imaginary $-\sqrt{k}$ terms are shown. 64
20 The graph illustrating the critical point k at which $\omega(k) = 0$ showing both cases of attractive and repulsive 65
21 The graph illustrating the critical point k at which $\omega(k) = 0$ showing both cases of attractive and repulsive, critical value only exist for attractive case not for repulsion. Above the critical value k the Bose-Einstein condensate of gravitons is possible. . . . 67

List of Tables

1 The table is showing degeneracy with the energy levels of electron confined in a square box with corresponding quantum numbers n_x, n_y. 6
2 shows four fundamental forces of nature and its corresponding properties [35] 44

PREFACE

In order to understand the content of my thesis I have divided this document into three parts. They are Introduction, Black Hole Literature Review and finally my thesis are discussed in this part III: MASTERS BY RESEARCH in PHYSICS.

Part I-INTRODUCTION

Introduction part I of this document deals with the very basic undergraduate level Physics concepts and some are introductory concepts to the topics Quantum mechanics, Quantum field theory, Statistical Mechanics, Condensed matter physics and Thermodynamics. The reason to give introduction topics is any reader could follow my thesis mostly even undergraduate level physics student. It has been made easier for any reader to understand.

Part II- BLACK HOLE LITERATURE REVIEW

This part is based on my previously submitted work that is based on literature review of Black hole physics, all the important facts are discussed from my literature review. In this part is is presented in new perspective. The origin of Black Hole physics and topics of general relativity.

PartIII- MASTERS BY RESEARCH in PHYSICS

In this part, I present my work for my academic year of 2013-2014 one year Master by research work in physics on "*Bose Einstein condensate of Gravitons*" under the supervision of Dr. Vadim Cheianov at Lancaster University. This is based on the research work of Prof. Gia Dvali who is professor of physics at New York University's Center for Cosmology and Particle Physics and at LMU Munich, and is a director at the Max Planck Institute for Physics, Munich. and Prof. Cesar Gomez (Institute of Theoretical Physics, Madrid). In my thesis I will be investigating the different approach to understand Bose Einstein condensate of gravitons.

Part I

INTRODUCTION

My Research is based on one of the analogue models of Black hole, which is Bose-Einstein condensate of gravitons. In the published scientific literature as it is proposed by authors Gia Dvali and Cesar Gomez in their paper [19] [20] says that the Black holes can be understood as self-sustained Bose Einstein condensate of N-gravitons where hawking radiations can be understood as leakage of gravitons from the condensate. In order to investigate this I would like to make different approach to see if I can reach the same conclusion as in order to reach that point. Firstly I would like to review some of the basic introduction topics in physics in this introduction chapter. In this chapter I discuss all the basics that are needed in order to understand the content of my thesis and support my derivations logically. In this part-I Introduction, The topics that are reviewed are quantum mechanics, quantum field theory, condensed matter physics, thermodynamics and statistical mechanics.

1 REVIEW OF THE QUANTUM MECHANICS

The quantum mechanics is the branch of science that deals with physical behaviour at close to Planck length (l_p). It is dramatically different from the consistency of classical laws of physical observations and measurement. The issue arises in quantum mechanics when observation is being made or a measurement of atomic or sub atomic length scales. The difference between classical laws of physics and quantum laws of physics are difference in abstractions that is the idea of the state of the quantum system. These states can be mathematical representations have differences in their logical structure. The states and measurements in classical law are mostly fathomable. The state of the system and measurements are two distinctive concepts in quantum mechanics. As for the state of the quantum system could be position and momentum of a particle, the measurement of the state of the system cannot give the information about position and momentum of particle at same time. In classical world one can do the experiments and get the results as this is obvious for an observer. Some of the features of quantum mechanics are discussed below.

1.1 Particle wave duality

The wave-particle duality is the quantum objects exhibiting the dual properties of both particle and wave nature. Firstly the nature of light being wave and particle has been argued throughout history. Sir. Isaac Newton's corpuscular particle theory of light describes light travels in the form of particles, then Huygens's wave theory of light tell as light wave propagating through the hypothetical medium called ether. Later on after several centuries, Young's double slit experiment which is very first experimental evidence providing the observable fact that light indeed travels in the form of waves. The interference pattern of wave fronts can be seen on the screen. Wave theory of light was widely accepted, and then theoretically proved by Scottish physicist and mathematician James Clerk Maxwell.[38] Using partial differential equations and integrating the discoveries made in relationship with electricity and magnetism by Lorentz, Faraday, Gauss and Ampere. As four equations prove that light travels in the form of electromagnetic wave and also speed of light was calculated. The particle nature of light was not popular anymore until the discovery of Max Planck [50]. In his research of black body radiation, as the radiation from black body was emitted in the form of oscillators, which later on Planck called it as energy quanta. These indicated that radiation from the black body was not continuous but emitted in the form of discrete sets of packets of energy called quanta. Later on this was proved by Albert Einstein from his research on photoelectric effect [21] proving that energy quanta is actually a physical particle which is later on called as photon. The Planck relation or also known as Planck-Einstein energy relation which tells us that frequency of light is proportional to the energy is given by

$$E = h\nu, E = \hbar\omega \tag{1.1}$$

- $h = 6.62606957(29)\text{x}10^{34} j.s$ is Planck constant
- $\hbar = \dfrac{h}{2\pi} = 1.054571726(47)\text{x}10^{34} j.s$ is reduced Planck constant
- ν -photon's frequency
- ω -photon's angular frequency

Light has dual nature both as particles-waves, particle as photon and wave as electromagnetic wave. Light can also transfer energy and momentum to the matter, the same can be seen in sub atomic particles behaviour as this has been tested by experiments such as double slit experiments. In 1924, Louis De Broglie in his Ph.D. thesis [17] suggested that the electrons can also quantum mechanically be both wave and particle as he developed pilot-wave theory of matter wave nature which is dual nature of sub atomic particles. The de Broglie wavelength can be formulated as with planck constant h and the corresponding momentum is p.

$$\lambda = \frac{h}{p}, p = \hbar k \tag{1.2}$$

- k-wave vector
- p-momentum
- λ- wavelength

De Broglie's predication of pilot-wave theory was verified by Davisson-Germer experiment Diffraction of electrons by a crystal of nickel [16] in 1927.

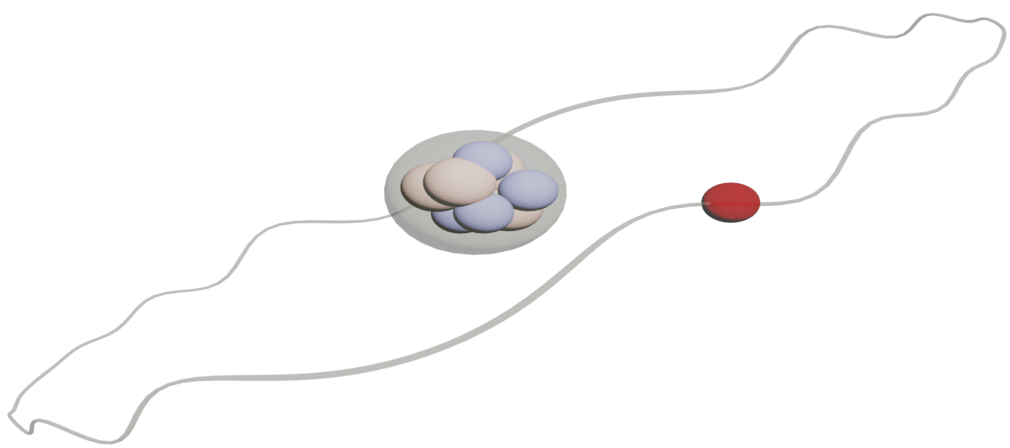

Figure 1: The 3D model illustrates an electron orbiting around the atom, as the electron can be both wave and a particle according to De Broglie's pilot-wave.

1.2 Schrödinger equation

Basically, sound waves can be described by classical mechanics, light waves can be described by Maxwell equations and matter waves for non-relativistic particles can be described Schrödinger wave equations. Austrian physicist Erwin Schrödinger used De Broglie's matter wave theory to develop the mathematical representation of charge of an electron in the form of wave equation. Consider a simple wave that is travelling along the string or it could be just a sound wave. [28] The wave nature of the light wave can be described as $\overrightarrow{E}(x, y, z, t)$ the electric field component of the wave. When an observation being made in a location of space and time. There should be a varying quantity that should be used in order to describe nature of the matter wave as its property to carry momentum, charge and matter such as electron dual nature. The wave function $\Psi(x, y, z, t)$ as this can describe the natural wave nature. Which gives us the space dependent factor $\psi(x, y, z)$ and time dependent in exponential $e^{-i\omega t}$.

$$\Psi(x, y, z, t) = \psi(x, y, z)e^{-i\omega t} \tag{1.3}$$

The $\omega = 2\pi f$ denotes the angular frequency of the matter wave. As for the probability of a matter wave being detected by an experimental physicist in the lab with specific time it would be proportional to $|\psi|^2$ as this is real and positive value, which is also known as probable density. When an experimental physicist is sure about finding particle with absolute way at a point in space is denoted with wave function $|\psi|$ as this is a complex quantity. Using the potential energy $U(x)$ of moving particle in x-direction and its corresponding total mechanical energy E. Applying the concept of De Broglie's matter wave theory the Schrödinger wave equation was derived as

$$-\frac{\hbar}{2m}[\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}] + U(x, y, z)\Psi(x, y, z) = E\Psi(x, y, z) \tag{1.4}$$

This is non-relativistic, as m is mass term and it is time independent equation for three-Dimensional case. As for the time dependent in 3D case is derived as

$$-\frac{\hbar}{2m}[\nabla^2\Psi(r, t)] + U(r, t)\Psi(r, t) = i\hbar\frac{\partial}{\partial t}\Psi(r, t) \tag{1.5}$$

Rewriting equation 1.4.

$$\frac{\hbar}{2m}[\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}] + (E - U(x, y, z))\Psi(x, y, z) = 0 \tag{1.6}$$

The general solutions to this 3D equation can be written as

$$\psi_x = Ae^{ikx} + Be^{-ikx} \tag{1.7}$$

$$\psi_y = Ce^{ipy} + De^{-ipy} \tag{1.8}$$

$$\psi_z = E^{'}e^{iqz} + Fe^{-iqz} \tag{1.9}$$

$A, B, C, D, E^{'}, F$ are constants. These solutions are further used in my research. For the relativistic case as this is discussed in this section 2.3.

1.2.1 Particle in a box

Infinite potential wall or a particle in a box is a model in the quantum mechanics which distinguishes the particle in a box with classical example. As the wall itself is a barrier where particle cannot escape from it, this particle may never have zero energy it may either be in lower energy levels or higher energy levels. Consider a particle such as an electron trapped in box. As for the square box the energy of the electron in the box can be derived by Schrödinger's equation, using equation 1.4 and substituting it's corresponding solutions 1.7 1.8 1.9 based on the quantum numbers for which is

n_x, n_y, n_z and L_x, L_y, L_z are corresponding to the length of the sides of the box as it is for the three dimensional case. Schrödinger's equation shows us that we can write the energy of the electron as

$$E_{nx,ny,nz} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) \tag{1.10}$$

As illusterated in the 1.2.1 the matter-wave of an electron enters into the ground state energy level. Once the electron gets trapped in the box, the quantum numbers is n_x, n_y, n_z must fit in with corresponding lengths of L_x, L_y, L_z. This solution for energy E is further used in my research to calculate the energy assuming the particle to be a graviton. In for particle in a box we have condition for wave vector $k = \frac{n\pi}{L}$ are allowed values of k for volume V as $n = (1, 2, 3, 4....)$ as L is size of the box and n is a quantum number.

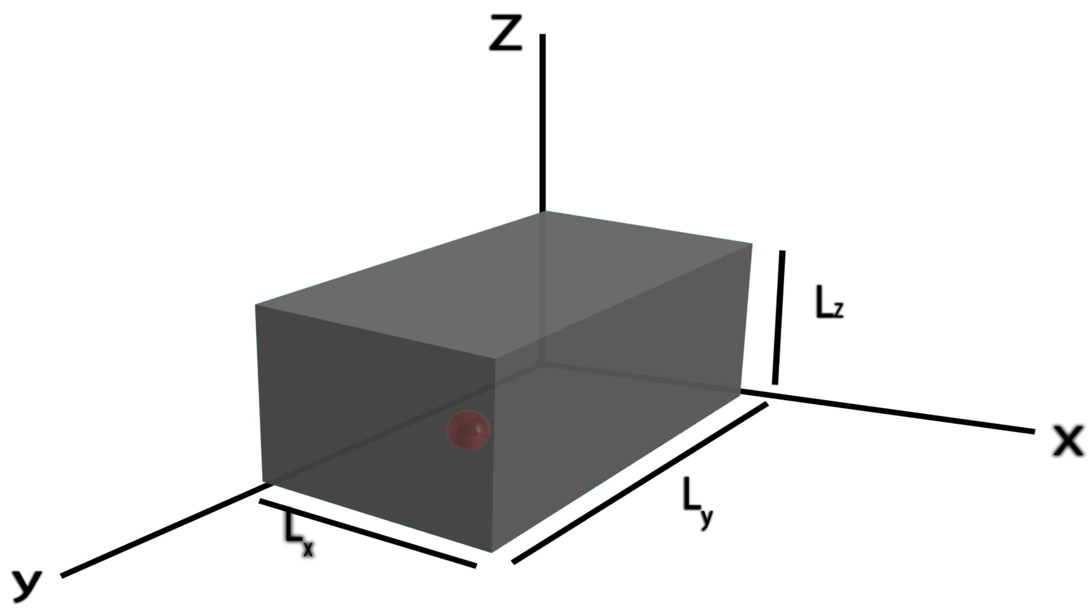

Figure 2: The 3D model of the box with particle such as electron inside the box, as the rectangular box with lengths L_x, L_y, L_z.

1.3 Energy levels and Degeneracy

When an electron or ion or a sub-atomic particle is in bound state or confined in a box or when an electron moves around the nucleus, this particle will have a discrete values of energy as this is known as energy levels. An electron or particle exists in the lowest possible energy level is called the ground state. The electron can also have high energy state which is excited state. We may have situation to that two particles or more may have same energy as these energy levels are said to be degenerate or degeneracy also known as degenerate energy levels. If we have quantum numbers or wave function of $n_x = 1$ and $n_y = 2$ also with quantum numbers or wave function $n_x = 2$ and $n_y = 1$ both are in same energy level 5 as the two degenerate wave functions have same energy level as we have two degeneracies.

n_x	n_y	$E\left(\frac{h^2}{8mL^2}\right)$	Degeneracy
1	1	2	1
1	2	5	2
2	1	5	
2	2	8	1
3	1	10	2
1	3	10	
4	1	17	2
1	4	17	

Table 1: The table is showing degeneracy with the energy levels of electron confined in a square box with corresponding quantum numbers n_x, n_y.

1.4 Heisenberg's Uncertainty Principle

When we try to make a measurement of a quantum particle it is impossible to have information about the particle for its corresponding position and momentum at the same time. This was proposed by German physicist Werner Heisenberg in 1927, it states that the one cannot measure the momentum of particle and its position more both precisely at the same time. The Δx measures the position with uncertainty and Δp measures the uncertainty in momentum always greater than or equal to $\frac{\hbar}{2}$.

$$\Delta x \, \Delta p \geq \frac{\hbar}{2} \tag{1.11}$$

2 REVIEW OF THE QUANTUM FIELD THEORY

Quantum field theory is one of the leading branches in theoretical physics which is a theoretical model for understanding the quantum mechanical models of subatomic particles in particle physics also often related to quasi particles 5.1 in condensed matter physics. The quantization is a process of describing the quantum mechanics which is from a classical understanding of the physical phenomena. Using classical field theory the quantum field theory is constructed. Quantisation of the field refers to the describing the field in terms of quantum mechanics. For example, we have electromagnetic field which can be described by Maxwell equations. For the fields in quantum field theory can be in quantum superposition of states, as quantization for this field would be the field of quanta which is the photon. However, photons may be both particle and wave nature following the fundamental rules of quantum mechanics. That is mostly used in condensed matter physics and many other fields such as nuclear physics research. The quantum field theory is well known for being difficult theory in modern physics. In this section I am going to discuss the topics which are related to my research.

2.1 Coupling constant

The coupling constant [47] it is a dimensionless number that represents the force exerted in an interaction. The role of coupling constant is that it determines the strength of the interaction with respect to corresponding kinetic energy. For instance electric charge could be coupling constant. This constant number represents how strong or weak the interaction could be Newton's gravitational constant is G_N this can be seen both Newton's laws and Einstein's general theory of relativity. This is coupling constant for gravitational interactions. The interaction results in the motion of particles as a consequence the strength of interaction depends on particle's coupling constant. The dynamics of particles are being decided by their coupling constant. The coupling constants are reintroduced in my calculation in my research for interaction between bosons using GrossPitaevskii equation 5.4.

2.2 Ultrarelativistic Particle

When a particle approaches close to the speed of light 3x10$^8 m/s = c$ the particle is called ultra-relativistic. The relationship between mass energy for the rest mass m and p for the momentum is given by the equation

$$E^2 = m^2c^4 + p^2c^2 \tag{2.1}$$

As for the relativistic particle which is close to the speed of light, most of the energy of the particle is due to its momentum p . The $pc >> mc^2$ as pc is very much greater than mc^2 the approximation is normally taken as $E^2 = 0 + p^2c^2$, which is $E^2 = 0 + p^2c^2$ this equation becomes $E = pc$. Here in this case it is derived for the massless particle such as photon. The system of photons may have zero invariant mass as it is referred as massless and it is zero rest mass.

2.3 Klein-Gordon Equation

The Klein Gordon equation is the relativistic version of the Schrödinger equation and follows Einstein's special relativity. The solution of Klein Gordon equation involves the quantum field, the field quanta are spin-less particles as the scalar field represent zero spin particles. The any solution to the Dirac equation is also a solution to the Klein-Gordon equation as the converse is not the case. Klein Gordon equation for a free particle is represented as

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi = \frac{m^2c^2}{\hbar^2}\psi \tag{2.2}$$

- ψ- is wave function
- $\hbar$ - reduced Planck constant
- c - Speed of light
- m - mass of a free particle
- ∇^2 -Laplace operator

3 REVIEW OF THE THERMODYNAMICS

Thermodynamics is the branch of physics which relates the energy and work with heat and temperature as it defines it as macroscopic variables such as understanding the internal energy, pressure and natural disorder of the system. Generally there are four laws of thermodynamics that is zeroth law, first law , second law , third law of thermodynamics. These laws explain the behaviour of molecules or gas in large scale. These laws can be further investigated in statistical mechanics. These well understood principles and set of laws are applied in science and engineering. In applications such as to improve the efficiency and power output of an engines, this can be understood in the concept of Carnot engine [10], Which states that the most efficient heat engine is the one that can operate reversibly.

3.1 Laws of thermodynamics

3.1.1 Zeroth law of thermodynamics

The Zeroth law of thermodynamics states that two bodies with thermodynamic systems are said to be in thermal equilibrium with the third then all the three bodies are said to be in thermal equilibrium with each other.

3.1.2 First law of thermodynamics

The first law of thermodynamics states the relationship between work and heat. The change in internal energy of the closed system is equal to difference between the amount of heat supplied to the system and work done by the system on its surroundings. This defines why we do not have perpetual motions in machines is impossible because of the law of conservation of the energy. The energy neither can be created nor can be destroyed. The equation for first law is generally represented as

$$dE = \delta Q - \delta W \tag{3.1}$$

$$\mathrm{d}U = \delta Q - P\,\mathrm{d}V \tag{3.2}$$

- δQ denotes the increment of heat supplied to system from its surroundings.
- P pressure, V volume
- W work done by the system

3.1.3 Second law of thermodynamics

The entropy of an isolated system never decreases, thus system normally evolves towards thermodynamic equilibrium this conflicts with idea of perpetual motion this law makes it impossible. Entropy of an isolated system can never decrease. The meaning of entropy as increase in disorder sometimes physicists relate it to the flow of “ time” as increase in disorder from the past to the future as increase in entropy [1] [13]. The second law of thermodynamics can be represented by the

[1] which I do not want to agree with it completely

equation of entropy.

$$dS = \frac{\delta Q}{T} \tag{3.3}$$

The small change in entropy or increase in entropy (dS) of the system is equal to the heat supplied to closed system (δQ) from its surroundings to inverse of the temperature of the system.

3.1.4 Third law of thermodynamics

The third law of thermodynamics states that at absolute zero temperature the entropy of the perfect crystal is zero, it states that the atoms remain motionless without having any kinetic energy. It was also stated as in finite number of operations no matter how idealized the conditions may be it is always impossible for any system to have absolute zero entropy.

3.2 Entropy

The thermodynamics defines entropy as the measurable number of specific ways the thermodynamic system can be arranged as this it also known as measure of the disorder. The second law states that entropy of an isolated system never decreases thus the system always evolves to thermal equilibrium. The third law states that it is impossible for any finite operations to have any system at absolute zero temperature in order to have zero entropy.

3.3 Violation of laws of thermodynamics

3.3.1 Maxwell Demon

Maxwell Demon is the thought experiment, which was imagined by James Clerk Maxwell. This is to describe the way to violate the second law of thermodynamics. Maxwell imagined a container which is divided into two sections or parts named Box A and Box B. Now, both the parts are said to be filled with some gas at same temperature which is placed side by side separated by the wall. The hypothetical demon which is placed between the separating wall guards and observes the gas molecules. When demon notices a gas molecule moving faster than the average molecule from A side the demon opens the door temporarily or for short period of time for molecule with high kinetic energy to pass through between A Box and B Box will have faster moving molecule and average moving molecules as illustrated in the images. Then the demon closes the doors instantly, this gives the partitions for molecules with high average velocity separated with molecules to low velocity in B Box.Since the Low average velocity of molecules are in B Box, when high velocity molecules moved towards A Box the system changes the average molecules speed in B Box and A Box. This average speed actually corresponds to the temperature. In the end the temperature increases in A Box containing all the particles which are denoted in red particle with vector notation for high kinetic energy and temperature drops in B, thus creating the perfect order. This is against the second law of thermodynamics. Which states that entropy of an isolated system never decreases, this is to eliminate the increase in disorder in the system by making it in a controlled order. The problem with this thought experiment is for real life demon to exist, it has to do with the information that demon needs to collect about the motion of particles as there could be millions of molecules moving in the container. How would demon have the information about position and movement of all the molecules? This act of processing the information would require huge amount of energy that needs to be performed. Perhaps supercomputers may have possibility of having measurement it may also have in accuracy since the system is microscopic. This involves the complications to isolate even for supercomputers. The demon needs to erase the information at some point in order to process new informations about moving molecules. This is irreversible process that increases the entropy of the universe. The erase of information by finite process of demons memory can increase entropy of the system.

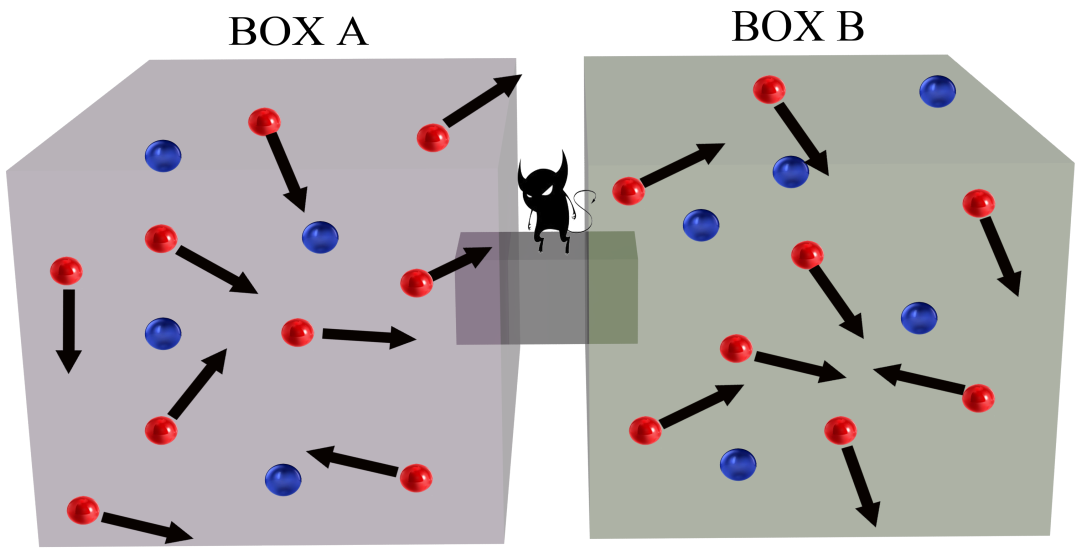

Figure 3: The Maxwell's Demon, as illustrated in my image that two containers with A and B are placed side by side with the barrier. A and B are in thermal equilibrium with each other and have same temperature. Molecules with red are with high kinetic energy (denoted with velocity vectors) and molecules with blue are low in average velocity.

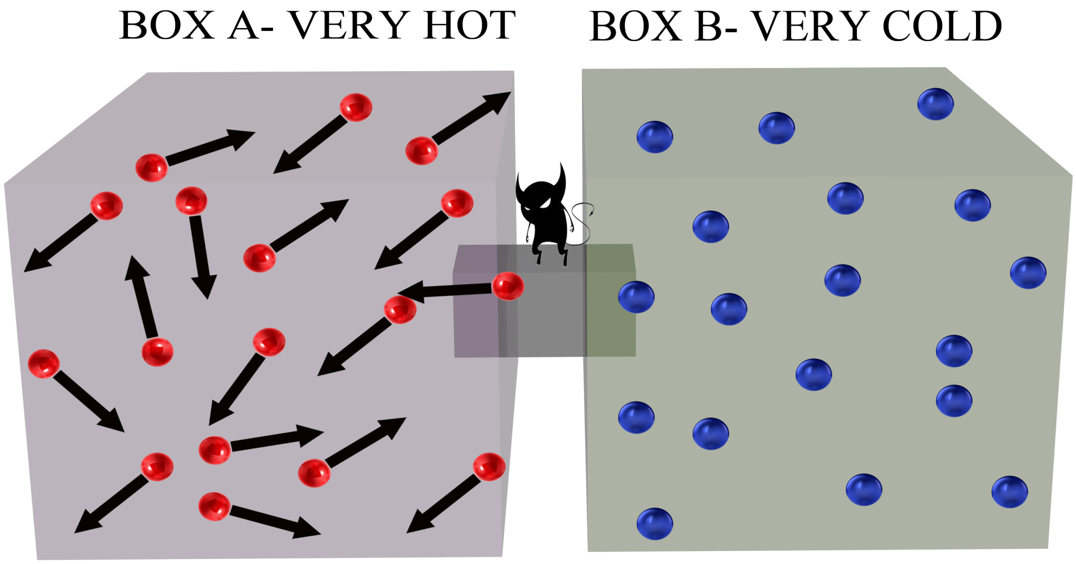

Figure 4: The Maxwell's Demon, as illustrated in my image that two containers with very hot Box A and B being very cold are placed side by side with the barrier. A and B are no longer in thermal equilibrium with each other and have different temperature. Molecules with red are with high kinetic energy (denoted with velocity vectors) enclosed in Box A creating the perfect order and molecules with blue are low in average velocity enclosed in Box B. This separation is caused by the ability of Maxwell's demon to move the particle through the barrier creating the perfect order, Entropy of isolated system decreases violating the second law of thermodynamics. There is relationship with The information plays key role, Demon uses information to create the order in a box.

3.4 Absolute zero

The absolute zero is when we have temperature T=0 Kelvin that is to say $-273°$C. This is to say that entropy of the system is zero. It is the lowest temperature that exists, in the quantum mechanics we have the description of the matter at absolute zero is said to be in lowest possible energy level which is in the ground state. If there exists such system at absolute zero such system can still have quantum mechanical free energy known as zero-point energy. The experimental physicists have reached very close to absolute zero temperatures such as close to $100pK$ that is 1.0 x 10^{-10} K. At this temperature one can find the quantum effects such as superfluity and superconductivity.

3.4.1 Lowest temperature in the universe

Due to the statements from laws of thermodynamics it is impossible to reach absolute zero, even if it is possible hypothetically in thought experiment, the laws of thermodynamics may be needed to be questioned. The laws of thermodynamics play fundamental role in the physics. These laws cannot be questioned, it is impossible to have absolute zero.

In an empty space as space not really empty due to the fact that quantum activity such as creation and annihilation or quantum mechanical activity taking place all over the universe. The lowest temperature ever measured in the universe is nearly 3 Kelvin, The temperature left over by cosmic microwave background radiation in space would be around 2.73 Kelvin. It is hard to imagine where we could look for absolute zero temperature.

NASA's new project called Cold Atom Lab (CAL) funded project which is scheduled to launch to the international space station in the year 2016 [52]. This mission is meant to achieve even coldest temperature below 100pk.

3.5 Zero point energy and Vacuum Energy

The Zero-point energy or a quantum vacuum energy is the lowest possible energy that one can have in ground state. The entire quantum mechanical systems have their own respective associated zero point energy which is lowest energy level. At ground state level we have small motion in quantum particles, as it is stated by Heisenbergs uncertainty principle which requires the physical system must have minimum energy. [7]

[15] Vacuum energy is the set of all fields in space, the fields such as electromagnetic field, gauge fields , fermion fields or higgs fields. This is the energy from the vacuum of space quantum mechanically it can be described in terms of quantum field theory. The quantum field theory defines space as set of fields not as empty space. Here zero point field can be understand as lowest energy state of a field.

[24] The quantum activity in empty space can be understood by the effect called Casimir effect arising from quantized field. When two metallic plates are brought closer together separated apart around the length close to nanometres. Due to quantum fluctuations in the background the plates are moved towards each other and pushed together. Thus the space itself is not completely empty, due to background activity it can force objects to move. The net force generated by the virtual particles in the background push the plates. This can be understood by the concept of zero point energy of the quantized field.

4 REVIEW OF THE STATISTICAL MECHANICS

In order to understand the state of a system of particle or mechanical behaviour of the average particles one can predict the probabilistic nature of the particles the study of state of system using mathematics of probability theory is known as Statistical Mechanics. It is the branch of theoretical physics that deals with how particles behave in mechanical system using probabilistic predictions for uncertain system. The statistical mechanics used in understanding of thermodynamic behaviour of such as large system. The concepts of heat, entropy as disorder in the system and temperature are not provided in the perspective of microscopic mechanical laws. However, statistical mechanics shows and provides the origin of heat, entropy and temperature from uncertainty of a system. The statistical mechanics method actually combines the thermodynamic quantities for such system of microscopic behaviour. Statistical mechanics is used to solve the problems in real system such as many particles. For many particles system can be represented by the wave function Ψ.

$$\psi_{total} = \text{linear combination of } \psi_a(1)\psi_b(2)\psi_c(3).... \tag{4.1}$$

ψ_a particle in a state "a" its corresponding energy would be E_a For particle energy in a box is defined by equation derived from Schrödinger's equation that is 1.10.

In a gas, such as ideal gas for non-interacting system gas molecules or particles do not interact. when this is the case the particles energy levels are not affected by neighbouring particles. As each particle has its own energy. In order to describe the system for the particles distributed over different energy levels statistical mechanics is used. For temperature $T > 0$ system would have the total amount of energy as E. In order to understand how E is distributed among all the particles. The application of statistical mechanics would give us the understanding of energy distribution. The thermodynamics gives the description of heat and direction of heat flow. In terms of statistical mechanics gives a good microscopic view of heat as structure of the matter as it gives the thermal properties of matter such as heat capacity.

4.1 Macrostates and Microstates

[26] The Microstate can be understood as a state of the system where all the parameters of particles or its constituents are specified. In a system such as many microstates exists for each state specified in macroscopic variables. The parameters such as Energy, number of particles N as there could be many parameters for a given state. This can be viewed from the two ways one is classical perspective that is the positions (x, y, z) of particles and momentum P_x, P_y, P_z this results in 6N degrees of freedom. However, In quantum mechanical perspective the energy levels and the states of the particles in terms of quantum numbers are normally used for parameters of a microstate. The macrostate of the system deals with the specific properties such as pressure (P), temperature (T). It describes the state of the system where the distribution of particles over the energy levels is actually spaced. In macrostate as it deals with the different energy levels and number of particles of the system. Macrostate can be described as the probabilistic distribution of possible micro states. The probability distribution of all possible states, across the statistical ensemble of all the microstates is one of the characteristic features of macrostate. Depending up on how given system is defined the distribution laws and ensemble approaches are applied.

There are three types of Ensemble approaches

- **Micro canonical Ensemble**: this is for isolated system with fixed energy E, The ensemble is when total energy of the system and its corresponding number is fixed. The requirement for members of ensemble is basically required to have the number of particles and total energy. The system is an isolated system which cannot exchange energy or particle number from its surroundings for system to be in statistical equilibrium.
- **Canonical Ensemble**: The canonical ensemble is statistical ensemble energy of system is unknown however the number of particles is actually fixed. To replace the unknown factor of energy the temperature is specified. The statistical canonical ensemble approach is valid for the closed system which has weak thermal contact. The system must be closed due to the conditions required for statistical equilibrium.
- **Grand canonical ensemble:** In grand canonical ensemble nothing is fixed, neither energy nor the particle number. Here under these circumstances replacing unknown term such as energy and number of particles, the chemical potential (μ) and temperature (T) can be specified. The grand canonical ensemble is more applicable for describing the open system, as the partial contact with the thermal contact or a chemical potential or electric field contact etc. As for the ensemble for the system of statistical equilibrium in the case of system in weak contact with other surrounding system this can be described by the ensembles with the same chemical potential of the system with same temperature.

The disorder in the system can be denoted by Ω which is microstates available to macrostate. This is mostly related to entropy as it is nature of disorder, at equilibrium disorder number is maximum. The distinguishability of particles is often questioned, so in order to describe particles which are indistinguishable every microstate becomes a macrostate. Particles can be distinguished which are fixed and localised in space. For the isolated system the volume V, energy E and the total number of particles N are fixed as this helps to distinguish the particles. For the non-localised particles or while they are involved in interactions or collisions the system of small particles or identical particles become indistinguishable. Since this involves collisions among particles thus making it non-localised particles. Some of the notations used in statistical mechanics

➔ Ω micro-states the disorder number which is related to entropy

➔ Each level j has energy ε_j,

➔ degeneracy g_j

➔ occupation n_j

➔ k = 1.3806488x10 $^{-23} JK^{-1}$ is the Boltzmann constant

➔ T is absolute temperature.

➔ i or j states representations

➔ N is the number of particles

➔ $< N_j >$ is the number of particles in i state represents the states with energy ε_j

4.2 Maxwell-Boltzmann statistics

In statistical mechanics in order to describe the ideal gas system of particles that are distinguishable, which move randomly and freely in non-interacting over different energy states in thermal equilibrium, quantum effects are neglected when we consider a system of particle density is low or when temperature is high. For different energy states while the system is in thermal equilibrium, the number of particles with energy in a state i, ϵ_i . The Maxwell-Blotzmann statistics is defined for $\langle N_i \rangle$. The anticipated number of particles with energy ε_i for Maxwell - Boltzmann statistics is $\langle N_i \rangle$

$$\langle N_i \rangle = \frac{g_i}{e^{\frac{\epsilon_\nu - \mu}{\kappa T}}} \tag{4.2}$$

- k = 1.3806488x10 $^{-23} JK^{-1}$ is the Boltzmann constant
- g_j degeneracy
- T- Temperature
- i- occupational state of single particle
- μ - Chemical potential

4.3 Bose-Einstein statistics

The Bose-Einstein statistics was actually introduced for Photons during 1924 by Bose and then the concept of Bose statistics was adapted to atoms by Einstein during 1924 to 1925. This statistics only applies to Bosons; the bosons do not follow Paulis exclusion principal. At very low temperatures the characteristics of bosons are different then fermions. This is to say that Bosons act differently from fermions in behaviour. The unique property of Bosons is that the N or unlimited number of bosons can occupy the same energy level this unusual property of bosons give rise to new state of matter which forms condensate close to absolute zero temperature. The newly formed condensate or new state of matter known as Bose Einstein condensate. The expected number of particles for i energy state ϵ_i is given as

$$n_i(\epsilon_i) = \frac{g_i}{e^{\frac{\epsilon_i - \mu}{\kappa T}} - 1} \tag{4.3}$$

- k = 1.3806488x10 $^{-23} JK^{-1}$ is the Boltzmann constant
- g_j degeneracy
- T- Temperature
- i- occupational state of single particle
- μ - Chemical potential

4.4 Fermi-Dirac statistics

Fermi Dirac statistics is normally used to describe the distribution of particles in such a system of particles which obey Pauli's exclusion principle. Fermi-Dirac statistics can be applied for the fermions with half integer spin particles under thermodynamic equilibrium. The particles in the system can have extremely small mutual interaction which can be neglected. Since the interactions are extremely weak this can be described in terms of single particle energy state. The Pauli's exclusions principle states that no two electrons can have same state thus this affects the system's property.

To the system of identical fermions the average number of fermions in a single particle state i is given by Fermi-Dirac Distribution.

$$n_i(\epsilon_i) = \frac{g_i}{e^{\frac{\epsilon_i - \mu}{\kappa T}} + 1} \tag{4.4}$$

- k = 1.3806488x10 $^{-23} JK^{-1}$ is the Boltzmann constant
- g_j degeneracy
- T- Temperature
- i- occupational state of single particle
- μ - Chemical potential

This statistics was first developed by Fermi and Dirac in 1926. Fermi-Dirac Statistics was actually applied to understand the collapse of white dwarf state a star In order to understand the dense matter problems in astrophysics by Fowler in 1926 which will be further discussed in this chapter6 6.10.5.

4.5 Bose-Einstein Condensate

The Bose Einstein condensate is a condition for state of matter for bosons with integer spin which is not constrained by Pauli's exclusion principle, which states that no two fermions can have four quantum numbers same. It is the state of matter gained by cooling the bosons close to absolute zero. At close to absolute zero ($T = -273.15\,°\mathrm{C}$, 0 K), when cooled close to the absolute zero all the bosons enter into ground state level or lowest quantum state, at this point the quantum effects are seen more like macroscopic scale, this is also known as macroscopic scale at which the quantum mechanical behaviour is seen in macroscopic scale. Most importantly all the quantum mechanical behaviour actually happens in the scale of microscopic realm of quantum world at Planck scale l_p. When we make an observation of condensate such as Bose Einstein Condensate the matter that is seen is actually in macroscopic scale. Some of the examples of macroscopic quantum phenomenon can be super fluidity and superconductivity. These macroscopic natures are experimentally can be observed at very low temperature. This concept was first proposed by both physicists Satyendra Nath Bose and Albert Einstein in 1924-25.

The concept of the Bose Einstein condensate actually first instigated by Indian physicist Satyendra Nath Bose. It was started with the concept of quantum statistics [49]. When a system has thermodynamically established equilibrium, in any thermodynamic equilibrium state there is no flow of matter or energy with outside system. For an Isolated system of when there is no change in macroscopic property such as entropy, energy, temperature of the system, such system is said to be in equilibrium thermodynamically.

The description of particles in same state which only follows the Bose-Einstein statistics such as a streaming photons from laser light and superfluid helium with frictionless behaviour. The distribution or collection of such particles such as bosons with integer spin can be distributed. Bose and Einstein both collaboration produced the concept of Bose-Einstein condensate. The Bose Einstein condensate applies to the particles which does not obey Pauli Exclusion Principle, were actually named Bosons for particles with integer spin. For such particles there is not much required amount of the interaction between the particles. The statistical description of the quanta or bosons can be understood as the phase transition which is associated with the condensate of the atoms at ground state or lowest energy state of atoms. Due to the lack of technology the Bose-Einstein condensate remained a theoretical work and predications of new state of matter for decades.

Until, the combinations of the different cooling techniques were successfully developed after several years of the prediction of Bose Einstein condensate. Finally it got tested by MIT and University of Colorado at Boulder NIST-JILA lab, physicists Cornell, Wieman and Ketterle this own them the Noble prize in physics for the achievement of Bose Einstein Condensation in dilute gases of alkali atoms it is mostly a fundamental study of the properties of the condensate.[48] The alkali atoms which are mostly boson composite atoms such as rubidium atoms were cooled extremely close to absolute zero which is at $T = 170$ nanokelvin (nK). Later on, this was tested on different elements such as So far, the atoms ^1H,^7Li,^{23}Na,^{39}K,^{41}K,^{52}Cr,^{85}Rb,^{87}Rb,^{133}Cs and ^{170}Yb as for the ^4He it normally be the helium atom in an excited state have been demonstrated to undergo BoseEinstein condensation.

For non-interacting bosons, the Bose distribution function is

$$f^o(\epsilon_\nu) = \frac{1}{e^{\frac{\epsilon_\nu - \mu}{\kappa T}} - 1} \tag{4.5}$$

- T- Temperature
- ν- occupational state of single particle
- μ - Chemical potential

[26]For N bosons under Bose Einstein Condensate, the equation for BEC at absolute zero requires the condition $T < T_B$. As T_B is Bose-Einstein temperature, The Bose-Einstein gas: excited and ground states population is given by

$$n_0 = N[1 - \left(\frac{T}{T_B}\right)^{\frac{3}{2}}] \tag{4.6}$$

population of particles N_0 in the ground state, At $T = 0$ for absolute zero temperature , we have $n_0 = N$ and all the particles are in the ground state. Bogoliubov theory is used for weakly interacting Bose gas this will be discussed in the section 5.4 which is required discussion for my thesis.

5 REVIEW OF CONDENSED MATTER PHYSICS

The condensed matter physics [2] is the branch of physics that deals with the condensed matter states of matter. Condensed matter states are different phases of matter achieved under suitable controlled temperatures naturally or in laboratory conditions. Physicists who study this field investigate on the behaviour of phases by sing laws of physics, laws mainly from field of study of on quantum mechanics and statistical mechanics. Phase change of matter commonly seen in solids and liquids. In right conditions under suitable temperatures the new phase of matter can be seen as superconductivity. When cooled to very low temperature matter such as composite Bose gas, The composite boson particles such as atoms, nuclei or hadrons can be bosons or fermions. The particle containing the even number of fermions could become boson due to the nature of the spin being integer. For example carbon 12 atoms contains 6 protons and 6 neutrons or another example would be helium 4 which consists of 2 protons 2 neutrons and 2 electrons. Here one can apply Bose-Einstein statistical for Bose gas and can see the Bose Einstein condensate of matter, as the condensed matter is collection of bosons in ground state level. The concept of superconductivity is used when in specific materials when they are cooled down to a characteristic critical temperature as a consequence the zero electrical resistance and magnetic fields occurring can be seen. This was discovered by Dutch physicist Heike Kamerlingh Onnesin 1911. Later on the different characteristics were observed on superconducting materials such as Meissner effect as it is ejection of magnetic field lines from superconducting material as it is a transition into a superconducting state. Superconductivity is divided into two types, the type-I superconductivity is when a superconductivity is destroyed in the under strong magnetic fields. In type-II superconductor one can find the magnetic vortices in applied magnetic field. The vortex density increases with increasing in the magnetic field, once it crosses the critical field the superconductivity is completely vanishes and cannot be seen. When the vortex density becomes too large the material becomes non-superconducting this is due to higher critical field. Super fluidity is a state of matter when a fluid matter has zero viscosity it behaves like self-propel to defy forces of gravity and its own surface tension. This was actually first seen in liquid helium. Later on it was also studied in quantum gravity, astrophysics. It was discovered theoretically that super fluidity exists inside neutron stars [34] as this is further discussed here 6.10.6. It is anticipated that the nucleons in the neutron stars at high density and low temperature can actually from cooper pair due to the nuclear force can lead to superconductivity and super fluidity.

5.1 Quasiparticle

Quasiparticles are an emergent phenomenon that exists inside solids. Consider a particle moving through the system and it interacts with neighbouring particles either it tries to pull or push these particles, as a result this forms an emergent phenomenon like cloud of particles moving across the medium. [37] The movement of these particles coexists with a real particle along with its cloud is nothing but a quasi-particle. A simple analogy can be considered describing this emergent phenomenon is we can think of quasi particle as a bubble in a carbonated water, as the bubble itself is not independently existing object. The volume of the water can push the bubbles as bubbles are raised to surface, this temporary existing phenomenon which is emergent phenomenon can be seen. This analogy is similar to the emergent phenomenon of quasi-particles. The cloud

[2] I am new to this field of physics, however for the thesis I would like to discuss some topics in condensed matter physics in order to relate it to my thesis.

of particles the actually surrounds the real particle as a result these particles interact weakly among themselves. This system of interaction creates many-body system, as it is an elementary excitation of the interacting system. Quasi particles can have properties like object such as shape, volume, momentum, particle's lifetime and energy, as these physical properties allows us for direct experimental observation. The effective mass of these quasiparticles are different from the mass of real particle. The actual particle is shielded by the quasi particles as a consequence weakly interactions can be seen. Quasi particles can be illustrated by examples.

5.1.1 Bogolons

As the nature of being part electrons and part holes, the linear combination of an electron in state (+k,$\uparrow$) and (-k,$\downarrow$) as this is representing hole, as this is the elementary excitations in a superconductor. The Bogoliubov transformations actually combines hole operator and electron operator with opposite spins, as quasi particles are like part electrons and holes. [57] These quasiparticles are known as Bogoliubov quasi particles or Bogolons. Bogolons are created and annihilated represented by the corresponding operators which are elementary excitations above the ground state of superconductors. Bogolons are not similar to cooper pairs, cooper pairs are not excitations, as they can be ground state of superconductor.

5.1.2 Quasi ion in an electrolyte liquid

Consider a electrolyte as it is a substance or a solution, this solution got the property of ionization, as Ionization is the process by which a molecule or an atom loses its electrons or gains an electron to become positively charged or negatively charged to form ions. As this could form a solution or substance in ionizing solvents such as water, salts, acids and bases. Some of the examples of ions of electrolytes are Sodium (Na^+), potassium (K^+), Chloride (Cl^-) and magnesium (Mg^{2+}). [37] Now let us consider the electrolyte solution with equal number of negative and positive ions moving and colliding with each other. In the figure the conditions are illustrated, as the positive ion in the system being introduced. The positive ion moves through the system where this ion experiences columbic interactions with the surrounding negative (-) ions. When this happens the (+) ion will form a cloud as the (-) ions stick around for short period of time due to collisions these (-) ions will be replaced by the other neighbouring (-) ions. The coat or shield of opposite charge is represented as blue shaded field in the diagram 5.1.2.

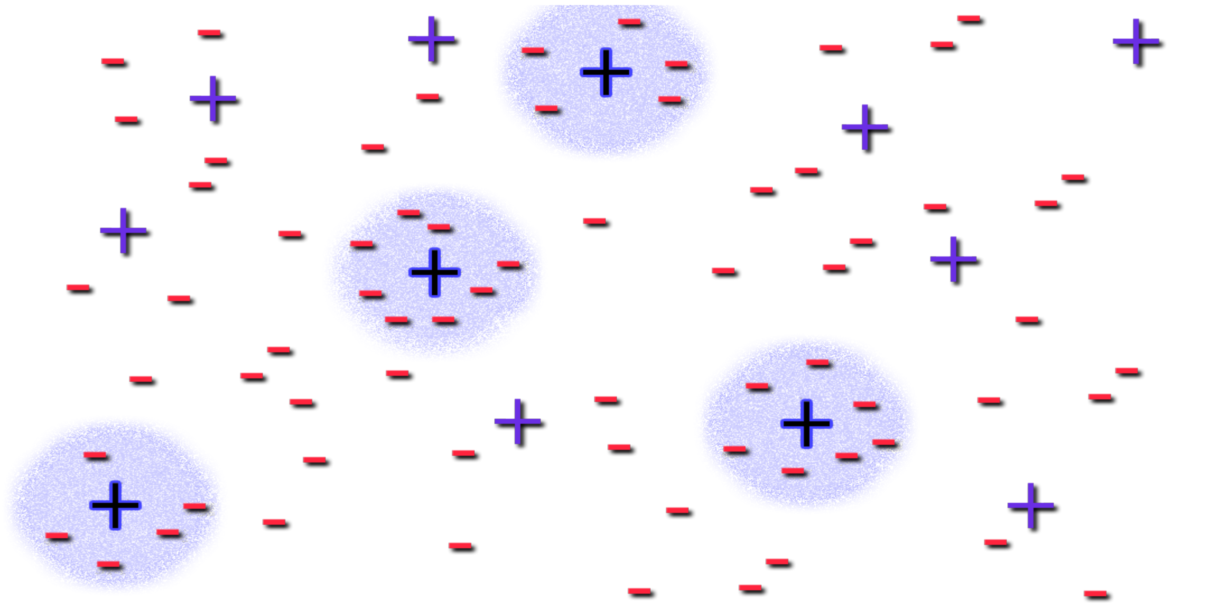

Figure 5: The group of positive and negative ions forming quasi particles as blue clouds representing the formation

The (-) Ion individuals may also have shield of (+) ions. This process of shielding or coating may result in weakly interaction with surrounding ions when compared to the unshielded or uncoated interactions. The quasi particles may be understood as the combination of the real particle along with its corresponding coat or shield of other particles. We also have a system of bare or clothed particle interactions which can be understood by quantum electrodynamics. Consider an electron which is not covered or surrounded by any other particles as this is a bare particle. When a bare electron interacts with photons this electron may be dressed or clothed by virtual photons. When the interaction involves in with the real particle is known as bare interaction, while quasi particles represent weakly interactions.

Considering the experimental observation of observing the quasiparticles, one can observe how the system is affected by sending an extra test particle through the same system. The behaviour of neighbouring particles can be observed as the test particles moves through the system and the properties of quasiparticles are predictable by the tiny interactions made.

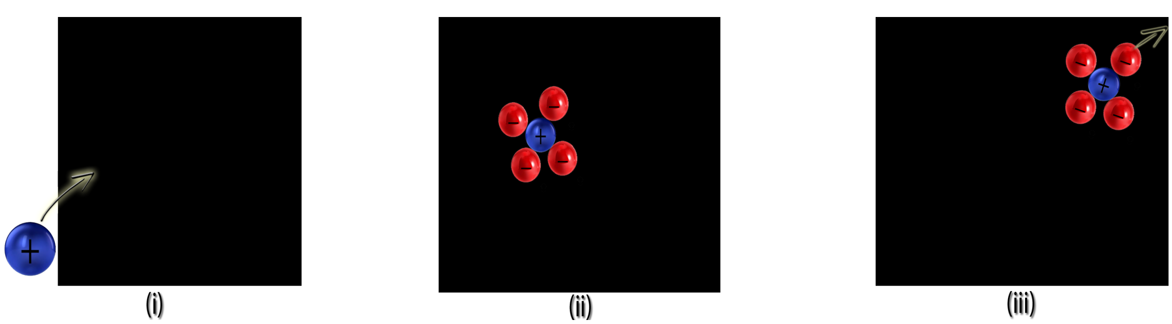

Figure 6: (i). Shows the introduction of (+) ion into the system, (ii). Shows as (+) ion gains cloud or shield of (-) ions making it as quasi particle, (iii). The quasi ion moves through the system

The figure 5.1.2 illustrates that the particle moves with in the system as this will result in momentum p for an average time τ_p. This can be understood as the life time of the quasi particle τ_p. In order for every quasi particle to exist there life time must be long enough in order to be identified in a system. Due to the fact that a quasi-particle surrounded by the shield of particles this may have effective mass as this difference the energy of total mass for this system the new energy law can be defined. The energy of a particle for corresponding momentum p

$$\epsilon = \frac{p^2}{2m} \tag{5.1}$$

The new energy law is defined as

$$\epsilon' = \frac{p^2}{2m^*} \tag{5.2}$$

The equation represents the effective mass of quasi particle m^*. The difference of energy in quasi particle by the bare particle gives self-energy of the quasi particle. The self-energy of the quasi particle is due to the many body interactions made by the particle with the system and interaction with itself this in variable energy of the particle.

5.1.3 Quasi particle as electron

Although the concept of quasi particles may arise due to the fact that it is many body interactions of the system. However it may also arise from an external particle moving through a system with potential, in the system such as an electron moving through metal. In microscopic view of metallic substance one can see the lattice structure of the positive ions. As the electron enters and moves through the lattice structure of (+) ions there exist a periodic force field due to the columbic interactions between the electron and the ions. In an event of electron entering into the impure metal the variation in periodic forces can be seen in fig (i)5.1.3, similar effect may also happen when an ion is displaced in lattice as seen in fig (ii) 5.1.3.

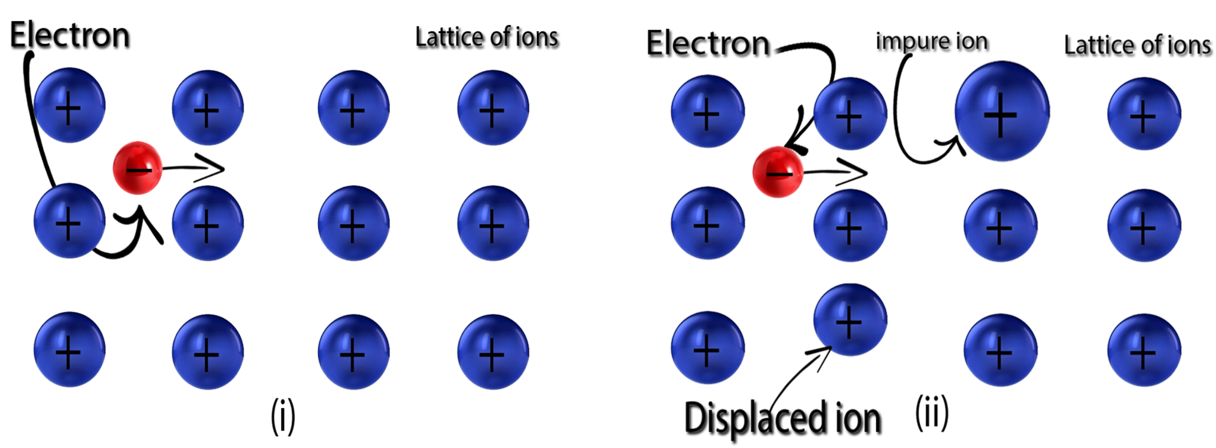

Figure 7: (i).Conduction of an electron through pure metal of (+) ions, (ii).Electron passes through impure metal

The lattice structure is a static structure, which means ions will remain in their respective positions. Since we have a static lattice structure, we cannot have moving cloud or shielded. However, it is possible to have a clothing or cloud of ions around the electron with effective mass m* and infinite lifetime due to imperfection in the structure of lattice.

5.2 Collective excitations

These many body system interactions can also be elementary excitations of the system [37]. This is due to the fact that quasi particle is in an excited energy level of the many body system. As quasi particles are emergent phenomenon there exist different types of fictitious particles in many body systems as this is known as collective excitations. These collective excitations may act very much wavelike motion of all particles in a system collectively at the same time, as these particles do not surround around the individual particles. The collective excitations can be understood in following examples of particles which act as collective excitations.

5.2.1 Phonons

As Phonons are collective excitations in elastic arrangement of molecules in condensed matter. It actually represents quantization of vibrations of in an elastic medium of interacting particles and mostly viewed as quantum mechanical excited state. Sometimes phonons are also referred as the quasi particles, as these particles act as modes of vibration this represents the excited state. Collective excitations can act like sound waves in solid crystal lattice in a solid, this can be quantized into the collective excitations which is known as phonons.

5.2.2 The collective excitations of plasmons

When we observe the microscopic structure of metal foil one can be the high energy elections filling up the foil. A plasma wave can be generated within this medium of electron gas in the foil the resultant sinusoidal wave as illustrated in the figure5.2.2. The sinusoidal quanta oscillations are called plasmons. Whenever there is a change in the neutral state of the medium of the system, such as disturbance in the metal the waves are propagate across the system [57]. The plasma wave has its own frequency ω_p and its corresponding wave length is λ_p. This wave has its high density regions and low density regions. The figure 5.2.2 shows the difference in density regions. This difference may create holes in low density regions and additional electrons in high density regions. These waves can be quantised into energy units as $E = \hbar\omega$ which is representing plasma waves. Furthermore, this can be $E = \hbar\omega_p$ which is for plasmons.

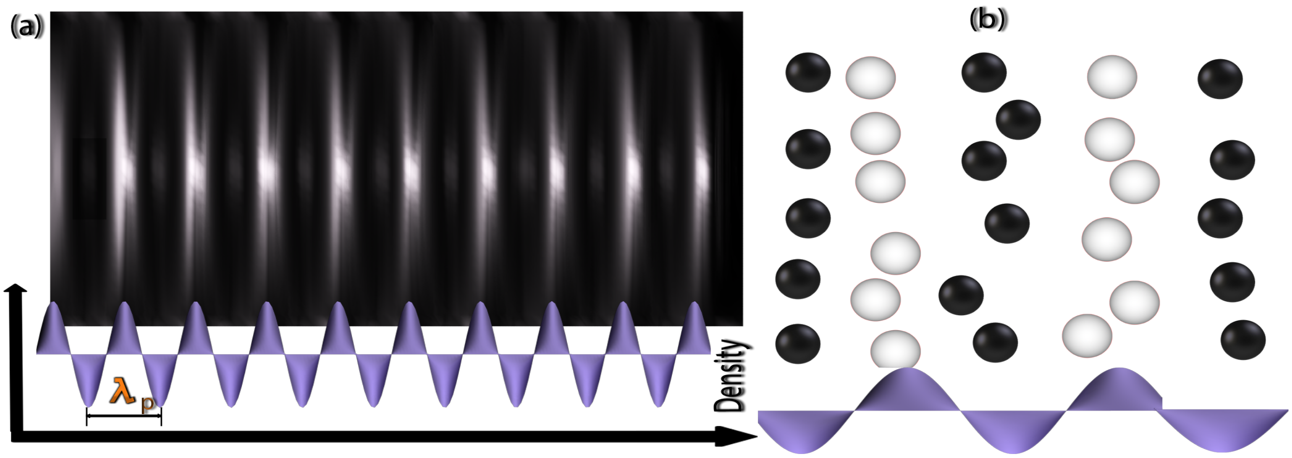

Figure 8: (a) Shows plasma wave in electron gas with high and low density regions, (b) Shows particle-hole picture of plasma wave as black represents holes and white as particle

5.3 Bogoliubov theory for weakly interacting Bose gas

In 1947 Nikolay Bogolyubov developed theories for microscopic theory of interacting models of Bose gas using the concept of Bose-Einstein condensate. The interaction between the particles affect the system in dramatic way [49]. In order to understand particles under interactions new method was needed to be developed and traditional techniques will not work for this kind of system. The new theory developed by Bogoliubov theory is actually based on the new perturbation technique thus provides modern approach to dilute gases of Bose-Einstein condensate.

5.4 Gross Pitaevskii equation

The Gross- Pitaevskii equation [48]describes the ground state quantum system of bosons which are identical under interactions in the system. Using Schrödinger's equation for a free quantum particle can be described. Considering the average spacing between the particles is greater than that of scattering length a_s as this is known as dilute limit. The Gross- Pitaevskii equation is derived for time dependent and time independence.

The time-independent GrossPitaevskii equation

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + V(\mathbf{r}) + \frac{4\pi\hbar^2 a_s}{m}|\psi(\mathbf{r})|^2\right)\psi(\mathbf{r}) = \mu\psi(\mathbf{r}), \tag{5.3}$$

This equation is the time-dependent Gross-Pitaevskii equation,

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + U_0|\Psi(\mathbf{r},t)|^2\right)\Psi(\mathbf{r},t). \tag{5.4}$$

In order to find the elementary excitations of Bose Einstein condensate, one can use Bogoliubov methods for the Gross - Pitaevskii equation. Considering the wavefuntions with small perturbations

$\partial\psi$. When there is no external potential, the dispersion law of interacting Bose-Einstein-condensed particles is given by Bogoliubov spectrum at $T = 0$:

$$\omega_p = \sqrt{\frac{p^2}{2m}\left(\frac{p^2}{2m} + 2U_0 n\right)} \tag{5.5}$$

- $U_0 = \dfrac{4\pi\hbar^2 a_s}{m}$ is coupling constant a_s is scattering length
- $n = \dfrac{N}{V}$ Density of gas, N number of particle by V it's corresponding volume
- $V(r)$ is potential
- $\boldsymbol{p}$ momentum of plane waves

Part II

BLACK HOLE LITERATURE REVIEW

6 REVIEW OF BLACK HOLES

The concept of Black hole[3] is one of the results from Einstein's General theory of relativity. In this chapter I would like to review the background topics of Black holes.

6.1 Newton's Classical view of 3D Space "and" Universal Time

In 1689 [45] Sir. Isaac Newton pictured space and time as absolute using the idea of bucket and the water analogy. In Newtonian mechanics the concept of absolute space and time is well known. The space itself should be regarded as absolute inertial reference frame where all the motions of an object's velocity, acceleration and momentum are relative to inertial reference frame which is empty space itself. The progress of time is absolute according to Newton; any clock in the universe for given point in space, would tick in the same way so there is universal time. The measurable duration of time itself is same every ware in an absolute space. The space itself is a stage "backdrop where all the action takes place under progress of time.Later on Galileo and other physicists made changes in our understanding of space.

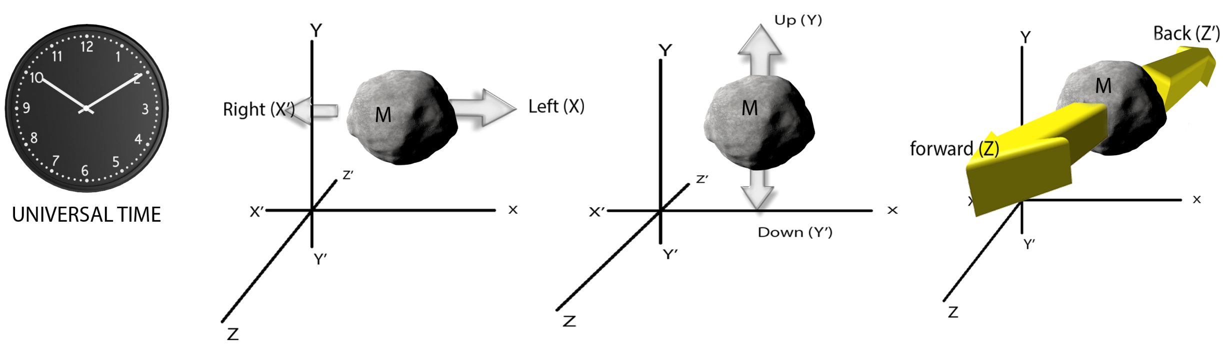

Figure 9: shows fundamental degrees of freedom for mass M in 3D Space and Universal Time

6.2 Newton's Gravity

In Newton's Principia, newton explained the universal law of gravity as, The gravitational force between two masses is directly proportional to the product of the mass and inversely proportional to the square of the distance separating the two masses.

$$F = G\frac{m_1 m_2}{r^2} \tag{6.1}$$

[3]Previously I have reviewed some of the topics of Black Hole and its origin in my literature review [32]. Here I would like to review and discuss additional topics which are more relevant to my further discussions. This is different from my first literature report.

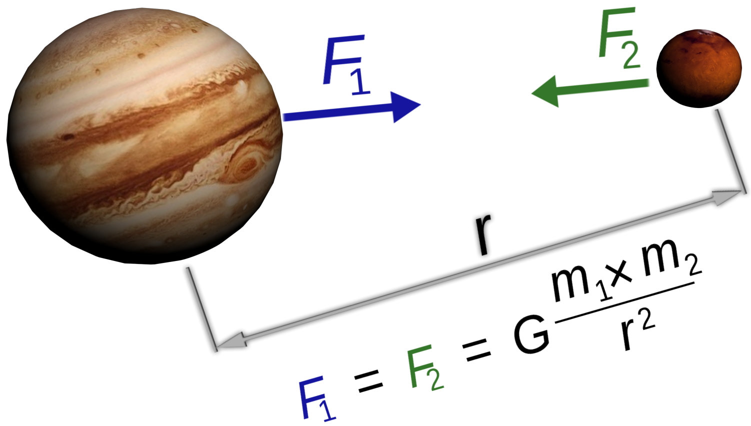

Figure 10: shows Newton's Universal law of gravity

6.3 Limitations of Newton's Universal gravity

There are some limitations to Newton's law of gravity. such as explaining how gravity actually works ? other one of the limitation as I pointed out for the systems of "isolated mass" in my report 1 literature review.

6.3.1 Universally Isolated mass system of M_1, one body problem

I would like to review my point which I noticed independently, here in this section again consider a isolated mass in space M_1 in empty space where there no other relative mass such as $M_2, M_3, M_4, M_5 ... M_N$ does not exist, this is my own thought experiment so I want my readers to imagine the universe were there is no relative mass other than M_1, In this lonely universe with only M_1 existing in space, now when I try to use Newton's universal law of gravity without any other relative mass such as M_2 in order for Newton's gravity to work within 2 body system. The force of attraction is directly proportional to product of two mass and inversely proportional to square of distance separating there masses. When I try to apply Newton's Universal law of gravity it fails to describe gravity for one body or isolated body problem. Using Newton's equation

$$F = G\frac{m_1 m_2}{r^2}$$

for $M_2 = 0$ as this is an isolated system in imagined universe.

$$F_g = G\frac{m_1(0)}{r^2} \tag{6.2}$$

$$F_g = 0 \tag{6.3}$$

$F_g = 0$ is the obvious solution to this, this tell us that Newton's gravity actually can not tell us how gravity actually works. However, it does work well describing the motion of planets. More additional limitations which are discussed here [32].

6.3.2 Mercury's perihelion shift

Urbain Le Verrier, French mathematician who discovered the planet Neptune, made several observations on planet motion of planet mercury in the early 1859. [33] He was first to report the slow precession of the planet Mercury's orbit around the Sun as this was completely unexpected. This could not be explained by Newtonian mechanics or law of gravity. New theory was needed in order to understand the this effect. Which is known as Perihelion precession of Mercury.

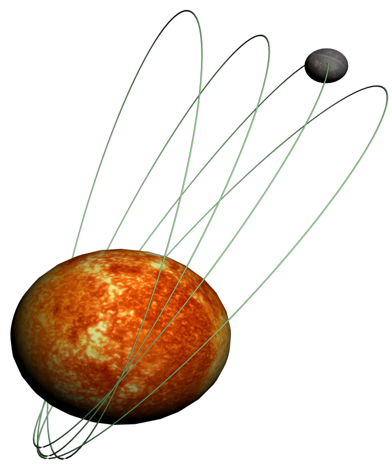

Figure 11: shows Perihelion precession of Mercury around the sun, the orbit size, planet and sun size are not to scale

6.3.3 Incompatibility of Newton's gravity with special theory of relativity

Newton's law of gravity does not explain under which frame of reference the gravitational force of attraction works such as non-inertial frame of references. It is also incompatible with special theory of relativity [23]. The following example can be considered in order to understand why Newton's picture of gravity fails to satisfy Einstein's special relativity. Consider our solar system where planets are elliptically revolving around the sun. It takes nearly 8 minutes for light to reach earth where sun is separated from the distance of 93 million miles. If sun if the sun ceases to exist instantaneously, according to Newton's picture of gravity the planets in the elliptical orbits and the earth should feel instantaneous effects of losing sun's gravitational attraction exceeding the speed of light as this violates postulates of special relativity. As consequence the planets should fly off from the elliptical orbits immediately after sun's ceases to exist as a result of change in mutual attraction of gravity. Special relativity states that nothing can travel faster than light that is 3x10$^8 m/s$ as this applies to any kind of communication signals or material objects. Newton's views of gravity are in direct conflict with Einstein's Special relativity. In Einstein's view the planet earth should feel the effect after 8 minutes with the same speed as light.

6.4 General Relativity

After publication of Special relativity Albert Einstein looked into the problems with Newton's laws of gravity associated with Mercury's perihelion shift and disagreement with special theory of relativity. [54] Einstein imagined in his though experiments that an accelerating rocket in space and objects experiencing the gravity on planet in stationary position has similar effects. That is to say force experienced locally by an observer on massive body such as mass of a planet has similarities with the accelerating observer in a rocket. This is known as principle of equivalence. By understanding how speed of light would get affected by this phenomenon, Einstein defined gravity in new way, as it nothing but a force which is consequence of geometry of space-time. Here space and time are not different aspects but the same thing. The reason for existence of gravity for any massive objects is due to the fact that it is nothing but distortion of space and time. Mass tells how space should bend and space tells how mass should move. Einstein's famous field equation is given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{C^4}T_{\mu\nu} \tag{6.4}$$

Einstein's field equation represents tensor notations of space time curvature from the left side of the equation represents the curvature of space: For the terms

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}$$

- $R_{\mu\nu}$ -Ricci Tensor
- R - Scalar curvature
- $g_{\mu\nu}$ - Metric tensor
- Λ- Cosmological constant

For the mass term

$$\frac{8\pi G}{C^4}T_{\mu\nu}$$

- $T_{\mu\nu}$ - Energy momentum tensor
- C - Velocity of light
- G - Newton's gravitational constant

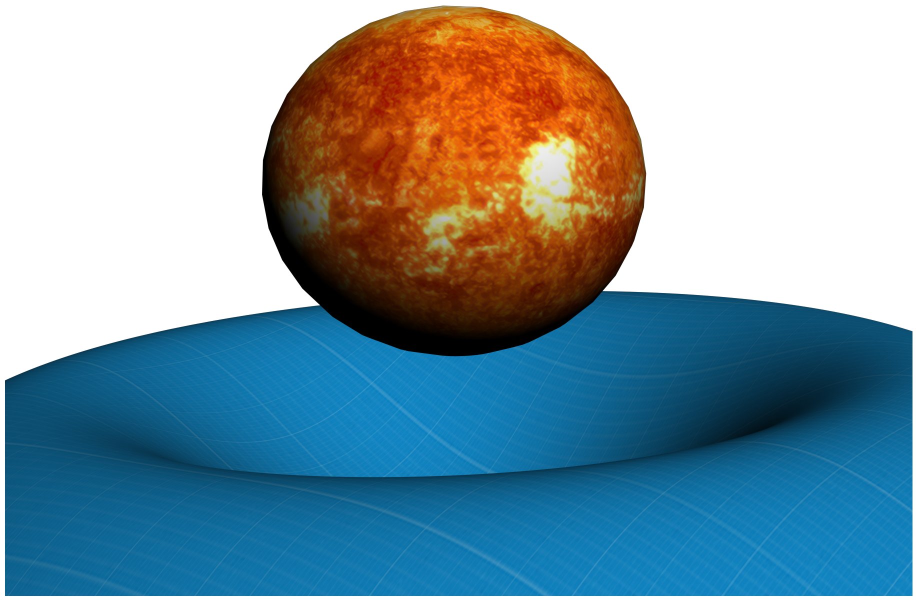

Figure 12: Illustrates the concept of general relativity, massive object such as sun can distort space-time geometry.

6.5 The gravitational effects of general relativity

Einstein's general theory of relativity predicts following effects when massive objects results in distortion of space time:

1. **The geodesic effect:** Einstein's general theory of relativity predicts that space-time around the any massive object such as mass of sun or planet earth can cause distortion. This curvature of space-time represents Geodesic effect.

2. **Frame dragging effect:** Another prediction by general relativity is as a rotating massive object can result in dragging space-time along with it. This effect is known as frame dragging effect. It is directly proportional to angular momentum of massive object.

3. **Gravitational time dilation:** In special relativity the time dilation actually caused by objects travelling with high speeds as they get close to speed of light, as photon in the light clock bouncing off between two mirrors needs to take longer path which demonstrates the concept of time dilation. General relativity predicts gravitational time dilation as another consequence of distorted space-time from gravitational potential of massive objects. The actual time elapsed between two events as measured by observer are differed. The observer notices clock tick slower close to gravitational source when compared to clocks ticking away from the massive objects such as events in empty space. Time actually ticks slower when measured close to massive objects when compared to clocks tick faster which are away from massive bodies.

4. **Gravitational Waves:** For a massive object when they spin or move through space this causes the ripples due to the curvature of space-time to form a wave outward from the source which can actually propagate through space . This is one of the predictions made by Einstein's general relativity. Gravitational waves are nothing but the transport of the

gravitational radiation. The actual source can be from the massive objects such as binary systems or white dwarfs or from black holes.It is important to note that in Newton's view of gravity 6.2 there is no such thing as gravitational waves.

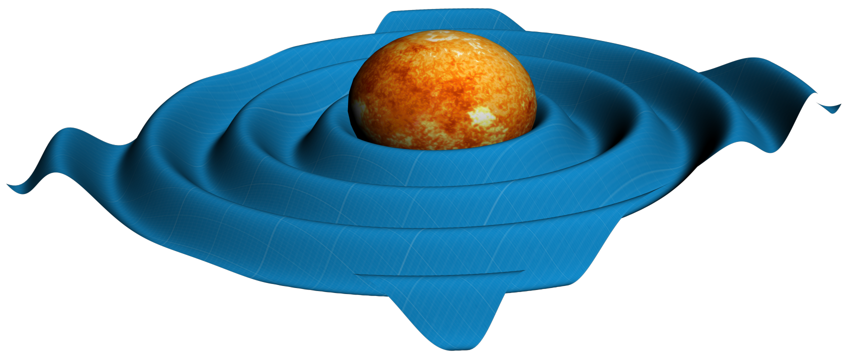

Figure 13: Illustrates the gravitational waves from the massive state

5. **Gravitational lensing:** The gravitational lens is caused by the distortion in space time geometry this is due to distribution of the matter such as a galaxies clusters between the sources of light such as background stars and the observer. This mass or huge matter distribution can gravitationally bend light and act as lens. The NASA's Hubble space telescope captured the actual naturally occurring lensing effects [44]. This is like a lens in space which sometimes magnifies the background image and sometimes shows us same source in multiple ways. The Einstein ring is another effect of gravitational lensing effect, Light from distant blue galaxy got bent and lensed by the Luminous Red Galaxies LRG 3-757 thus forming almost ring which is known as Einstein ring image named "The Cosmic Horseshoe" [9]. furthermore, Black holes can case the same effect but with intensive distortion.

6.6 Tests of general relativity and gravitational waves

Einstein's general relativity verified by NASA's gravity probe A [42] for gravitational time dilation and gravity probe B [43] for frame dragging effect and geodesic effect. Additionally, it was also proved during solar eclipse as sun distorts space-time the background stars were viewed which were actually hidden from the view of stars completely blocked by the sun in photography plate provided by Arthur Eddington. Nevertheless, General relativity correctly calculated the precession of the perihelion shift of planet Mercury, this completely agrees with observation also it unifying Newton's law of universal gravity with special relativity6.3.3. Using the concept of gravitational waves when sun vanishes from the center of solar system the gravitational waves would reach the planets obits after 8 minutes then the planets would fly off from its orbits after gravitational waves reach the planets, there is no such thing as gravitational waves in Newton's picture. Einstein's General relativity also solves the problems with isolated mass system 6.3 as when Isolated system in the universe, it would actually distort space and time around it thus explaining gravity for one body system problem.

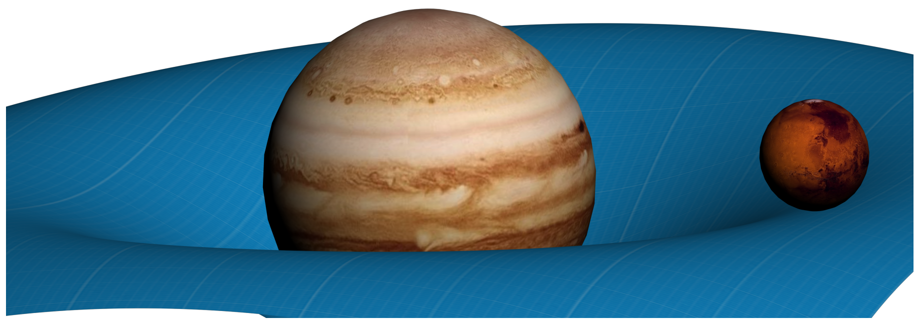

Figure 14: My image illustrates the concept of General relativity replacing the concept of the Newton's view of gravity ,consider an imaginary situation where the planet Mars orbits around planet Jupiter, the curvature of space itself determines the orbit of the planet Mars around Jupiter.

Moreover, The Gravitational radiation or gravitational waves not been observed directly yet. Hulse-Taylor binary pulsar system[2] gives the possibilities for studying gravitational waves via studying new type of pulsar binary system which is also known as Hulse-Taylor binary pulsar also known as PSR B1913+16 showed possibilities that Gravitational wave really exist.

In 17th March 2014, astronomers who are working at HarvardSmithsonian Center for Astrophysics made an announced claiming the discovery of primordial gravitational waves which were actually produced during big bang and the gravitational radiations left in as fossil record in cosmic microwave background radiations this would be strong evidence on inflation theory which happened after big bang [29]. The inflation theory which states that universe accelerated its expansion after the big bang. Using BICEP2 telescope in Antarctica were there were able to make some measurements. The peer review went on these claims. However, In around 19th September 2014, while comparing the data with the European Space Agency's Planck spacecraft have shown that the radiation from the dust in our galaxy. [6] However, peer review is on-going on this announcement comparing date with the ESAs Planck spacecraft. However, this announcement still remains questionable since it is not consistent with Planck spacecraft they found difference in results are taken through relatively dusty sky.

6.7 Schwarzschild metric

[14] Karl Schwarzschild in 1916, looked into Einstein's field equations and investigated it further. While he was calculating the gravitational field outside a spherical mass by taking into consideration of corresponding electric charge, angular momentum and mass of a massive spherical object.The metric for the at space generally represented with space-time coordinates with $R^4(x, y, z, t)$

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 \tag{6.5}$$

This space metric alternatively called as Minkowski metric can be represented by η in the form of matrix representation.

$$\eta = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6.6}$$

As it is defined for $R^4(x, y, z, t)$ replacing terms with (t, r, θ, ϕ) for spherical coordinate system we normally get.

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\Omega^2 \tag{6.7}$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2 \tag{6.8}$$

This metric is known as Schwarzschild metric. This gives the solution to static black holes with no change and no rotational characteristic system of black hole.

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1}dr^2 + r^2d\Omega^2 \tag{6.9}$$

$$c^2d\tau^2 = \left(1 - \frac{r_s}{r}\right)c^2dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{6.10}$$

- r_s Schwarzchild radius $R_s = \frac{2GM}{C^2}$
- c - velocity of light
- r - Radius
- $d\tau$ - proper time

This Schwarzchild metric or solution is also useful as approximation to describe slow non-rotating astronomical objects such as planets and stars. Furthermore, this is meant for static systems. As for the combination of charged and non-charged, rotating and non-rotating systems different matrices used. such systems can be classified by the type of black holes, Reissner-Nordström metric solution used to understand the charged, non-rotating systems. Kerr metric is used for uncharged and rotating solution. Kerr-Newman metric used for charged and rotating system.

6.8 Schwarzschild radius as Event Horizon

The Schwarzchild radius is actually describing the radius of a black hole which is called the event horizon. Event Horizon is like the region or point where once light or a photon enters into it, it can not escape from it. Due to intense gravitational pull of the collapsed star since all the matter compressed into single radius R_s. Below this radius the object is said to become a black hole with event horizon as surface of the sphere. Even light it self cannot escape from this region, in order to escape this region in space, a imiginary space ship must have velocity faster than speed of light were special relativity forbids it. Event horizon is the region in space where nothing can not escape once anything passing this boundary.

$$R_s = \frac{2GM}{C^2} \tag{6.11}$$

- c is the speed of light in vacuum
- m is the mass of the object or mass of collapsed star
- R_s is the Schwarzschild radius
- G is the gravitational constant

If Radius of sun which is $R = 700,000Km$ Schwarzschild radius R_s would be around $3Km$ when compressed below this radius if Sun has enough gravity to compress the mass below this radius the sun would theoretically will become black hole, however our sun lacks enough gravity for formation of black hole.

6.9 Gravitational Singularity

Singularity is seen in the solutions of Schwarzschild metric which was unexpected. After crossing the event horizon region one finds that the spacetime curvature becomes infinitely dense. This does not make much sense in physical reality, mathematically it is described as if $r = 0$ and $r = r_s$[14].

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{6.12}$$

As some of the metric in the equation is 0 as most of the terms vanishes from the equation. If $r = r_s$. if r is greater than r_s there is no problem with the equation. When r becomes smaller then r_s when r=0 hits the singularity. The formation of Black hole from life cycle of stars and more about types of singularity been discussed in here [32].

6.10 Life cycle of stars

The Stellar evolution or Life [4] cycle of a star undergoes dramatic changes in its entire lifespan. The life time of any star depends on the mass of the star itself, the supermassive stars like VY Canis Majoris [36] which is a red hypergiant star, astronomers can easily spot it in the constellation Canis Major since it is the brightest when compared to all other stars. If this hypergiant star is placed in the center of our solar system its radius would extend more than Jupiter's orbit. Perhaps, this is the largest known star in the visible universe. This hyper-giant is nearly $17 \pm 8\ M_\odot$ [5] I would like to use this star as example for formation of black hole. As for the average star like sun the life cycle would be different.

[4] its time for physical existence, stellar evolution
[5] times the solar mass

6.10.1 Stellar evolution-The Protostar

[3] [51] The Protostar or the first step for formation of star begins with the stellar evolution of gravitational collapse of the huge molecular cloud. To comprehend the size of this molecular cloud it would be around 100 of light years across. Due to gravitational potential energy the and total internal attractions in the different regions of the clouds results in the division of this huge cloud into smaller pieces. Each of these fragments of the clouds releases the heat though its gravitational potential energy. During this process the fragments of the clouds condensates to the rotating object forming the superhot gas sphere of a protostar. After the formation of protostar continues to grow by the accretion of gas and dust with in the molecular cloud. Further developments takes place within the star until the star fully formed which can be comparable to that of a sun's mass $M_{\odot}$.

6.10.2 Young star, it's Balance of Life and Death

The protostar progresses to its final formations the core of the star which begins to run out of hydrogen supply. The gravity takes over the young star however to counter the gravity the electron degeneracy becomes essential. When this happens due to pull of gravity the core itself becomes hot enough as the helium fusion kicks in the star. This process continuous though out the life cycle of the star until the its final years. As the process of fusion already had been instigated the pressure caused by nuclear fusion balances with the force of gravity as long as the fusion continuous the nuclear pressure pushing the sun away from its own centre, this opposes the gravitational pull.

6.10.3 The Red Giant

As star gets older, the average star like sun or larger star runs out of hydrogen fuel. The core of red giant stars unable to balance with gravity collapse. The resultant collapse releases energy however the electron degeneracy stops the complete core collapse but the gravity tries to take over as the fusion of hydrogen to helium takes place even faster in the core. This results in appearance of the bright star as its luminosity increases and outer layers tend to expand. During this process the temperature decreases. Some stars do not become red giant star due to the nature of being massive they take different path.

6.10.4 Supernova

The supernova explosion is formed when there is a sudden re-ignition of the nuclear fusion in a degenerate star or by the gravitational core collapse. The degenerate white dwarf may have sufficient material via accretion in this process the rise in core temperature ignite carbon fusion this might trigger the nuclear fusion this whole process might disrupt the star. This way supernova explosion takes place as there are different types of supernova explosions. Some stars even may turn into planetary nebula.

6.10.5 Chandrasekhar limit for White Dwarf

[3] The degenerate dwarf or also known as the white dwarf is the stellar remnant which is full of electron degenerate matter. Although the volume can be close to the volume of the earth the

density is comparable to that of sun. The emitted luminosity is not bright its source is the thermal energy left in white dwarf. The few observable white dwarfs are Sirius B and Sirius binary.

As I discussed in this section 4.4 earlier, The Fermi Dirac Statistics was actually applied to understand the white dwarf in 1926 by Fowler. [1] By the application of quantum mechanics and Fermi Dirac statistics in white dwarf electron degenerate. In lower temperature the electrons can occupy lowest possible energy level. William Alfred Fowler and Subramanyan Chandrasekhar worked on white dwarf as Chandrasekhar showed that when white dwarf exceeded the Chandrasekhar limit the nuclear reaction did not take place. Electron degenerate pressure cannot balance with gravity in supernova explosion the white dwarf is destroyed.

The maximum mass that is required for white dwarf to act up on the electron degeneracy before collapsing into a neutron star or a black hole the limit is 1.44 $M_{\odot}$ this limit is known as Chandrasekhar limit. The Paulis exclusion principle states that no two electrons can have same quantum numbers that is half integral spin of electron cannot sit in the same energy level. When this is forced causes degenerate pressure within the star.

6.10.6 Tolman Oppenheimer Volkoff limit/TOV limit for Neutron star

5 previously discussed that It was discovered theoretically that super fluidity exists inside neutron stars [34] It is expected that the nucleons in the neutron stars at high density and low temperature can actually from cooper pair due to the nuclear force can lead to superconductivity and super fluidity.

Julius Robert Oppenheimer , George Volkoff used work of Richard Chance Tolman in 1939. The research progressed on Tolman's work. The Neutron star is balanced by short range repulsive neutron to neutron interactions [56] actually mediated by the strong force. This is quantum degeneracy pressure of neutrons preventing the collapse of the dying star. Above the Tolman Oppenheimer Volkoff limit the star would fall below the Schwarzschild radius R_s. Furthermore if not black hole this would become new kind of degeneracy which is known as quark degeneracy. In this case the quark to quark interaction which forms degeneracy. [27] In the neutron star, a star which is mostly filled with neutrons, this can be understood as neutron degeneracy due to pull of gravitational collapse. The other model describes the core in the neutron star as super-fluid neutron degenerate matter. It is strange that the degeneracy goes beyond the neutron degeneracy to the ultra-dense quark degenerate matter [27].

6.10.7 The Black holes: The battle of Quantum mechanics Vs. Classical physics

After understanding the very short review on life cycle of star, considering the facts and existence of degeneracy while the star is trying to gravitational collapse. However, after crossing the limits of Chandrasekhar limit for white dwarf and Tolman Oppenheimer Volkoff limit/TOV limit for Neutron star. after further theoretical degeneracies the star can fall below the Schwarzschild radius R_s with dense mass and become black hole. The super-massive stars like VY Canis Majoris have enough mass and gravity theoretically to become black hole. Here, its mostly philosophically interesting that how the quantum degeneracy battling against gravity which decides the fate of the star. When black hole is formed it predicted to take form of spherical shaped black sphere with event horizon and corresponding singularity. Quantum mechanics actually prevents the existence of singularity. If the curvature of space is densely distorted to infinity is to say that it is possible that it may go smaller than plank length (L_p) and beyond the strings from string theory. How do singularity exist in physical existence ? it is hard to imagine.

6.11 No-hair conjecture

When an observer makes an observation outside the black hole, all the physical parameters that he can measure on the surface of event horizon is mass of the black hole with corresponding angular momentum and change. The No-hair conjecture postulates that all the collapsed stars which form black holes satisfying the equations of general relativity for gravitation and electromagnetism, only parameters that are can be measured outside the black hole would be mass, electric charge and its corresponding angular momentum. Moreover all the other information which falls into black hole event horizon would vanishes; the external observer may not be able to make any measurements other measurements about an object falling into the black hole.[41] The only observable parameters that an observer can make for rotating and non-rotating black holes are following:

- Q - electric charge (charge on event horizon)
- J - angular momentum (along three components of black hole)
- M - mass-energy, mass of black hole
- P - linear momentum (three components along x y z axis)
- X - position (three components)

6.12 The Thermodynamic laws of black hole mechanics

Black hole thermodynamics is the one of the way to understand the how a law of thermodynamics works in the existence of event horizon. The four laws of black hole mechanics are the physical laws of black holes. These laws are similar to the laws of thermodynamics previously discussed here 3.1. These laws were proposed by Brandon Carter, Stephen Hawking and James Bardeen. [25] The laws of black hole mechanics can be expressed in geometrized units such as taking constant values and equating it to unity.

6.12.1 The Zeroth Law

The Zeroth law of thermodynamics states that , the surface gravity of the black hole is constant all over the event horizon. This is similar to the statement made in zeroth law of thermodynamic 3.1 as long as body with thermal equilibrium with other bodies the temperature remains constant. here in case of black hole the surface gravity k remains constant.

$$k \textit{ is constant over the horizon of a black hole}$$

6.12.2 The First Law

The first law is similar to that of the first law of the thermodynamics 3.1.ΩdJ is the work performed upon a black hole when its spin changes by dJ

The first law of black hole thermodynamics states that the for stationary black hole solutions of Einstein's equations has been derived as

$$dE = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ \tag{6.13}$$

- $\Omega$is angular velocity
- $J$is the angular momentum
- E is the energy
- κ is the surface gravity
- A is Horizon area
- $\Phi$is electrostatic potential
- Q is electric charge

The rate of change in dE in the energy as this is proportional to the mass. The change in energy due to electromagnetism Φ and the corresponding angular momentum J this is denoted with the second term and third term. The energy conservation includes the term Tds in the equation.

6.12.3 The Second Law

The horizon area of the black hole is when assumed with weak energy conditions which is non decreasing function of time.

No classical process can make the horizon area A of a black hole decrease.

$$\frac{dA}{dt} \geq 0 \tag{6.14}$$

The second law of thermodynamics of black hole is similar to the second law in classical thermodynamics. The change in the system's entropy of an isolated system is greater than or equal to 0, this is the link between area of event horizon and entropy. It violates the second law when matter falls into black hole thus decreasing the entropy. The generalized version of the law can be stated as the entropy of black hole with outside event horizon entropy is equal to the total entropy. This law triggered the idea of Hawking's radiation [30].

6.12.4 The Third Law

The Black holes with minimum possible mass that can be compatible with the change and angular momentum, the smallest possible black hole that can exist while it is rotating at the given fixed constant speed this kind of black holes is also known as external black holes. These black holes have vanishing surface gravity. κ cannot go to zero is analogy from the third law of thermodynamics as it has been discussed here 3.1 . The entropy of system at absolute zero is well defined constant. The system temperature exists in the lowest quantum level which is ground state level. When ΔS which is change in entropy approaches to $T = 0$ Kelvin, S becomes 0. According to the third law of thermodynamics the temperature can not go to absolute zero, this is analogus to this law for surface gravity κ.

It is not possible to form a black hole with vanishing surface gravity. $\kappa = 0$ *is not possible to achieve.*

6.12.5 Understanding the four laws

Based on the laws of Black hole mechanics or Black hole thermodynamics we have to understand that the analogy is backed on the key facts:

- Surface gravity κ related with temperature T of classical thermodynamics
- The area of event horizon A related with Entropy E from the classical thermodynamics

By no hair theorem it is possible to have zero temperature. However, the laws of black hole mechanics still remain analogy. When viewed with the quantum mechanical perspective the conclusion becomes the emission of hawking radiation or thermal radiation.

6.13 Entropy of Black hole

[22] In order to satisfy the second law of thermodynamics it is important for black holes to have entropy. According to the second law of thermodynamics, as discussed here 3.1.3, the entropy of the isolated system never decreases. Anything gets inside event horizon the object is said to be taken into the black hole thus there is decrease of entropy for the object swallowed by the black hole.

In order to save the second law of thermodynamics, Jacob Bekenstein and Hawking predicted that black holes must have entropy and the entropy of the black holes must be proportional to that of surface of the event horizon. The area of the event horizon is divided into the planck area [53].

$$S_{BH} = \frac{\kappa A}{4L_p^2} \tag{6.15}$$

- l_p This is Planck length which is $l_p = \sqrt{\frac{G\hbar}{c^3}}$
- A- Area of event horizon which is calculated at $4\pi R^2$
- κ - Boltzmann's constant

the multiplicative constant of the Bekenstein-Hawking entropy is

$$S_{BH} = \frac{A}{4} \tag{6.16}$$

Entropy of black hole is surface area of black hole event horizon (A) divided by 4.

6.14 Hawking radiation

Stephen Hawking proposed the concept of thermal radiation by understood the quantum mechanics, that the black hole can actually emit thermal radiation which is now known as Hawking radiation at a temperature T [30].

$$T_H = \frac{\kappa}{2\pi} \tag{6.17}$$

The thermal radiation T_H is the κ surface gravity of event horizon. By applying the quantum field theory to the curved space-time the black hole actually emits the electromagnetic radiation with a temperature. Unruh temperature is derived by William Unruh in 1976. The temperature measured by an accelerating frame of observer or an accelerating detector in a vacuum field.

$$T = \frac{\hbar a}{2\pi c \kappa_B} \tag{6.18}$$

- c -Speed of light
- $\hbar$- Reduced planck constant
- κ_B - Boltzmann constant
- a - Local acceleration

If the local acceleration is about $a = 3$ x $10^{20} ms^{-2}$ the temperature would be around approximately $T = 1.21$ Kelvin. Unruh effect and hawking radiation have lot similarities although it was discovered independently, for Temperature of black hole is given by

$$T = \frac{\hbar g}{2\pi c \kappa_B} \text{ as g is } g = \frac{c^4}{4GM} \tag{6.19}$$

- g- Acceleration parameter of surface gravity in black hole case
- M- Mass of Black hole

The radiation from black hole is black-body radiation with temperature T. By considering all the constants the equation is equal to

$$T = \frac{\hbar c^3}{8\pi GM\kappa_B} \approx \left(\frac{1.227\text{x}10^{23}kg}{M} K\right) \tag{6.20}$$

According to Hawking, as he imagined that empty space is not completely empty and there is lot of quantum activity goes. Mostly the virtual particle or pair production of matter and anti-matter which is produced in space can also be produced around the event horizon of a black hole. When the pair is produced one of the pair falls into the black hole say antiparticle which can annihilate particle that exist inside event horizon thus reducing the mass of black hole when one of the pair gets separated. The separated particle will act as hawking radiation as it escapes away from the event horizon. This radiation is said to be thermal radiation from the black explained using quantum mechanics.

When one of the pair which is anti-matter falls into black hole it reduces the mass. If the radiation continuous to emit, the black hole mass reduces as $E = mc^2$ this increases the possibility of evaporation of black hole.

6.15 Trans-Planckian problem

[11] The trans-planckian problem is commonly seen when theories refer to the scaling quantities beyond the Planck scale, when a theory predicts phenomenon beyond the Planck scale the physical laws are needed to be modified beyond the quantum measurable scale which is the Planck scale. This is the case where in the inflationary cosmology and Black hole physics are seen. The equations that are used to derive Hawking radiation involves in the derivations that are used to solve the problem. In case of analogue models of black holes such as sonic black holes which are discussed in this chapter 7.1. The Trans- Planckian problem is comfortably considered in formulating the analogue models of black hole. The Trans-planckian problem exist in inflation cosmology it involves in the consideration of the below Planck length.

6.16 What is the Minimum mass required to bend space ?

Based on the understanding of general relativity so far I would like to ask, since the mass of the object distorts the space-time geometry. What would be the least minimum mass required so that I could see the distortion of space-time ? Would the least mass of electron can distort space time ? So at what point gravitational force vanishes? This is the problem in modern physics we do have any experimental evidence and do not understand nor comprehend the force in terms of microscopic mass which is close to Planck mass. In order to understand this we have physicists working in the field called quantum gravity. Which is they work on models of general relativity and quantum mechanics with the help of quantum field theory. Sometime in other fields such as analogue models of gravity from condensed matter physics might give us hint to understand gravity in different way as this is further discussed in this chapter 7.1.

6.17 General relativity Vs. Quantum mechanics

Quantum theory is the theory of very small which is close to Planck scale. General relativity is the theory of very large which is describing the curvature of space time caused by massive object. Now, how would both fit together to explain the gravity at small scale? We may have quantum field theory to describe the special relativity version of quantum mechanics. These must be new theory needed to describe the general theory of relativity in terms of quantum mechanics. This may someday help us to understand the gravity at small scale. The general relativity which describes the large scale universe as theory implies that if there is no mass in the space then the theory tells us that space should be flat without having any curvature. Since the space-time is flat in general theory of relativity we must now focus on the quantum mechanical description of empty space. When we zoom into the order of magnitude of quantum world we get to see the fabric of space as emptiness as we zoom in furthermore the structure of space remains the basic form. This is reasoning with the classical perspective of general relativity observation. As we zoom in further more and more we enter into quantum world where we see quantum fluctuations that follow uncertainty principle. Even the gravitational field must follow the uncertainty principle and further more fields exist. On the average we may fine the values of 0 however due to quantum fluctuations exist in quantum space. This conflicts with general relativity when we compare with quantum fluctuations. This is incompatibility of general relativity with quantum mechanics. As general relativity actually predicts the flat space [23]. It is important to remember that classical laws are radically different from quantum laws there is a gap in our understanding this mystery in nature of reality.

6.18 Quantum gravity

The quantum gravity is the field of theoretical physics which makes an attempt to combine the theory of large which is general relativity and theory of small which is quantum mechanics. If successful it can describe the quantum nature of gravity. Gravity is poorly understood when we look from the perspective of quantum mechanics. The bond between the classical laws and quantum laws is something very difficult to formulate. The difficulty is that when one tries to use quantum field theory to that of force of gravity it is completely does not make meaningful. This is the field of effort to make gravity quantized field. Other approaches are from the field of loop quantum gravity and string theory which makes and attempt to unify all the fundamental forces in nature. The loop quantum gravity is one of the fields of research which attempts to understand the properties of gravity. This considers the space to be not continuous but granular which is direct consequence of the quantization. In this picture the space itself is discrete.

6.19 Why gravity is too weak?

Now consider that a material that is made up of iron which is magnetic substance. Consider that this iron substance is lying on the ground using bar magnet one can overtake the earth's gravitational pull. Gravity is so weak that it's easy for birds to fly off and having enough escape velocity we can send rockets to mars and moon all around solar system. One can ask why gravity is so weak compared to the other fundamental forces of nature such as strong, weak and electromagnetism. It is still a mystery, one of the possibilities is explained in theory such as string theory that it is weak due to the fact that we may live with extra dimensions and gravity can actually travel though these extra dimensions, as these dimensions are very large, perhaps making the gravity weak [5].

6.20 N-Gravitons

In the framework of the quantum field theory as for the mediating particle for gravity graviton was hypothetically proposed in theory. The graviton is different from other gauge particles that are gluons which are force carriers for strong interactions from strong nuclear force. W^+, W^-, Z^0 Bosons for the weak interactions. For electromagnetism the interactions are photons and graviton is the only particle with spin 2 and zero mass as photon is spin 1 and massless, gluon being massless with spin 1. It is illustrated in the table of fundamental gauge particles representing the fundamental forces of nature. The fundamental forces which is strong, weak, electromagnetism and gravity has been discussed here [32]. The graviton is yet to be discovered for now its a theoretical predication.

	Mediators	Strength	Helicity	Range	Live example
Strong force	8 gluons (g)	1	1	Short	Stars
Electromagnetism	Photon (γ)	10^{-2}	1	Long	Lightning or Light
Weak force	W^{+}, W^{-}, Z^{0}	10^{-5}	1	Short	Radio active decay
Gravity	Graviton (G)	10^{-40}	2	Long	Planets motion around the sun

Table 2: shows four fundamental forces of nature and its corresponding properties [35]

If gravitons are mediating particles between two masses. How would it explain the gravitational field of isolated system. 6.3 Gravitons are more satisfactory to the Newton's law of gravity when compared to Einstein's general theory of relativity. As for the N-number of gravitons is being refereed here 8.1.

7 ANALOGUE MODELS OF BLACK HOLES

Some of the features of Einstein's general theory of relativity can be seen in analogue models of condensed matter[46]. There are some restrictions of the analog models which do not obey Einstein's theory of gravity. Moreover, these analog models may not be accurately describing gravity as physical system but it may give significant features to understand the phenomenon of gravity in different perspective. The reason why these models are widely discussed in several scientific technical papers by physicists is because of its significant features and to look into possibility of experimental observations of analog versions of Black holes and Hawking radiation [22]. When we consider looking experimental possibilities for general relativity in the lab it has always been difficult to recreate gravity or a black hole in lab. Since this requires huge amount of mass and energy-momentum to recreate distortion of space-time comparable as planets and stars do. If ever performed on planet earth this would be risky and may interact with earth's gravity. Due to the lack of enough evidence of micro black holes in the lab or in particle collider such as LHC at CERN, effects such as Hawking radiation or Hawking evaporation of black holes are yet to be seen. The best way to understand Einstein's gravity is by using some of analog models of gravity or Black hole. Mostly the analogue models are based on condensed matter theories. If these models can be constructed in the lab we may have opportunity to understand black holes in new perspective, this is mostly anticipated by physics community. Physicists who are working on analog gravity models are investigating ways to relate condensed matter physics, statistical mechanics, quantum gravity and quantum field theory to describe in terms of general relativity. It also matters how much it can be related and how it can be experimentally tested. In the future, perhaps we may be able to demonstrate and experimentally test the analogue of Black hole, Gravity, Hawking radiation and many more possibilities of understanding the nature of gravity which are theoretically predicted. In these analog models geometry of space-time can be effectively related and can be used as important phenomenon.

The particles such as photons and quasiparticles can be described in terms of effective metric which involves in use of Einstein's general relativity [46]. By using the effects of quantum field theory researchers can construct the theories from condensed matter systems which can give us the analogues of curvature of spacetime geometry. This may resolve a problem that arises as curvature at shortest distances, how do we understand gravity close to Planck scale? If this problem could be solved this could give rise to new quantum theory of gravity, at the same time these new methods can be also be used for understanding condensed matter systems as these methods have been originally instigated for gravity.

7.1 Acoustic analogue model: The Dumb Hole

The acoustic analog model uses the equations for propagation of sound waves in the moving fluid. [22] Here sound waves are playing the role of the photons travelling in the empty space. Even photons can cause the curvature in space-time geometry. In this case, sound waves can cause the distortion in space time of a fluid. Sound waves have velocity just as light waves. The acoustic analog model of the black hole was first instigated by Bill Unruh. This model is also known as the dumb hole. Since we cannot test with real version of black hole in the lab we can always create the analog model of the system. In this model of acoustic or dumb-hole model or sonic black hole, experimentally it is possible to create in lab the way it is done is by producing the sound waves as it travels in a fluid. One can create a point in a region of space (which is in fluid) where the velocity of the fluid at that point is higher than the speed of sound. This forms two separate regions with supersonic and sub sonic speeds this from an analog of the black hole's event horizon where the region of space is denser like curved space of event horizon. In un-curved space is like subsonic fluid space and the curved space is like supersonic fluid space which is the point at which the sound travels in higher speeds and making the fluid highly denser. When any kind of subsonic sound wave enters this point of supersonic region the subsonic waves will not return thus these waves need to have wave velocity higher than that of the supersonic only then it is possible to have such escape velocity. This is lot similar to event horizon where in falling observer need to travel faster than speed of light in order to escape from inside horizon curvature. Here in the case of sound entering the reason of supersonic will never return back as this is creating the dumb effect were the sound is not heard outside the horizon [8]. Most importantly this analogy is has very much important significance in both classical and quantum effects. In case of fluids the particles phonons are assumed as I discussed the topics of quasi particles and phonons in this section 5.2.1. This can be understood as space-time in the fluid as granular.

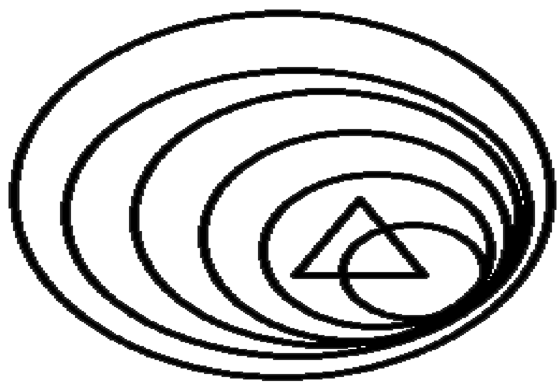

Figure 15: image illustrates the concept of the moving fluid which will drag sound pulses along with it. This may form supersonic and subsonic regions.

If experimentally performed in the lab, the phonons would in fact move in a curved space-time of the fluid as sound wave move across the system. As for the sound which is lot similar to that of the curvature in a scalar field equation of spacetime. This can be understood as that of the representation of quantum and classical effect of the field propagation as the spacetime background is fixed this can be reproduced in this analog model. In case an experimentalist anticipates to find if any thermal radiation being realised from supersonic region to subsonic in the form of if any phonons are close to event horizon of supersonic region phonons might be emitted from region itself this act as thermal emissions of phonons this effect is analogue for the hawking radiation.

It is important to notice in this analog model of gravity is that the solutions which are being derived from the laws of hydrodynamics using effective metric. These solutions are not similar to that which is derived for gravity in general relativity this is one of the restriction. The other limitation is that the sound wave which is used in fact is a collective motion of the media constituents which could be phonons; this is nothing but excitation of actually works with the large scales larger than that of atomic sizes. The corresponding acoustic metric being derived here in details [46].

Physicists are looking into the possibility that gravity may be an emergent phenomenon. Calculation for Hawking radiation and in study of inflationary universe was involved in dealing with the problem of Planckian and trans-planckian energies as it has been discussed here in this section 6.15. Understanding the quantum gravity in the analog models of gravity one can use trans-planckian, Events such as big bang and black holes require the quantum mechanical effect in order to have such large scale formations that we get to see in the current picture of the universe. In case of analog models of gravity the similar phenomenon can be used to show that large scale structures does not have to depend up on the high energy since sound wave with high energy does not have much sense quantum mechanically atomic properties are significant here.

7.2 Quantum Liquids

[8] [46] The quantum liquids are nothing but the ability of a matter to remain in super fluid state of matter at very low temperature even at absolute zero. The only known superfluid element is helium remains fluid even at temperatures close to absolute zero this is due to the zero point motion. This property being exhibited by the two kinds of liquids that is Helium-4 and Helium-3. These are similarities to identical properties of chemical structure of both the liquids. Here Helium 4 got integer spin its nuclei have zero spin which is Bose liquid. Whereas Helium 3 got $\frac{1}{2}$ integer spin, thus acts as fermions which are Fermi liquid. Both the liquids are very good to get consider this system to be described in terms of hydrodynamics. For analog model of gravity these liquids make good combination as quantum liquids. Their property is highly significant which the liquids viscosities are considered. In order to understand Hawking radiation in this analogy one can consider the zero dissipation for super-fluid while at temperature is very much close to zero.

Consider the N atoms in the quantum liquid. When we try to construct the number of atoms in N quantum liquid there exist zero point motion as there is ensemble of atoms in the ground state even without external pressure.

7.3 Shallow water waves

In order to represent the gravitational waves or gravity waves the analog model of shallow water waves is presented. In this analogy the effective metric is classical system when flow of the fluid or a liquid is not rotational that is to say irrotational and not considering the viscosity of the fluid, in this case the wave form can be represented by the relativistic scalar wave equation in 2+1 dimensional curved spacetime.it is important to note that the velocity of the wave depends up on the depth of the basin which is filled with liquid. It can represent gravitational wave effectively and moreover experiments can also be done for this analogue model.

Since the velocity of wave in this model can be changed by the depth of the basin the velocity can also be made too slow in order to represent the ergo regions where this is external field for event horizon. There is one disadvantage in this method while trying to look for hawking radiation the relative temperature is found to be very low. In order to look for the possibility to have observation of hawking radiation or hawking evaporation in this analogy one must have the low temperature fluids. This analogue serves as the best example for the gravitational waves. [55] This has been experimentally done by Weinfurtner.

7.4 Analogue gravity from Bose-Einstein condensates

Bose Einstein condensate is the quantum mechanical version for analogue models of gravity.[46] In condensed matter systems one can find effective geometry when considering the Bose Einstein condensate of the quasi particles. In Bose-Einstein condensate state of matter is considered to be diluted gas as there is weakly interaction among the bosons. Consider confining bosons in an external system when cooled down to temperature close to absolute zero the bosons sit in ground state level. Acoustic metric is introduced for the propagation of phonons or quasi particle in a condensate. The quantization works in a way where phonon are acting like back ground dynamics of acoustic metric. The dynamics of the acoustic metric is more like the scalar curvature in spacetime. The propagation of the quasiparticles in a condensate acts like acoustic metric. The Bose Einstein condensate is very good models to describe the quantum effects of analog model of the curved space geometry. In order to study this experimentally the Bose Einstein's condensate needed to be achieved at low temperature which is close to 10^{-7}K. This is the suitable conditions to test this experimentally and this is actually been tested in the lab [4]. The propagation of collective fluctuations of the phonons agrees with the general relativity description of spacetime metric. The wavelengths might correspond to the quantum field theory in a curved spacetime. The metric that is defined for phonons can actually describe the black hole as it is described in Einstein's general relativity. These are a possibility of phonon excited as thermal radiation which describes as hawking radiation. In the next section 8 I would like to discuss the new research field where phonons are replaced by gravitons.

8 BOSE EINSTEIN CONDENSATE OF GRAVITONS

I would like to acknowledge to my readers that I am master's student. I fully do not have all the knowledge fact about the advanced topics in this paper such as string theory, gauge theories, quantum gravity and advanced quantum field theory etc. I have yet to gain lot of experience in theoretical physics. But for now I will do my best to bring the general understands of these scientific papers [19] [20] for the literature review to give understanding for purpose of my thesis as my Master's research is based on these scientific papers.

8.1 Black Hole's Quantum N picture

As it is suggested by authors Gia Dvali and Cesar Gomez in their paper [19] [20] that the black hole is a self-sustained Bose Einstein condensate of gravitons. The condensate is characterised by wavelength, as occupational number of gravitons N is compatible with the given wave length characterised for gravitons. The black hole is maximum packed system of gravitons as this favours the quantum theory of black holes. The defined plancks length L_p along with the Planck mass M_p with the Newton's coupling constant G_N.

$$L_p \equiv \frac{\hbar}{M_p} \equiv \sqrt{\hbar G_N} \tag{8.1}$$

The characteristic of black hole was described in their paper as in terms of L_p and N as [19].

- occupational number of gravitons N
- Wave-length $=\sqrt{N} L_p$
- interaction coupling strength as $= \frac{1}{N}$
- Temperature $=\dfrac{\hbar}{\sqrt{N} L_P}$
- Entropy $= N$

The occupational number for gravitons is given as

$$N = \frac{M^2 G_N}{\hbar} = \frac{M^2}{M_P^2} \tag{8.2}$$

As described by their discussions that the Hawking radiation is understood as depletion of Bose Einstein condensate of gravitons. In classical theory the Schwarzschild radius R_s is derived as $R_s = \dfrac{2GM}{c^2}$ .In quantum N picture, the quantity Schwarzschild radius R_s has been derived as

$$R_S = \sqrt{N} L_P \tag{8.3}$$

The interaction between graviton to graviton is given by coupling constant α_{gr}. This is dimensionless quantity is given by

$$\alpha_{gr} = \frac{L_p^2}{\lambda^2} = \frac{L_p^2}{R_s^2} \; when \; \lambda(wave - length) = R_s \tag{8.4}$$

In [20] The black hole picture is described as the leaky bound state under cold Bose Einstein condensate gravitons. The total number of gravitons is taken as N occupational number, with the wave length $\sqrt{N}$. The existences of gravitons in the ground state may not always stay in the ground state, some may get depleted. This depletion is seen as Hawking radiation. [12] According to Hawking picture of radiation the pair production of sub atomic particles such as particle and antiparticle. One of the pair would fall into Black hole and reduce the mass of the Black hole and other pair which is matter would be like released radiation.

In this picture of Black hole model, Black holes do have "hair which is terms of Baryonic numbers which is Baryons (strongly interacting fermions, which can be explained by Fermi-Dirac statistics) and leptonic number (subatomic particle which do not undergo strong interactions with $-\frac{1}{2}$ spin) which gives good astrophysical importance [18].

The classical version of Black holes are viewed as the gravitational collapse of massive star as described by Schwarzschild metric and event horizon being Schwarzschild radius (R_s) with singularity. When viewed with the picture of Bose Einstein condensate of gravitons as gravitons are massless particles with spin 2, the analogue models do not show singularities. However they can show us the analogue of gravity. The findings in model of Bose Einstein condensate of gravitons as described in the scientific papers [19] [20].

This quantum picture of black holes follows the quantum theories which does not use any classical geometry. For the N weakly interacting gravitons has a property of the wave-length derived as the condition of bound state of Bose Einstein condensate of gravitons.

$$\lambda = \sqrt{N} L_P \tag{8.5}$$

- λ is the wavelength of N graviton,
- $L_P = \sqrt{\frac{\hbar G}{c^3}}$ is Planck length.

The interaction strength among gravitons is given by

$$\alpha = \frac{1}{N} \tag{8.6}$$

The weakly interaction among gravitons is described by coupling strength $\frac{1}{N}$. The leaky nature of condensate is described as gravitons with the wavelength $\lambda = \sqrt{N} L_P$, since the escape energy is above the escape energy of the Bose Einstein Condensate at absolute zero as some of the N gravitons cannot stay in the ground state for long time. This is the explanation given for Hawking radiation. The escaping energy from the condensate of gravitons is due to the fact the nature of being leaky. The Black hole temperature in classical theory being described as the temperature of Black hole is directly proportional to surface gravityκ which is $T = \frac{\kappa}{2\pi}$, In this quantum N picture, the thermal spectrum of Hawking radiation [20] is being derived as Temperature T:

$$T = \frac{\hbar}{\sqrt{N} L_P} \tag{8.7}$$

The Bekenstein entropy is explained as the natural degeneracy of N number of graviton states which scales corresponding to exponential growth with respective N, as e^N. The universality of N is described as the quantum foundation for describing black hole in terms of No-hair, Hawking radiation emerging thermally and as N being able to describe entropy comparatively giving description as classical version of Black hole. The reason why entropy is related to N , N relates as Bekenstein's entropy. Which it is related to the mass of black hole is that N is weakly interacting gravitons with number of quantum states.In classical theory of Black hole entropy or Black hole thermodynamics the entropy is proportional to $\frac{1}{4}$ area (A) of event horizon as $S = \frac{A}{4}$. I would like to imagine Black Hole with Bose Einstein Condensate of gravitons floating in empty space [6].

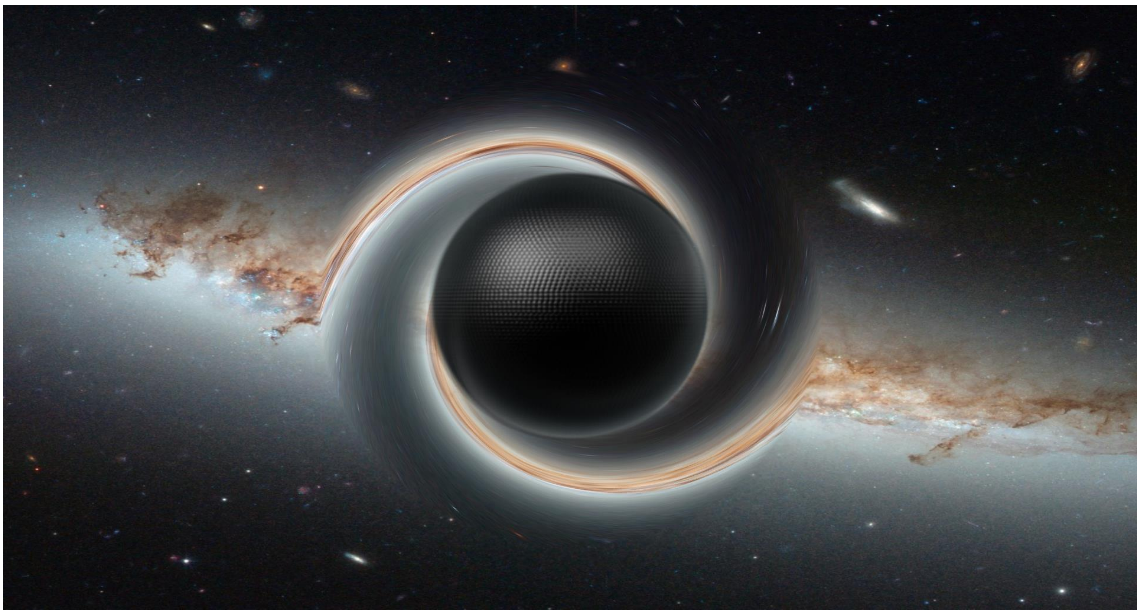

Figure 16: I would like to imagine from "artistic" perspective of Bose Einstein Condensate of N-gravitons, as gravitons are taken as black particles to represent black hole. This is non-scientific but based on the discussion on Bose Einstein condensate of gravitons, perhaps light may escape from event horizon as shown in the image which is not reflection.

[6] Background is taken from NASA's Hubble image under use of public domain according to NASA

Part III

MASTERS BY RESEARCH in PHYSICS

9 A GRAVITON IN A BOX

My research begins with looking into the conclusions of 8, Gia Dvali and Cesar Gomez in their paper [19] [20] that the black hole is a self-sustained Bose Einstein condensate of gravitons. My investigation started with particle in a box in order to look for equation for Schwarzschild radius of N gravitons. I would like to check the Schwarzschild radius for N gravitons to see if I can get the similar equation from quantum field theory approach or I might get different equation compared to the equation concluded on 8.3.

9.1 Particle in a box

The concept of particle in a box been discussed here in this section 1.10 Schrödinger's equation shows us that we can write the energy of the electron as 1.10

$$E_{nx,ny,nz} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) \tag{9.1}$$

9.2 Bose-Einstein Condensate energy distribution for graviton

Since I am looking for statistical distribution of gravitons I can looking into considering the energy distribution in a state of ν. BoseEinstein statistics For non-interacting bosons the concept of Bose Einstein condensate been discussed here4.3, the Bose distribution function is

$$f^o(\epsilon_\nu) = \frac{1}{e^{\frac{\epsilon_\nu - \mu}{\kappa T}} - 1} \tag{9.2}$$

- T- Temperature
- ν- occupational state of single particle
- μ - Chemical potential
- ϵ_ν is the energy of the ν state
- k is the Boltzmann constant

9.3 Particle in a square box

Consider a square box with length "a" on the each side of the box. Than the Energy of a particle in ground state. n_x, n_y, n_z three quantum numbers I can think of while considering the ground state, for 3 degrees of freedom. The energy of this ground state is given by $n_x = 1, n_y = 1, n_z = 1$ corresponding length of the cube is $L_x = a, L_y = a, L_z = a$. equation 1.10 becomes

$$E_{1,1,1} = \frac{h^2}{8m}\left(\frac{1^2}{a^2} + \frac{1^2}{a^2} + \frac{1^2}{a^2}\right) \tag{9.3}$$

The equation for Energy of particle in a box

$$E = \frac{3h^2}{8ma^2} \tag{9.4}$$

For N particle in ground state this would be

$$E = N\left(\frac{3h^2}{8ma^2}\right) \tag{9.5}$$

The energy of N graviton particles in a box being in the case of non interacting boson gas or a graviton gas. The energy distribution for non-interacting bosons in thermodynamical equilibrium. using equation 9.2.

$$f^o(\epsilon_\nu) = \frac{1}{e^{\frac{\frac{N3h^2}{8ma^2}-\mu}{\kappa T}} - 1} \tag{9.6}$$

Representing the above equation with new notation.

$$\xi(n_x, n_y, n_z) = \frac{1}{e^{\frac{Enx,ny,nz-\mu}{\kappa T}} - 1} \tag{9.7}$$

The total energy of the of this system can be written as

$$E_{Total} = N \sum_{n_x,n_y,n_z=1}^{\infty} \frac{E_{nx,ny,nz}\xi(n_x, n_y, n_z)}{\sum_{n_x,n_y,n_z} \xi(n_x, n_y, n_z)} \tag{9.8}$$

for mass term in equation 1.10, as this is relativistic case, since gravitons are massless, its good to use Klein-Gordon equation for the relativistic equation. Using Schrödingers equation we can get the following form, substituting it into Klein-gordon equation as this concept been discussed here 2.3 for massless boson which is in this case is graviton. Taking ϕ as plane wave solutions into Klein-Gordon equation.

$$\frac{d^2\Phi}{dt^2} - c^2\nabla^2\Phi = 0, \Phi = e^{i\omega t}\Phi(x) \tag{9.9}$$

$$[\omega^2 + c^2\nabla^2]\Psi(x, y, z) = 0 \tag{9.10}$$

The general solutions to Schrödingers equation can be used for wave function Ψ.

$$\Psi(x, y, z) = \psi_x\psi_y\psi_z \tag{9.11}$$

$$\psi_x = Ae^{ikx} + Be^{-ikx} \tag{9.12}$$

$$\psi_y = Ce^{ipy} + De^{-ipy} \tag{9.13}$$

$$\psi_z = E^{'}e^{iqz} + Fe^{-iqz} \tag{9.14}$$

$A, B, C, D, E^{'}, F$ are constants using equation 9.10 the frequency can be derived.

$$[\omega^2\Psi(x, y, z) + c^2\nabla^2\Psi(x, y, z)] = 0 \tag{9.15}$$

The equation becomes

$$\omega^2\Psi(x, y, z) = -c^2\nabla^2\Psi(x, y, z) \tag{9.16}$$

$$\omega^2\Psi(x,y,z) = -c^2[-[k^2(Ae^{ikx}+Be^{-ikx})+p^2(Ce^{ipy}+De^{-ipy})+q^2(E^{'}e^{iqz}+Fe^{-iqz})]] \quad (9.17)$$

Dividing by $\Psi(x,y,z)$ on both sides as $\Psi(x,y,z) = \psi_x\psi_y\psi_z$

$$\omega^2 = -c^2[-[k^2+p^2+q^2]] \quad (9.18)$$

$$\omega^2 = c^2[k^2+p^2+q^2] \quad (9.19)$$

$$\omega = \sqrt{c^2[k^2+p^2+q^2]} \quad (9.20)$$

$$\omega = c\sqrt{[k^2+p^2+q^2]} \quad (9.21)$$

frequency spectrum of the Klein-Gordon equation for hard-wall boundary conditions in a square box. As k,p,q are similar to the equation 1.10 and since $E = h\omega$ multiplying h on both sides. The energy spectrum is obtained by multiplying the frequencies by Planck's constant.

$$h\omega = hc\sqrt{\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right)} \quad (9.22)$$

As

$$k^2 = \frac{n_x^2}{L_x^2}$$

$$p^2 = \frac{n_y^2}{L_y^2}$$

$$q^2 = \frac{n_z^2}{L_z^2}$$

$$E = hc\sqrt{\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right)} \quad (9.23)$$

For massless boson particle inside box in a ground state, with each length of the cube being $L_x = L_y = L_z = a$. Quantum numbers for $n_x = n_y = n_z = 1$ for the ground state. Therefore, the energy of massless boson inside cube is

$$E = \sqrt{3}\frac{hc}{a} \quad (9.24)$$

For N massless bosons under BEC, the equation for BEC at absolute zero requires the condition $T < T_B$. As T_B is Bose-Einstein temperature, The Bose-Einstein gas: excited and ground states population is given by

$$n_0 = N[1-\left(\frac{T}{T_B}\right)^{\frac{3}{2}}] \quad (9.25)$$

4.3 This distribution of energy states of gravitons no longer can be used for absolute zero as explained here 4.6. Population N_0 gravitons in the ground state, At $T = 0$, we have $n_0 = N$and all the particles sit in the ground state. Which is to say we see gravitons fall into lowest energy level. Then the equation for energy becomes using the equation, the total energy equation becomes 9.8

$$E_{Total} = N\sqrt{3}\left(\frac{hc}{a}\right) \quad (9.26)$$

This is the total energy of bosons which is getting calculated for gravitons, in the ground state at absolute zero. As this is relativistic case mass terms can be defined, using mass energy relation from special relativity.

$$E = mc^2 \tag{9.27}$$

Rewriting it as

$$m = \frac{E_{Total}}{c^2} \tag{9.28}$$

using equation 9.26 into above equation gives.

$$m = \frac{N\sqrt{3}\frac{hc}{a}}{c^2} \tag{9.29}$$

$$m = \frac{\sqrt{3}Nh}{ac} \tag{9.30}$$

The value of a is $a = V^{\frac{1}{3}}$ corresponding to volume of the cube. Now applying this logic to the black hole Schwarzschild radius R_s to volume of the sphere to define Schwarzschild volume V_r

$$V_R = \frac{4}{3}\pi r_s^3 \tag{9.31}$$

Schwarzschild radius R_s is

$$R_s = \frac{2GM}{c^2} \tag{9.32}$$

$$V_R = \frac{4}{3}\pi\left(\frac{2GM}{c^2}\right)^3 \tag{9.33}$$

$$a = V^{\frac{1}{3}} = V_R^{\frac{1}{3}} = \left(\frac{4}{3}\pi\left(\frac{2GM}{c^2}\right)^3\right)^{\frac{1}{3}} = \left(\frac{4}{3}\pi\right)^{\frac{1}{3}}\left(\frac{2GM}{c^2}\right) = \left(\frac{2GM}{c^2}\right) \tag{9.34}$$

substituting above equation as a into mass term gives 9.30

$$m = \frac{\sqrt{3}Nh}{\left(\frac{2GM}{c^2}\right)c} \tag{9.35}$$

$$m = \frac{\sqrt{3}Nh}{\left(\frac{2GM}{c}\right)} \tag{9.36}$$

$$m = \frac{\sqrt{3}Nhc}{(GM)} \tag{9.37}$$

$$m = \frac{\sqrt{3}Nhc}{(GM)} \tag{9.38}$$

$$m = \frac{Nhc}{GM} \tag{9.39}$$

Defining the Schwarzschild radius R_s as function of N gravitons in ground state, denoted with N_g

$$R_s(N_g) = \frac{2GM}{c^2} \tag{9.40}$$

Using the mass term derived from the equations 9.38 into the equation 9.40.

$$R_s(N_g) = \frac{2G\left(\dfrac{Nhc}{GM}\right)}{c^2} \tag{9.41}$$

$$R_s(N_g) = \frac{Nh}{Mc} \tag{9.42}$$

Replacing h with $\hbar$ that is $\hbar = \dfrac{h}{2\pi}$, as $h = 2\hbar\pi$. rewriting the above equation,

$$R_s(N_g) = \frac{N\hbar\pi}{Mc} \tag{9.43}$$

- M - Mass of the black hole
- c- speed of light
- $\hbar$- reduced planck constant
- N- N number of gravitons in ground state
- N_g - Schwarzschild radius R_s as function of N number of gravitons.

as mass of the black hole can be rewritten as

$$M = \frac{c^2 R_s}{2G} \tag{9.44}$$

Substuting 9.44 in 9.43.

$$R_s(N_g) = \frac{N\hbar\pi}{\dfrac{c^2 R_s}{2G}c} \tag{9.45}$$

$$R_s(N_g) = \frac{2GN\hbar\pi}{c^3 R_s} \tag{9.46}$$

$$R_s(N_g) = N\frac{G\hbar}{c^3 R_s} \tag{9.47}$$

Since $R_s(N_g) = R_s$ The final equation becomes

$$R_s^2 = N\frac{G\hbar}{c^3} \tag{9.48}$$

$$R_s = \sqrt{N}\sqrt{\frac{G\hbar}{c^3}} \tag{9.49}$$

As $l_p = \sqrt{\dfrac{G\hbar}{c^3}}$ the final equation is

$$R_s = \sqrt{N}\,(l_p) \tag{9.50}$$

This equation concludes in similar way compared to 8.3. Prof. Gia Dvali and Prof. Cesar Gomez in their paper [19] [20] that the black hole is a self-sustained Bose Einstein condensate of gravitons. The equation 9.50 for Schwarzschild radius R_s for N gravitons can also be rewritten as, in this case

I am not going to consider constant terms in the calculation for this part which is 3.674547035. I am directly taking this equation $R_s = \sqrt{N}\,(l_p)$ The occupational number for gravitons is given as

$$R_s = \sqrt{N}\,(l_p)\,,\,\frac{R_s^2}{l_p^2} = N \tag{9.51}$$

$$\frac{R_s^2}{l_p^2} = N = \frac{(\frac{2GM}{c^2})^2}{\frac{G\hbar}{c^3}} = \frac{\frac{G^2M^2}{c^4}}{\frac{G\hbar}{c^3}} = \frac{GM^2}{c\hbar} \tag{9.52}$$

As Planck mass is $M_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}}$ which is for $M_{\mathrm{P}}^2 = \frac{\hbar c}{G}$

$$N = \frac{M^2G}{\hbar c} = \frac{M^2}{M_P^2} \tag{9.53}$$

This is the similar derivation to that of 8.2 published research on Bose-Einstein condensate of gravitons. Derivation for N gravitons in Schwarzschild radius R_s for N gravitons. Here the derivations are showing the same results that as it is agreeing with the solutions of Prof.Gia Dvali and Prof.Cesar Gomez in their paper [19] [20] that the black hole is a self-sustained Bose Einstein condensate of gravitons. These equations include fundamental constants such as Planck mass and Planck length with Gravitational constant.

10 PRESSURE INSIDE THE BLACK HOLE

By using the above conclusions on Schwarzschild radius R_s for N gravitons in Bose Einstein condensate.

Thermodynamically I can define the pressure inside box that is redefining the N as defined earlier , I get

$$N = \frac{R_s^2}{l_p^2} \tag{10.1}$$

Redefining the Energy equation using the equation 9.26, replacing N

$$E_{Total} = \frac{R_s^2}{l_p^2}\sqrt{3}\frac{hc}{a} \tag{10.2}$$

Since a is already been derived 9.34, as $a = V^{\frac{1}{3}} = \left(\frac{2GM}{c^2}\right) = R_s$.

$$E_{Total} = \frac{R_s^2}{l_p^2}\sqrt{3}\left(\frac{hc}{R_s}\right) \tag{10.3}$$

$$E_{Total} = \sqrt{3}\frac{R_s}{l_p^2}hc \tag{10.4}$$

Replacing equations to get volume from R_s, as $a = V^{\frac{1}{3}} = R_s$.
Therefore, $R_s = V^{\frac{1}{3}}$.

$$E_{Total} = \sqrt{3}\frac{V^{\frac{1}{3}}}{l_p^2}hc \tag{10.5}$$

$$E_{Total} = \frac{hc}{l_p^2}V^{\frac{1}{3}} \tag{10.6}$$

Since this is for the pressure inside box at absolute zero temperature as $T = 0$. The Fundamental thermodynamic relation becomes,fundamental thermodynamic relation is generally expressed as an infinitesimal change in internal energy in terms of infinitesimal changes. The laws of thermodynamics are discussed here 3.1.2

$$dU = Tds - Pdv \tag{10.7}$$

- U is internal energy
- T- Temperature is absolute zero
- P- Pressure
- V- Volume
- S- Entropy

Since Energy is denoted with E=U.

$$dE = -Pdv, p = -\frac{dE}{dV} \tag{10.8}$$

substituting the energy 10.7

$$p = -\frac{hc}{l_p^2}\frac{d}{dV}\left(V^{\frac{1}{3}}\right) \tag{10.9}$$

$$p = -\frac{1}{3}\frac{hc}{l_p^2}\left(V^{-\frac{2}{3}}\right) = -\frac{hc}{l_p^2}\left(\frac{1}{V^{\frac{2}{3}}}\right) \tag{10.10}$$

$$p = -\frac{hc}{l_p^2}\left(\frac{1}{(V^{\frac{1}{3}})^2}\right) \tag{10.11}$$

As as $a = V^{\frac{1}{3}} = R_s$

$$p = -\frac{hc}{l_p^2}\left(\frac{1}{(R_s)^2}\right) \tag{10.12}$$

$$p = -\frac{hc}{l_p^2}\left(\frac{1}{(R_s)^2}\right) \tag{10.13}$$

This is the pressure that can be expressed inside the condensed black hole with gravitons. this includes fundamental constants such as Planck length l_p with velocity of light c along with R_s, the schwarzschild radius.

11 ULTRA-RELATIVISTIC CASE

In order to understand the Black hole entropy. Jacob Bekenstein and Hawking predicted that black holes must have entropy and the entropy of the black holes must be proportional to that of surface of the event horizon. The entropy of black hole been discussed here 6.13, furthermore, the case of ultra relativistic particle such as graviton which is massless with spin 2 this is discussed here 2.2.

Classical expression for kinetic energy in terms of momentum

$$\varepsilon(p) = \frac{P^2}{2m} \tag{11.1}$$

Setting up $\frac{p}{2m} \to c$ for ultra relativistic case as c is velocity of light this logic can be applied to the relativistic particle such as graviton. The energy of an ultra-relativistic particle is almost completely due to its momentum as , thus can be approximated by $E = pc$

$$\varepsilon(p) = p\frac{p}{2m} = pc \tag{11.2}$$

Using the klein gordon and gross pitaevskii equation for ultra relativistic case becomes the equation becomes klein gordon-Gross pitaevskii equation. Using equation 9.9 and gross-pitaevskii equation which has been discussed here 5.4. The interaction term $U_o|\psi|^2\psi - \mu\psi$ which is combined with relativistic term $\frac{\partial^2\psi}{\partial t^2} - c^2\Delta\psi$ gives us the ultra relativistic description of particle which is massless for mutual interactions and weakly interaction between gravitons.

$$\frac{\partial^2\psi}{\partial t^2} - c^2\Delta\psi + U_o|\psi|^2\psi - \mu\psi = 0 \tag{11.3}$$

to describe the Bogoliubov condensate to include the interaction term, which is

$$U_o|\psi|^2\psi - \mu\psi = 0$$

The homogeneous solution would be Since Φ is constant ψ=ψ_0

$$|\psi_o|^2 = \mu/U_o, \psi_o = \sqrt{\mu/U_o} \tag{11.4}$$

For the corrections we need new term to denote this

$$\psi_o + \chi = \Phi$$

$$\bar{\psi_o} + \bar{\chi} = \Phi^*$$

$$\frac{\partial^2\psi}{\partial t^2} - c^2\Delta\chi + U_o|\psi_o + \chi|^2(\psi_o + \chi) - \mu(\psi_0 + \chi) = 0 \tag{11.5}$$

$$\bar{\psi_o} * \psi_o = |\psi_o|^2$$

Linearize in χ

$$\frac{\partial^2\chi}{\partial t^2} - c^2\Delta\chi + U_o[|\psi_o|^2 + \chi\psi_o + \bar{\chi}\psi_o](\psi_o + \chi) - \mu(\psi_o + \chi) = 0 \tag{11.6}$$

$$\frac{\partial^2\chi}{\partial t^2} - c^2\Delta\chi + U_o[|\psi_o|^2\chi + (\chi\psi_o + \bar{\chi}\psi_o)\psi_o + |\psi_o|^2\psi_o] - \mu(\psi_o + \chi) = 0 \tag{11.7}$$

$$\frac{\partial^2\chi}{\partial t^2} - c^2\Delta\chi + U_o[|\psi_o|^2\chi + (\chi\psi_o + \bar{\chi}\psi_o)\psi_o + |\psi_o|^2\psi_o] - \mu(\psi_o + \chi) = 0 \tag{11.8}$$

$$\frac{\partial^2\chi}{\partial t^2} - c^2\Delta\chi + U_o|\psi_o|^2\chi + U_o(\chi + \bar{\chi})\psi_o^2 + U_o|\psi_o|^2\psi_o - \mu(\psi_o + \chi) = 0 \tag{11.9}$$

$$\frac{\partial^2\chi}{\partial t^2} - c^2\Delta\chi + U_o|\psi_o|^2\chi + U_o(\chi + \bar{\chi})\psi_o^2 + U_o|\psi_o|^2\psi_o - \mu\psi_o - \mu\chi = 0 \tag{11.10}$$

using the condition 11.4 as $U_o|\psi_o|^2\psi_o = \mu\psi_o$,$\mu\psi_o$ gets cancelled and replacing $|\psi_o|^2 = \mu/U_o$

$$\frac{\partial^2\chi}{\partial t^2} - c^2\Delta\chi + U_o\frac{\mu}{U_o}\chi + U_o(\chi + \bar{\chi})\psi_o^2 - \mu\chi = 0 \tag{11.11}$$

The final linearised equation

$$\frac{\partial^2\chi}{\partial t^2} - c^2\Delta\chi + U_o(\chi + \bar{\chi})\psi_o^2 = 0 \tag{11.12}$$

Pairs of equation with complex conjugate

$$\frac{\partial^2\chi}{\partial t^2} - \Delta c^2\chi + U_o(\chi + \bar{\chi})\psi_o^2 = 0 \tag{11.13}$$

$$-\frac{\partial^2\bar{\chi}}{\partial t^2} - c^2\Delta\bar{\chi} + U_o(\chi + \bar{\chi})\psi_o^2 = 0 \tag{11.14}$$

Replacing $\frac{\partial}{\partial t} \to -i\omega$ and $\frac{\partial}{\partial x} \to ik$ as a result I get, $\frac{\partial^2}{\partial^2 t} \to -\omega^2$,$\frac{\partial^2}{\partial^2 x} \to -k^2$, equation 11.13

$$-\omega^2\chi - c^2(-k^2)\chi + U_o(\chi + \bar{\chi})\psi_o^2 = 0 \tag{11.15}$$

$$-\omega^2\chi + c^2(k^2)\chi + U_o(\chi + \bar{\chi})\psi_o^2 = 0 \tag{11.16}$$

The conjugate equation becomes 11.14

$$-(-\omega^2)\bar{\chi} - c^2(-k^2)\bar{\chi} + U_o(\chi + \bar{\chi})\psi_o^2 = 0 \tag{11.17}$$

$$(\omega^2)\bar{\chi} + c^2(k^2)\bar{\chi} + U_o(\chi + \bar{\chi})\psi_o^2 = 0 \tag{11.18}$$

using both equations to represent in 2x2 determinant matrix form 11.16 and 11.18.

$$Det\begin{bmatrix} c^2k^2 + U_o\psi_o^2 - \omega^2 & U_o\psi_o^2 \\ U_o\psi_o^2 & c^2k^2 + U_o\psi_o^2 + \omega^2 \end{bmatrix} = 0 \tag{11.19}$$

$$\left[(c^2k^2 + U_o\psi_o^2)^2 - (\omega^2)^2 - (U_o\psi_o^2)^2\right] \tag{11.20}$$

$$\left[(c^4k^4 + U_o^2\psi_o^4 + 2c^2k^2U_o\psi_o^2) - (\omega^4) - (U_o^2\psi_o^4)\right] \tag{11.21}$$

$$\left[(c^4k^4 + 2c^2k^2U_o\psi_o^2) - (\omega^4))\right] \tag{11.22}$$

$$\omega^4 = c^4k^4 + 2c^2k^2U_o\psi_o^2 \tag{11.23}$$

$$\omega(k) = (c^4k^4 + 2c^2k^2U_o\psi_o^2)^{\frac{1}{4}} \tag{11.24}$$

For the attractive case equation becomes as the above equation with positive coupling constant, gives the repulsive interactions between gravitons. For attraction negative coupling constant can be taken $-U_o$. This dispersion relation can be rewritten as.

$$\omega(k) = (c^4k^4 - 2c^2k^2U_o\psi_o^2)^{\frac{1}{4}} \tag{11.25}$$

Now considering the repulsive for postivie coupling constant case 11.24 as $+U_o$ among gravitons. For the large values of k as it is wave-vector. The term $2c^2k^2U_o\psi_o^2$ can be neglected as c^4k^4 increases linearly as $\omega(k) = ck$. $\omega(k)$ plotted against k on the graph.

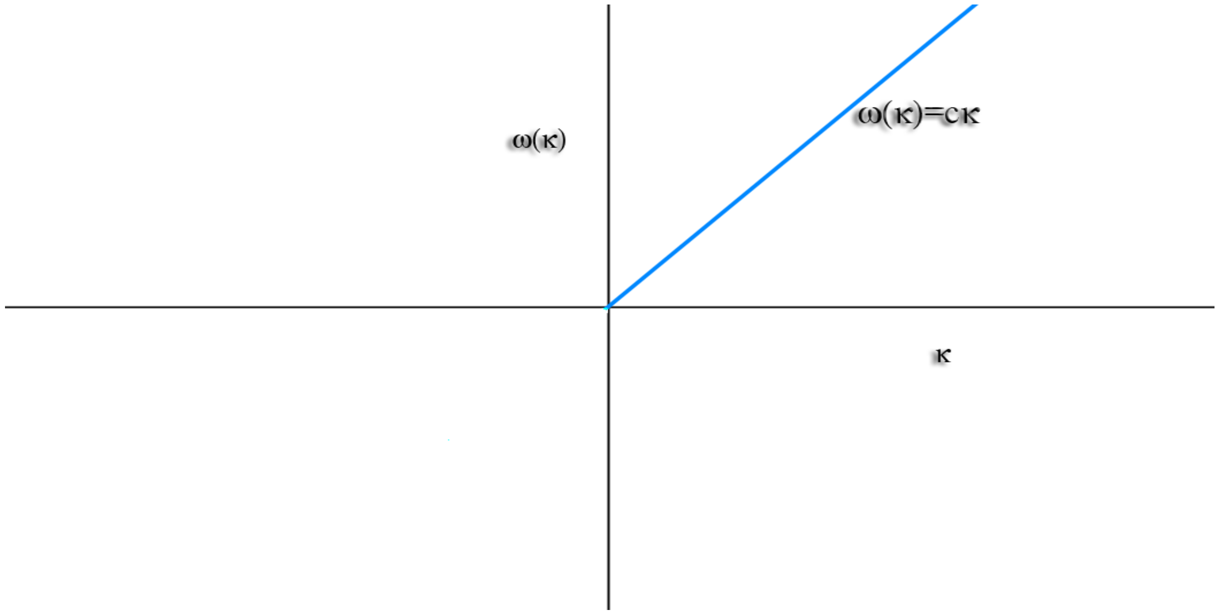

Figure 17: The graph illustrating the relation between $\omega(k)$ and k for positive value of large values of k in repulsive case as coupling constant $+U_o$ for the equation 11.24

When k is very small the term $2c^2k^2U_o\psi_o^2$ is considered and c^4k^4 becomes small. the term $\omega(k) = (2c^2k^2U_o\psi_o^2)^{\frac{1}{4}}$ as this becomes relation with $\omega(k)$ and $+\sqrt{ck\psi_o}$.

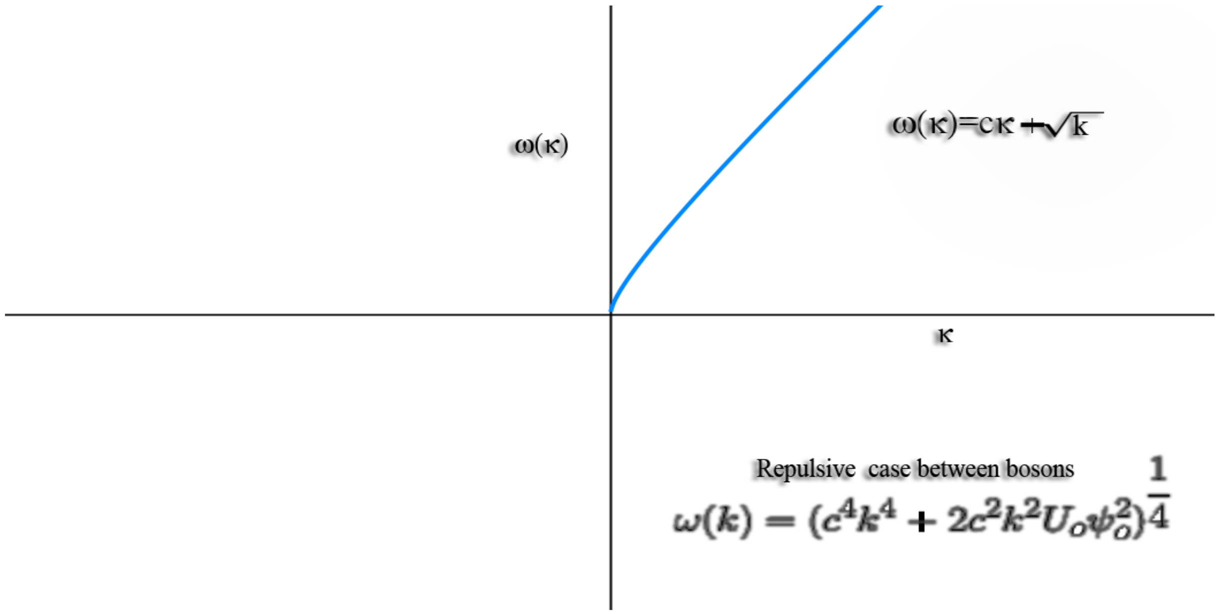

Figure 18: The graph illustrating the relation between $\omega(k)$ and k for positive value of large values of k in repulsive case as coupling constant $+U_o$ for the equation 11.24

In non-relativistic Bogoliubov theory small values of k for dispersion relation is sound waves as ω is proportional to k but here in relativistic case we have $\sqrt{k}$. Taking the attractive case between the bosons 11.25 for large values of k we have, For the large values of k as it is wave-vector. The term $2c^2k^2U_o\psi_o^2$ can be neglected as c^4k^4 increases linearly as $\omega(k) = ck$. $\omega(k)$ plotted against k on the graph. When both terms are summed as displayed on the graph, the curve tends to remain 0 as the value of k increases $\omega(k)$ increases.

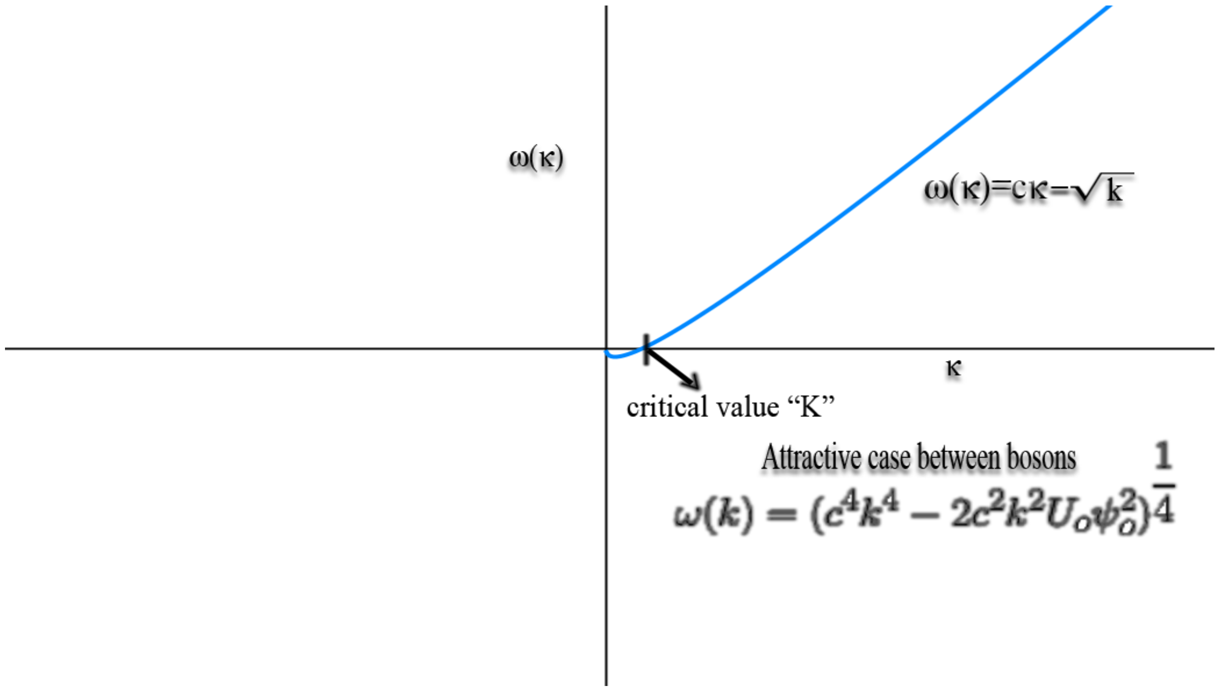

Figure 19: The graph illustrating the relation between $\omega(k)$ and k for positive value of large values of k in attractive case as coupling constant $-U_o$ for the equation 11.25 The imaginary $-\sqrt{k}$ terms are shown.

For the small values of k as imaginary frequency corresponds to the instability frequency, as it is not propagating mode. bogoliubov modes do not exist at small k as ω gets imaginary. If this modes are excited state BEC system would system blows up. The black hole would form when the value of k is grater than that of critical value.

As for the values of k at which $\omega = 0$ is for the equation 11.25.

$$0 = (c^4k^4 - 2c^2k^2U_o\psi_o^2) \tag{11.26}$$

$$(c^2k^2)^2 = (2c^2k^2U_o\psi_o^2) \tag{11.27}$$

$$(ck)^2 = (2U_o\psi_o^2) \tag{11.28}$$

$$ck = (\sqrt{2U_o}\psi_o) \tag{11.29}$$

$$k = \frac{\sqrt{2U_o}\psi_o}{c} \tag{11.30}$$

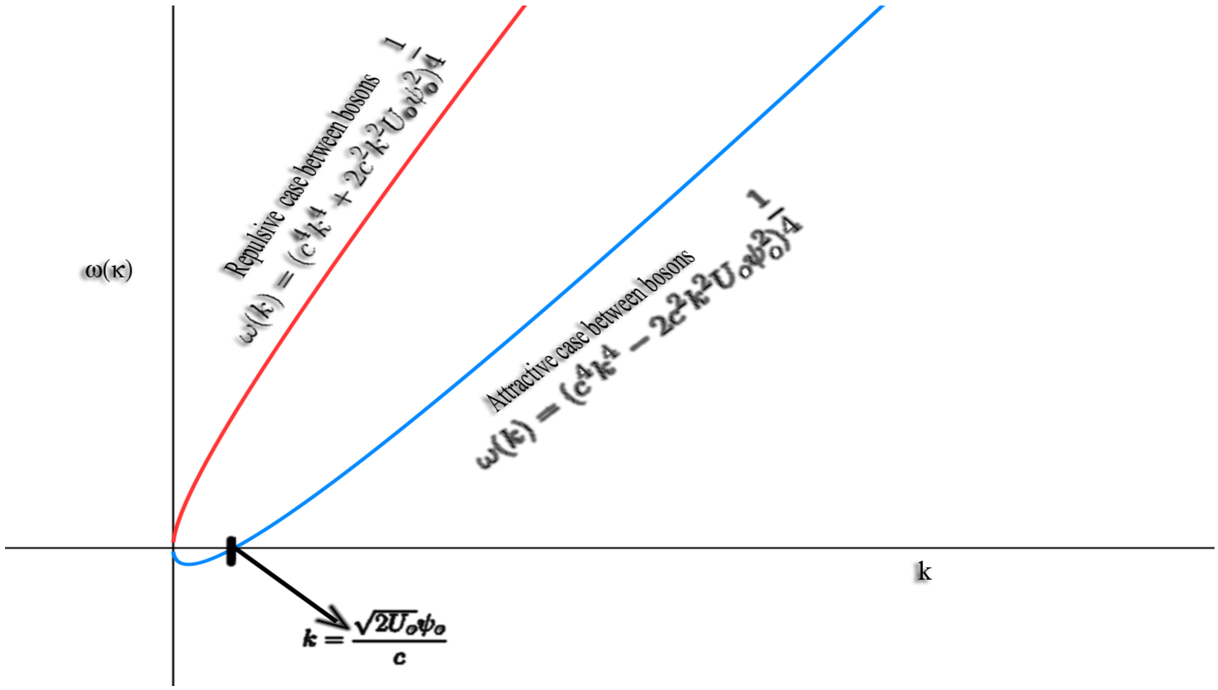

Figure 20: The graph illustrating the critical point k at which $\omega(k) = 0$ showing both cases of attractive and repulsive

In quantum mechanics for particle in a box we have condition for wave vector $k = \dfrac{n\pi}{L}$ are allowed values of k for volume V as $n = (1, 2, 3, 4....)$ as L is size of the box, here in the Black hole's case it's schwarzschild radius R_s as relationship becomes $k = \dfrac{n\pi}{R_s}$. Wave-number is restricted to certain number of values of k. As for the finite size system of Black hole the k value cannot be small. As for the Planck's relation $E = \hbar\omega(k)$ as $\omega(k) = 0$, the Planck's relation for energy becomes 0. The exciting Bogoliubov modes cost no energy. As Black Holes filled with gravitons can change states of system with no energy costs. Black Holes can be many different states of same energy that is entropy of black hole, as the zero energy level of ground state level entropy. Excited state has same energy as ground state because of Bogoliubov modes has zero frequency. The general equation for entropy is

$$S = k_B log N \tag{11.31}$$

N is maximum number of quanta (graviton) that can be squeezed into Bogoliubov modes. To define Bekenstein's entropy, N must be defined and how many gravitons can be fit at zero frequency. When $\omega = 0$ indicates that ground level is degenerate. If a quanta is being added into Bogoliubov modes the energy will not change by Planck's relationship due to the critical value of k. As it is ground level degeneracy. It is needed to be verified if this is true which is if Black hole is critical, such that system size is $\dfrac{1}{R_s}$ is critical momentum as this can be verified by coupling constant for graviton to graviton interaction. This has been already derived in Prof. Gia Dvail's papers 8.3 [19] [20]. The coupling constant between graviton to graviton interaction is taken as α_{gr} which is dimensionless quantity.

$$U_0 = \alpha_{gr} = \frac{L_p^2}{\lambda^2} \tag{11.32}$$

as λ is wavelength of gravitons.

$$U_0 = \alpha_{gr} = \frac{L_p^2}{\lambda^2} \tag{11.33}$$

as λ is wavelength of gravitons. Since $\lambda = \sqrt{N}L_P$ and $R_s = \sqrt{N}L_P$ from the derivation discussed 8.5

$$U_0 = \alpha_{gr} = \frac{L_p^2}{\lambda^2} = \frac{L_p^2}{R_s^2} \tag{11.34}$$

The coupling constant above is the dimensionless, therefore using dimension analysis I will try to find the value for $U_o\psi_o^2$ to equate it to the above coupling constant.

For α_{gr} using the dimension analysis I get $\alpha_{gr} = \frac{U_0}{c^2}$, rewriting it becomes $c^2\alpha_{gr} = U_0$. For N number of gravitons as the density would be $\rho \sim |\psi_0|^2 = \psi_0 * \psi_0$ dimensionally equalized to

$$|\psi_0|^2\frac{1}{L_p} = \rho \tag{11.35}$$

The density can be rewritten as N number

$$N = R_s^3|\psi_0|^2\frac{1}{L_p}, \frac{NL_p}{R_s^3} = |\psi_0|^2 \tag{11.36}$$

Coupling constant dimensionally equal to

$$c^2\alpha_{gr} = U_0 \tag{11.37}$$

Now replacing the coupling constant term in the equation we can find the critical point k substituting in equation 11.26. We get

as

$$U_0|\psi_0|^2 = \frac{NL_p}{R_s^3}c^2\alpha_{gr} = \frac{NL_p}{R_s^3}c^2\frac{L_p^2}{R_s^2} = c^2\frac{NL_p^3}{R_s^5} \tag{11.38}$$

taking the values into this equation in equation 11.26

$$0 = (c^4k^4 - 2c^2k^2U_o\psi_o^2)$$

$$0 = (c^4k^4 - 2c^4k^2\frac{NL_p^3}{R_s^5}) \tag{11.39}$$

$$c^4k^4 = 2c^4k^2\frac{NL_p^3}{R_s^5} \tag{11.40}$$

$$k^4 = 2k^2\frac{NL_p^3}{R_s^5} \tag{11.41}$$

$$k^2 = 2\frac{NL_p^3}{R_s^5} \tag{11.42}$$

$$k = \sqrt{\frac{2NL_p^3}{R_s^5}} \tag{11.43}$$

Black hole in this picture does have the critical point. This can be illustrated on the graph

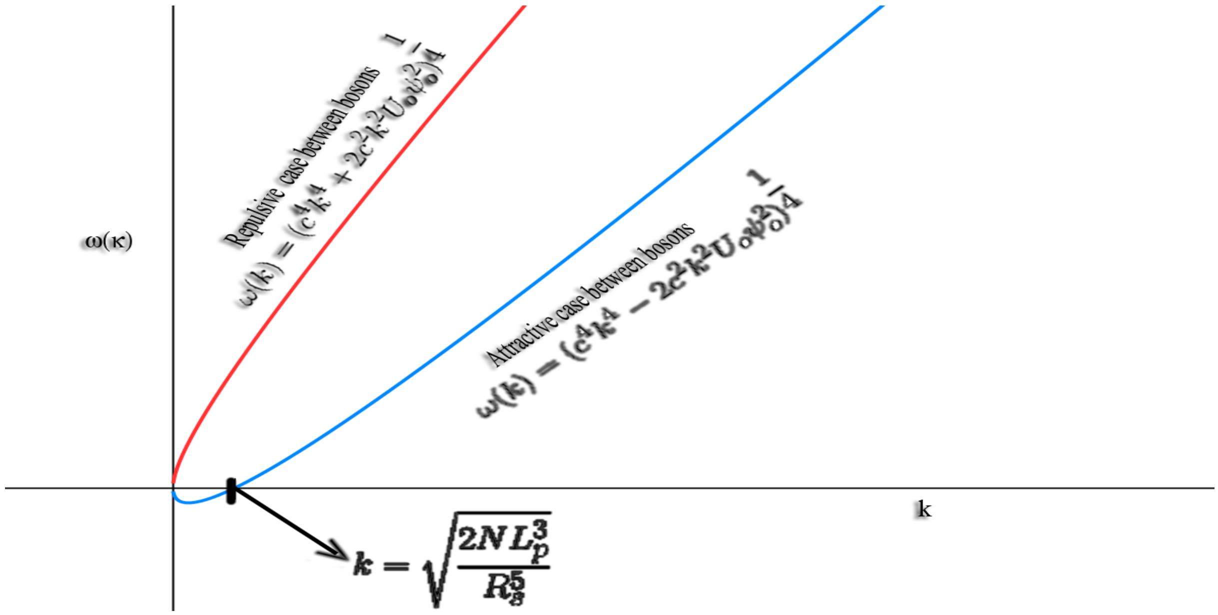

Figure 21: The graph illustrating the critical point k at which $\omega(k) = 0$ showing both cases of attractive and repulsive, critical value only exist for attractive case not for repulsion. Above the critical value k the Bose-Einstein condensate of gravitons is possible.

12 CONCLUSION

The Bose Einstein condensate of gravitons can be described by the corresponding R_s schwarzschild radius which is to say that the schwarzschild radius of the black hole is directly proportional to the square root of the N gravitons in Bose Einstein condensate. Planck length (L_p) acts as proportionality constant 9.50.

$$R_s = \sqrt{N}\,(L_p) \tag{12.1}$$

The above equation can also be rewritten as the value for N number of gravitons. 9.53

$$N = \frac{M^2 G}{\hbar c} = \frac{M^2}{M_P^2} \tag{12.2}$$

The Bose Einstein condensate can have internal pressure the pressure can be calculated by the equation 10.14

$$p = -\frac{hc}{l_p^2}\left(\frac{1}{(R_s)^2}\right) \tag{12.3}$$

This is the pressure that can be expressed inside the condensed black hole with gravitons. this includes fundamental constants such as Planck length l_p with velocity of light c along with R_s, the schwarzschild radius.

In order to understand the interactions among the gravitons as graviton is ultra-relativistic particle with spin-2, using GrossPitaevskii equation for interaction term and klein gordon equation for relativistic term11.24. The Bogoliubov dispersion relation for the elementary excitations of the weakly-interacting Bose gas is shown to hold for the case of the weakly-interacting Graviton gas or N weakly interacting gravitons. This equation is derived for repulsive interactions among gravitons. In case for repulsive interaction the coupling constant in the equation is $+U_o$.

$$\omega(k) = (c^4k^4 + 2c^2k^2U_o\psi_o^2)^{\frac{1}{4}} \tag{12.4}$$

When considered for the repulsive case the gravitons Bose Einstein condensate with weakly interactions shows the critical value for wave vector k. The coupling constant $+U_o$ becomes $-U_o$ for the case of attraction11.25.

$$\omega(k) = (c^4k^4 - 2c^2k^2U_o\psi_o^2)^{\frac{1}{4}} \tag{12.5}$$

In this negative coupling constant attraction case, the equations shows the Critical value for k at which the Bogoliubov dispersion relation $\omega(k) = 0$ where Black holes does not form the Bose Einstein condensate of gravitons at 11.43.

$$k = \sqrt{\frac{2NL_p^3}{R_s^5}} \tag{12.6}$$

Above this critical value the black hole forms but below the critical value gives the imaginary values for Bogoliubov dispersion relation $\omega(k)$ which mean Black hole can not be formed for unstable values of k that is below the critical value of $k = \sqrt{\frac{2NL_p^3}{R_s^5}}$. This is illustrated on the graph 11.

In order to understand Bekenstein's entropy, N must be defined and how many gravitons can be fit at zero frequency. When $\omega = 0$ indicates that ground level is degenerate. If a quanta or a graviton is being added into Bogoliubov modes the energy will not change by Planck's relationship due to the critical value of k as $\omega(k) = 0, E(k) = \hbar\omega(k), E(k) = 0$ as it is ground level degeneracy. The Bose Einstein Condensate of Black Holes can be filled with gravitons can change states of system with no energy costs. Black Holes can be many different states of same energy that is entropy of black hole, as the zero energy level of ground state level entropy. Excited state has same energy as ground state because of Bogoliubov modes has zero frequency. In this research it can be further investigated for Hawking radiation and quantum properties of black hole in order to understand how gravity would actually work in quantum mechanics perspective as analogue models might give us the hints.

12.1 Temperature and Bose Einstein condensate, Thinking in the practice perspective

The analogue model of the black hole Bose Einstein condensate of gravitons is theoretical perspective. This is theoretically based model although there are many theories and analogue gravity models to understand gravity. We have to understand that the concept of Bose-Einstein condensate exists only below the critical temperature required to form low temperature of the condensate. The black holes do experience the thermal bath from binary system or surroundings of the condensed gravitons such as in a system when black hole feeds on the mass of the sun, therefore increasing the temperature of the condensed system. When temperature increases as a consequence this breaks the condensate requirement for the Bose-Einstein statistics to work with gravitons. Furthermore, the concept of absolute zero is less likely seen in the universe due to the law of thermodynamics. The laws of thermodynamics used to in calculation of many theories in classical physics to theoretical physics of condensed matter physics. At the same time many calculations are done theoretically not taking second law of thermodynamics seriously. Mathematically it is always good to investigate the possibilities of absolute zero however it is not considering the second law of thermodynamics which is fundamental physics. The bose Einstein condensate was achieved with using a gas of rubidium atoms cooled to 170 Nano kelvin (nK) or $1.7107 Kelvin$. The photons Bose Einstein condensate was also achieved in 2010 [31]. The problems with the real observation of Bose Einstein condensate of gravitons is that the role played by temperature in empty space as I discussed in this section 3.4.1. The space is full of cosmic microwave background radiation which involves in the temperature around temperature slightly less than 3 degrees Kelvin (about 2.76 K), from micro wave background radiation. It is possible the background radiation might prevent existence of black hole as the entropy of black holes can be increased from its surrounding radiation. Existence of gravitons are yet unknown, perhaps maybe we may discover the graviton in Large hadron collider CERN. The theoretically there are predicted by still remains theoretical predictions.

If discovered one day we may not know how to capture gravitons in a box or closed container. Like photons it may or may not even be condensed in the lab. According to string theories gravitons may be able to escape to higher dimensions. Possible it may not be possible to so the gravitons being collected in the lab. Why is Hawking radiation as leakage of gravitons?, if we consider the definition of event horizon or Schwarzschild radius any observer or object falls into black hole would need to have escape velocity faster than that of speed of light. However, in this picture of Bose-Einstein condensate of gravitons the gravitons would leak from the condensate and emit radiation as hawking radiation. Here there is no pair production of virtual particles from quantum mechanics is used. We know from special relativity that nothing can move after than speed of light, thus gravitons can only move with the speed of light. How gravitons would escape when it is viewed from classical perspective of the black hole physics. Mostly while looking into two different perspectives of black hole theories. One is the classical-quantum version and other one is from analogue models of condensed matter physics. They both seem to be radically different approaches which have their own perspectives; furthermore they both seem to view black holes mechanics in different way. It is always important to look for all possibilities. However, which models would actually describe the nature of physical reality of black hole. In condensed matter physics analogue models of Black hole there is no such things as singularity but one must ask can singularity fundamentally allowed in nature ? If it is allowed in nature how one would explain singularity from classical perspective in condensed matter physics of analogue models of Black Holes. I was actually considering to look for the possibilities to describe the singularities such as considering the possibility of infinite compressibility in the analogue model of Bose Einstein condensate of gravitons.

12.2 No Black holes ? The current research

During my research on Black hole analogue models I have been reading disappointing news about questioning the existence of Black hole. In the press releases there are lot of articles that are released based on Prof. Stephen Hawking claims that Black holes do not exist in his research papers which was not been peer reviewed during the time and later on other physicists disagreed with Prof. Stephen Hawking. Again, recently on 24th Sep 2104 yet another claims that Black hole do not exist and they may never be formed during the super massive collapse [40] [39], again these papers are not peer reviewed yet. The existence are Black hole being questioned time to time it is said that Albert Einstein did not want to believe in the existence of black hole however his theories predict, Unless we have observational evidence from astronomical date or Hubble space telescope. We had indirect evidence from Cygnus X-1 which is well known for black hole candidate. There is no direct observational evidence yet we just have indirect evidence. We know that the concept of Black hole is well-established concept it should not be overlooked until if we would have any well published theories against the existence of black hole and until scientific community accepts those claims. I personally would not want to believe in non-existence of black holes, it is sad and disappointing if they don't exist considering such a beautiful predictions from theory of General Relativity.

References

[1] The nobel prize in physics 1983 @ONLINE, 1983. http://www.nobelprize.org/nobel_prizes/physics/laureates/1983/.

[2] The nobel prize in physics 1993 @ONLINE, 1993. http://www.nobelprize.org/nobel_prizes/physics/laureates/1993/index.html.

[3] Stellar structure and evolution 1993 @ONLINE, 1993. http://www.astro.umd.edu/~miller/teaching/astr606/.

[4] Hawking radiation glimpsed in artificial black hole @ONLINE, 2010. http://www.newscientist.com/article/dn19508-hawking-radiation-glimpsed-in-artificial-black-hole.html?page=2#.VCcqNcnp_FY.

[5] Theories of the brane: Lisa randall @ONLINE, 2013. http://www.pbs.org/wgbh/nova/blogs/physics/2012/09/why-is-gravity-such-a-weakling/.

[6] Bicep2 gravitational wave result bites the dust thanks to new planck data @ONLINE, 2014. http://physicsworld.com/cws/article/news/2014/sep/22/bicep2-gravitational-wave-result-bites-the-dust-thanks-to-new-planck-data.

[7] BEISER ARTHUR. Concepts of modern physics, 1995.

[8] Carlos Barceló, Stefano Liberati, Matt Visser, et al. Analogue gravity. *Living Rev. Rel*, 8(12):214, 2005.

[9] V Belokurov, NW Evans, A Moiseev, LJ King, PC Hewett, M Pettini, L Wyrzykowski, RG McMahon, MC Smith, G Gilmore, et al. The cosmic horseshoe: Discovery of an einstein ring around a giant luminous red galaxy. *The Astrophysical Journal Letters*, 671(1):L9, 2007.

[10] Claus Borgnakke and Richard Edwin Sonntag. *Fundamentals of thermodynamics*. Wiley New York, 2009.

[11] Robert H Brandenberger. Introduction to early universe cosmology. *arXiv preprint arXiv:1103.2271*, 2011.

[12] Bradley W Carroll, Dale A Ostlie, and M Friedlander. *An introduction to modern astrophysics*, volume 523. Addison-Wesley New York, 1996.

[13] Sean Carroll. *From eternity to here: the quest for the ultimate theory of time*. Penguin, 2010.

[14] Sean M Carroll. *Spacetime and geometry. An introduction to general relativity*, volume 1. 2004.

[15] Paul Davies. *The new physics*. Cambridge University Press, 1992.

[16] Clinton Davisson and Lester H Germer. Diffraction of electrons by a crystal of nickel. *Physical Review*, 30(6):705, 1927.

[17] Louis Victor Pierre Raymond de Broglie. Ondes et quanta;waves and quanta, on the theory of quanta. *CR Acad. Sci. Paris*, 177:507, 1923.

[18] Gia Dvali. Composition of space-time, 2013. `http://www.youtube.com/watch?v=hqlduaHzPtw`.

[19] Gia Dvali and Cesar Gomez. Landau–ginzburg limit of black holes quantum portrait: Self-similarity and critical exponent. *Physics Letters B*, 716(1):240–242, 2012.

[20] Gia Dvali and Cesar Gomez. Black hole's quantum n-portrait. *Fortschritte der Physik*, 61(7-8):742–767, 2013.

[21] Albert Einstein. The photoelectric effect. *Ann. Phys*, 17:132, 1905.

[22] Valeri P Frolov and Andrei Zelnikov. *Introduction to black hole physics*. Oxford University Press, 2011.

[23] Brian Greene. *The elegant universe: Superstrings, hidden dimensions, and the quest for the ultimate theory*. Random House LLC, 1999.

[24] Brian Greene. The fabric ofthe cosmos. *The Fabric According to String*, 2004.

[25] Øyvind Grøn and Sigbjorn Hervik. *Einstein's general theory of relativity: with modern applications in cosmology*. Springer, 2007.

[26] Tony Guénault. *Statistical Physics*. Springer, 2007.

[27] Paweł Haensel, Aleksander Yu Potekhin, and Dmitry G Yakovlev. *Neutron stars 1: Equation of state and structure*, volume 326. Springer, 2007.

[28] David Halliday, Robert Resnick, and Jearl Walker. *Fundamentals of physics extended*. John Wiley & Sons, 2010.

[29] IOPBISEP2. Cosmic inflation @ONLINE, May 2014. `http://www.iop.org/resources/topic/archive/cosmic-inflation/index.html`.

[30] Ted Jacobson. Introductory lectures on black hole thermodynamics. *Given at Utrecht U. in*, 1996.

[31] Jan Klaers, Julian Schmitt, Frank Vewinger, and Martin Weitz. Bose-einstein condensation of photons in an optical microcavity. *Nature*, 468(7323):545–548, 2010.

[32] Kiran Kolishetty. Black holes revisited-(literature review). Master's thesis, Lancaster university, 2013-2014.

[33] Urbain Le Verrier. Lettre de m, le verrier à m: Faye sur la théorie de mercure et sur le mouvement du périhélie de cette planète, english: Letter from m. le verrier to faye on the theory of mercury and the motion of the perihelion of the planet. *Comptes rendus hebdomadaires des séances de lAcadémie des sciences*, 49:379–383, 1859.

[34] Umberto Lombardo and Hans-Josef Schulze. Superfluidity in neutron star matter. In *Physics of Neutron Star Interiors*, pages 30–53. Springer, 2001.

[35] Robert Mann. *An introduction to particle physics and the standard model*. CRC Press, 2011.

[36] Philip Massey, Emily M Levesque, and Bertrand Plez. Bringing vy canis majoris down to size: An improved determination of its effective temperature. *The Astrophysical Journal*, 646(2):1203, 2006.

[37] Richard D Mattuck. *A guide to Feynman diagrams in the many-body problem*. Courier Dover Publications, 2012.

[38] J Clerk Maxwell. A dynamical theory of the electromagnetic field. *Proceedings of the Royal Society of London*, 13:531–536, 1863.

[39] Laura Mersini-Houghton. Backreaction of hawking radiation on a gravitationally collapsing star i: Black holes? *Physics Letters B*, 2014.

[40] Laura Mersini-Houghton and Harald P Pfeiffer. Back-reaction of the hawking radiation flux on a gravitationally collapsing star ii: Fireworks instead of firewalls. *arXiv preprint arXiv:1409.1837*, 2014.

[41] Charles W Misner, Kip S Thorne, and John Archibald Wheeler. Gravitation, 1973. *Freeman, San Francisco, CA, Moffat JW, Phys. Rev. D*, 56:6264, 1997.

[42] NASA. Gravity probe a @ONLINE, May 2009. `http://funphysics.jpl.nasa.gov/technical/grp/grav-probea.html`.

[43] NASA. Gravity probe b @ONLINE, May 2009. `http://funphysics.jpl.nasa.gov/technical/grp/grav-probeb.html`.

[44] NASA. Gravitational lensing @ONLINE, May 2014. `http://imagine.gsfc.nasa.gov/docs/features/news/grav_lens.html`.

[45] Isaac Newton. *Philosophiae naturalis principia mathematica*. sumptibus Societatis, 1714.

[46] Mário Novello, Matt Visser, and Grigory E Volovik. *Artificial black holes*, volume 5. World Scientific, 2002.

[47] Michael E Peskin and Daniel V Schroeder. *An introduction to quantum field theory*. Westview, 1995.

[48] Christopher Pethick and Henrik Smith. *Bose-Einstein condensation in dilute gases*. Cambridge university press, 2002.

[49] Lev Pitaevskii, Sandro Stringari, and Anthony Leggett. Bose-einstein condensation. *Physics Today*, 57(10):74–76, 2004.

[50] Max Planck. On the law of distribution of energy in the normal spectrum. *Annalen der Physik*, 4(553):1, 1901.

[51] Dina Prialnik. *An introduction to the theory of stellar structure and evolution*. Cambridge University Press, 2000.

[52] Dr. Robert Thompson. Cold atom lab (cal, 2014. `http://coldatomlab.jpl.nasa.gov/`.

[53] Paul K Townsend. Black holes. *arXiv preprint gr-qc/9707012*, 1997.

[54] Robert M Wald. *General relativity*. University of Chicago press, 2010.

[55] Silke Weinfurtner, Edmund W Tedford, Matthew CJ Penrice, William G Unruh, and Gregory A Lawrence. Classical aspects of hawking radiation verified in analogue gravity experiment. In *Analogue Gravity Phenomenology*, pages 167–180. Springer, 2013.

[56] Stan E Woolsey, A Heger, and TA Weaver. The evolution and explosion of massive stars. *Reviews of Modern Physics*, 74(4):1015–1071, 2002.

[57] A Zagoskin. Quantum theory of many-body systems: techniques and applications, ser. graduate texts in contemporary physics, 1998.