Economics Working Paper Series

2014/024

Coarse Correlated Equilibria in an Abatement Game

Herve Moulin, Indrajit Ray and Sonali Sen Gupta

The Department of Economics
Lancaster University Management School
Lancaster LA1 4YX
UK

Coarse Correlated Equilibria in an Abatement Game*

Herve Moulin[†], Indrajit Ray[‡] and Sonali Sen Gupta[§]

November 2014

Abstract

We consider the well-analyzed *abatement game* (Barrett 1994) and prove that correlation among the players (nations) can strictly improve upon the Nash equilibrium payoffs. As these games are potential games, *correlated equilibrium* – CE – (Aumann 1974, 1987) cannot improve upon Nash; however we prove that *coarse correlated equilibria* – CCE – (Moulin and Vial 1978) may do so. We compute the largest feasible total utility and hence the efficiency gain in any CCE in those games: it is achieved by a lottery over only two pure strategy profiles.

Keywords: Abatement game, Coarse correlated equilibrium, Efficiency gain.

JEL Classification Numbers: C72, Q52.

*Moulin's research was supported by the NSF under Grant CCF 1101202 while Ray's research was supported by the British Academy Small Grant on "Mediation in Environmental Games". We wish to thank all seminar and conference participants at Birmingham, Royal Holloway, São Paulo and Surrey and in particular, Matthew Cole, Colin Rowat, Herakles Polemarchakis and Aurélie Slechten for many helpful comments. Ray also acknowledges the hospitality he received from Rice University and Brown University where part of the work was originally carried out.

[†]Department of Economics, Adam Smith Business School, University of Glasgow, Glasgow G12 8QQ, UK. E-mail: Herve.Moulin@glasgow.ac.uk; Fax: +44.141.330.4939.

[‡]*Author for Correspondences.* Economics Section, Cardiff Business School, Cardiff University, Colum Drive,Cardiff CF10 3EU, UK. E-mail: rayi1@cardiff.ac.uk; Fax: +44.2920.874419.

[§]Department of Economics, Management School, Lancaster University, Lancaster LA1 4YX, UK. E-mail: s.sengupta@lancaster.ac.uk; Fax: +44.1524.594244.

1 INTRODUCTION

Many environmental problems can be suitably investigated and analyzed using different game theoretic models and solution concepts (see Folmer *et al* 1998, Finus 2001, Missfeldt 2002 and Wood 2011 for books and surveys on this area). For example, think of a problem faced by a few countries, each of which emits a pollutant that damages a shared environmental resource. A well-studied model from the literature called the *abatement game* (Barrett 1994) studied this issue as a non-cooperative game played by several countries choosing the level of abatement. In this set-up, any country's payoff is the benefit from abatement minus the cost of abatement where each country's benefit is assumed to depend on the total abatement while the abatement cost is assumed to depend on its own abatement level.

For a non-cooperative model such as the abatement game, it is fairly natural and acceptable to analyze the Nash equilibrium of the game and then compare this equilibrium outcome with the efficient outcome of the model (that maximizes the joint payoff). Indeed, the literature confirms that in this class of games, the Nash equilibrium is suboptimal although it actually can be very close to the efficient outcome. However, considering the magnitude of costs and payoffs involved in these games, even a small efficiency loss can indeed be a huge amount in practice. Therefore, it is important to look for other solution concepts within the non-cooperative framework to improve upon the Nash outcome.

Several (non-cooperative as well as cooperative) solutions have already been analyzed for the abatement game; for instances, Barrett (2001) and McGinty (2007) studied asymmetric versions of the abatement game. Barrett (1994) also considered the Stackelberg model of abatement which was later analyzed by Rubio and Ulph (2006). Finus (2003) presented generalization of Barrett's results in terms of the number of countries in a stable equilibrium. However, the impact of correlation has not been analyzed.[1]

We believe that the concept of correlation for normal form games has a very natural interpretation in the abatement game. A correlation device is a lottery over the outcomes (strategy profiles) of a given normal form game. A *correlated equilibrium* (Aumann, 1974, 1987; thereafter CE) is *implemented*[2] by a mediator who selects strategy profiles according to a publicly known probability distribution and sends to each player the private recommendation to play the corresponding realized strategy. The equilibrium property is that each player finds it optimal to follow this recommendation. In a *coarse correlated equilibrium*[3] (Moulin and Vial 1978; thereafter CCE), the mediator requires more commitment from the players: it asks the players, *before running the lottery*, to either commit to the future outcome of the

[1] Forgó, Fülöp and Prill (2005) and Forgó (2011) recently used (modified versions of) Moulin and Vial's notion of (coarse) correlation in other environmental games. Baliga and Maskin (2003) surveyed some models of mechanisms in this literature.

[2] However, not *fully*, as shown by Kar, Ray and Serrano (2010).

[3] In their paper, Moulin and Vial (1978) called this equilibrium concept a *correlation scheme*. Young (2004) and Roughgarden (2009) introduced the terminology of *coarse correlated equilibrium* that was later adopted by Ray and Sen Gupta (2013) and Moulin, Ray and Sen Gupta (2014), while Forgó (2010) called it a *weak correlated equilibrium*.

lottery or play any strategy of their own without learning anything about the outcome of the lottery. The equilibrium property is that each player finds it optimal to commit *ex ante* to use the strategy selected by the lottery.

In the context of climate change negotiation, in particular for the abatement game, a correlation device can be interpreted as an independent agency providing a recommendation to all relevant countries towards the ultimate goal of global emission reduction. In a CCE of the abatement game, each country remains free to revert to a non-cooperative emission, but does not benefit from doing so as long as other countries commit to the policy selected by the agency.

To the best of our knowledge, in the literature on game theoretic applications in environmental economics, correlation, in either the CE or the CCE format, has been mostly ignored. It was recently discovered that CE cannot help improving upon the Nash equilibrium in many important microeconomic games. Liu (1996) and Yi (1997) proved that the only correlated equilibria in a large class of oligopoly games are mixtures of pure Nash equilibria, a result later on generalized by Neyman (1997) and Ui (2008) to all potential games with smooth and concave potential functions. The abatement game is also a smooth potential game and hence its only CE is the (unique) Nash equilibrium.

In this paper, we show that the abatement game has many CCEs. In the cases that has been identified below, some of these CCEs are strictly more efficient than the Nash equilibrium outcome. We apply the general methodology introduced in Moulin, Ray and Sen Gupta (2014) to compute the most efficient CCE in a symmetric 2-person abatement game with quadratic payoff functions. We show that the optimal CCE is a symmetric mixture of two pure outcomes. Thus, a mediator by using such a CCE can help the countries to choose abatement levels that will generate higher (expected) payoffs than that in the Nash outcome.

To understand our contribution, consider for example the following numerical model of an abatement game with two countries where country i (= 1 and 2) chooses a non-negative abatement level q_i with payoffs

$$u_1(q_1, q_2) = (q_1 + q_2) - 2(q_1 + q_2)^2 - q_1^2;\ u_2(q_1, q_2) = u_1(q_2, q_1).$$

The Nash equilibrium for this game is $(\frac{1}{10}, \frac{1}{10})$, with corresponding payoff (for either country) of $\frac{11}{100} = 0.11$. For this particular numerical example, the efficient (total) payoff that the countries can jointly achieve is $\frac{2}{9} \approx 0.2222$. Thus the Nash equilibrium (total) payoff 0.22 is 99% efficient, that is, the ratio of the total payoff in the Nash equilibrium and the efficient (total) payoff is 0.99. In this game, consider a mediator using the lottery that chooses two outcomes $(\frac{11+\sqrt{3}}{104}, \frac{11-\sqrt{3}}{104})$ and $(\frac{11-\sqrt{3}}{104}, \frac{11+\sqrt{3}}{104})$ each with probability $\frac{1}{2}$. That is, the mediator asks the countries to commit to the future outcome of the lottery in which, with equal probability, one country abates $q_i = \frac{11+\sqrt{3}}{104}$ while the other chooses $\frac{11-\sqrt{3}}{104}$.

The above lottery is clearly not a CE because $(\frac{11+\sqrt{3}}{104}, \frac{11-\sqrt{3}}{104})$ is not a Nash equilibrium of this game.

But it is a CCE: if country 1 chooses any q_1 and assumes country 2 is choosing either $\frac{11+\sqrt{3}}{104}$ or $\frac{11-\sqrt{3}}{104}$ with equal probability, its expected payoff $[\frac{15}{26}q_1 - 3q_1^2 + \frac{11}{104} - (\frac{11+\sqrt{3}}{104})^2 - (\frac{11-\sqrt{3}}{104})^2]$ is maximized at $q_1 = \frac{5}{52}$ and gives $u_1 = \frac{299}{2704}$, precisely the same as by committing to follow the outcome of the above lottery L, that generates the expected utility of $u_1(L) = (z + z') - 2(z + z')^2 - \frac{1}{2}(z^2 + z'^2)$, where $z, z' = \frac{11\pm\sqrt{3}}{104}$. Thus $u_1(L) = \frac{299}{2704} \approx 0.1105$.

More importantly, we prove below that this lottery is actually the optimal CCE, with the total payoff $\pi^{CC} = 2u_1(L) = \frac{598}{2704} = \frac{23}{104} \approx 0.2211$. Clearly, it is an improvement over the Nash outcome. The optimal CCE in this example has an improvement ratio $\frac{\pi^{CC}}{\pi^{Neq}}$ of $\frac{575}{572} \approx 1.0052$, yielding just about $\frac{1}{2}\%$ increase over and above the Nash equilibrium payoff. So the optimal CCE only incurs about $\frac{1}{2}\%$ of efficiency loss compared to the efficient outcome. As mentioned earlier, given the magnitudes involved, $\frac{1}{2}\%$ gain can indeed be a big achievement.

We generalize below the argument presented in this example and formally characterize the optimal CCE for any 2-player abatement game (Theorem 1) under the assumption that the benefit parameter (b) is bigger than the cost parameter (c) in the payoff function in the model, after showing that the inequality $b > c$ is actually necessary to allow any improvement at all.[4] The total payoff at the optimal CCE is very close to the efficient payoff for this class of games, with a clear improvement above the Nash equilibrium total payoff.

As already mentioned, we apply the general algorithm in Theorem 1 of Moulin, Ray and Sen Gupta (2014) to compute the most efficient CCE in a general symmetric 2-person game with quadratic payoff functions. This algorithm is too complex to deliver a general closed form solution; in our earlier work, we derived closed form solutions for two special cases, respectively, for a Cournot duopoly and a public good provision game. The class of abatement games is another subset of the quadratic games, for which Theorem 1 below identifies the optimal CCE in a closed form.[5] The optimal CCE is a 2-dimensional anti-diagonal symmetric lottery similar to those studied by Ray and Sen Gupta (2013) who called such a lottery a Simple Symmetric Correlation Device (SSCD) as introduced in Ganguly and Ray (2005) to discuss correlation.

The contribution of this paper is two-fold. First as a theoretical exercise, our result is perhaps the first attempt of characterizing the benefit from (coarse) correlation in choosing abatement levels by countries. Second, as the importance of enforcing agreements is an important theme in the environmental literature, our characterization suggests why and how a mediator (an independent agency) could be used for agreements and commitments in abatement games in practice; a mediator can improve upon the

[4] Gerard-Varet and Moulin (1978) proved that Nash equilibrium can be *locally improvable* by using a concept similar to CCE under a condition, which for this game, perhaps not surprisingly, also turns out to be $b > c$.

[5] Note that the class of public good provision and that of abatement games differ only in the cost term which is linear there and quadratic here. This difference however changes the entire analysis; for instance, in the public good provision game, the support of the optimal CCE is on the axis, which never happens here.

Nash equilibrium outcome by using the optimal CCE which is just a lottery over two outcomes that the countries would agree to commit to.[6] Of course, there are a few limitations of our result; admittedly, our analysis is restricted to two countries only with a specific payoff function. However, as mentioned above, full characterization of the optimal improvement by (coarse) correlation is not easy to achieve and thus marks a contribution in this literature. We discuss further limitations of our work at the end of this paper.

We formally present the two-person abatement game in Subsection 2.1 and define CCEs for normal form games in Subsection 2.2. Section 3 presents the main result characterizing the optimal CCE for the 2-player abatement game while Section 4 concludes with some remarks.

2 MODEL

2.1 Abatement Game

We present below the model proposed in Barrett (1994) with two countries ($n = 2$).

The payoff function of a country is a function of the abatement level chosen by both countries q_1 and q_2. Let us write the total abatement as Q ($Q = q_1 + q_2$) and therefore we have the benefit function[7] of country i as

$$B_i(Q) = \frac{B}{2}(AQ - \frac{Q^2}{2}).$$

The cost function of each country is a function of its own abatement level q_i and is given as

$$C_i(q_i) = \frac{Cq_i^2}{2}.$$

The payoff function of country 1 (and similarly for country 2) is thus given by

$$u_1(q_1, q_2) = \frac{AB}{2}(q_1 + q_2) - \frac{B}{4}(q_1 + q_2)^2 - \frac{C}{2}q_1^2, \text{ where } A, B \text{ and } C \text{ are all positive.}$$

We now set $a = \frac{AB}{2}$, $b = \frac{B}{4}$, $c = \frac{C}{2}$ for simplicity and rewrite the above payoff function in the following form:

$$u_1(q_1, q_2) = a(q_1 + q_2) - b(q_1 + q_2)^2 - cq_1^2; \; u_2(q_1, q_2) = u_1(q_2, q_1). \tag{1}$$

We call the above game an *abatement game*.

[6] Although we cannot point to a precise example in real life, our abstract mediator embodies in spirit the kind of commitment shown in the 1992 United Nation Framework Convention on Climate Change (UNFCC) that several authors have analysed (see for example Slechten 2013).

[7] Note that the benefit function in the published version of Barrett (1994) has a typo that we have corrected here.

Given q_2, the best response of country 1 (similarly, for country 2) is $BR_1(q_2) = \frac{\partial u_1(q_1,q_2)}{\partial q_1} = a - 2b(q_1 + q_2) - 2cq_1$.

Thus the Nash equilibrium (q_1^{Neq}, q_2^{Neq}) and the corresponding (total) payoff π^{Neq} are given by

$$q_1^{Neq} = q_2^{Neq} = \frac{a}{2(2b+c)};\ \pi^{Neq} = \frac{a^2(4b+3c)}{2(2b+c)^2}.$$

We now compute the efficient abatement levels (q_1^{eff}, q_2^{eff}). To maximize the total payoff $u_1(q_1,q_2) + u_2(q_1,q_2) = 2a(q_1+q_2) - 2b(q_1+q_2)^2 - c(q_1^2+q_2^2)$, we clearly need to choose $q_1 = q_2$; it is easy to prove that

$$q_1^{eff} = q_2^{eff} = \frac{a}{4b+c};\ \pi^{eff} = \frac{2a^2}{4b+c}.$$

Therefore, the relative efficiency ratio of the Nash outcome is $\frac{\pi^{Neq}}{\pi^{eff}} = \frac{(4+3\lambda)(4+\lambda)}{4(2+\lambda)^2}$, where $\lambda = \frac{c}{b}$, which can be viewed as a function of λ. Note that this ratio decreases slowly from 1 (when $\lambda = 0$) to $\frac{3}{4}$ (at $\lambda = \infty$); however, in the region relevant to our analysis ($0 \leq \lambda \leq 1$, as we will explain later), the ratio $\frac{\pi^{Neq}}{\pi^{eff}}$ decreases only from 1 to $\frac{35}{36} = 0.9722$ (see Figure 1 below).

2.2 Coarse Correlation in Games

We present here the notations and definitions used in Moulin, Ray and Sen Gupta (2014), for the sake of consistency and completeness.

Consider a two-person normal form game, $G = [X_1, X_2; u_1, u_2]$, where the strategy sets, X_1 and X_2, are closed real intervals and the payoff functions $u_i : X_1 \times X_2 \to \mathbb{R}$, $i = 1, 2$, are continuous.

Let $\mathcal{L}(X_1 \times X_2)$ with generic element L and $\mathcal{L}(X_i)$ with generic element ℓ_i denote the sets of probability measures on $X_1 \times X_2$ and X_i respectively. Let the mean of $u_i(x_1, x_2)$ with respect to L be denoted by $u_i(L)$.

The deterministic distribution at z is denoted by δ_z, and for product distributions such as $\delta_{x_1} \otimes \ell_2$ we write $u_i(\delta_{x_1} \otimes \ell_2)$ simply as $u_i(x_1, \ell_2)$.

Definition 1 *A coarse correlated equilibrium (CCE) of the game G is a lottery $L \in \mathcal{L}(X_1 \times X_2)$ such that*

$$u_1(L) \geq u_1(x_1, L^2) \text{ and } u_2(L) \geq u_2(L^1, x_2) \text{ for all } (x_1, x_2) \in X_1 \times X_2. \quad (2)$$

Following Ray and Sen Gupta (2013), Definition 1 above can be presented for any finite n-person normal form game, $[N, \{X_i\}_{i\in N}, \{u_i\}_{i\in N}]$, with set of players, $N = \{1, ..., n\}$, finite pure strategy sets, $X_1, ..., X_n$ with $X = \prod_{i\in N} X_i$, and payoff functions, $u_1, ..., u_n$, $u_i : X \to \Re$, for all i. For such a game, a probability distribution L over X is a CCE if for all i, for all $x_i' \in X_i$, $\sum_{x\in X} L(x)u_i(x) \geq \sum_{x_{-i}\in X_{-i}} L^i(x_{-i})u_i(x_i', x_{-i})$, where $L^i(x_{-i}) = \sum_{x_i\in X_i} L(x_i, x_{-i})$ is the *marginal* probability distribution over $x_{-i} \in X_{-i}$, for any deviant $i \in N$ while the others commit to L.

3 RESULTS

The abatement game is a potential game for the potential function $P(q_1, q_2) = a(q_1 + q_2) - b(q_1 + q_2)^2 - c(q_1^2 + q_2^2)$, which is smooth and concave. Therefore, the only correlated equilibrium (*a la* Aumann) is the Nash equilibrium q^{Neq} (Neyman 1997).

Our goal in this paper is to compute for the abatement game the CCE that maximizes the total payoff $u_1 + u_2$ and to compare this joint payoff with the efficient payoff and the Nash equilibrium payoff.

As the abatement game is symmetric, we can limit our search to symmetric lotteries L only (as explained in Moulin, Ray and Sen Gupta 2014, when one identifies an optimal symmetric CCE, one also captures an optimal CCE among all CCEs, symmetric or otherwise). We denote the set of symmetric lotteries by $\mathcal{L}^{sy}(\mathbb{R}^2_+)$.

We first characterize the equilibrium condition (2) presented in Definition 1 in terms of three moments of L. If L is the distribution of the symmetric random variable (Z_1, Z_2), these are respectively the expected values of Z_i, Z_i^2, and $Z_1 \cdot Z_2$ as denoted below.

$$\alpha = E_L[Z_1];\ \beta = E_L[Z_1^2];\ \gamma = E_L[Z_1 \cdot Z_2]$$

Proposition 1 *A symmetric lottery $L \in \mathcal{L}^{sy}(\mathbb{R}^2_+)$ is a CCE of the abatement game if and only if*

$$\max_{z \geq 0}\{(a - 2b\alpha)z - (b + c)z^2\} \leq a\alpha - (b + c)\beta - 2b\gamma \tag{3}$$

and the corresponding utility (for a country) is

$$u_1(L) = 2a\alpha - (2b + c)\beta - 2b\gamma.$$

Proof. First note that the expected utility (for a country) from any lottery $L \in \mathcal{L}^{sy}(\mathbb{R}^2_+)$ can be written as

$$u_1(L) = aE_L[Z_1] + aE_L[Z_2] - bE_L[Z_1^2] - bE_L[Z_2^2] - 2bE_L[Z_1 \cdot Z_2] - cE_L[Z_1^2],$$

which by symmetry is

$$\begin{aligned} u_1(L) &= 2aE_L[Z_1] - (2b + c)E_L[Z_1^2] - 2bE_L[Z_1 \cdot Z_2] \\ &= 2a\alpha - (2b + c)\beta - 2b\gamma. \end{aligned}$$

We write the expected payoff when country 1 plays a pure strategy z and country 2 commits to L, as

$$\begin{aligned} u_1(z, L^2) &= az + aE_L[Z_2] - bz^2 - bE_L[Z_2^2] - 2bzE_L[Z_2] - cz^2 \\ &= (a - 2b\alpha)z - (b + c)z^2 + a\alpha - b\beta. \end{aligned}$$

Hence, L is a CCE if and only if

$$\max_{z\geq 0}\{(a-2b\alpha)z-(b+c)z^2\}+a\alpha-b\beta\leq 2a\alpha-(2b+c)\beta-2b\gamma,$$

which, after rearranging, gives us the condition in the statement. ■

The next subsection presents our main result.

3.1 Optimal CCE for the Abatement Game

The following theorem characterizes the utility maximizing CCE for the abatement game.

Theorem 1 *i) If $b \leq c$, the Nash equilibrium of the abatement game is its only CCE.*

ii) If $b > c$, setting $\lambda = \frac{c}{b}$, the optimal values of the three moments of the utility maximizing L are given by $(\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\gamma})$:

$$\widetilde{\alpha} = \frac{a}{b}\frac{2+2\lambda-\lambda^2}{2(4+5\lambda)},$$

$$\widetilde{\beta} = \frac{a^2}{b^2}\frac{4+8\lambda+\lambda^2-4\lambda^3}{4(4+5\lambda)^2} \text{ and } \widetilde{\gamma} = \frac{a^2}{b^2}\frac{4+8\lambda-\lambda^2-4\lambda^3+2\lambda^4}{4(4+5\lambda)^2};$$

while the optimal CCE is $\widetilde{L} = \frac{1}{2}\delta_{(z,z')} + \frac{1}{2}\delta_{(z',z)}$, with

$$z, z' = \frac{a}{b}\frac{2+2\lambda-\lambda^2 \pm \lambda\sqrt{1-\lambda^2}}{2(4+5\lambda)}.$$

Using Theorem 1, one can compute the maximum payoff obtained by the CCE $\widetilde{L}$ (when $b > c$).

Corollary 1 *The payoff function (of a country) at $\widetilde{L}$ is*

$$u_1(\widetilde{L}) = \frac{1}{b^2-c^2}\left[\frac{a^2}{b^2}\frac{(2+2\lambda-\lambda^2)^2}{4(4+5\lambda)} - \frac{a^2c}{4b^2}\right] = \frac{a^2}{b}\frac{4+4\lambda-\lambda^2}{4(4+5\lambda)}.$$

3.1.1 Example

We illustrate our main result above by revisiting the example in the Introduction more formally. Consider the following values of the parameters, $a = 1$, $b = 2$ and $c = 1$ in the abatement game. Here, $\lambda = \frac{c}{b} = \frac{1}{2} < 1$ and the payoff function is given by $u_1(q_1, q_2) = (q_1 + q_2) - 2(q_1 + q_2)^2 - q_1^2$, with Nash equilibrium abatement levels, $q^{Neq} = \frac{a}{2(2b+c)} = \frac{1}{10}$.

From Theorem 1, the corresponding optimal values of the moments are:

$\widetilde{\alpha} = \frac{11}{104} \approx 0.1057$,

$\widetilde{\beta} = \frac{31}{2704} \approx 0.0114$ and

$\widetilde{\gamma} = \frac{59}{5408} \approx 0.0109$.

Thus the optimal CCE is the lottery $\widetilde{L} = \frac{1}{2}\delta_{(z,z')} + \frac{1}{2}\delta_{(z',z)}$, where $z, z' = \frac{11\pm\sqrt{3}}{104}$, that chooses two outcomes $(\frac{11+\sqrt{3}}{104}, \frac{11-\sqrt{3}}{104})$ and $(\frac{11-\sqrt{3}}{104}, \frac{11+\sqrt{3}}{104})$ each with probability $\frac{1}{2}$, as mentioned in the Introduction.

From Corollary 1, the corresponding expected payoff (for one country) derived by playing this CCE is $u_1(\widetilde{L}) = \frac{299}{2704} \approx 0.1105$.

3.2 Efficiency Performance of the Optimal CCE

We now compare the (total) payoff from the optimal CCE $\pi^{CC} = 2u_1(\widetilde{L})$, with both the efficient and the Nash equilibrium (total) profits. To analyze the performance of (coarse) correlation, we are going to use the terminologies presented in Ashlagi *et al* (2008).

We call the ratio between the total payoff from the optimal CCE to the total payoff obtained in the Nash equilibrium the *mediation value*; similarly, let the *enforcement value* denote the ratio between the maximum total payoff obtained in a CCE to the maximal welfare (total payoff) in the efficient outcome.

We can now calculate the enforcement and mediation values for the abatement game recalling

$$\pi^{eff} = \frac{2a^2}{4b+c} = \frac{a^2}{b}\frac{2}{4+\lambda} \text{ and } \pi^{Neq} = \frac{a^2(4b+3c)}{2(2b+c)^2} = \frac{a^2}{b}\frac{4+3\lambda}{2(2+\lambda)^2}.$$

Corollary 2 *For the abatement game, the enforcement and mediation values depend only upon* $\lambda = \frac{c}{b}$*, as follows:*

$$\textit{Enforcement Value} = \frac{\pi^{CC}}{\pi^{eff}} = \frac{(4+\lambda)(4+4\lambda-\lambda^2)}{4(4+5\lambda)} \textit{ for } 0 \leq \lambda \leq 1,$$

$$\textit{Mediation Value} = \frac{\pi^{CC}}{\pi^{Neq}} = \frac{(2+\lambda)^2(4+4\lambda-\lambda^2)}{(4+5\lambda)(4+3\lambda)} \textit{ for } 0 \leq \lambda \leq 1; \ \frac{\pi^{CC}}{\pi^{Neq}} = 1 \textit{ for } \lambda \geq 1.$$

Corollary 2 on the enforcement and mediation values is shown in Figures 1 and 2. In Figure 2, we have plotted the ratios $\frac{\pi^{CC}}{\pi^{eff}}$ (as in Corollary 2 above) and $\frac{\pi^{Neq}}{\pi^{eff}}$ (as stated earlier in Subsection 2.1) together to show that indeed coarse correlation improves upon Nash in the relevant range of the parameter ($0 \leq \lambda \leq 1$).

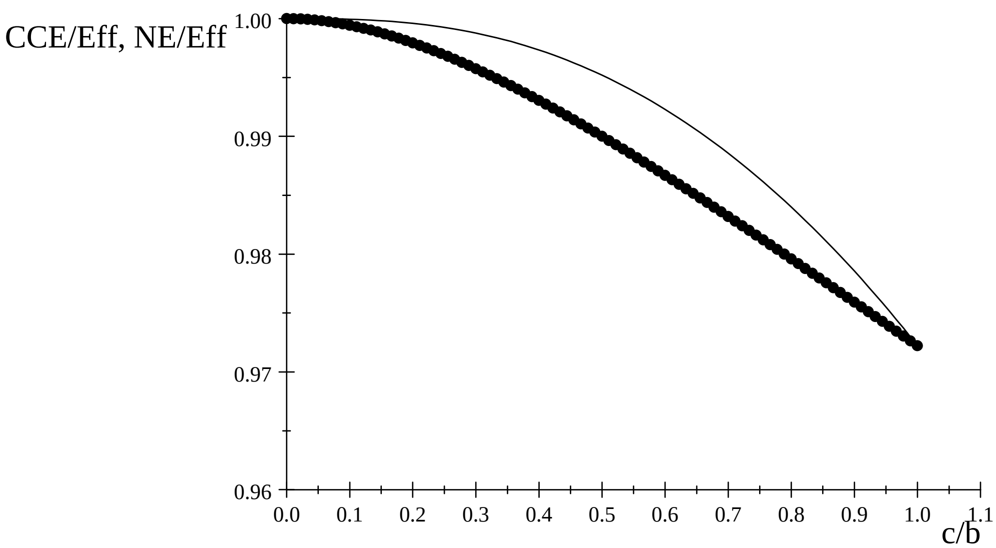

Figure 1: Enforcement value and $\frac{\pi^{Neq}}{\pi^{eff}}$ in the abatement game

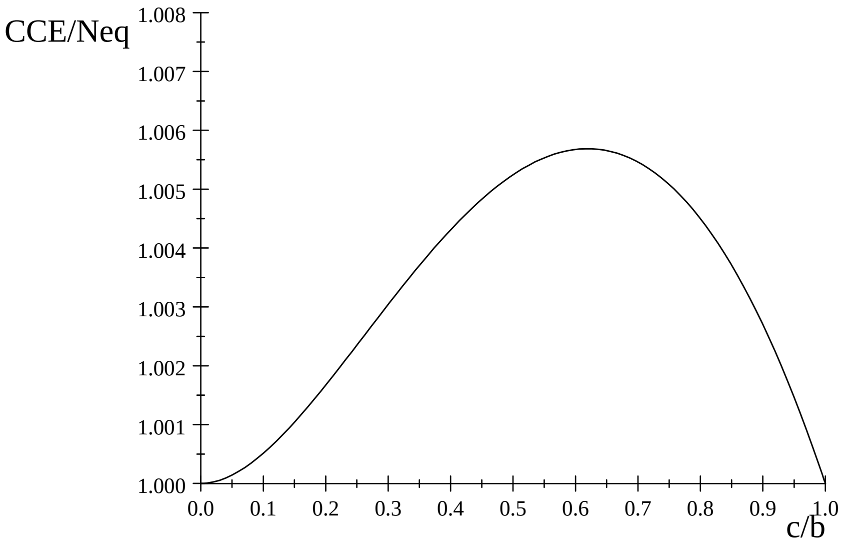

Figure 2: Mediation value in the abatement game.

3.2.1 Example (continued)

We revisit again our baseline example. In the example, the optimal CCE (total) payoff is $\pi^{CC} = 2u_1(\widetilde{L}) = \frac{23}{104} \approx 0.2211$, while the efficient (total) payoff is $\pi^{eff} = \frac{2}{9} \approx 0.2222$ and the Nash equilibrium (total) payoff is $\pi^{Neq} = \frac{11}{50} \approx 0.22$ (and hence $\frac{\pi^{Neq}}{\pi^{eff}} = 0.99$, as mentioned earlier).

Using Corollary 2, the corresponding values here are: enforcement value $= \frac{\pi^{CC}}{\pi^{eff}} = \frac{207}{208} \approx 0.9951$ and mediation value $= \frac{\pi^{CC}}{\pi^{Neq}} = \frac{575}{572} \approx 1.0052$.

4 REMARKS

We have analyzed coarse correlated equilibria in a class of 2-person symmetric games called the abatement game where correlation *a la* Aumann does not offer anything more than the Nash equilibrium. Incorporating the techniques introduced by Moulin, Ray and Sen Gupta (2014), we have characterized the utility maximizing CCE and have shown that they have a very simple support with only four deterministic strategy profiles. Moreover, as we mentioned already, the benefit of using the optimal CCE in the abatement game can be huge even though in percentage terms the achievable improvement may not seem significant. Such a computation is the first of its kind for coarse correlated equilibria for the abatement game and, this is why we regard this exercise as an interesting first step towards more sophisticated computations to understand mediation in general for such games.

Some remarks are in order.

Clearly, there are limitations of our approach. First, we have used a quadratic payoff function, and not any general differentiable concave function. This is not just because it enables us to use the techniques identified in Moulin, Ray and Sen Gupta (2014). This choice has been justified in the literature (such as the RICE model in Nordhaus *et al* 2000) that tries to set up abatement cost functions fitting real data. Quadratic approximation is indeed a natural choice for payoffs as shown in the models by Bosetti *et al* 2009, Finus *et al* 2005, Klepper *et al* 2006.

Also, we have worked with the assumption of identical nations for simplicity. We postpone the work on asymmetric countries for future research.

Our characterization is only for a 2-player game. Although it is unclear how our main result could be generalized in a game with n players, our conjecture is that CCE can improve upon the Nash equilibrium outcome in an abatement game with n countries. It is of course true that the efficiency of the results depends heavily upon the number of nations. Consequently, our paper does not address the important issues of participation decisions and abatement levels.[8]

Finally, we do not relate our work to the important issues of coalition formation and applications of coalitional form games, which are now perhaps standard approaches in the literature on the International Environmental Agreements (IEAs).[9] We are aware of the issues on the structures of (self-enforcing) IEAs to analyze the interaction among countries and their behaviors to arrive at a final outcome (Barrett 2003, Finus 2008, McGinty 2007 for example) which are beyond the scope of our current paper.

[8] Finus (2003) showed that full participation and the efficient outcome is obtained with only two players.

[9] See, for example, Tulkens (1998) and the references therein.

5 PROOF OF THE MAIN THEOREM

Proof of Theorem 1 involves a couple of known lemmata that we present below for the sake of completeness. The first is due to Moulin, Ray and Sen Gupta (2014). It identifies the range of the vector (α, β, γ) when $L \in \mathcal{L}^{sy}(\mathbb{R}^2_+)$ and also shows that this range is covered by two families of very simple lotteries with at most four strategy profiles in their support.

Let $\mathcal{L}^*$ be the subset of $\mathcal{L}^{sy}(\mathbb{R}^2_+)$ containing the simple lotteries of the form $L = \frac{q}{2}(\delta_{z,z} + \delta_{z',z'}) + \frac{p}{2}(\delta_{z,z'} + \delta_{z',z})$, where z, z', q and p are non-negative and $q + p = 1$. Let $\mathcal{L}^{**}$ be the subset of $\mathcal{L}^{sy}(\mathbb{R}^2_+)$ of the form $L = q \cdot \delta_{z,z} + q' \cdot \delta_{0,0} + \frac{p}{2}(\delta_{0,z} + \delta_{z,0})$, where z, q, q' and p are non-negative and $q + q' + p = 1$.

Lemma 1 *i) For any $L \in \mathcal{L}^{sy}(\mathbb{R}^2_+)$ and the corresponding random variable (Z_1, Z_2), we have*

$$\alpha, \gamma \geq 0; \ \beta \geq \gamma; \ \beta + \gamma \geq 2\alpha^2; \tag{4}$$

ii) Equality $\beta = \gamma$ holds if and only if L is diagonal: $Z_1 = Z_2$ (a.e.);

iii) Equality $\beta + \gamma = 2\alpha^2$ holds if and only if L is anti-diagonal: $Z_1 + Z_2$ is constant (a.e.);

iv) For any $(\alpha, \beta, \gamma) \in \mathbb{R}^3_+$ satisfying inequalities (4), there exists $L \in \mathcal{L}^* \cup \mathcal{L}^{**}$ with precisely these parameters.

Note that (4) implies $\beta \geq \alpha^2$, with equality $\beta = \alpha^2$ if and only if L is deterministic, because $\beta = \alpha^2$ implies both $\beta = \gamma$ and $\beta + \gamma = 2\alpha^2$.

The proof of Lemma 1 can be found in Moulin, Ray and Sen Gupta (2014) and thus is omitted here.

Lemma 2 below states a two-step algorithm to find the utility maximizing CCEs using Lemma 1 and Proposition 1 (stated earlier in this paper).

Lemma 2 *Given the abatement game, the following nested programs generate the utility maximizing CCEs:*

Step 1: Fix α non-negative, and solve the linear programme

$$\min_{\beta,\gamma}\{(2b + c)\beta + 2b\gamma\} \text{ under constraints}$$

$$\beta \geq \gamma \geq 0; \ \beta + \gamma \geq 2\alpha^2; \ (b + c)\beta + 2b\gamma \leq a\alpha - \max_{z \geq 0}\{(a - 2b\alpha)z - (b + c)z^2\}.$$

Step 2: With the solutions $\beta(\alpha), \gamma(\alpha)$ found in Step 1, solve

$$\max_{\alpha}\{2a\alpha - (2b + c)\beta(\alpha) - 2b\gamma(\alpha)\} \text{ under constraints}$$

$$\alpha \geq 0; \ \max_{z \geq 0}\{(a - 2b\alpha)z - (b + c)z^2\} \leq a\alpha - (b + c)\beta(\alpha) - 2b\gamma(\alpha).$$

Moreover, there is a utility maximizing CCE in $\mathcal{L}^* \cup \mathcal{L}^{**}$.

Lemma 2 is similar to Theorem 1 in Moulin, Ray and Sen Gupta (2014) and hence the proof is omitted.

We are now ready to prove Theorem 1. First, consider the equilibrium condition (3) as in Proposition 1. Note that if $a - 2b\alpha < 0 \iff \alpha > \frac{a}{2b}$, the L.H.S. of that inequality (the maximum over $z \geq 0$) is zero; therefore, (3) becomes

$$a\alpha \geq (b+c)\beta + 2b\gamma = b(\beta+\gamma) + c\beta + b\gamma > b(\beta+\gamma) \geq 2b\alpha^2,$$

which is a contradiction. So, we must have $\alpha \leq \frac{a}{2b}$; then the L.H.S. of (3) is $\frac{(a-2b\alpha)^2}{4(b+c)}$. The equilibrium condition is now

$$(b+c)\beta + 2b\gamma \leq a\alpha - \frac{(a-2b\alpha)^2}{4(b+c)} = -\frac{b^2\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}}{b+c}. \tag{5}$$

We now fix α and solve Step 1 in Lemma 2: we must minimize $(2b+c)\beta + 2b\gamma$ in the polytope $\Psi = \{(\beta,\gamma) | \beta \geq \gamma, \beta + \gamma \geq 2\alpha^2\}$ under the additional constraint (5). Note that Ψ is unbounded from above and bounded from below by the interval $[P, Q]$, where $P = (\alpha^2, \alpha^2)$ and $Q = (2\alpha^2, 0)$. We distinguish two cases here.

Case 1 ($b \leq c$)**:** In this case, the minimum in Ψ of both $(2b+c)\beta + 2b\gamma$ and $(b+c)\beta + 2b\gamma$ is achieved at P. Therefore, if P meets (5) it is our optimal pair $(\beta(\alpha), \gamma(\alpha))$; otherwise, there is no CCE for this choice of α. Now, P meets (5) if and only if $(3b+c)\alpha^2 \leq -\frac{b^2\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}}{b+c}$, which reduces to $[a-(2b+c)\alpha)]^2 \leq 0 \iff \alpha = \frac{a}{2(2b+c)} = q_i^{Neq}$. By Lemma 1, the optimal CCE L is diagonal ($\beta = \gamma$) and deterministic ($\beta = \alpha^2$). It is simply the Nash equilibrium $L = \delta_{q^{Neq}}$ of our game.

Case 2 ($b > c$)**:** Here, the minimum of $(b+c)\beta + 2b\gamma$ in Ψ is achieved at Q; so, if Q fails to meet the constraint (5) there is no hope to meet it anywhere in Ψ. Thus, we must choose α such that

$$2(b+c)\alpha^2 \leq -\frac{b^2\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}}{b+c} \iff \Lambda(\alpha) = (3b^2 + 4bc + 2c^2)\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4} \leq 0 \tag{6}$$

The discriminant of the right-hand polynomial $\Lambda(\alpha)$ is $a^2(b^2 - c^2)$; therefore, (6) restricts α to an interval $[\alpha_-, \alpha_+]$, between the two positive roots of $\Lambda(\alpha)$. For such a choice of α, the constraint (5) cuts a subinterval $[R, Q]$ of $[P, Q]$, where R meets (5) as an equality. Note that $R = P$ only if $\alpha = q_i^{Neq}$ (from Case 1 and the fact that $\Lambda(q_i^{Neq}) < 0$), otherwise $R \neq P$. Clearly, R is our optimal choice $(\beta(\alpha), \gamma(\alpha))$ and it solves the system

$$\beta + \gamma = 2\alpha^2; \ (b+c)\beta + 2b\gamma = -\frac{b^2\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}}{b+c}.$$

Therefore,

$$\beta(\alpha) = \frac{1}{b^2 - c^2}\left[b(5b+4c)\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}\right] \text{ and}$$

$$\gamma(\alpha) = \frac{1}{b^2 - c^2}\left[-(3b^2 + 4bc + 2c^2)\alpha^2 + a(2b+c)\alpha - \frac{a^2}{4}\right].$$

Now in Step 2 of Lemma 2, we must maximize $2a\alpha - (2b+c)\beta(\alpha) - 2b\gamma(\alpha)$ under the constraints $\alpha \geq 0$ and $\Lambda(\alpha) \leq 0$. Developing this objective function yields the program

$$\frac{1}{b^2-c^2}\max_{\alpha}\{-b^2(4b+5c)\alpha^2 + a(2b^2+2bc-c^2)\alpha - \frac{a^2c}{4}\} \tag{7}$$

under the constraints

$$\alpha \geq 0 \text{ and } \Lambda(\alpha) = (3b^2+4bc+2c^2)\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4} \leq 0.$$

The unconstrained maximum of the objective function is achieved at $\widetilde{\alpha} = \frac{a(2b^2+2bc-c^2)}{2b^2(4b+5c)}$.

We now show that $\Lambda(\widetilde{\alpha}) \leq 0$. With the change of variable $\lambda = \frac{c}{b}$, this amounts to

$$\frac{(3+4\lambda+2\lambda^2)(2+2\lambda-\lambda^2)^2}{4(4+5\lambda)^2} - \frac{(2+\lambda)(2+2\lambda-\lambda^2)}{2(4+5\lambda)} + \frac{1}{4} \leq 0$$

$$\iff 4+8\lambda-5\lambda^2-12\lambda^3+3\lambda^4+4\lambda^5-2\lambda^6 \geq 0$$

The above polynomial is 0 at $\lambda = 1$; it is also easy to check, numerically, that it is non-negative on $[0,1]$. The proof of is complete if we now express $\widetilde{\alpha}$, $\widetilde{\beta}$ and $\widetilde{\gamma}$ in terms of λ. This is indeed easy for $\widetilde{\alpha}$. One may also verify, using the expression for $\widetilde{\alpha}$ that

$$\begin{aligned}\widetilde{\beta} &= \beta(\widetilde{\alpha}) = \frac{1}{b^2-c^2}\left[b(5b+4c)\widetilde{\alpha}^2 - a(2b+c)\widetilde{\alpha} + \frac{a^2}{4}\right] \\ &= \frac{a^2}{b^2}\frac{4+8\lambda+\lambda^2-4\lambda^3}{4(4+5\lambda)^2} \text{ and}\end{aligned}$$

$$\begin{aligned}\widetilde{\gamma} &= \gamma(\widetilde{\alpha}) = \frac{1}{b^2-c^2}\left[-(3b^2+4bc+2c^2)\widetilde{\alpha}^2 + a(2b+c)\widetilde{\alpha} - \frac{a^2}{4}\right] \\ &= \frac{a^2}{b^2}\frac{4+8\lambda-\lambda^2-4\lambda^3+2\lambda^4}{4(4+5\lambda)^2}.\end{aligned}$$

Finally, we construct the optimal CCE $\widetilde{L}$. From $\widetilde{\beta}+\widetilde{\gamma} = 2\widetilde{\alpha}^2$ and Lemma 1(iii), we see that $\widetilde{L}$ is an anti-diagonal lottery of the form $\widetilde{L} = \frac{1}{2}\delta_{(z,z')} + \frac{1}{2}\delta_{(z',z)}$, where z and z' are non-negative numbers such that $z+z' = 2\widetilde{\alpha}$ and $z^2+z'^2 = 2\widetilde{\beta}$. This implies $2zz' = (2\widetilde{\alpha})^2 - (2\widetilde{\beta}) = 2\widetilde{\gamma}$, hence z, z' solve $Z^2 - 2\widetilde{\alpha}Z + \widetilde{\gamma} = 0$. The discriminant is $\widetilde{\alpha}^2 - \widetilde{\gamma} = \widetilde{\beta} - \widetilde{\alpha}^2 = \frac{a^2}{b^2}\frac{\lambda^2(1-\lambda^2)}{4(4+5\lambda)^2}$; thus the expressions for z and z' follow.

Corollary 2 can also be proved now. From the expression (7) of the payoff $u_1(\widetilde{L})$ and the expression of $\widetilde{\alpha}$ in Theorem 1ii), straightforward computations provide the expressions presented in Corollary 2.

6 REFERENCES

1. Ashlagi, I., D. Monderer and M. Tennenholz (2008), "On the value of correlation," *Journal of Artificial Intelligence*, 33, 575-613.

2. Aumann, R. J. (1974), "Subjectivity and correlation in randomized strategies," *Journal of Mathematical Economics*, 1, 67-96.

3. Aumann, R. J. (1987), "Correlated equilibrium as an expression of Bayesian rationality," *Econometrica*, 55, 1-18.

4. Baliga, S. and E. Maskin (2003) "Mechanism design for the environment," in K. Maler and J. Vincent (eds.) *Handbook of Environmental Economics*, Volume 1, Elsevier Science.

5. Barrett, S. (1994), "Self-enforcing international environmental agreements," *Oxford Economic Papers*, 46, 878-894.

6. Barrett, S. (2001), "International cooperation for sale," *European Economic Review*, 45, 1835-1850.

7. Barrett, S. (2003), *Environment and statecraft: the strategy of environmental treaty making*, Oxford University Press.

8. Bosetti, V., C. Carraro, E. De Cian, R. Duval, E. Massetti and M. Tavoni (2009), "The incentives to participate in, and the stability of, international climate coalitions: a game-theoretic, analysis using the WITCH model," *Sustainable Development Series*, FEEM.

9. Finus, M. (2001), *Game theory and international environmental cooperation*, Edward Elgar.

10. Finus, M. (2003), "Stability and design of International Environmental Agreements: the case of transboundary pollution," In H. Folmer and T. Tietenberg (eds.) *International Yearbook of Environmental and Resource Economics*, 2003/04, Edward Elgar.

11. Finus, M. (2008), "Game theoretic research on the design of international environmental agreements: insights, critical remarks, and future challenges," *International Review of Environmental and Resource Economics*, 2, 29-67.

12. Finus, M., J-C. Altamirano-Cabrera and E. C. Van Ierland (2005), "The effect of membership rules and voting schemes on the success of international climate agreements," *Public Choice*, 125, 95-127.

13. Folmer, H., N. Hanley and F. Missfeldt (1998), "Game-theoretic modeling of environmental and resource problems: an introduction," In N. Hanley and H. Folmer (eds.) *Game Theory and the Environment*, Edward Elgar.

14. Forgó, F. (2010), "A generalization of correlated equilibrium: a new protocol," *Mathematical Social Sciences*, 60, 186-190.

15. Forgó, F. (2011), "Generalized correlated equilibrium for two-person games in extensive form with perfect information," *Central European Journal of Operations Research*, 19, 201-213.

16. Forgó, F., J. Fülöp and M. Prill (2005), "Game theoretic models for climate change negotiations," *European Journal of Operations Research*, 160, 252-267.

17. Ganguly, C. and I. Ray (2005), "Simple mediation in a cheap-talk game," *Discussion Paper* 05-08, Department of Economics, University of Birmingham.

18. Gerard-Varet, L. A. and H. Moulin (1978), "Correlation and duopoly," *Journal of Economic Theory*, 19, 123-149.

19. Kar, A., I. Ray and R. Serrano (2010), "A difficulty in implementing correlated equilibrium distributions," *Games and Economic Behavior*, 69, 189-193.

20. Klepper G. and S. Peterson (2006), "Marginal abatement cost curves in general equilibrium: the influence of world energy prices", *Resource and Energy Economics*, 29, 1-23.

21. Liu, L. (1996), "Correlated equilibrium of Cournot oligopoly competition," *Journal of Economic Theory*, 68, 544-548.

22. McGinty, M. (2007), "International environmental agreements among asymmetric nations," *Oxford Economic Papers*, 59, 45-62.

23. Missfeldt, F. (2002), "Game-theoretic modeling of trans-boundary pollution," *Journal of Economic Surveys*, 13, 287-321.

24. Moulin, H., I. Ray and S. Sen Gupta (2014), "Improving Nash by coarse correlation," *Journal of Economic Theory*, 150, 852-865.

25. Moulin, H. and J. P. Vial (1978), "Strategically zero-sum games: the class of games whose completely mixed equilibria cannot be improved upon," *International Journal of Game Theory*, 7, 201-221.

26. Neyman, A. (1997), "Correlated equilibrium and potential games", *International Journal of Game Theory*, 26, 223-227.

27. Nordhaus W. D. and J. Boyer (2000), *Warming the world: economic models of global warming*, MIT Press.

28. Ray, I. and S. Sen Gupta (2013), "Coarse correlated equilibria in linear duopoly games," *International Journal of Game Theory*, 42, 541-562.

29. Roughgarden, T. (2009), "Intrinsic robustness of the price of anarchy," *STOC* '09.

30. Rubio, S. J. and A. Ulph (2006), "Self-enforcing international environmental agreements revisited," *Oxford Economic Papers*, 58, 233-263.

31. Slechten, A. (2013), "Environmental agreements under asymmetric information," Mimeo.

32. Tulkens, H. (1998), "Cooperation versus free-riding in international environmental affairs: two approaches," In N. Hanley and H. Folmer (eds.) *Game Theory and the Environment*, Edward Elgar.

33. Ui, T. (2008), "Correlated equilibrium and concave games," *International Journal of Game Theory*, 37, 1-13.

34. Yi, S. S. (1997), "On the existence of a unique correlated equilibrium in Cournot oligopoly," *Economic Letters*, 54, 235-239.

35. Young, H. P. (2004), *Strategic learning and its limits*, Oxford University Press.

36. Wood, P. J. (2011), "Climate change and game theory: a mathematical survey," *Working Paper* 2.10, Crawford School of Economics and Government, Australian National University.