Search for Two-Scale Localization in Disordered Wires in a Magnetic Field

H. Schomerus and C. W. J. Beenakker
Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands
(Received 1 November 1999)

A recent paper [A. V. Kolesnikov and K. B. Efetov, Phys. Rev. Lett. **83**, 3689 (1999)] predicts a two-scale behavior of wave function decay in disordered wires in the crossover regime from preserved to broken time-reversal symmetry. We have tested this prediction by a transmission approach, relying on the Borland conjecture that relates the decay length of the transmittance to the decay length of the wave functions. Our numerical simulations show no indication of two-scale behavior.

PACS numbers: 72.15.Rn, 05.60.Gg, 73.20.Fz

In a remarkable paper [1], Kolesnikov and Efetov have predicted that the decay of wave functions in disordered wires is characterized by two localization lengths, if time-reversal symmetry is partially broken by a weak magnetic field. Using the supersymmetry technique [2], it was demonstrated that the far tail of the wave functions decays with the length ξ_2 characteristic for completely broken time-reversal symmetry—even if the flux through a localized area is much smaller than a flux quantum. At shorter distances the decay length is $\xi_1 = \frac{1}{2}\xi_2$. It was suspected that previous studies by Pichard *et al.* [3] found single-scale decay because of the misguiding theoretical expectation of such behavior. This expectation was also the basis for the interpretation of the experiments by Khavin, Gershenson, and Bogdanov [4] on submicron-wide wires.

The prediction of Kolesnikov and Efetov calls for a test by means of a dedicated experiment or computer simulation. It is the purpose of this work to provide the latter. We target the key feature of the two-scale localization phenomenon, which is the doubling of the asymptotic decay length at infinitesimally weak magnetic fields.

Our numerical simulations are based on a transmission approach. We rely on the Borland conjecture [5] (believed to be true generally [6]) that relates the asymptotic decay of the transmittance T with increasing wire length L to the asymptotic decay of the wave function $\psi(L)$. According to the Borland conjecture, the Lyapunov exponent $\alpha = -\lim_{L\to\infty}\frac{1}{2}L^{-1}\ln T$ is identical to the inverse localization length $\xi^{-1} = -\lim_{L\to\infty}L^{-1}\ln|\psi(L)|$. Moreover, ξ and α are self-averaging, meaning that the statistical fluctuations become smaller and smaller as $L\to\infty$. Our numerical simulations show that the crossover from $\xi = \xi_1$ to $\xi = \xi_2$ does not occur until the flux Φ_ξ through a wire segment of length ξ_1 is of the order of a flux quantum $\Phi_0 = h/e$. For our longest wires ($L \gtrsim 150\xi_1$), the crossover according to Ref. [1] should have occurred at $\Phi_\xi/\Phi_0 \simeq \exp(-L/8\xi_1) \simeq 10^{-8}$. We consider various possible reasons for the disagreement, and suggest that the quantity considered in Ref. [1] is dominated by anomalously localized states.

Our first set of results is obtained from the numerical calculation (by the technique of recursive Green functions [7]) of the transmission matrix t for a two-dimensional Anderson Hamiltonian with on-site disorder. In units of the lattice constant $a \equiv 1$, the width of the wire is $W = 13$ and the wavelength of the electrons is $\lambda = 5.1$, resulting in $N = 5$ propagating modes through the wire. The localization lengths $\xi_1 = (N+1)l$ and $\xi_2 = 2Nl$ are determined by the scaling parameter l of quasi-one-dimensional localization theory, which differs from the transport mean-free path by a coefficient of order unity [8]. The average of the transmittance $T = \mathrm{tr}\, tt^\dagger$ in the metallic regime, fitted to $\langle T\rangle = N(1 + L/l)^{-1}$, yields $l = 65$. This gives a localization length $\xi_1 = 390$ for preserved time-reversal symmetry (symmetry index $\beta = 1$) and a localization length $\xi_2 = 650$ for broken time-reversal symmetry ($\beta = 2$).

Figure 1 shows the ensemble-averaged logarithm of the transmittance $\langle \ln T\rangle$ as a function of wire length L for various values of the magnetic field B (or flux $\Phi_\xi = W\xi_1 B$). We find a smooth transition between the theoretical expectations for preserved and broken time-reversal symmetry. Most importantly, we find an asymptotic slope $s(B) = \lim_{L\to\infty}L^{-1}\langle\ln T\rangle$ that interpolates smoothly between the values $s = -2/\xi_1$ for $B = 0$ and $s = -2/\xi_2$ for large B. There is no indication of a crossover to the slope $s = -2/\xi_2$ for smaller values of B, even for very long wires ($L \gtrsim 150\xi_1$). According to the theory of Ref. [1], the crossover should occur at a length L_{cross} given by

$$L_{\mathrm{cross}}/\xi_1 = 8\ln(\sqrt{12}\,\Phi_0/4\pi\Phi_\xi) + \mathcal{O}(1), \qquad (1)$$

which is well within the range of our simulations ($L_{\mathrm{cross}} \simeq 14\xi_1$ for $\Phi_\xi \simeq 0.05\Phi_0$). The absence of two-scale behavior in the transmittance of an individual, arbitrarily chosen realization is demonstrated in the inset of Fig. 1, for $\Phi_\xi = \frac{1}{2}\Phi_0$. The self-averaging property of the Lyapunov exponent is evident.

The asymptotic decay length $\xi(B) = -2/s(B)$ is plotted versus magnetic field in Fig. 2, together with the weak-localization correction $\delta T = T(B=\infty) - T(B)$ at $L = \xi_1$. For both quantities, breaking of time-reversal symmetry sets in when Φ_ξ is comparable to Φ_0. The transition from $\beta = 1$ to $\beta = 2$ is completed for $\Phi_\xi \approx 100\Phi_0$.

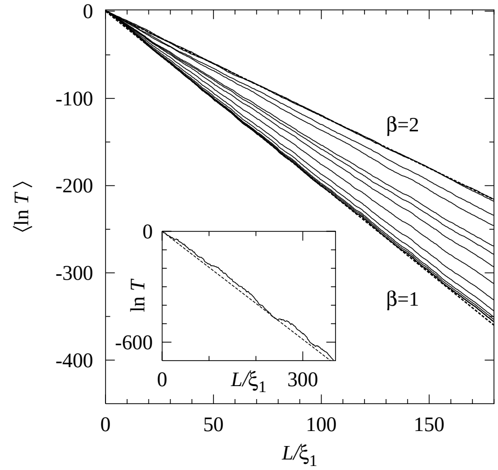

FIG. 1. Average logarithmic transmittance $\langle \ln T \rangle$ as a function of wire length L for the Anderson model with $N = 5$ propagating modes. The two dashed lines have the slopes predicted for preserved ($\beta = 1$) and broken ($\beta = 2$) time-reversal symmetry. From bottom to top the data correspond to fluxes $\Phi_\xi/\Phi_0 = 0$, 0.0005, 0.005, 0.05 (four indistinguishable solid curves), 0.5, 1, 2.5, 5, 10, 15, 20, 25, 40, 50, 75, 125 (two indistinguishable solid curves). The inset shows $\ln T$ for an individual realization with $\Phi_\xi = \frac{1}{2}\Phi_0$ (solid curve) and the slope of the ensemble-averaged result (dashed line). The statistical error is of the order of the wiggles of the curves.

Our second set of results is obtained from a computationally more efficient model of a disordered wire, consisting of a chain of chaotic cavities (or quantum dots) with two leads attached on each side. This so-called "domino" model [9] is similar to Efetov's model of a granulated metal [2] and to the Iida-Weidenmüller-Zuk model of connected slices [10]. The length L is now measured in units of cavities, and the mean-free path $l = 1$. The scattering matrices of each cavity are randomly drawn from an ensemble (proposed by Życzkowski and Kuś [11]) that interpolates (by means of a parameter δ) between the circular orthogonal ($\beta = 1$, $\delta = 0$) and unitary ($\beta = 2$, $\delta = 1$) ensembles of random-matrix theory. The relationship between δ and Φ_ξ/Φ_0 is linear for $\delta \ll 1$.

We increased the number of propagating modes to $N = 50$, because it is conceivable that the two-scale localization becomes manifest only in the large N limit, or that only in this limit the critical flux Φ_ξ for the transition from ξ_1 to ξ_2 becomes $\ll \Phi_0$. (In the experiments of Ref. [4] $N \approx 10$, so our simulations are in the experimentally relevant range of N.) Because of the much larger value of N, we restricted ourselves for larger values of the magnetic flux to $L \simeq 25\xi_1$, which should be sufficient to observe the localization length ξ_2 for $\Phi_\xi/\Phi_0 \gtrsim 10^{-2}$. For smaller values of the flux, we increased the wire length to $L \simeq 100\xi_1$. The data are presented in Fig. 3. It is qualitatively similar to the results for the $N = 5$ Anderson model. Instead of two-scale behavior, we see only a single decay length which crosses over smoothly from ξ_1 to ξ_2 with increasing δ. Again, the crossover of ξ coincides with the crossover of the weak-localization correction, so there is no anomalously small crossover flux for the localization length.

The logarithmic average $\langle \ln T \rangle$ is the experimentally relevant quantity since it is representative for a single realization (see Fig. 1, inset). The average transmittance

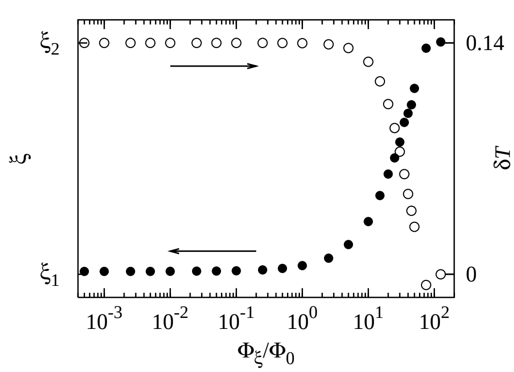

FIG. 2. Asymptotic decay length (solid circles) and weak-localization correction δT (open circles) as a function of flux for the $N = 5$ Anderson model. The statistical error is of the order of the size of the circles.

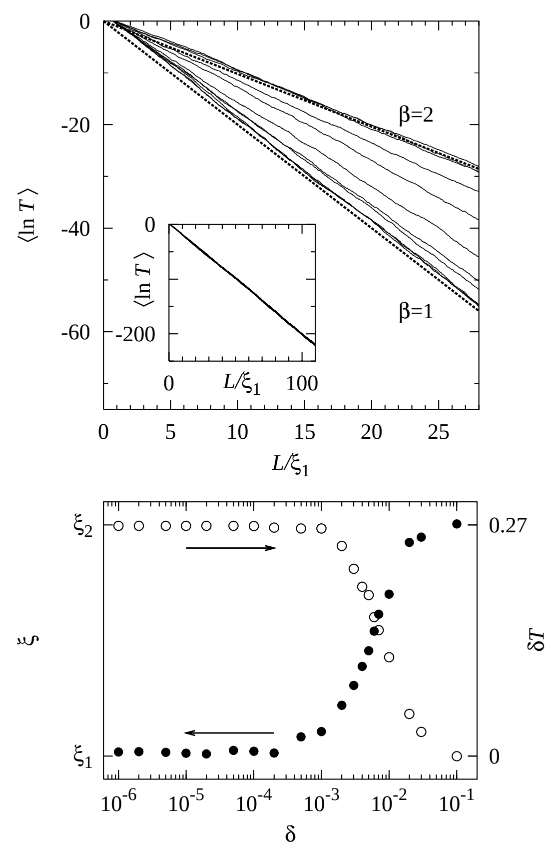

FIG. 3. Same quantities as in Figs. 1 and 2, but now for the $N = 50$ domino model. In the upper panel, the magnetic flux parameter $\delta = 0$, 0.0001, 0.0002, 0.0005, 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, and 0.1. In the inset, $\delta = 0$, 0.000 01, and 0.0001 (indistinguishable curves).

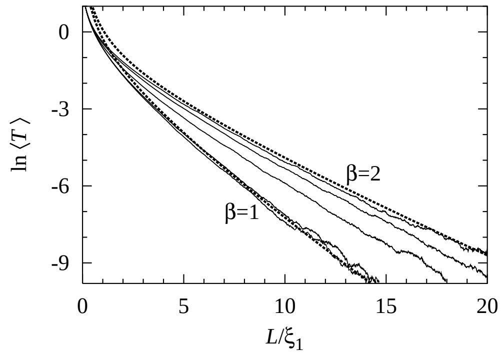

FIG. 4. Logarithm of the average transmittance $\ln\langle T\rangle$ as a function of wire length L for the $N = 5$ Anderson model at various values of the magnetic field (solid curves; from bottom to top, $\Phi_\xi/\Phi_0 = 0$, 5, 25, 50, 125). The dashed curves are the theoretical prediction of Refs. [13,14] for zero and large magnetic field.

$\langle T\rangle$ itself is not representative, because it is dominated by rare occurrences of anomalously localized states [12]. Since Kolesnikov and Efetov [1] studied the average of wave functions themselves, rather than the average of logarithms of wave functions, it is conceivable that their findings are the result of such rare occurrences. For completely broken or fully preserved time-reversal symmetry the average transmittance is given by [13]

$$\ln\langle T\rangle = -L/2\xi_\beta - \tfrac{3}{2}\ln L/\xi_\beta + \mathcal{O}(1). \qquad (2)$$

The order 1 terms are also known [13,14] and contribute significantly for $L \lesssim 30\xi_1$. (This is the numerically accessible range, because anomalously localized states become exponentially rare with increasing wire length.) We have plotted the full expressions in Fig. 4 (dashed curves), together with the numerical data for the $N = 5$ Anderson model. Again we find a smooth crossover between preserved and broken time-reversal symmetry. There is no transition with increasing wire length to a behavior indicative of completely broken time-reversal symmetry, even though the flux Φ_ξ is much larger than required [according to Eq. (1)] to observe this crossover for the wave functions.

In conclusion, we have presented a numerical search for the two-scale localization phenomenon predicted by Kolesnikov and Efetov [1], with a negative result: The asymptotic decay length of the transmittance is found to be given by ξ_1 and not by ξ_2, as long as the flux through a localization area is small compared to the flux quantum. How can one reconcile this numerical finding with the result of the supersymmetry theory? We give three possibilities: (i) One might abandon the Borland conjecture and permit the asymptotic decay length of the transmittance (Lyapunov exponent) to differ from the asymptotic decay length of the wave function (localization length). Since the Borland conjecture has been the cornerstone of localization theory for more than three decades, this seems a too drastic solution. (ii) One could argue that the wires in the simulation are too narrow or too short—although they are in the experimentally relevant range of N and L, as well as in the range of applicability of the theory of Ref. [1]. (iii) One could attribute the two-scale localization phenomenon to anomalously localized states that are almost fully transmitted but become exponentially rare with increasing length and are irrelevant for a typical wire. This seems to be the most likely solution. The decay due to anomalously localized states is solely due to their exponentially decreasing fraction among all states, and is not directly related to the localization length. For the limiting cases of fully preserved or totally broken time-reversal symmetry, the decay is by a factor of 4 slower than the localization length, but a two-scale behavior for partially broken time-reversal symmetry is conceivable.

A discussion with P. G. Silvestrov motivated us to look into this problem. We acknowledge helpful correspondence with A. V. Kolesnikov and support by the Dutch Science Foundation NWO/FOM.

[1] A. V. Kolesnikov and K. B. Efetov, Phys. Rev. Lett. **83**, 3689 (1999).
[2] K. B. Efetov, *Supersymmetry in Disorder and Chaos* (Cambridge University, Cambridge, England, 1997).
[3] J.-L. Pichard, M. Sanquer, K. Slevin, and P. Debray, Phys. Rev. Lett. **65**, 1812 (1990).
[4] Yu. B. Khavin, M. E. Gershenson, and A. L. Bogdanov, Phys. Rev. Lett. **81**, 1066 (1998); Phys. Rev. B **58**, 8009 (1998).
[5] R. E. Borland, Proc. R. Soc. London A **274**, 529 (1963).
[6] A. Crisanti, G. Paladin, and A. Vulpiani, *Products of Random Matrices* (Springer-Verlag, Berlin, 1993).
[7] H. U. Baranger, D. P. DiVincenzo, R. A. Jalabert, and A. D. Stone, Phys. Rev. B **44**, 10 637 (1991).
[8] C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
[9] T. Dittrich, E. Doron, and U. Smilansky, J. Phys. A **27**, 79 (1994).
[10] S. Iida, H. A. Weidenmüller, and J. A. Zuk, Ann. Phys. (N.Y.) **200**, 219 (1990).
[11] K. Życzkowski and M. Kuś, Phys. Rev. E **53**, 319 (1996).
[12] A. D. Mirlin, Phys. Rep. **326**, 259 (2000).
[13] M. R. Zirnbauer, Phys. Rev. Lett. **69**, 1584 (1992).
[14] A. D. Mirlin, A. Müller-Groeling, and M. R. Zirnbauer, Ann. Phys. (N.Y.) **236**, 325 (1994).