Shot Noise in Ballistic Quantum Dots with a Mixed Classical Phase Space

H.-S. Sim and H. Schomerus
Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Strasse 38, 01187 Dresden, Germany
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We investigate shot noise for quantum dots whose classical phase space consists of both regular and chaotic regions. The noise is systematically suppressed below the universal value of fully chaotic systems, by an amount which varies with the positions of the leads. We analyze the dynamical origin of this effect by a novel way to incorporate diffractive impurity scattering. The dependence of the shot noise on the scattering rate shows that the suppression arises due to the deterministic nature of transport through regular regions and along short chaotic trajectories. Shot noise can be used to probe phase-space structures of quantum dots with generic classical dynamics.

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The phenomenon of shot noise, the time-dependent fluctuations in electrical currents caused by the discreteness of the electron charge e, has been extensively studied in recent years in mesoscopic systems (for a review, see [1]). The focus of interest was on classically chaotic ballistic quantum dots (electron billiards) and on diffusive quantum wires, which can be investigated theoretically by applying random-matrix theory [2–4] as well as semiclassical methods [5–7]. For incoherent transport through a chaotic quantum dot the shot noise would assume the Poissonian value $P_0 = 2eG_0V$, where $G_0 = Ne^2/(2h)$ is the serial conductance of the two quantum point contacts which connect the dot to electronic reservoirs (maintained at a voltage difference V) by N channels. For low temperatures correlations of electrons due to Fermi statistics suppress the noise P by a factor of $\mathcal{F} = P/P_0$ relative to this value of uncorrelated electrons. In chaotic quantum dots the suppression factor $\mathcal{F} = \mathcal{F}_{\rm ch} = 1/4$ is universal [3], i.e., independent of the details of the system, which also has been confirmed by an experiment [8]. The origin of the low-temperature noise is the *probabilistic* nature of quantum transport, arising from attempts to transmit charge carriers between electronic reservoirs with a finite success probability $\in [0, 1]$. Very recently, nonuniversal corrections to the shot noise due to residual signatures of classically *deterministic* scattering [9] have been discussed by Agam, Aleiner, and Larkin [10]. They predicted that shot noise in a chaotic dot can be further reduced below $\mathcal{F}_{\rm ch}$ when electrons pass the dot without sufficient diffraction. This has been verified in a recent experiment [11].

In this Letter we address the shot noise of *generic* ballistic quantum dots. They are not classically chaotic [12] but possess a mixed phase space, where regular islands are separated from chaotic seas by impenetrable dynamical barriers. Signatures of the mixed phase space in quantum transport have been found in the conductance, which exhibits fractal fluctuations [13] and isolated resonances [14]. Shot noise can be seen as the second cumulant of charge counting statistics, with the conductance being the first cumulant. As we will demonstrate, shot noise carries valuable dynamical information which can be extracted systematically. For generic quantum dots the suppression factor $\mathcal{F}$ is found to be systematically reduced below the universal value $\mathcal{F}_{\rm ch}$ for fully chaotic systems, not only due to the deterministic nature of transport along short chaotic trajectories (this mechanism of Ref. [10] will be confirmed), but also because quantum diffraction is strongly reduced for transport through regular regions, as well.

Our investigation of the dynamical origin of the additional shot-noise reduction is based on a novel procedure to analyze shot noise (which we obtain from a numerical simulation) with help of the Poisson kernel [15–17], a statistical ensemble of random-matrix theory [4]. In the Poisson kernel one averages the scattering matrix over an energy range $E_{\rm av}$, in this way eliminating the system-specific details of the dynamics with time scales longer than $t_{\rm av} = \hbar/E_{\rm av}$, and replaces these by random dynamics of the same universality class as elastic diffractive impurity scattering (equivalently, fully chaotic dynamics). (Reference [10] also employed diffractive impurity scattering; however, there it was used to mimic the nondeterministic processes of quantum chaos within a theory that cannot account for them, while in our case all quantum effects are fully included from the beginning.) The effective mean free scattering time $t_{\rm av}$ can be tuned by changing the energy-averaging window $E_{\rm av}$. The dependence of the suppression factor $\mathcal{F}$ on $t_{\rm av}$ then allows one to analyze the properties of trajectories with classical dwell times $t_{\rm dwell} \simeq t_{\rm av}$ (up to a possible factor of order unity), over the large range of dwell times that is typically involved in the classical transport through generic quantum dots (in contrast, chaotic dots are characterized only by a single time scale, the mean dwell time). The analysis via the Poisson kernel is supplemented by a semiclassical estimation of the shot noise, obtained from classically deterministic motion course grained over Planck cells while preserving Fermi statistics, in the spirit of Refs. [6,7]. It follows that shot noise is suppressed most strongly if the leads are well coupled to classical regular regions (with area larger than a Planck cell).

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A representative model for mixed regular and chaotic classical dynamics is the two-dimensional annular billiard [18], Fig. 1(a), which consists of the region between two circles with radii R, r, and eccentricity δ. Two leads (openings) of width W are attached opposite to each other at an angle θ with respect to the axis through the two circle centers. The phase space of the closed annular billiard for $r = 0.6R$, $\delta = 0.22R$ is shown in Fig. 1(b), which displays two regular whispering-gallery (WG) regions, a large regular island, neighboring satellite islands, and a chaotic sea. Figures 1(c) and 1(d) show the phase space of the open annular billiard, which includes only trajectories of particles that are injected into the billiard through leads attached at $\theta = 0$ and $\theta = \pi/2$, respectively (the width of the leads is $W = 0.222R$). The large island is well coupled to one of the openings for $\theta = 0$, and is completely decoupled from both openings at $\theta = 0.5\pi$. The chaotic sea and WG regions are well coupled for arbitrary position of the openings, while the coupling of the satellite islands depends on θ and W. In this way, one can select regions in phase space by varying θ and W.

We computed the dimensionless conductance $T = \mathrm{tr}t^\dagger t$ and the shot-noise suppression factor $\mathcal{F} = (2/N)\mathrm{tr}t^\dagger t(1 - t^\dagger t)$ from the transmission matrix t (the dimension of this matrix is given by the number of channels N in each lead) [1]. The transmission matrix is obtained numerically by the method of recursive Green functions [19], for which space is discretized on a square lattice. In terms of the lattice constant a, $R = 144a$, $r = 86.4a$, $\delta = 31.7a$, and $W = 32a$. Energy E will be measured in units of $\hbar^2/(2ma^2)$ and time in units of $2ma^2/\hbar$, with m the mass of the charge carriers. In these units the mean level spacing $\Delta \simeq 0.0003$. We will work in the energy window $E \in (0.408, 0.433)$, in which the Fermi wavelength $\lambda_F \simeq 9.5a$, resulting in $N = \mathrm{Int}(2W/\lambda_F) = 6$.

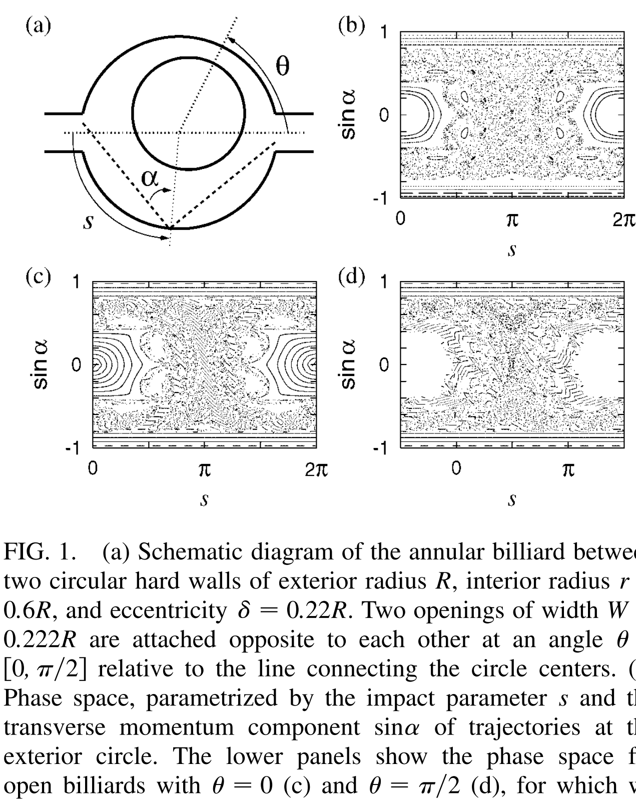

FIG. 1. (a) Schematic diagram of the annular billiard between two circular hard walls of exterior radius R, interior radius $r = 0.6R$, and eccentricity $\delta = 0.22R$. Two openings of width $W = 0.222R$ are attached opposite to each other at an angle $\theta \in [0, \pi/2]$ relative to the line connecting the circle centers. (b) Phase space, parametrized by the impact parameter s and the transverse momentum component $\sin\alpha$ of trajectories at the exterior circle. The lower panels show the phase space for open billiards with $\theta = 0$ (c) and $\theta = \pi/2$ (d), for which we only record trajectories that are injected through the openings (until they leave the billiard again).

The two panels of Fig. 2(a) show T and $\mathcal{F}$, as a function of energy, for the case that the two leads are attached at $\theta = \pi/2$. The dependence of the energy-averaged suppression factor $\mathcal{F}$ on θ is shown in Fig. 2(b). Our first observation is that for all values of θ, the suppression factor is smaller than the universal value $1/4$ for fully chaotic motion. Moreover, the shot noise is more suppressed for the cases that one opening couples to the large regular island ($0 < \theta \lesssim 0.3\pi$) than for the cases in which the regular island is decoupled from the openings ($\theta \gtrsim 0.3\pi$). This behavior indicates that electrons injected into the large regular island contribute less to the shot noise.

In order to determine in detail which processes are responsible for the additional shot-noise suppression, we introduce a function $f(t_{av})$ which can be interpreted as the probability distribution function of deterministic processes with dwell time $t_{dwell} \simeq t_{av}$. This will be achieved by monitoring the rate of change of the shot noise,

$$f(t_{av}) = -(d/dt_{av})\mathcal{F}(t_{av})/\mathcal{F}_{ch}, \tag{1}$$

as we successively replace the dynamics on time scales $\gtrsim t_{av}$ by fully chaotic dynamics (equivalent to diffractive impurity scattering). The function f is normalized to the total shot-noise suppression $\int_0^\infty dt_{av} f(t_{av}) = 1 - \mathcal{F}/\mathcal{F}_{ch}$, taken relative to the universal value of chaotic dynamics.

The transmission matrix t determining T and $\mathcal{F}$ is a subblock of the scattering matrix

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}, \tag{2}$$

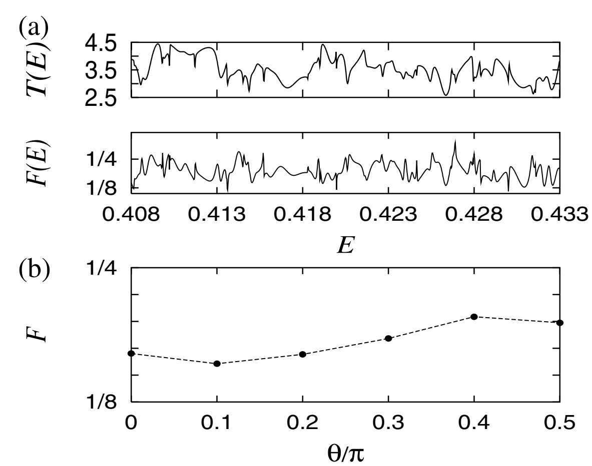

FIG. 2. (a) Dimensionless conductance $T(E)$ and shot-noise suppression factor $\mathcal{F}(E)$ for leads attached at $\theta = 0.5\pi$. (b) Energy-averaged suppression factor as a function of lead position θ.

where r and r' are reflection coefficients and the transmission matrix t' contains the same information as t. The elimination of the system-specific details on time scales $\gtrsim t_{\rm av}$ is achieved by averaging the scattering matrix S over an energy range $[E_0 - E_{\rm av}/2, E_0 + E_{\rm av}/2]$ of width $E_{\rm av} = \hbar/t_{\rm av}$ (which will be taken inside the total energy range [0.408,0.433] of our numerical simulation),

$$\overline{S}(E_{\rm av}; E_0) = E_{\rm av}^{-1} \int_{E_0 - E_{\rm av}/2}^{E_0 + E_{\rm av}/2} dE S(E). \tag{3}$$

Here the information on processes with longer time scales than $t_{\rm av}$ is lost, because it is encoded into the short-range energy correlations (fluctuations) of the scattering matrix [20], while the information on the dynamics on shorter time scales than $t_{\rm av}$ modulates the scattering matrix on larger energy scales and hence is retained. Chaotic processes are then introduced to substitute the eliminated ones by coupling to an auxiliary chaotic system with scattering matrix S_0, taken from the appropriate circular ensemble of random-matrix theory (observing the same symmetries as the original scattering matrix, as time-reversal or spatial parities [21]), resulting in

$$S'(E_{\rm av}; E_0; S_0) = \overline{S}(E_{\rm av}; E_0) + \mathcal{T}'(1 - S_0 \mathcal{R})^{-1} S_0 \mathcal{T}. \tag{4}$$

The ensemble of scattering matrices (4) is the so-called Poisson kernel [4,15–17], with $\overline{S}$ the so-called optical scattering matrix. The coupling matrices $\mathcal{T}$, $\mathcal{T}'$, and $\mathcal{R}$ must be chosen such that S' is a unitary matrix, but the invariance of the circular ensemble guarantees that results do not depend on their specific choice.

We now can calculate the mean suppression factor $\mathcal{F}(E_{\rm av})$ for fixed $E_{\rm av}$ (and hence fixed $t_{\rm av}$), first by averaging the noise within each Poisson kernel (fixing also E_0), and then averaging these values over $E_0 \in (0.408, 0.433)$. The result for different positions of the leads is shown in Fig. 3. The value $\mathcal{F}(E_{\rm av} = 0)$ is identical to the suppression factor $\mathcal{F}$ shown in Fig. 2(b), because the Poisson kernel becomes a delta function at $S(E_0)$ as $\overline{S}$ approaches this unitary matrix for $E_{\rm av} \to 0$. For increasing $E_{\rm av}$ the suppression factor $\mathcal{F}(E_{\rm av})$ approaches the universal value $\mathcal{F}_{\rm ch}$ of random-matrix theory, because $\overline{S} = 0$ in this limit. The function $\mathcal{F}(E_{\rm av})$ is monotonically increasing almost everywhere (negative values of the slope are within the statistical uncertainty in the numerical simulation), indicating that the noise is *enhanced* by the replacing the system-specific deterministic details of the transport by the indeterministic transport of random-matrix theory. Consequently, as shown in Fig. 3(b), the function $f(t_{\rm av})$ introduced in Eq. (1) is positive and hence allows the interpretation of a probability distribution function of deterministic scattering.

The most striking feature in Fig. 3 is that $\mathcal{F}(E_{\rm av})$ rises quickly in the three cases $\theta = 0$, 0.1π, 0.2π, in which the leads couple to the large regular island in phase space, while the slope of $\mathcal{F}(E_{\rm av})$ almost vanishes up to

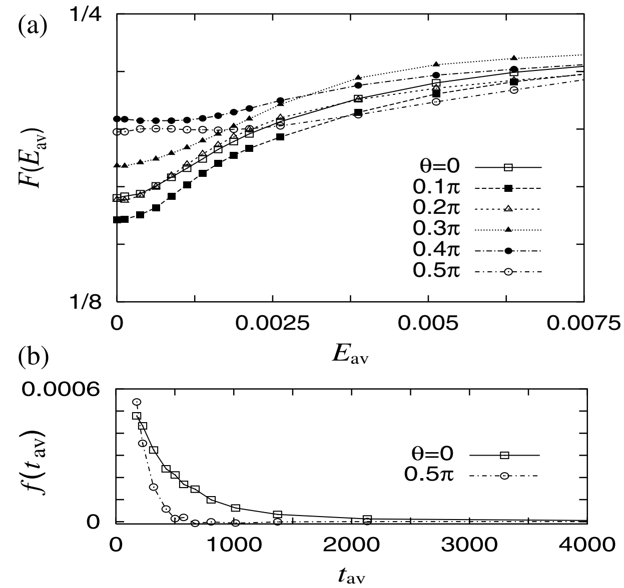

FIG. 3. (a) Shot-noise suppression factor $\mathcal{F}$ as a function of the strength of diffractive impurity scattering with a rate $\simeq E_{\rm av}/\hbar$, for different positions θ of the leads. (b) Distribution function $f(t_{\rm av})$ of deterministic processes.

an energy $E_{\rm indet} \approx 0.002$ for $\theta = 0.4\pi$, 0.5π (where the regular region is decoupled). The distribution function $f(t_{\rm av})$ decays only slowly in the former case, while it vanishes almost identically beyond a time $t_{\rm indet} \approx 500$ in the latter case. This difference can be interpreted as follows: Wave packets are trapped for long times in regular regions, but show only little dispersion because of the stability of regular trajectories [22]. Hence, the scattering is deterministic and strongly affected by introducing diffractive scattering even at these long time scales. On the other hand, wave packets disperse quickly in the chaotic regions, where the transport is already indeterministic for $t_{\rm av} \gtrsim t_{\rm indet}$, and this is not modified by adding diffractive scattering. The shot-noise suppression for $\theta = 0.4\pi$ and $\theta = 0.5\pi$ hence arises from short chaotic trajectories with dwell times $\lesssim t_{\rm indet}$. This is in accordance with the predictions of Ref. [10]; chaotic quantum transport is deterministic up to the Ehrenfest time $(1/\lambda)\ln(2\pi R/\lambda_{\rm F}) \simeq 500$, estimated from the Lyapunov exponent $\lambda \simeq v_{\rm F}/R$ and the Fermi velocity $v_{\rm F}$. Thus $f(t_{\rm av})$ reveals system-specific time scales as the Ehrenfest time in chaotic regions and long dwell times in regular regions. Moreover, $f(t_{\rm av})$ vanishes for $t_{\rm av} \lesssim \min t_{\rm dwell} \simeq 50$ (not resolved in Fig. 3).

Our interpretation is confirmed by a semiclassical estimate which discriminates the contributions to the shot noise from different regions in phase space. We divide the phase space into Planck cells of area $2W/(NR)$ and calculate the fluctuations in the semiclassical occupation numbers of these cells. The average occupation number $\overline{f}_j = T_{jL} f_L + T_{jR} f_R$ is obtained from the classical transmission probabilities $T_{jL,R}$ from cell j to the left (L) and right (R) opening, respectively, where $f_{L,R}$ is the Fermi distribution in the electronic reservoirs attached to each

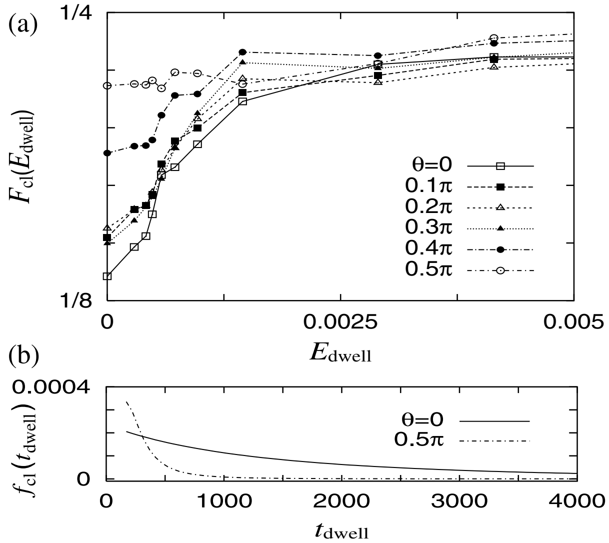

FIG. 4. Semiclassical estimates of $\mathcal{F}$ and f of Fig. 3, obtained by introducing indeterminism into the classical dynamics beyond a given dwell time $t_{\text{dwell}} = \hbar/E_{\text{dwell}}$.

opening. Within the model of minimal correlations of Ref. [7], the shot-noise suppression factor is then approximated as $\mathcal{F}_{\text{cl}} = \langle \overline{f}_j(1 - \overline{f}_j)\rangle$, where $\langle \cdots \rangle$ denotes the average over the phase space cells at the openings.

Diffractive scattering can be introduced into the semiclassical theory by replacing the classical transmission probability of trajectories longer than a given dwell time t_{dwell} by the value $1/2$ of ergodic dynamics. [The universal value of shot noise for chaotic dynamics follows from the ergodic value $\overline{f}_j = 1/2$.] Figure 4 shows the resulting $\mathcal{F}_{\text{cl}}(E_{\text{dwell}})$ and $f_{\text{cl}}(t_{\text{dwell}})$, with $E_{\text{dwell}} = \hbar/t_{\text{dwell}}$. The same relative time scales for the different positions of the leads are found as in the exact results in Fig. 3. The agreement is reasonable given that we are not deep in the semiclassical limit and accounting for the fact that the model of minimal correlations in Ref. [7] originally was devised for chaotic dynamics. The suppression factor can be further decomposed into contributions of different regions in phase space. For $\theta = 0$, $\mathcal{F}_{\text{cl}}(E_{\text{dwell}})$ initially increases due to the large regular island and then due to deterministic processes in the chaotic sea, while for $\theta = 0.5\pi$ it is constant up to $E_{\text{dwell}} \simeq E_{\text{indet}}$. Unlike the large regular island, the regular WG regions (which we have ignored so far in our discussion) contribute to shot noise with the universal value, because for the present system parameters Planck cells do not yet resolve reflected from transmitted trajectories which are injected into these regions. One can see from the model of minimal correlations that shot noise eventually will be suppressed also in the WG regions when the semiclassical limit is approached. The same holds for the hierarchical structure of small regular regions embedded in the chaotic part of phase space. A pronounced shot-noise suppression should arise once that these trapping regions are resolved, because in the vicinity of stable structures the Ehrenfest time depends only algebraically on $\hbar$ [22].

In conclusion, we have demonstrated that shot noise is a measure of the amount of deterministic transport through generic quantum dots. The dynamical analysis developed in this work may be used to study time scales in a large variety of systems. Experimentally, these time scales could be probed by measuring shot noise while tuning the indeterministic scattering rate by adjusting a gate voltage [23].

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