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Ramsey Monetary and Fiscal Policy: the Role of Consumption Taxation *

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Abstract

We study Ramsey monetary and fiscal policy in a small scale New Keynesian model where government spending has intrinsic value, public debt is state-noncontingent and the fiscal authority is constrained by using distortive taxation. We show that Ramsey policy is remarkably altered when consumption taxation is considered as a source of government revenues alongside or as an alternative to labour income taxes. First, we show that the optimal steady-state size of the public spending is, ceteris paribus, greater under consumption taxation than under labour income tax. We further show that adopting consumption taxation has enourmous long run welfare gains and that these gains are increasing in the level of outstanding public debt. These welfare gains are not limited to the steady-state, but they are also present in the dynamic stochastic equilibrium. The reason is that the dynamic nature of consumption taxation enables the policy-maker to affect the stochastic discount factor via modifications of the marginal utility of consumption. This extra wedge impacts on the pricing decisions of firms, and hence on inflation stabilization, and greatly improves welfare in the stochastic equilibrium.

Keywords: Ramsey policy, Optimal public spending, Consumption tax, Distortionary taxation.

JEL: E30,E61, E62.

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1 Introduction

Following the recent financial crisis, governments around the globe implemented massive fiscal plans and a large number of fiscal reforms. This has contributed to increase the interest of the economic discipline in fiscal policy issues. Along this line, this paper tries to address several policy questions. What are the consequences for the setting of optimal monetary and fiscal policies of different tax arrangements? In particular, what are the welfare consequences of different tax instruments? What is optimal size of the public sector and how this optimal size is affected by different fiscal arrangements and different levels of public debt?

In order to address these questions, this paper studies the optimal mix of monetary and fiscal policy in a New Keynesian model where public spending has intrinsic value and the fiscal authority can use consumption and labour income taxation in order to finance public spending and to finance public debt.

The economic environment considered in this paper features three inefficiencies. First, firms have market power in the good markets which allows them to charge a mark-up over marginal costs. This causes output to be below the efficient level. Second, sticky prices in the good market prevent firms from fully adjusting their prices in response to shocks. Third, fiscal policy has to use distortionary consumption and labor income taxes to finance public spending and interest payments on outstanding government debt. Public spending and government debt thus have additional adverse effects on economic activity.

In order to address our policy questions, we analyze what is the best way of jointly setting the different policy instruments, i.e. consumption and labour income taxation, government spending, public debt and the short term interest rate, in order to deal with the distortions described in the previous paragraph. In turn, as we will discuss in details later, this model generates several channels for which there exist non trivial interactions between monetary and fiscal policy instruments.

The policy analysis is divided into two main parts. In the first one, we analyze Ramsey policy at steady state. We start by showing that in order to replicate long run efficiency monetary policy implements a zero inflation policy. This means that the Ramsey Planner does not find it optimal to use steady state inflation in order to decrease the real values of profits or to erode the real value of any outstanding debt/asset position. Furthermore, efficiency requires, independently from the tax instrument adopted, the Ramsey Planner to accumulate large asset positions against the private sector. The income from these assets are then used to balance the government budget constraint and to correct the distortions generated by monopolistic competition in the good market. This result implies that when public debt is positive, or at least not too negative, the Ramsey Planner cannot replicate the efficient allocation.

In this second best scenario with positive public debt, we find that optimal Ramsey policy calls for extremely high consumption taxation and labour subsidy, well above 100%, as the difference between consumption and wage income is usually small relative to government spending.

However, this tax scheme may easily lead to two type of problems. First, a wage subsidy of the order of 500%, say, seems impractical, as it would lead to tremendously high costs associated in verifying hours worked. Applying the same logic, a 600% consumption tax rate would probably lead to a large amount of unreported barter. Therefore we leave the Ramsey Planner free to tax jointly labour income and consumption when large asset positions can be accumulated against the public, while in the second best scenario where public debt is allowed to take an arbitrary (and rather large) value, we restrict the policy maker to tax either labour or consumption.

Under both scenarios, we find that optimal policy implies zero steady state inflation. This result does not dependent on the long run level of public debt, nor on the degree of monopolistic power in the good markets. Therefore, the Ramsey Planner never finds optimal to use inflation in order to erode the real value of profits and public debt.

Furthermore we find that under both fiscal scenarios, the Ramsey Planner actively uses government spending as a policy instrument. In particular, we study the optimal size of government consumption relative to total output. By setting a certain ratio of government spending to GDP, the Ramsey Planner can influence private sector behavior and therefore can reduce inefficiency and increase welfare. We show that in the present setting which considers homothetic preferences over public and private consumption, the incentive for the Ramsey Planner to increase the government spending to GDP ratio increases with the households' risk aversion. The intuition for this result is the following. Consider, for instance, the case where only labour income taxes are available to the policy-maker and assume there is an increase in inefficiency, due, for example, to more monopolistic market power or higher outstanding debt. This would push economic activity away from the first best. The Ramsey Planner by adjusting government spending to GDP ratio and therefore the level of taxation, can influence households labour supply and private consumption and potentially reduce inefficiency. For example, by imposing a government to GDP ratio lower than in the first best allocation, the policy maker can sustain a given level of outstanding debt with lower taxation. Lower taxation increases labour supply. We show that when consumers' risk aversion is 'low' the increase in the labour supply that this policy brings about boosts private consumption and hence reduces inefficiency. On the other hand, when the degree of risk aversion is 'high', an increase in labour supply decreases private consumption. In this case, the Ramsey Planner has the incentive to set the government spending to GDP ratio above its first best counterpart. This policy decreases labour supply and pushes private consumption towards its efficient level.

A similar logic can be applied to the case when only consumption taxation is available. In this case, government size, and therefore taxation, directly impacts on private consumption so that lower tax rates are associated with higher consumption levels. Ceteris paribus, we show that when the degree of risk aversion is 'low', higher consumption implies a higher labour supply. In this case, we find that the Ramsey Planner sets the government spending to GDP ratio below its first best counterpart. This policy boosts both consumption and labour supply and in turn pushes the economy closer to efficiency. On the other hand, when consumers' risk aversion is high, higher consumption is associated with lower labour supply. In this case optimal policy sets the government spending to GDP ratio above its first best counterpart.

When we compare the steady state optimal allocation under the two tax schemes, i.e. labour income and consumption taxation, we find that under consumption taxation the optimal share of government spending is always higher then under labour income taxation. Furthermore, we find that, for a given level of outstanding debt-to-GDP ratio, there is an enormous welfare gain in taxing consumption rather than labour income. In turn, we show that these gains are increasing in the level of steady state public debt and in the risk aversion parameter.

In the second part of the paper we analyze the dynamic behavior of the economy under technology shocks. First we study the Ramsey dynamics under the assumption that the government is allowed to accumulate large assets positions against the public so that the steady state allocation is efficient.

When the policy maker has access to both labour income and consumption taxation, the Ramsey solution perfectly coincides with the efficient equilibrium along the transition path. Hence, the Ramsey Planner can commit to a state contingent policy that perfectly replicates the first best allocation. In doing so, the two tax instruments need to move in opposite directions, i.e. if consumption tax increases, the labour income tax rate must decrease of the same amount. This policy does not distort the marginal rate of substitution between consumption and leisure and the real wage is free to adjust according to the technology process.[1] Furthermore, the Ramsey Planner must use consumption taxation and the nominal interest rate in order to replicate the state contingent return in the bond market via the real stochastic discount factor in the Euler equation, and thereby eliminate any incentive for firms to change prices. At the same time government spending has to move so that the marginal utility of public and private consumption are the same.

Then we study a scenario where the Ramsey Planner is constrained to using one tax instrument while the other tax rate is fixed at zero but accumulates assets such that the steady state is efficient. By losing consumption or labour income taxation, the Ramsey dynamics are not fully efficient any longer, and, as a consequence, households suffer welfare losses in the stochastic equilibrium. For example, the policy-maker cannot offset anymore the distortive consequences of a tax change on the marginal rate of substitution between consumption and leisure. As a result, optimal policy allows for deviations from full price stability that in turn modify the return in the bond market. This policy is meant to push the dynamic equilibrium towards the efficient allocation. While this cannot be completely obtained, under consumption taxation this policy is much more effective. The reason is that the dynamic nature of consumption taxation enables the policy-maker to affect the real stochastic discount factor via modifications of the marginal utility of consumption. This extra wedge helps the Ramsey Planner to push the dynamic allo-

[1] As it is well known, e.g. Benigno and Woodford (2003) and Schitt-Grohé and Uribe (2004), when prices are sticky, optimal policy abandons the traditional tax smoothing behaviour.

cation closer to the efficient one. In turn, the welfare loss of being constrained of keeping labour income tax fixed at zero are roughly 1.6% as big as when the policy-maker is constrained in keeping fixed (at zero) consumption taxation. Furthermore, it is interesting to note that there is very little use of government spending as a stabilization tool in the stochastic equilibrium.

Similarly, when the policy maker is constrained to use only one tax instrument and public debt is positive, the welfare losses under consumption taxation are much smaller than under labour income tax. As in the previous case, the superiority of consumption taxation is due to the dynamic nature of this policy instrument.

This paper links to the existing literature in several ways. First of all, it is closely related to Benigno and Woodford (2003), Schmitt-Grohé and Uribe (2004, 2007) and Adam (2011). These works also analyze optimal Ramsey policy in a New Keynesian setting and show, as here, that nominal rigidities prevent the government from using price level changes as an important source of state-contingent taxation in the presence of nominal government debt. As a result, government debt optimally follows a near random walk, as in Barro (1979) and Aiyagari et al. (2002). However, none of these works consider the possibility of consumption taxation. An unfortunate aspect of this restriction, though, is that it rules out a tax rate that is clearly used by many countries. As reported by Gordon and Li (2009), the average consumption tax rate in developed countries is around 16% and it reaches 25% in Hungary and Denmark.

Secondly, this paper is related to the optimal policy literature on consumption taxation. Coleman (2000) studies Ramsey policy under consumption taxation in a neoclassical growth model. He finds that replacing income taxes with a constant consumption tax leads to a welfare gain that is only slightly lower than that attained by a dynamic policy that taxes consumption and income. Correia (2010) extends this result in a setting with heterogenous agents and also finds large welfare gains in adopting consumption taxation. Correia, Nicolini and Teles (2008) show that in a New Keynesian model with consumption and labour taxation, full price stability can be achieved via an appropriate mix of labour and consumption taxation. Furthermore, Correia, Farhi, Nicolini and Teles (2013) show that when the economy is constrained at the zero lower bound on the nominal interest rate, consumption taxation may replace the monetary policy instrument as demand management tool. However, unlike here, they study a framework in which government spending is exogenous and hence they do not analyze how different taxations and public debt levels affect optimal government spending.

We believe that analyzing optimal fiscal policy with endogenous government spending may be important for two reasons. First, government spending represents an important share of GDP of all industrialized countries and it is one of the main fiscal instruments used by policy-makers. It is therefore hard to understand how it can be treated as exogenous from a normative perspective. Second, as Teles (2013) points out, different assumptions about the endogeneity of government spending may lead to sharp differences in optimal policy prescriptions and welfare calculations.

The paper is organized as follows. Section 2 sets up the model. Section 3 presents the optimal policy exercises. Section 4 concludes.

2 The model

We add a fiscal policy block to the standard sticky-price cashless DSGE framework, similar to the workhorse model in e.g. Clarida et al. (1999) or Woodford (2003).

Public consumption has intrinsic value for the agents as in Galí and Monacelli (2008), Adam and Billi (2008) and Adam (2011). Government spending must be financed with linear labour income and/or consumption taxes. Lump-sum taxes or transfer are ruled out. We restrict public nominal debt to be of one-period maturity and to be state-noncontingent as in Schmitt-Grohé and Uribe (2004) and Correia, Nicolini and Teles (2008).

Besides presenting the model ingredients, this section derives the implementability constraints characterizing optimal private sector behavior, i.e., it derives the optimality conditions determining households' consumption and labor supply decisions, firms' price setting decisions, as well as the government's budget constraint.

2.1 Private Sector

2.1.1 Households

There is a continuum of infinitely-lived households with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, h_t, g_t\right), \tag{1}$$

where β is the discount factor, c_t represents individual consumption, h_t denotes hours worked and g_t is government spending. E_t identifies the rational expectations operator. We impose that utility is separable in its three arguments and $u_c > 0, u_{cc} < 0, u_g > 0, u_{gg} < 0, u_h < 0$ and $u_{hh} \leq 0$. Where u_x defines the derivative of the utility function with respect to the generic variable x. Furthermore, we assume homothetic preferences over public and private consumption, so that $-\frac{g u_{gg}}{u_g} = -\frac{c u_{cc}}{u_c}$.

There is a continuum of goods, indexed by i. Each i good enters with the same weight in the Dixit-Stiglitz aggregator. This can be written as

$$c_t = \left[\int_0^1 \left(c_{i,t}\right)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} ; \quad i \in [0,1], \eta > 1, \tag{2}$$

while the aggregate consumption price index is

$$P_t = \left[\int_0^1 P_{i,t}^{1-\eta} di\right]^{\frac{1}{1-\eta}}. \tag{3}$$

Hence, the demand for good i follows

$$c_{i,t} = c_t \left(\frac{P_{i,t}}{P_t} \right)^{-\eta}, \tag{4}$$

where $\eta \in (1, \infty)$ is the price elasticity for differentiated goods.

Households maximize (1) subject to the following period budget constraint

$$P_t c_t \left(1 + \tau_t^c\right) + R_t^{-1} B_{t+1} + E_t \Lambda_{t,t+1} Q_{t+1} = B_t + Q_t + P_t d_t + P_t w_t h_t \left(1 - \tau_t^h\right). \tag{5}$$

In each time period t, households can purchase any desired state-contingent nominal payment Q_{t+1} in period $t+1$ at the dollar cost $E_t \Lambda_{t,t+1} Q_{t+1}$. The variable $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period t and $t+1$. Here the only role of state-contingent securities is to define state-contingent prices. We assume that state-contingent claims are in zero net supply. Real dividends are denoted by d_t, while B_t is the quantity of risk-less nominal bonds purchased in period t at price R_t^{-1} and paying one unit of the consumption numeraire at period $t+1$. Taxes on consumption and labour income are, respectively, τ_t^c and τ_t^h, and w_t is the real wage.

The solution for the optimizing household problem is standard and it can be written as:

$$u_{c,t} = \lambda_t \left(1 + \tau_t^c\right), \tag{6}$$

where λ_t stands for the Lagrangian multiplier associated with this programme. Labor supply is determined by

$$-\frac{u_{h,t}}{u_{c,t}} = w_t \frac{\left(1 - \tau_t^h\right)}{\left(1 + \tau_t^c\right)}, \tag{7}$$

while the Euler equation is

$$\frac{u_{c,t}}{1 + \tau_t^c} = \beta E_t \left[\frac{u_{c,t+1}}{\left(1 + \tau_{t+1}^c\right)} \frac{R_t}{\pi_{t+1}} \right], \tag{8}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate. The stochastic discount factor is defined as $E_t \Lambda_{t,t+1} = \beta E_t \frac{u_{c,t+1}}{P_{t+1}} \frac{P_t}{u_{c,t}} \frac{\left(1+\tau_t^c\right)}{\left(1+\tau_{t+1}^c\right)}$, and absence of arbitrage profits in the asset markets implies that $E_t \Lambda_{t,t+1} = R_t^{-1}$.

2.1.2 Firms

A generic good i is produced in a monopolistically competitive market with technology

$$y_{i,t} = a_t h_{i,t}, \tag{9}$$

where a_t is a common exogenous technology process. Firm i's real marginal costs are:

$$mc_t = \frac{w_t}{a_t}. \tag{10}$$

We assume that firms, when resetting their prices, incur in quadratic adjustment costs a lá Rotemberg (1983), i.e.,

$$\frac{\varphi}{2} P_t \left(\frac{P_{i,t+s}}{P_{i,t+s-1}} - 1 \right)^2, \tag{11}$$

where φ represents the degree of price stickiness. The profit maximizing generic firm i's problem can be expressed as

$$\max_{\{P_{i,t}\}} E_t \sum_{s=0}^{+\infty} \beta^s \left(\frac{u_{c,t+s}}{u_{c,t}} \frac{(1+\tau_t^c)}{(1+\tau_{t+s}^c)} \right) \left[\left(\frac{P_{it+s}}{P_{t+s}} y_{it+s} - w_{t+s} \frac{y_{it+s}}{a_{t+s}} \right) - \frac{\varphi}{2} \left(\frac{P_{i,t+s}}{P_{i,t+s-1}} - 1 \right)^2 \right]$$
$$s.t. \; y_{i,t+s} = \left(\frac{P_{i,t+s}}{P_{t+s}} \right)^{-\eta} y_{t+s}.$$

We focus on the symmetric equilibrium in which $P_{it} = P_t$ holds. Therefore, substituting for mc_t, the condition for optimal pricing decision is

$$[(1-\eta) a_t + \eta w_t] (h_t) - \varphi (\pi_t - 1)(\pi_t) + \varphi \beta E_t \left\{ \left[\frac{u_{c,t+1}}{u_{c,t}} \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} \right] (\pi_{t+1} - 1)(\pi_{t+1}) \right\} = 0. \tag{12}$$

2.2 Aggregation

In the symmetric equilibrium $h_{it} = h_t$. Hence the economy-wide production function is

$$y_t = a_t h_t. \tag{13}$$

Furthermore, using the household's budget constraint, we can obtain the expression for aggregate profits as

$$d_t = a_t h_t - w_t h_t - \frac{\varphi}{2} (\pi_t - 1)^2. \tag{14}$$

Combining the government budget constraint, the definition of profits and the households budget constraint, one can obtain the aggregate resource constraint as

$$y_t = a_t h_t = c_t + g_t + \frac{\varphi}{2} (\pi_t - 1)^2 \tag{15}$$

2.3 Government sector

Macroeconomic policies are implemented by two authorities. There is a Central Bank which sets the nominal interest rates on short-term nominal bonds. Furthermore, there is a government choosing the level of public expenditures, labor income tax, consumption tax and public debt. The government finances current expenditure by raising linear labor income and consumption taxes and by issuing new one-period state-noncontingent debt, i.e.,

$$\frac{b_t}{\pi_t} + g_t = \frac{b_{t+1}}{R_t} + w_t h_t \tau_t^h + c_t \tau_t^c. \tag{16}$$

The fiscal authority credibly commits to repaying its debt. In what follows we assume that government debt and tax policies are such that the no-Ponzi constraint

$$\lim_{s \to +\infty} E_t \left[\left(\prod_{i=0}^{t+s-1} \frac{1}{R_i} \right) B_{t+s} \right] = 0, \tag{17}$$

and the transversality constraint

$$\lim_{s \to +\infty} \left[\beta^{t+s} \left(\frac{u_{c,t+s}}{1+\tau_{t+s}^c} \right) \frac{B_{t+s}}{P_{t+s}} \right] = 0, \tag{18}$$

are both satisfied.

2.4 Rational Expectation Equilibrium

Definition 1 ***(Rational Expectations Equilibrium)*** *A Rational Expectations Equilibrium (REE) is defined by a sequence of private sector decisions* $\{c_t,\ y,\ h_t,\ w_t,\ \pi_t\}_{t=0}^{\infty}$ *and economic policies* $\left\{\tau_t^h,\ \tau_t^c,\ R_t,\ g_t,\ b_{t+1}\right\}_{t=0}^{\infty}$ *that, given the initial level of public debt* b_0 *and the evolution of technology, solves equations (7), (8), (12), (13), (15), (16), (17) and (18).*

3 Ramsey Policy

3.1 First Best Allocation

Definition 2 ***(Social Planner's Program)*** *The Social Planner's Program defines the first best allocation and consists in choosing* $\{c_t^*, h_t^*, g_t^*\}_{t=0}^{\infty}$ *taking as given the technology process* $\{a_t\}_{t=0}^{\infty}$*, in order to maximize the utility function of households as in (1) subject to the constraints imposed by the production technology.*

Proposition 3 *The Social Planner's allocation is*

$$(u_{c,t})^* = -\frac{(u_{h,t})^*}{a_t} = (u_{g,t})^* \tag{19}$$

Proof. Please refer to the *Appendix.* ■

In the first best equilibrium the marginal utilities of private and public consumption must equate to the marginal disutility of labour, where the latter is scaled by total productivity. This simple allocation rule is optimal because it is equally costly to produce public and private consumption goods.

Definition 4 ***(Ramsey Problem):*** *The Ramsey Problem is to maximize (1) over Rational Expectation Equilibria. A Ramsey outcome is a Rational Expectation Equilibrium that attains the maximum of (1).*

In this policy problem, the Lagrangian multipliers associated with forward looking variables, i.e. $E_t\pi_{t+1}$, $E_t u_{c,t+1}, E_t\tau^c_{t+1}$ are additional state variables. Assuming these states initial values equal to zero implies transitory non-stationary components in the solution to the Ramsey problem, even in a non-stochastic environment. This is because in the initial period the policy-maker may have the temptation to temporary increase taxes or generate inflation so as to erode the real value of any outstanding government debt.[2] We do not analyze these non-stationary deterministic components and concentrate instead on the time-invariant deterministic long-run outcome. This is what we define as the *Ramsey Steady State* (RSS henceforth). This means that we are imposing an initial commitment on the policy-maker not to generate 'surprise' movements in taxes, government spending, or nominal interest rates in period zero. This is standard practice in the optimal taxation literature, see e.g. Schmitt-Grohè and Uribe (2004).[3]

3.2 Analytical Results

As it is well known, there exists a continuum of RSS, each of which associated with an outstanding level of public debt. In other words, without specifying the steady state level of public debt, the model displays an indeterminacy of degree one.

Proposition 5 ***(Optimal Inflation)****: Despite this indeterminacy, all the RSS are characterized by*

$$\pi = 1 \tag{20}$$

and

$$R = \frac{1}{\beta} \tag{21}$$

Proof. See Appendix. ■

[2] However contrary to a flexible price economy, the Ramsey Planner does not find optimal to set $P_0 = \infty$.

[3] Furthermore, given the presence of sticky prices and state-noncontingent govenrment bond, our primal form of the Ramsey problem can no longer reduced to a unique intertemporal budget constraint in period 0 and a feasibility constraint holding in every period. However we could still write our policy problem in terms of a sequence of intertemporal implementability constraints. For a detailed discussion, see Aiyagari et. al. (2002) and Schmitt-Grohé and Uribe (2004).

It is therefore suboptimal to use steady-state inflation to reduce the real value of any outstanding level of public debt or to erode the real value of profits. This is a common result in the literature that considers this class of models, e.g. Adam and Billi (2008), Adam (2011).

In order to identify the indeterminacy problem, it sufficies to recall the first order condition of the Ramsey problem with respect to b_{t+1}, i.e.

$$-\beta E_t \frac{\gamma_{4,t+1}}{\pi_{t+1}} + \frac{\gamma_{4,t}}{R_t} = 0. \tag{22}$$

At steady-state, the Euler equation implies $R = \frac{1}{\beta}$. Therefore, at steady-state, (22) is satisfied for any values of γ_4. Therefore, in order to pin down the RSS one has to fix the outstanding level of public debt such that (22) becomes redundant, i.e. public debt becomes exogeneous at steady-state. This means that in order to find the RSS, one can rewrite the (16) as

$$\tilde{x} + g_t = w_t h_t \tau_t^h + c_t \tau_t^c \tag{23}$$

where $\tilde{x} = (1-\beta)\, b_0$ represents the steady-state level of real government liabilities. As in Adam (2011), this modified version of the Ramsey problem in leads to the same RSS as the general problem in Definition (4) and is used to obtain a number of analytical results.[4]

Proposition 6 ***(First Best Decentralized Equilibrium)****: the Ramsey Steady State and the Social Planner's allocation at steady state coincide under the following necessary conditions*

$$\pi = 1 \tag{24}$$

$$\frac{1-\tau^h}{1+\tau^c} = \frac{\eta}{\eta-1} \ \textit{with}\ \tau^h \in (-\infty, 1) \ \textit{and}\ \tau^c \in (-1, \infty) \tag{25}$$

$$\frac{\tilde{x}^{opt}}{h^*} = \frac{c^*}{h^*}(1+\tau^c) + \frac{\eta-1}{\eta}\tau^h - 1 \ \textit{with}\ \tilde{x}^{opt} < 0 \tag{26}$$

where $\tilde{x}^{opt} = b_0^{opt}(1-\beta)$

Proof. See Appendix. ■

Corollary 7 *The Ramsey Planner cannot replicate the Social Planner Allocation for a generic level of outstanding public debt with* $\tau^h \in (-\infty, 1)$ *and* $\tau^c \in (-1, \infty)$.

A few things are worth stressing. First, from (25), the first best allocation requires at least one fiscal instrument between τ^h and τ^c to be a subsidy, i.e. to be negative.[5] Second, the first best allocation requires negative public debt. Put it differently, the Ramsey Planner cannot

[4] Formal proof of this can be found in the Appendix B, Subsection 6.5.

[5] In particular, if $\tau^h > 0$, then first best allocation requires $\tau^c < 0$. However it is possible to find negative values of τ^h for which the first best requires $\tau^c < 0$. At the same time, if $\tau^c > 0$, the efficient allocastion requires $\tau^h < 0$.

obtain the first best allocation with a generic level of debt, as this would imply either $\tau^c < -1$ or $\tau^h > 1$.[6]

Moreover, using numerical solutions which we describe in details below, we find that the policy maker would always find a marginal welfare advantage in increasing consumption taxation and subsidy labour when the level of outstanding public debt is greater than $\tilde{x}^{opt}$. However, while this policy will never replicate the first best allocation, it would create extreme tax and subsidy positions.[7] Hence, this policy would be extremely difficult to implement due, for example, to the high costs associated in verifying hours worked. Similarly, a very high consumption tax rate would perhaps lead to a significant amount of unreported barter. Therefore, we allow the policymaker to use both tax instruments when $\tilde{x} = \tilde{x}^{opt}$, while under a generic level of steady-state debt(asset)-to-GDP, we study the cases where the Ramsey Planner is constrained in using only one tax instrument, i.e. either $\tau^h = 0$ or $\tau^c = 0$.[8]

Proposition 8 *Under labour income taxation, the Ramsey Planner sets*

$$u_g \geq -u_h \tag{27}$$

and

$$u_g \geq u_c \underbrace{\left(\frac{\eta - 1}{\eta} - \frac{g + \tilde{x}}{h} \right)}_{\in(0,1)} \tag{28}$$

Proof. See Appendix. ■

This proposition shows that it is optimal to set public consumption below the *level* suggested by the Social Planner. This is because there exists a wedge between the marginal utility of leisure and the marginal utility of consumption. This wedge is composed of two components. First, the monopolistic power that firms hold. Second, the distortive nature of fiscal policy used to finance public spending and public debt. By reducing government spending, the Ramsey Planner can lower the tax rate and hence shrinks the wedge between consumption and leisure. Moreover, while (28) does not give a precise analytical mapping between u_g and u_c, we can nevertheless provide some intuitions about this relationship. Consider, for instance, any increase in inefficiency, due, for example, to more market power or higher outstanding debt. This would push down economic activity. The Ramsey Planner by adjusting government size relative to

[6] Some readers may correctly wonder if this result relies on the absence of profit taxation. As it is well known, the optimal profit taxation in this class of models is generally equal to 100%, i.e. the Ramsey planner finds it optimal to confiscate all the profits in the economy, see for instance, Correia, Nicolini and Teles (2008) and Correia, Farhi, Nicolini and Teles (2013). We proof in the Appendix that even with the introduction of full profit taxation, i.e. $\tau^d = 1$, the efficient level of public debt is negative.

[7] For example Coleman (2000) finds that in a perfectly competitive economy with capital accumulation, the Ramsey planner would set $\tau^c = 692\%$ and $\tau^h = -692\%$. Monopolistic distortion amplifies even further these fiscal positions, i.e. within standard parametrisations consumption taxation is higher than 1000%.

[8] This is common practice in the literature when optimal policy implies extreme tax positions, e.g. Coleman (2000), Correia (2010) and Martin (2010).

GDP and therefore the level of taxation, can influence households labour supply and private consumption and potentially reduce inefficiency. For example, by imposing $u_g > u_c$, i.e. a government to consumption ratio lower than the first best, the policy maker can sustain a given level of outstanding debt with lower taxation. Lower taxation would increase labour supply.[9] The sign of $\frac{\partial c}{\partial h}$ determines whether a higher labour supply implies a higher or lower consumption level. In the Appendix we show that

$$\frac{\partial c}{\partial h} \geq 0 \iff \frac{c}{\left(c - \tilde{x} - \frac{1}{\eta}h\right)} \geq -\frac{cu_{cc}}{u_c}. \tag{29}$$

The sign of (29) depends, inter alia, on the Arrow-Prat measure of risk aversion $-\frac{cu_{cc}}{u_c}$, the degree of monopolistic competition η and the level of outstanding debt $\tilde{x}$. When (29) is positive, i.e. when risk aversion is low, shrinking the government size (relative to GDP) below the first best, allows the Ramsey Planner to increases labour supply and private consumption, thus reducing inefficiency. The opposite is true as (29) turns negative, i.e. when risk aversion is high. In this case the policy-maker has a strong incentive to set the government spending-to-GDP ratio greater than the first best allocation, i.e. $u_g < u_c$. With such a policy the Ramsey Planner can induce an increase in consumption and therefore reduce the wedge between the marginal utility of private consumption and the marginal utility of leisure. The desire for such a policy, i.e. $u_g < u_c$, is decreasing both in the degree of monopolistic competition and in the size of public debt. In the particular case of perfect competition and no public debt, i.e. $\eta \to \infty$ and $\tilde{x} = 0$, the RHS of (29) collapses to 1. In this case, if the utility is logaritmic in private and public consumption, the Ramsey Planner finds it optimal to set the share of government spending over total output as in the first best.[10]

Next we analyze the scenario where the Ramsey Planner has access only to consumption taxation, i.e. $\tau^h = 0$.

Proposition 9 *Under consumption taxation the Ramsey Planner sets*

$$u_g > -u_h. \tag{30}$$

Furthermore, if

$$-\frac{cu_{cc}}{u_c} \begin{cases} > 1 \Longrightarrow u_c > u_g \\ = 1 \Longrightarrow u_c = u_g \\ < 1 \Longrightarrow u_c < u_g \end{cases} \tag{31}$$

and $\frac{u_c}{u_g}$ *is decreasing (increasing) in* $\tilde{x}$ *if* $-\frac{cu_{cc}}{u_c} > (<)\ 1$. *In the special case where* $-\frac{cu_{cc}}{u_c} = 1$, $\frac{u_c}{u_g} = 1$ *for any outstanding level of public debt.*

[9] We are implicitly assuming that the substituion effect on labour supply always prevails, which is consistent with a Laffer curve in government revenues.

[10] In the case of log utility and positive public debt and/or monopolistic competition, the Ramsey planner finds it optimal to set the share of public spending-to-GDP lower than the first best, i.e. $\frac{\partial c}{\partial h} > 0$.

Proof. See Appendix. ■

As under labour income taxation, the Ramsey Planner finds it optimal to set the *level* of government spending below the Social Planner level. This is due to the monopolistic features of the good markets and the distortive nature of fiscal policy. Furthermore (31) clarifies whether the optimal allocation requires the *share* of government over total output to be above or below the first best level. Interestingly, this is now only function of the consumers' risk aversion. If this is higher (lower) than 1, the optimal government spending-to-GDP *ratio* is set above (below) the Social Planner level. By allocating a high share of government spending, the Ramsey Planner is imposing, for a given level of outstanding debt, a higher tax rate. Assuming that consumption is a normal good, higher taxation implies lower consumption. In order to understand the implications of lower consumption on the labour supply schedule we need to study the sign of

$$\frac{\partial h}{\partial c} = -\frac{\left(-\frac{cu_{cc}}{u_c} - 1\right)\left[-u_h u_c \left(\frac{h+\tilde{x}}{c}\right)\right]}{\frac{1}{c}\left[-\left(h\frac{u_{hh}}{u_h} + 1\right)u_h u_c - u_{hh}u_c\tilde{x}\right]} \tag{32}$$

In the Appendix we show that the above expression is negative when $-\frac{cu_{cc}}{u_c} > 1$ and positive otherwise. This means that when $-\frac{cu_{cc}}{u_c} > 1$, the lower consumption generated by higher taxation implies an increase in labour supply, which in turn pushes the economy closer to the first best. On the contrary, when $-\frac{cu_{cc}}{u_c} < 1$, the Ramsey Planner sets $u_c < u_g$. The resulting lower taxation pushes consumption and therefore labour supply upward, i.e. $\frac{\partial h}{\partial c} > 0$.

For the same reason, the optimal policy calls for an increase in the government spending-to-GDP ratio when public debt increases. Moreover, for the particular case in which risk aversion is 1, the Ramsey Planner, independently of the outstanding level of public debt, sets the marginal utility of private consumption equal to the marginal utility of public spending, as prescribed by the Social Planner.

A simple comparative static exercise can show that for a given degree of risk aversion and a positive (or at least not too negative) outstanding debt, the optimal share of government spending-to-GDP is greater under consumption taxation than under labour income tax.[11]

3.3 Computational issues

As discussed above, we assume that in period 0 the economy is in the RSS. When an analytical expression is missing, we rely on non-linear numerical solution algorithms.[12] Firstly, we compute the exact non-linear RSS by using the OLS projection approach proposed by Schmitt-Grohè and Uribe (2004, 2007, 2012). This consists in exploiting the insight that the Ramsey equilibrium conditions are linear in the vector of Lagrange multipliers, γ_i. Then, using the perturbation method proposed by Schmitt-Grohé and Uribe (2004), we compute the accurate second-order

[11] Take for instance the case of logaritmic preferences, i.e. $-\frac{cu_{cc}}{u_c} = 1$. Under labour income taxation $u_g > u_c$, while under consumption taxation $u_g = u_c$.

[12] The different Ramsey problems are described in detail in the Appendix.

approximation of the Ramsey's FOCs around the non-stochastic steady state of these conditions. We then use this solution to simulate the Ramsey equilibrium in the face of a technology shock.[13] The shock realizations and all the other structural parameters used for the simulation are kept constant through the different fiscal scenarios. This means that any differences between fiscal arrangements are attributable entirely to the properties of the economic policies. Moreover, we measure welfare in terms of percentage of consumption units, i.e. ϖ required by a generic policy a to reach the same level of utility as under policy b, i.e.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t^a\left(1+\varpi\right), h_t^a, g_t^a\right)\right] = E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t^b, h_t^b, g_t^b\right)\right]. \tag{33}$$

3.4 Parametrization

We specify preferences to satisfy the conditions stated in Section 2, i.e.

$$u\left(c_t, h_t, g_t\right) = \frac{c_t^{1-\sigma}}{1-\sigma} - \omega_h \frac{h_t^{1+\phi}}{1+\phi} + \omega_g \frac{g_t^{1-\sigma}}{1-\sigma} \text{ with } \sigma, \phi > 0.$$

Each period represents a quarter with the discount factor, β, set to 0.9913. The elasticity of demand is chosen in order have a steady state gross markup of 1.2 ($\eta = 6$), which is in line with the macro literature. Given the importance of the CRRA parameter for our results, i.e. $\sigma = -\frac{c u_{cc}}{u_c}$, we solve the model with a set of values of risk aversion that are generally found in the literature, i.e. $\sigma \in (0.8, 2)$. However, we set $\sigma = 1$, i.e. log utility, as a benchmark value. The utility parameter ω_h is chosen so that the households supply between one fifth and one third of their time to work in the decentralized equilibrium when the steady-state level of public debt is zero, i.e. $\omega_h = 19.792$. We further fix ω_g in order to have in the first best allocation, i.e. the Social Planner Equilibrium, a ratio of government spending over total output of 30% , i.e. $\omega_g = 0.2641$. We set the inverse Frisch elasticity of labour supply ϕ to 1, a value generally used as a benchmark in the macroeconomic literature, e.g. Adam and Billi (2008). The price stickiness parameter is selected such that the log-linearized version of the Phillips curve (12) is consistent with the estimates of Sbordone (2002), ($\varphi = 17.4$). The quarterly standard deviation of the technology shocks is 0.6% and it has a quarterly persistence equal to 0.7. Furthermore, we show the implications of varying the long run level of public debt. Table 1 collects the parametrization adopted.

[13] In the case of efficient steady-state, i.e. when the government is allowed to accumulate a large asset position, the second order approximation approach gives the same welfare ranking of the correspondent linear-quadratic (LQ) representation of the problem, see Woodford (2003). For this reason, part of our results are readily comparable with the literature that adopts the LQ framework.

3.5 Steady State Results

This section explores the quantitative implications of different fiscal scenarios for the RSS allocations (with particular focus on the Ramsey public spending) and welfare. In particular, here we show how the various fiscal arrangements interact with government spending and long-run level public debt in determining the Ramsey allocations.

Figure (1) presents the RSS allocations when public debt is allowed to vary from -100% to 200% of GDP. Increasing the level of outstanding public debt implies an increase in the (distortive) tax rate and hence a loss of efficiency. Higher taxes imply a lower after tax real wage. However, despite the consumption tax being higher than the labour income tax, it has a lower impact on the real wage. This is because consumption is generally more inelastic than leisure. Hence households respond less, *ceteris paribus*, to a variation in consumption tax than to a variation in labour income tax. As a consequence, under consumption taxation the increase in public debt implies a lower response of the gap variables.

Figure (2) quantifies, in percentage of permanent steady state consumption units, the loss suffered by households as steady state public debt increases. Under both fiscal scenarios, higher values of steady state public debt imply an increase in inefficiency and hence a deterioration of welfare. However there is a surprisingly high gain from using consumption taxation over labour income taxation as a fiscal instrument. This gain is increasing in the debt-to-GDP ratio, passing from 24% when government debt to GDP is -100% to 42% when public debt is 200% of total income. As public debt increases, the fiscal authority has to devote more and more resources to pay its burden. Therefore, the lower distortive effects of consumption taxation generate a relative gain as the inefficiency generated by the burden of public debt increases.

Figure (3) shows the difference between the Ramsey government spending to GDP ratio and the Social Planner allocation when public debt is at its benchmark value, i.e. 80% of GDP and we allow σ to vary in the interval $[0.8, 2]$ while all the other parameters are kept at their benchmark values. This figure may be seen as a graphical presentation of Proposition 7 and 8. A negative (positive) number identifies a scenario where the Ramsey Planner sets the ratio between public spending and total output below (above) the first best, i.e. $\frac{g}{y} < (>) \frac{g^*}{y^*}$. Under both fiscal scenarios, this difference depends critically on the parameter controlling the risk aversion in the CRRA utility function. In particular, as consumers become more risk adverse, optimal policy calls for reducing private consumption in favor of public spending. By directly affecting the size of government spending and therefore the level of distortive taxation, the policy-maker can influence labour supply and private consumption and potentially reduce inefficiency. As discussed above, for a give level of risk aversion, optimal policy under consumption taxation implies a higher share of government spending over GDP than under labour income taxation.[14]

Figure (4) presents the optimal public spending rule as public debt increases for different

[14] In particular, under labour income tax $\frac{g}{y} = \frac{g^*}{y^*}$ for $\sigma = 1.3868 = \frac{c}{\left(c - x - \frac{1}{\varepsilon} h\right)}$, i.e. $\frac{\partial c}{\partial h} = 0$.

degrees of risk aversion. Under labour income tax, the optimal share of public spending is decreasing in public debt. This is because the strong distortionary effects of higher tax rates prevail over the incentive of the Ramsey Planner to increase g/y as the degree of risk aversion increases. This is consistent with (29), i.e. for a given level of risk aversion a higher level of steady state public debt generates an incentive to reduce the *share* of public consumption. Differently, under consumption taxation, the optimal spending rule depends crucially on whether risk aversion is below or above unity, as presented in Proposition 7. Therefore when $\sigma = 0.8$, i.e. $-\frac{cu_{cc}}{u_c} < 1$, the Ramsey Planner, by cutting the share of government as public debt increases, can boost both consumption and labour supply. On the contrary, when $\sigma = 2$, i.e. $-\frac{cu_{cc}}{u_c} > 1$, the optimal policy increases the government spending-to-GDP ratio in order to increase hours worked and therefore total output. Finally, as presented in Proposition 7, in the benchmark case of log utility in consumption, i.e. $-\frac{cu_{cc}}{u_c} = 1$, the optimal share of government spending is always at the first best, independently of the level of steady state public debt.

3.6 Ramsey Dynamics

3.6.1 First Best Steady State

We now turn our attention to the optimal policy in a stochastic setting. To this end we perturb the model with a technology shock as described in 3.4. Studying Ramsey policy in the face of this type of shock is standard practice in the literature (e.g. Schmitt-Grohé and Uribe (2004), Correia et al. (2008), Adam (2011), Leith and Wren Lewis (2013)) and allows us to better disentangle our contribution. We start our analysis by considering a situation where the policy-maker implements first best at steady-state. This implies that the government is allowed to accumulate large asset positions (i.e. negative debt) and that one (or both) taxes can be negative. Both these assumptions will be dropped later. We refer the reader to Proposition 5 in Section 3.3 on how a Ramsey Planner can achieve the first best in this environment. In practise, we fix the steady state value of consumption taxation to 16%. This value corresponds to the average level of consumption taxation in the industrialized countries found by Gordon and Li (2009). Then, the labour income tax rate and the level of the government assets are pinned down by (25) and (26) respectevely. Here we run three policy exercises. In the first one the policy maker can respond to shocks by using both taxes. In the other two, we restrict the fiscal authority to use only one tax instrument in the face of shocks. The three scenarios share the same steady state allocation so that any difference can be attributed entirely to the dynamic properties of the tax instruments adopted. Results are reported in Figure (5).

When the policy maker has access to both taxes, first best is attainable in the stochastic economy under consideration. In other words, the Ramsey Planner can commit to a state-contingent policy such that in face of a technology shock the response in the decentralized economy coincides with the Social Planner's allocation. Absence economic policy, in a sticky-price environment a negative technology shock would imply an increase of inflation and a positive output gap, i.e. price would increase but not as much as required in a flexible price world, thus imply an inefficiently high level of output. As it is well know from Galí (2001), a welfare maximizing policy-maker would therefore tighten aggregate demand as to push output towards the efficient level. This policy would also stabilize inflation. Here the Ramsey Planner looses monetary policy on impact and promises to keep it above the steady state level for some periods after the shock. At the same time, she increases the consumption tax and promises to cut it tomorrow. As we discuss in more details later, this contingent tax policy represents one of the main advantages of using consumption taxation as a demand management tool. Indeed, the combined optimal use of monetary and fiscal instruments allows the policy-maker to fully control the stochastic discount factor and hence offset firms' desire to changing prices, thus replicating the flexible price equilibrium.

Furthermore, in order to mimic the Social Planner solution, the Ramsey Planner has to avoid distorting households' consumption-leisure choice. In other words, the policy-maker moves the consumption and the labour income tax rate in the same measure and in opposite direction so that they do not create a wedges between the marginal utility of leisure and that of consumption. Finally the Ramsey equates the marginal utility of private and public consumption as required by the first best equilibrium by varying her assets positions. In other words, if the government is allowed to take large assets position and can use both consumption and labour income taxation, she can offset both the static and the dynamic inefficiencies of the model.

Table 1 reports the model's campionary moments of this exercise.[15] Given the shape of the utility function ($\sigma = 1$), dynamic efficiency requires output, consumption, government spending and real wages to be perfectly correlated with each other and with the technology process, while hours worked and inflation should remain at their steady state values. These results are closely related to Correia et al. (2008). They show that with consumption and labour income taxation, full profit taxation and exogenous government spending, the Ramsey Planner can mimic the flexible price equilibrium, so that any real allocation is independent from the degree of nominal rigidities. Here we show that the first best allocation can be implemented without profit taxation and with endogenous government spending as long as the government is allowed to take large asset positions.

Next, we analyze fiscal scenarios in which the government is constrained in using only one tax instrument. We start with the case where only consumption taxation is available. IRF's are presented in Figure 4. In this case, while the policy-maker can obtain long run efficiency via asset accumulation, the absence of labour income taxation makes it impossible to reach dynamic efficiency. In particular, the optimal tax policy required to balanced the government budget constraint distorts the leisure-consumption decision, thus creating an inefficiency wedge in the

[15] Given the quasi-random walk properties of public debt, unconditional moments are not available. Hence, like Schmitt-Grohé and Uribe (2004), Arseneau and Chugh (2008) and Nieman and Pichler (2011), we calculate the campionary moments of the model. In particular we simulate the model 5000 times for 100 periods. For each simulation we calculate the statistics of interest. Then we report the median value.

labour supply and therefore in the real wages. At the time of the shock, output, consumption and public spending all drop by roughly the same amount as the case where the Ramsey Planner has access to both taxation thus resulting in very small movements of gap variables.[16] This policy, coupled with an initial cut in the nominal rate and an increase in consumption taxation, is aimed to generate an increase on impact of inflation. This helps to reduce the cut in the government assets necessary to balanced the government budget constraint. As for the case with two tax instruments, this policy is reverted in the period after the shock. Fiscal policy commits in reducing consumption taxation while monetary policy increases the nominal rate above its steady state value. These combined policies affect the real stochastic discount factor and imply a one-period deflation episode, which in turn pushes the price level near its steady state value.

Table 1 reports the implied simulated moments of this policy experiment. Compared to the previous scenario, the inefficiency generated by the absence of labour income taxation generates an incentive for the policy maker to reduce the volatility of output, consumption, government spending and real wages. However, with the exception of real wages, consumption and government spending are perfectly correlated with output and almost perfectly correlated with technology. The desire to inflate the system at the time of the shock and deflate it in the period after the shock implies a necessary increase in the volatility of inflation. This policy generates also an increase in the volatility of the nominal rate and an increase of its correlation with output and technology. Interestingly, within this policy experiment, consumption taxation displays a behavior that is very similar to the case where both taxes are available. Overall, the welfare cost of loosing labour income taxation is very small, around 0.0015% of the efficient steady state consumption units.

Finally we analyze the scenario where the Ramsey Planner is allowed to use only labour income taxation. A novelty of this policy experiment with respect to similar contributions, e.g. Schmitt-Grohé and Uribe (2004), Adam (2011) and Leith and Wren-Lewis (2013), is that we consider optimal labour income tax policy with a first best steady state obtained via government assets accumulation. As before, the IRF's analysis is displayed in Figure (6). As under consumption taxation, under this scenario, the fiscal instrument generates an inefficient distortion in the marginal rate of substitution between leisure and consumption. However, now the policy-maker has less power to influence the intertemporal allocation via the real stochastic discount factor. This has dramatic consequences on the optimal policy functions as well as on the overall welfare. At the time of the shock, consumption, output and government spending all decrease. However, while consumption and output decrease less than in the efficient allocation, thus resulting in positive output and consumption gaps, the government finds it optimal to cut government spending more than in the previous scenarios. This is reflected in lower standard

[16] As in Benigno and Woodford (2003) and Adam (2011), consumption, output, government spending, taxes and debt all follow a near ramdom walk pattern. However, given the optimal consumption tax policy, the long run values of these variables are very close to their steady state counterparts.

deviations of output and consumption and a higher volatility of government spending (Table 2). This, coupled with the negative wealth effect on labour supply, allows the policy-maker to cut taxes and the nominal rate on impact thus inflating the system in the period of the shock. This is done in order to decrease the loss generated by lower public assets. However, the lack of control on the intertemporal households decisions that consumption tax brings about, forces the Ramsey Planner to move inflation away from steady state only in the first period and by a smaller amount, hence generating a lower volatility of inflation in equilibrium, and it excludes deflationary episodes. The welfare loss of using only labour income taxation amounts to 0.0914% of steady state efficient consumption, which is roughly 60 times larger than the scenario where the Ramsey Planner could use only consumption taxation.

3.6.2 Generic Steady State

In this section we study the optimal policy when the policy-maker faces a negative technology shock and the economy is affected by a positive level of outstanding public debt. All the parameters are set at their benchmark values and the public-debt to GDP ratio is set to 80%. As discussed in details above, when public debt is positive, even with two tax instruments, the Ramsey Planner cannot replicate the first best allocation. Furthermore, the second best policy would imply extreme (and unrealistic) tax positions. Therefore in this section we analyze the scenarios where the policy-maker can use only one tax instrument. Results of this policy exercise are reported in Figure (6) and (7). At the time of the shock, under both fiscal scenarios, output consumption and government spending all decrease. The dynamic behavior of these variable are very similar under consumption and labour income taxation. Notable differences across the two scenarios emerge, however, when considering the optimal responses of inflation, nominal interest rates, taxes, hours and government debt. Similar to the previous case, the optimal path of consumption taxation implies a sharp increase in the first period and a decrease in the second period, while under labour income taxation, taxes increase only in the first period and by a much smaller amount. As a consequence, the optimal level of public debt under consumption taxation increases by less than under labour income tax, thus reflecting a smaller incentive of the policy-maker to smooth taxes across states with variation in public debt positions.

Under both scenarios, inflation increases in the first period. This helps to reduce the real value of maturing debt. However, the dynamic properties of consumption taxes imply a deflationary episode after the shock which is completely absent under labour income taxation. As it is well known from Schmitt-Grohé and Uribe (2004) and Benigno and Woodford (2004), it is suboptimal in the presence of even small amounts of nominal rigidities to bring about large price level changes, i.e. to use nominal bonds as a state contingent source of taxation and hence to fully smooth taxes across states. This result shows up here in the form of rather small movements of inflation.

Table (3) reports the simulated moments and the welfare calculation of this policy experiment. As in the case where public debt is at first best, the optimal volatility of real as well nominal variables is closer to the efficient response to shock, thus confirming consumption taxation as a better policy instrument even under a positive level of public debt.

Welfare calculations presented in the Table (3) are formed by two elements, the steady state loss of each scenario and the loss at business cycle frequencies. Two things are worth noticing. First of all, under both scenario, positive public debt translates in a huge inefficiency in the optimal response to shocks. Second, even with positive public debt, consumption taxation remains a better policy instrument when compared to labour income taxation. The loss under the former amounts to 0.19% of permanent consumption units, while under the latter fiscal scenario the system suffer a loss of 0.44%.

4 Conclusions

This paper studies the optimal mix of monetary and fiscal policy in a New Keynesian model where public spending has intrinsic value and the fiscal authority can use consumption and labour income taxation in order to finance public spending and to repay the burden of public debt.

We show that the optimal policy mix is markably altered when consumption taxation is considered as a source of government revenues alongside or as an alternative to labour income taxation. We find interesting differences both in the deterministic and in the stochastic allocations. First, we show that the optimal steady state size of the public sector is, ceteris paribus, greater under consumption taxation than under labour income tax. We further show that adopting consumption taxation has enormous long run welfare gains and that these gains are increasing in the level of outstanding public debt. Then we find that these welfare gains are not limited to the steady state, but extend in the stochastic equilibrium. The reason is that the dynamic nature of consumption taxation enables the policy-maker to affect the stochastic discount factor via modifications of the marginal utility of consumption. This extra wedge greatly improves the welfare in the stochastic equilibrium whether the level of public debt is efficient or not.

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Figures

Figure 1: Steady state gap variables with respect to the first best

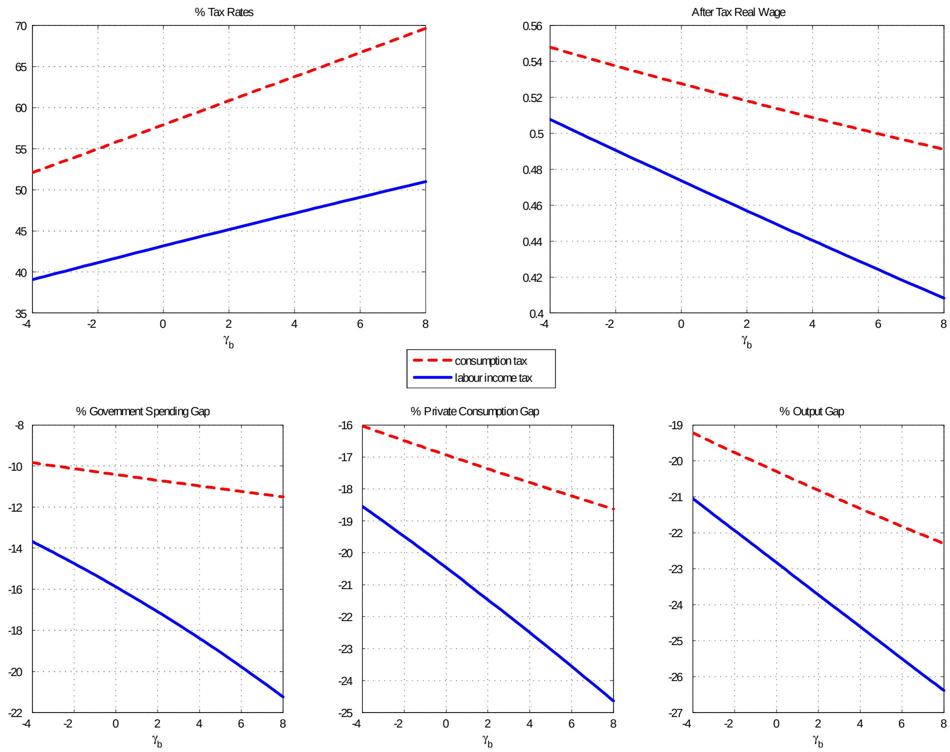


Figure 2: Welfare measure of steady state permanent consumption units.

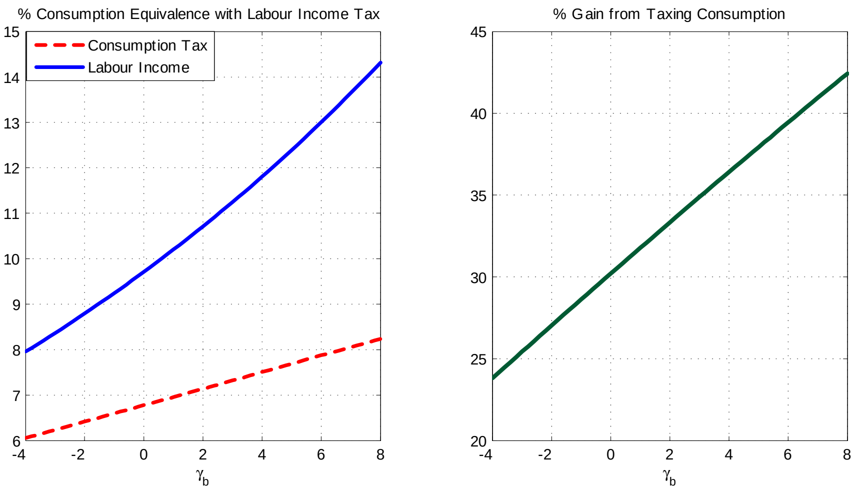


Figure 3: Ramsey vs First Best Government Share of Total Output.

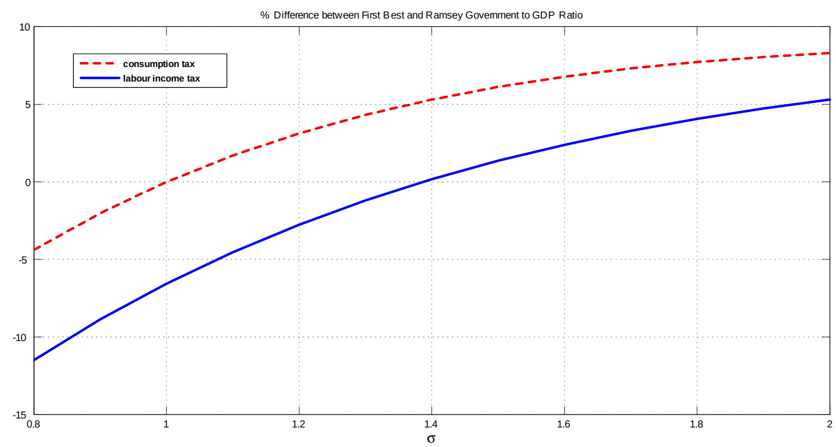


Figure 4: Share of government spending to total output as a function of the steady-state public debt.

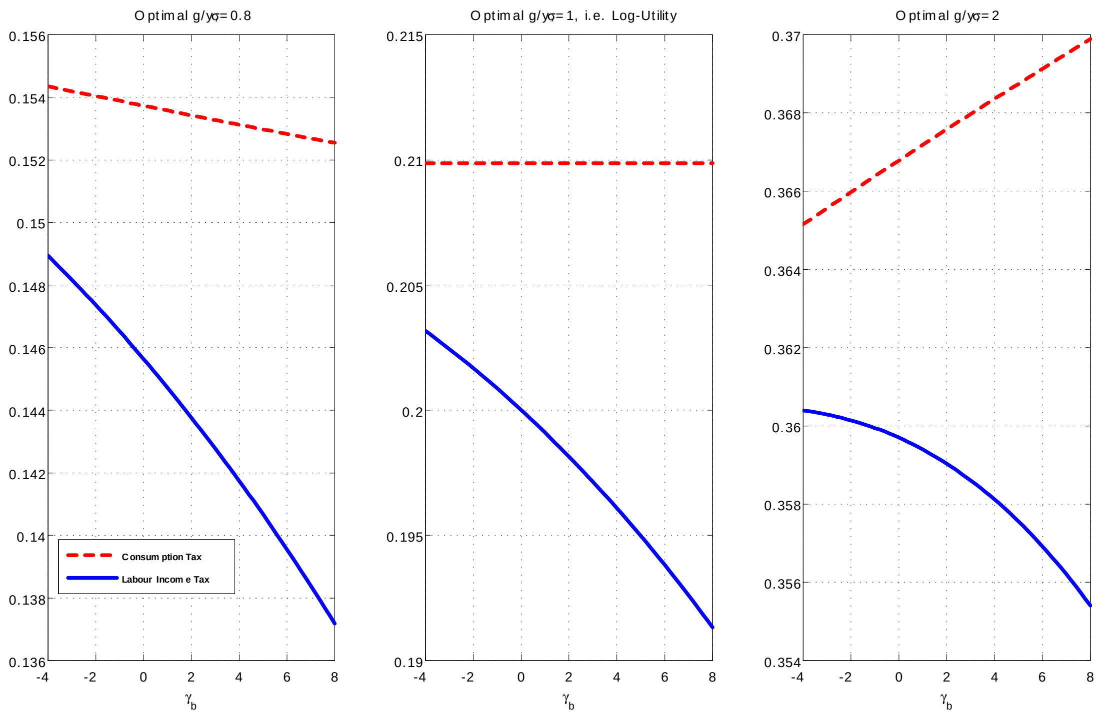


Figure 5: IRF's to one standard deviation negative technology shock in the efficient steady state.

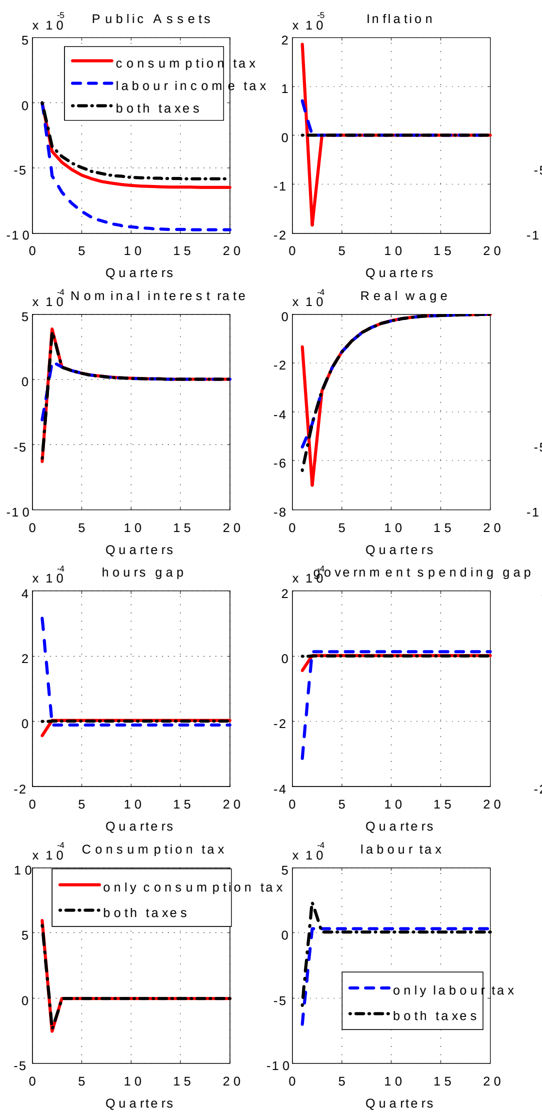


Figure 6: IRF's to one standard deviation negative technology shock with debt-to-GDP at 80%.

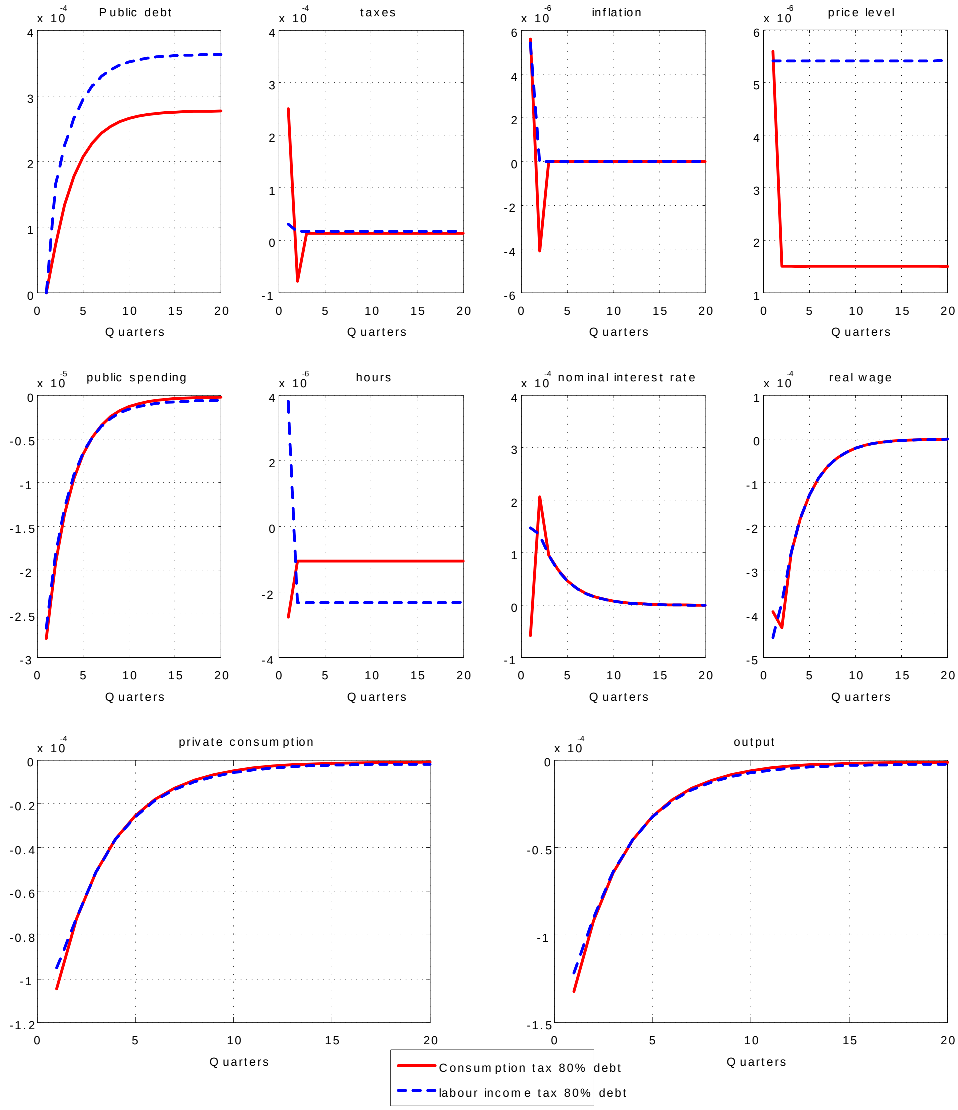
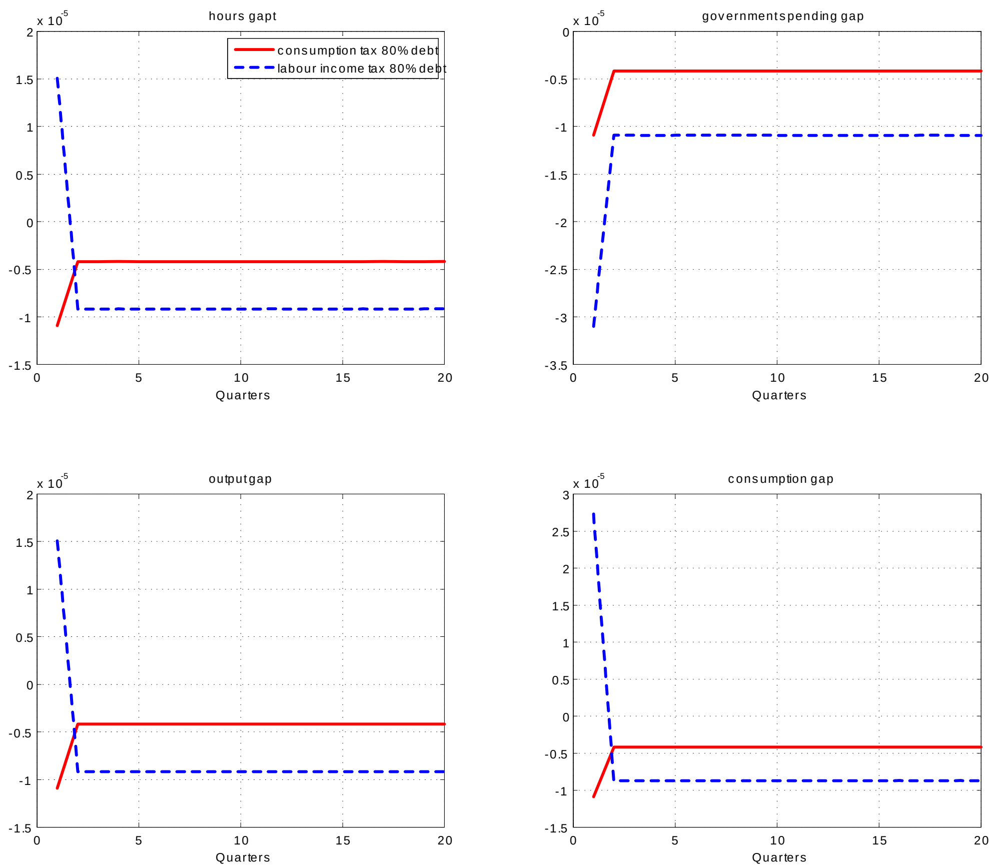


Figure 7: IRF's to one standard deviation negative technology shock with debt-to-GDP at 80%. Gap variables only



Tables

Table 1: PARAMETRIZATION OF THE MODEL. BV stands for benchmark value.

Parameter	Values
β	0.9913
η	6
σ	BV=1. Values range from 0.8 to 2.
ω_h	19.792
ω_g	0.2641
ϕ	1
ρ_A	0.7
σ_A	0.6%
b/y	BV=80%. Values range from-100%-200%. First best, see below

Table 2: SIMULATED MOMENTS												
	Both taxes (Welfare cost=0)				Only consumption tax (Welfare cost=0.0015)				Only labour income tax (Welfare cost=0.0914)			
x	$\bar{x}$	σ_x	σ_{xa}	σ_{xy}	$\bar{x}$	σ_x	σ_{xa}	σ_{xy}	$\bar{x}$	σ_x	σ_{xa}	σ_{xy}
y	0.2530	0.1078	1	1	0.2530	0.0284	0.9992	1	0.2530	0.0204	0.9283	1
c	0.1999	0.1078	1	1	0.1999	0.0224	0.9992	1	0.1999	0.0148	0.7885	0.9597
π	1	0	-0.1661	-0.1661	1	0.0036	-0.1804	-0.2140	1	0.00096	-0.7187	-0.4139
h	0.2530	0	0.6755	0.6755	0.2530	0.0016	0.6741	0.7034	0.2530	0.0113	-0.7447	-0.4376
w	0.8333	0.1078	1	1	0.8335	0.0858	0.7241	0.6978	0.8336	0.0826	0.9957	0.9579
r	1.0078	0.0901	0.2703	0.2703	1.0077	0.1059	0.3069	0.3434	1.0077	0.0488	0.3112	-0.0563
b	-11.5944	0.7736	-0.9683	-0.9683	-11.2171	0.8161	-0.9669	-0.9766	-11.3841	0.9198	-0.8610	-0.8132
τ^l	-0.3920	0.0759	0.4636	0.4636	- -	- -	- -	- -	-0.2001	0.1189	0.7424	0.4351
τ^c	0.16	0.0759	-0.4636	-0.4636	-0.1666	0.0750	-0.4672	-0.5002	- -	- -	- -	- -
g	0.0530	0.1078	1	1	0.0530	0.0060	0.9992	1	0.0530	0.0075	0.9737	0.8283

Note: Efficient Steady-State. Moments extracted as the median of simulations conducted via replicating the model for 100 periods, 5000times. Welfare loss represented as percentage of efficient steady-state consumption units conditional to a normalized technology shock.

Table 3: SIMULATED MOMENTS								
	Only consumption tax (total welfare cost=5.2316% of which 0.19% due to business cycle fluctuations)				Only labour income tax (total welfare cost=6.7705 of which 0.44% due to business cycle fluctuations)			
x	$\bar{x}$	σ_x	σ_{xa}	σ_{xy}	$\bar{x}$	σ_x	σ_{xa}	σ_{xy}
y	0.2027	0.0221	0.9998	1	0.1960	0.0209	0.9984	1
c	0.1601	0.0175	0.9998	1	0.1570	0.0166	0.9981	0.9999
π	1	0.0009	-0.2893	-0.2935	1	0.0006	-0.7185	-0.6954
h	0.2027	0.0005	0.5774	0.5945	0.1961	0.0012	-0.1939	-0.1376
w	0.8333	0.0827	0.9799	0.9787	0.8333	0.0829	0.9965	0.9976
r	1.0078	0.0307	-0.4758	-0.4719	1.0077	0.0287	-0.9903	-0.9930
b	0.6554	0.1201	-0.1482	-0.1293	0.6307	0.1545	-0.0347	-0.0813
τ^l	- -	- -	- -	- -	0.2686	0.0079	-0.4807	-0.5237
τ^c	0.2971	0.0330	-0.5390	-0.5459	- -	- -	- -	- -
g	0.0425	0.0046	0.9998	1	0.0390	0.0044	0.9984	0.9989

Note: Generic Steady-State, Public Debt at 80% of Total Output. Moments extracted as the median of simulations conducted via replicating the model for 100 periods, 5000times. Welfare loss represented as percentage of efficient steady-state consumption units conditional to a normalized technology shock.

5 Appendix A-Various Formulation of the Ramsey Programs

5.1 Ramsey Problem: General Formulation both Consumption and Labour Income Taxes

Following the definition given in the main text, the Lagrangian of the Ramsey Problem when both tax instruments are available can be represented as:

$$
\begin{aligned}
\mathcal{L} = & \max_{\left\{\begin{array}{c} c_t, h_t, \pi_t, g_t, \\ R_t \geq 1, \tau_t^h, \tau_t^c, b_t \end{array}\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t, h_t, g_t\right)\right] + \\
& + \gamma_{1,t} \beta^t \left[\begin{array}{c} u_{c,t}\left(\pi_t - 1\right)\pi_t - \left(\left(1-\eta\right) u_{c,t} - u_{h,t} \frac{\left(1+\tau_t^c\right)}{\left(1-\tau_t^h\right)} \frac{\eta}{a_t} \right) \frac{h_t a_t}{\varphi} - \\ \beta E_t \left[\frac{\left(1+\tau_t^c\right) u_{c,t+1}}{\left(1+\tau_{t+1}^c\right)} \left(\pi_{t+1} - 1\right)\left(\pi_{t+1}\right) \right] \end{array} \right] + \\
& + \gamma_{2,t} \beta^t \left[\frac{u_{c,t}}{\left(1+\tau_t^c\right)} - \beta E_t \left(\frac{u_{c,t+1}}{\left(1+\tau_{t+1}^c\right)} \frac{R_t}{\pi_{t+1}} \right) \right] + \\
& + \gamma_{3,t} \beta^t \left[a_t h_t - c_t - g_t - \frac{\varphi}{2}\left(\pi_t - 1\right)^2 \right] + \\
& + \gamma_{4,t} \beta^t \left[\begin{array}{c} -\frac{b_t}{\pi_t} - g_t + \frac{b_{t+1}}{R_t} - \\ \frac{u_{h,t}}{u_{c,t}} \frac{\left(1+\tau_t^c\right)}{\left(1-\tau_t^h\right)} h_t \tau_t^h + c_t \tau_t^c \end{array} \right],
\end{aligned}
\tag{34}
$$

where we substituted the wage with its representation in (7), i.e.

$$
-\frac{u_{h,t}}{u_{c,t}} \frac{\left(1+\tau_t^c\right)}{\left(1-\tau_t^h\right)} = w_t. \tag{35}
$$

The first order conditions with respect to the decision variables $c_t, \tau_t^h, h_t, \pi_t, R_t, b_t, \tau_t^c$ and g_t are respectively in order:

$$
\begin{aligned}
c_t \quad : \quad & 0 = u_{c,t} + \gamma_{1,t} \left[u_{cc,t}\left(\pi_t - 1\right)\pi_t - \frac{\left(1-\eta\right)}{\varphi} h_t a_t u_{cc,t} \right] + \\
& -\gamma_{1,t-1} \left(\frac{\left(1+\tau_{t-1}^c\right) u_{cc,t}}{\left(1+\tau_t^c\right)} \left(\pi_t - 1\right)\left(\pi_t\right) \right) + \\
& + \gamma_{2,t} \left(\frac{u_{cc,t}}{\left(1+\tau_t^c\right)} \right) - \gamma_{2,t-1} \left(\frac{u_{cc,t}}{\left(1+\tau_t^c\right)} \frac{R_{t-1}}{\pi_t} \right) + \\
& -\gamma_{3,t} + \gamma_{4,t} \left(\tau_t^c + \frac{u_{cc,t} u_{h,t}}{u_{c,t}^2} \left(1+\tau_t^c\right) h_t \frac{\tau_t^h}{\left(1-\tau_t^h\right)} \right);
\end{aligned}
\tag{36}
$$

$$h_t \quad : \quad 0 = u_{h,t} + \gamma_{1,t}\left(u_{hh,t}\frac{(1+\tau_t^c)}{\left(1-\tau_t^h\right)}\frac{\eta}{1}\frac{h_t}{\varphi} - \left((1-\eta)\, u_{c,t} - u_{h,t}\frac{(1+\tau_t^c)}{\left(1-\tau_t^h\right)}\frac{\eta}{a_t}\right)\frac{a_t}{\varphi}\right) + \\ +\gamma_{3,t}a_t - \gamma_{4,t}\frac{(1+\tau_t^c)\,\tau_t^h}{\left(1-\tau_t^h\right)u_{c,t}}\left(u_{h,t} + u_{hh,t}h_t\right); \tag{37}$$

$$\pi_t \quad : \quad 0 = \gamma_{1,t}\left(2\pi_t - 1\right)u_{c,t} - \gamma_{1,t-1}\left(2\pi_t - 1\right)\frac{\left(1+\tau_{t-1}^c\right)u_{c,t}}{(1+\tau_t^c)} + \\ +\gamma_{2,t-1}\left(\frac{u_{c,t}}{(1+\tau_t^c)}\frac{R_{t-1}}{\pi_t^2}\right) - \gamma_{3,t}\varphi\left(\pi_t - 1\right) + \gamma_{4,t}\frac{b_t}{\pi_t^2}; \tag{38}$$

$$\tau_t^h \quad : \quad 0 = \gamma_{1,t}\frac{u_{h,t}\,(1+\tau_t^c)}{\left(1-\tau_t^h\right)^2}\frac{\eta h_t}{\varphi} - \gamma_{4,t}\frac{u_{h,t}}{u_{c,t}}\left(1+\tau_t^c\right)h_t\left(\frac{1}{\left(1-\tau_t^h\right)^2}\right); \tag{39}$$

$$R_t \quad : \quad 0 = -\gamma_{2,t}\frac{u_{c,t+1}}{\left(1+\tau_{t+1}^c\right)}\frac{\beta}{\pi_{t+1}} - \gamma_{4,t}\frac{b_{t+1}}{R_t^2}; \tag{40}$$

$$b_t \quad : \quad 0 = -\beta E_t\frac{\gamma_{4,t+1}}{\pi_{t+1}} + \frac{\gamma_{4,t}}{R_t}; \tag{41}$$

$$\tau_t^c \quad : \quad 0 = \gamma_{1,t}\left(\frac{\eta}{\varphi}\frac{h_t u_{h,t}}{\left(1-\tau_t^h\right)} - \beta\frac{u_{c,t+1}\left(\pi_{t+1}-1\right)\left(\pi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)}\right) + \\ +\gamma_{1,t-1}\left(\frac{\left(1+\tau_{t-1}^c\right)u_{c,t}}{(1+\tau_t^c)^2}\left(\pi_t - 1\right)\left(\pi_t\right)\right) + \\ -\gamma_{2,t}\left(\frac{u_{c,t}}{(1+\tau_t^c)^2}\right) + \gamma_{2,t-1}\left(\frac{u_{c,t}}{(1+\tau_t^c)^2}\frac{R_{t-1}}{\pi_t}\right) + \\ -\gamma_{4,t}\left(\frac{u_{h,t}}{u_{c,t}}\frac{h_t\tau_t^h}{\left(1-\tau_t^h\right)} - c_t\right); \tag{42}$$

$$g_t \quad : \quad 0 = u_{g,t} - \gamma_{3,t} - \gamma_{4,t}. \tag{43}$$

The first order conditions with respect to the Lagrangian multipliers, $\gamma_{i,t}$ with $i = 1, 2, 3, 4$ are:

$$\gamma_{1,t} \quad : \quad 0 = u_{c,t}\left(\pi_t - 1\right)\pi_t - \left[(1-\eta)\, u_{c,t} - u_{h,t}\frac{(1+\tau_t^c)}{\left(1-\tau_t^h\right)}\frac{\eta}{a_t}\right]\frac{h_t a_t}{\varphi} + \\ -\beta E_t\left[\frac{(1+\tau_t^c)\,u_{c,t+1}}{\left(1+\tau_{t+1}^c\right)}\left(\pi_{t+1} - 1\right)\pi_{t+1}\right]; \tag{44}$$

$$\gamma_{2,t} \quad : \quad 0 = \frac{u_{c,t}}{(1+\tau_t^c)} - \beta E_t\left(\frac{u_{c,t+1}}{\left(1+\tau_{t+1}^c\right)}\frac{R_t}{\pi_{t+1}}\right); \tag{45}$$

$$\gamma_{3,t} \quad : \quad 0 = a_t h_t - c_t - g_t - \frac{\varphi}{2}\left(\pi_t - 1\right)^2; \tag{46}$$

$$\gamma_{4,t} \quad : \quad 0 = \frac{b_{t+1}}{R_t} - \frac{b_t}{\pi_t} - g_t - \frac{u_{h,t}}{u_{c,t}}\frac{(1+\tau_t^c)}{\left(1-\tau_t^h\right)}h_t\tau_t^h + c_t\tau_t^c. \tag{47}$$

At steady-state we can rewrite the first order conditions of the endogenous variables, the policy instruments and the Lagrangian multipliers as:

$$\tau^c \quad : \quad 0=\gamma_1\left(\frac{\eta}{\varphi}\frac{hu_h}{(1-\tau^h)}-\beta\frac{u_c\,(\pi-1)\,\pi}{(1+\tau^c)}\right)+\gamma_1\left(\frac{u_c}{(1+\tau^c)}\,(\pi-1)\,\pi\right)+$$
$$-\gamma_2\left(\frac{u_c}{(1+\tau^c)^2}\right)+\gamma_2\left(\frac{u_{c,t}}{(1+\tau_t^c)^2}\frac{R}{\pi}\right)-\gamma_4\left(\frac{u_h}{u_c}\frac{h\tau^h}{(1-\tau^h)}-c\right); \tag{48}$$

$$c \quad : \quad 0=u_c+\gamma_1\left[u_{cc}\,(\pi-1)\,\pi-\frac{(1-\eta)}{\varphi}hau_{cc}\right]-\gamma_1 u_{cc}\,(\pi-1)\,\pi+\gamma_2\left(\frac{u_{cc}}{(1+\tau^c)}\right)$$
$$-\gamma_2\left(\frac{u_{cc}}{(1+\tau^c)}\frac{R}{\pi}\right)+\gamma_4\left(\tau^c+\frac{u_{cc}u_h}{u_c^2}\,(1+\tau^c)\,h\frac{\tau^h}{(1-\tau^h)}\right)-\gamma_3; \tag{49}$$

$$h \quad : \quad 0=u_h+\gamma_1\left(u_{hh}\frac{(1+\tau^c)}{(1-\tau^h)}\frac{h\eta}{\varphi}-\left((1-\eta)\,u_c-u_h\frac{(1+\tau^c)}{(1-\tau^h)}\frac{\eta}{a}\right)\frac{1}{\varphi}\right)$$
$$+\gamma_3-\gamma_4\frac{(1+\tau^c)\,\tau^h}{(1-\tau^h)\,u_c}\,(u_h+u_{hh}h); \tag{50}$$

$$\tau^h \quad : \quad 0=\gamma_1\frac{u_h\,(1+\tau^c)\,\eta h}{(1-\tau^h)^2\,\varphi}=\gamma_4\frac{u_h}{u_c}\,(1+\tau^c)\,h\frac{1}{(1-\tau^h)^2}; \tag{51}$$
$$R \quad : \quad 0=-\gamma_2\frac{u_c}{(1+\tau^c)}\frac{\beta}{\pi}-\gamma_4\frac{b}{R^2}; \tag{52}$$
$$g \quad : \quad 0=u_g-\gamma_3-\gamma_4; \tag{53}$$
$$b \quad : \quad 0=-\frac{\gamma_4}{\pi}+\frac{\gamma_4}{\beta R}; \tag{54}$$
$$\pi \quad : \quad 0=\gamma_1\,(2\pi-1)\,u_c-\gamma_1\,(2\pi-1)\,u_c+\gamma_2\left(\frac{u_c}{(1+\tau^c)}\frac{R}{\pi^2}\right)-\gamma_3\varphi\,(\pi-1)+\gamma_4\frac{b}{\pi^2}; \tag{55}$$

$$\gamma_1 \quad : \quad u_c\,(\pi-1)\,\pi\,(1-\beta)-(1-\eta)\,u_c\frac{h}{\varphi}+u_h\frac{(1+\tau^c)}{(1-\tau^h)}\eta\frac{h}{\varphi}=0; \tag{56}$$
$$\gamma_2 \quad : \quad \frac{u_c}{(1+\tau^c)}-\beta\left(\frac{u_c}{(1+\tau^c)}\frac{R}{\pi}\right)=0; \tag{57}$$
$$\gamma_3 \quad : \quad h-c-g-\frac{\varphi}{2}\,(\pi-1)^2=0; \tag{58}$$
$$\gamma_4 \quad : \quad \frac{b}{R}-\frac{b}{\pi}-g-\frac{u_h}{u_c}\frac{(1+\tau^c)}{(1-\tau^h)}h\tau^h+c\tau^c=0. \tag{59}$$

5.2 Ramsey Problem with Labor Income Taxation

The Lagrangian of the Ramsey problem when the policy-maker is constrained to keep consumption taxation fixed at zero, i.e. $\tau_t^c = 0 \; \forall t$, is given by:

$$\begin{aligned}
\mathcal{L} = & \max_{\left\{ \begin{array}{c} c_t, h_t, \pi_t, g_t, \\ R_t \geq 1, \tau_t^h, b_t \end{array} \right\}_{t=0}^{\infty}} E_0 \sum_{t=o}^{\infty} \beta^t \left[u\left(c_t, h_t, g_t\right) \right] + \\
& + \gamma_{1,t} \beta^t \left[\begin{array}{c} u_{c,t} \left(\pi_t - 1\right) \pi_t - \left(\left(1-\eta\right) u_{c,t} - u_{h,t} \frac{1}{\left(1-\tau_t^h\right)} \frac{\eta}{a_t} \right) \frac{h_t a_t}{\varphi} - \\ \beta E_t \left[u_{c,t+1} \left(\pi_{t+1} - 1\right) \left(\pi_{t+1}\right) \right] \end{array} \right] + \\
& + \gamma_{2,t} \beta^t \left[u_{c,t} - \beta E_t \left(u_{c,t+1} \frac{R_t}{\pi_{t+1}} \right) \right] + \\
& + \gamma_{3,t} \beta^t \left[a_t h_t - c_t - g_t - \frac{\varphi}{2} \left(\pi_t - 1\right)^2 \right] + \\
& + \gamma_{4,t} \beta^t \left[-\frac{b_t}{\pi_t} - g_t + \frac{b_{t+1}}{R_t} - \frac{u_{h,t}}{u_{c,t}} \frac{h_t \tau_t^h}{\left(1-\tau_t^h\right)} \right].
\end{aligned} \tag{60}$$

where we have substituted the wage with

$$-\frac{u_{h,t}}{u_{c,t}} \frac{1}{\left(1-\tau_t^h\right)} = w_t.$$

The first-order conditions with respect to the decision variables $c_t, \tau_t^h, h_t, \pi_t, R_t, b_t$ and g_t respectively are:

$$\begin{aligned}
c_t \quad : \quad 0 = & \, u_{c,t} + \gamma_{1,t} \left[u_{cc,t} \left(\pi_t - 1\right) \pi_t - \frac{\left(1-\eta\right)}{\varphi} h_t a_t u_{cc,t} \right] + \\
& - \gamma_{1,t-1} \left(\frac{u_{cc,t}}{\left(1+\tau_t^c\right)} \left(\pi_t - 1\right) \left(\pi_t\right) \right) + \gamma_{2,t} \left(u_{cc,t}\right) - \gamma_{2,t-1} \left(u_{cc,t} \frac{R_{t-1}}{\pi_t} \right) + \\
& - \gamma_{3,t} + \gamma_{4,t} \left(\frac{u_{cc,t} u_{h,t}}{u_{c,t}^2} h_t \frac{\tau_t^h}{\left(1-\tau_t^h\right)} \right);
\end{aligned} \tag{61}$$

$$\tau_t^h : 0 = \gamma_{1,t} \frac{u_{h,t}}{\left(1-\tau_t^h\right)^2} \frac{\eta h_t}{\varphi} - \gamma_{4,t} \frac{u_{h,t}}{u_{c,t}} h_t \left(\frac{1}{\left(1-\tau_t^h\right)^2} \right); \tag{62}$$

$$\begin{aligned}
h_t \quad : \quad 0 = & \, u_{h,t} + \gamma_{1,t} \left(u_{hh,t} \frac{1}{\left(1-\tau_t^h\right)} \frac{\eta}{1} \frac{h_t}{\varphi} - \left(\left(1-\eta\right) u_{c,t} - u_{h,t} \frac{1}{\left(1-\tau_t^h\right)} \frac{\eta}{a_t} \right) \frac{a_t}{\varphi} \right) + \\
& + \gamma_{3,t} a_t - \gamma_{4,t} \frac{\tau_t^h}{\left(1-\tau_t^h\right) u_{c,t}} \left(u_{h,t} + u_{hh,t} h_t\right);
\end{aligned} \tag{63}$$

$$\pi_t \quad : \quad 0 = \gamma_{1,t}\left(2\pi_t - 1\right)u_{c,t} - \gamma_{1,t-1}\left(2\pi_t - 1\right)u_{c,t} + \tag{64}$$

$$+\gamma_{2,t-1}\left(u_{c,t}\frac{R_{t-1}}{\pi_t^2}\right) - \gamma_{3,t}\varphi\left(\pi_t - 1\right) + \gamma_{4,t}\frac{b_t}{\pi_t^2}; \tag{65}$$

$$R_t : 0 = -\gamma_{2,t}\beta\frac{u_{c,t+1}}{\pi_{t+1}} - \gamma_{4,t}\frac{b_{t+1}}{R_t^2}; \tag{66}$$

$$b_t : 0 = -\beta E_t\frac{\gamma_{4,t+1}}{\pi_{t+1}} + \frac{\gamma_{4,t}}{R_t}; \tag{67}$$

$$g_t : 0 = u_{g,t} - \gamma_{3,t} - \gamma_{4,t}; \tag{68}$$

and the first order condition with respect to the Lagrangian multipliers $\gamma_{i,t}$ with $i = 1, 2, 3, 4$ are:

$$\gamma_{1,t} \quad : \quad 0 = u_{c,t}\left(\pi_t - 1\right)\pi_t - \left(\left(1-\eta\right)u_{c,t} - u_{h,t}\frac{1}{\left(1-\tau_t^h\right)}\frac{\eta}{a_t}\right)\frac{h_t a_t}{\varphi} + \tag{69}$$

$$-\beta E_t\left[u_{c,t+1}\left(\pi_{t+1} - 1\right)\pi_{t+1}\right]; \tag{70}$$

$$\gamma_{2,t} : 0 = u_{c,t} - \beta E_t\left(u_{c,t+1}\frac{R_t}{\pi_{t+1}}\right); \tag{71}$$

$$\gamma_{3,t} : 0 = a_t h_t - c_t - g_t - \frac{\varphi}{2}\left(\pi_t - 1\right)^2; \tag{72}$$

$$\gamma_{4,t} : 0 = \frac{b_{t+1}}{R_t} - \frac{b_t}{\pi_t} - g_t - \frac{u_{h,t}}{u_{c,t}}\frac{h_t\tau_t^h}{\left(1-\tau_t^h\right)}. \tag{73}$$

At steady state we can write the first order conditions of this policy problem as:

$$\tau^h \quad : \quad 0 = \gamma_1\frac{u_h}{\left(1-\tau^h\right)^2}\frac{\eta h}{\varphi} - \gamma_4\frac{u_h}{u_c}h\frac{1}{\left(1-\tau^h\right)^2}; \tag{74}$$

$$R \quad : \quad 0 = -\gamma_2 u_c\frac{\beta}{\pi} - \gamma_4\frac{b}{R^2}; \tag{75}$$

$$g \quad : \quad 0 = u_g - \gamma_3 - \gamma_4; \tag{76}$$

$$b \quad : \quad 0 = -\frac{\gamma_4}{\pi} + \frac{\gamma_4}{\beta R}; \tag{77}$$

$$c \quad : \quad 0 = u_c + \gamma_1\left[u_{cc}\left(\pi - 1\right)\pi - \frac{\left(1-\eta\right)}{\varphi}hau_{cc}\right] - \gamma_1 u_{cc}\left(\pi - 1\right)\pi + \gamma_2 u_{cc} + \tag{78}$$

$$-\gamma_2\left(u_{cc}\frac{R}{\pi}\right) + \gamma_4\left(\frac{u_{cc}u_h}{u_c^2}h\frac{\tau^h}{\left(1-\tau^h\right)}\right) - \gamma_3; \tag{79}$$

$$h \quad : \quad 0 = u_h + \gamma_1\left(u_{hh}\frac{1}{\left(1-\tau^h\right)}\frac{h\eta}{\varphi} - \left(\left(1-\eta\right)u_c - u_h\frac{1}{\left(1-\tau^h\right)}\frac{\eta}{a}\right)\frac{1}{\varphi}\right) + \tag{80}$$

$$+\gamma_3 - \gamma_4\frac{\tau^h}{\left(1-\tau^h\right)u_c}\left(u_h + u_{hh}h\right); \tag{81}$$

$$\pi \quad : \quad 0 = \gamma_1\left(2\pi - 1\right)u_c - \gamma_1\left(2\pi - 1\right)u_c + \gamma_2\left(u_c\frac{R}{\pi^2}\right) - \gamma_3\varphi\left(\pi - 1\right) + \gamma_4\frac{b}{\pi^2}; \tag{82}$$

$$\gamma_1 \quad : \quad u_c (\pi - 1) \pi (1-\beta) - (1-\eta) u_c \frac{h}{\varphi} + u_h \frac{1}{(1-\tau^h)} \eta \frac{h}{\varphi} = 0; \tag{83}$$

$$\gamma_2 \quad : \quad u_c - \beta u_c \frac{R}{\pi} = 0; \tag{84}$$

$$\gamma_3 \quad : \quad h - c - g - \frac{\varphi}{2} (\pi - 1)^2 = 0; \tag{85}$$

$$\gamma_4 \quad : \quad \frac{b}{R} - \frac{b}{\pi} - g - \frac{u_h}{u_c} \frac{1}{(1-\tau^h)} h \tau^h = 0. \tag{86}$$

5.3 The Ramsey Problem with Consumption Taxation

The Lagrangian of the Ramsey Problem when only consumption tax is available, i.e. $\tau_t^h = 0 \ \forall t$, can be represented as:

$$\begin{aligned} \mathcal{L} = & \max_{\left\{ \begin{array}{c} c_t, h_t, \pi_t, g_t, \\ R_t \geq 1, \tau_t^c, b_t \end{array} \right\}_{t=0}^{\infty}} E_0 \sum_{t=o}^{\infty} \beta^t \left[u(c_t, h_t, g_t) \right] + \\ & + \gamma_{1,t} \beta^t \left[\begin{array}{c} u_{c,t} (\pi_t - 1) \pi_t - \left((1-\eta) u_{c,t} - u_{h,t} (1+\tau_t^c) \frac{\eta}{a_t} \right) \frac{h_t a_t}{\varphi} - \\ \beta E_t \left[\frac{(1+\tau_t^c) u_{c,t+1}}{(1+\tau_{t+1}^c)} (\pi_{t+1} - 1)(\pi_{t+1}) \right] \end{array} \right] + \\ & + \gamma_{2,t} \beta^t \left[\frac{u_{c,t}}{(1+\tau_t^c)} - \beta E_t \left(\frac{u_{c,t+1}}{(1+\tau_{t+1}^c)} \frac{R_t}{\pi_{t+1}} \right) \right] + \\ & + \gamma_{3,t} \beta^t \left[a_t h_t - c_t - g_t - \frac{\varphi}{2} (\pi_t - 1)^2 \right] + \\ & + \gamma_{4,t} \beta^t \left[-\frac{b_t}{\pi_t} - g_t + \frac{b_{t+1}}{R_t} + c_t \tau_t^c \right]. \end{aligned} \tag{87}$$

Where, as before, we substitute the wage with

$$-\frac{u_{h,t}}{u_{c,t}} (1+\tau_t^c) = w_t. \tag{88}$$

The first order conditions with respect to the decision variables $c_t, h_t, \pi_t, R_t, b_t, \tau_t^c$ and g_t respectively are in order:

$$\begin{aligned} c_t \quad : \quad & 0 = u_{c,t} + \gamma_{1,t} \left[u_{cc,t} (\pi_t - 1) \pi_t - \frac{(1-\eta)}{\varphi} h_t a_t u_{cc,t} \right] + \\ & -\gamma_{1,t-1} \left(\frac{\left(1+\tau_{t-1}^c\right) u_{cc,t}}{(1+\tau_t^c)} (\pi_t - 1)(\pi_t) \right) + \\ & + \gamma_{2,t} \left(\frac{u_{cc,t}}{(1+\tau_t^c)} \right) - \gamma_{2,t-1} \left(\frac{u_{cc,t}}{(1+\tau_t^c)} \frac{R_{t-1}}{\pi_t} \right) + \\ & -\gamma_{3,t} + \gamma_{4,t} \tau_t^c; \end{aligned} \tag{89}$$

$$h_t \quad : \quad 0 = u_{h,t} + \gamma_{1,t}\left(u_{hh,t}\left(1+\tau_t^c\right)\frac{\eta}{1}\frac{h_t}{\varphi} - \left(\left(1-\eta\right)u_{c,t} - u_{h,t}\left(1+\tau_t^c\right)\frac{\eta}{a_t}\right)\frac{a_t}{\varphi}\right) + \\ +\gamma_{3,t}a_t; \tag{90}$$

$$\pi_t \quad : \quad 0 = \gamma_{1,t}\left(2\pi_t - 1\right)u_{c,t} - \gamma_{1,t-1}\left(2\pi_t - 1\right)\frac{\left(1+\tau_{t-1}^c\right)u_{c,t}}{\left(1+\tau_t^c\right)} + \\ +\gamma_{2,t-1}\left(\frac{u_{c,t}}{\left(1+\tau_t^c\right)}\frac{R_{t-1}}{\pi_t^2}\right) - \gamma_{3,t}\varphi\left(\pi_t - 1\right) + \gamma_{4,t}\frac{b_t}{\pi_t^2}; \tag{91}$$

$$\tau_t^c \quad : \quad 0 = \gamma_{1,t}\left(\frac{\eta}{\varphi}h_t u_{h,t} - \beta\frac{u_{c,t+1}\left(\pi_{t+1}-1\right)\left(\pi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)}\right) + \\ +\gamma_{1,t-1}\left(\frac{\left(1+\tau_{t-1}^c\right)u_{c,t}}{\left(1+\tau_t^c\right)^2}\left(\pi_t - 1\right)\left(\pi_t\right)\right) + \\ -\gamma_{2,t}\left(\frac{u_{c,t}}{\left(1+\tau_t^c\right)^2}\right) + \gamma_{2,t-1}\left(\frac{u_{c,t}}{\left(1+\tau_t^c\right)^2}\frac{R_{t-1}}{\pi_t}\right) + \\ +\gamma_{4,t}c_t; \tag{92}$$

$$R_t \quad : \quad 0 = -\gamma_{2,t}\frac{u_{c,t+1}}{\left(1+\tau_{t+1}^c\right)}\frac{\beta}{\pi_{t+1}} - \gamma_{4,t}\frac{b_{t+1}}{R_t^2}. \tag{93}$$

$$b_t \quad : \quad 0 = -\beta E_t\frac{\gamma_{4,t+1}}{\pi_{t+1}} + \frac{\gamma_{4,t}}{R_t}. \tag{94}$$

$$g_t \quad : \quad 0 = u_{g,t} - \gamma_{3,t} - \gamma_{4,t}. \tag{95}$$

The first order conditions with respect to the Lagrangian multipliers $\gamma_{i,t}$ with $i = 1, 2, 3, 4$ are:

$$\gamma_{1,t} \quad : \quad 0 = u_{c,t}\left(\pi_t - 1\right)\pi_t - \left(\left(1-\eta\right)u_{c,t} - u_{h,t}\left(1+\tau_t^c\right)\frac{\eta}{a_t}\right)\frac{h_t a_t}{\varphi} + \tag{96}$$

$$-\beta E_t\left[\frac{\left(1+\tau_t^c\right)u_{c,t+1}}{\left(1+\tau_{t+1}^c\right)}\left(\pi_{t+1}-1\right)\left(\pi_{t+1}\right)\right]; \tag{97}$$

$$\gamma_{2,t} \quad : \quad 0 = \frac{u_{c,t}}{\left(1+\tau_t^c\right)} - \beta E_t\left(\frac{u_{c,t+1}}{\left(1+\tau_{t+1}^c\right)}\frac{R_t}{\pi_{t+1}}\right); \tag{98}$$

$$\gamma_{3,t} \quad : \quad 0 = a_t h_t - c_t - g_t - \frac{\varphi}{2}\left(\pi_t - 1\right)^2; \tag{99}$$

$$\gamma_{4,t} \quad : \quad 0 = \frac{b_{t+1}}{R_t} - \frac{b_t}{\pi_t} - g_t + c_t\tau_t^c. \tag{100}$$

At steady-state, we can write these first order conditions as:

$$\tau^c \quad : \quad 0 = \gamma_1 \left(\frac{\eta}{\varphi} \frac{hu_h}{(1-\tau^h)} - \beta \frac{u_c (\pi - 1)(\pi)}{(1+\tau^c)} \right) + \gamma_1 \left(\frac{u_c}{(1+\tau^c)} (\pi - 1)(\pi) \right) + \tag{101}$$
$$-\gamma_2 \left(\frac{u_c}{(1+\tau^c)^2} \right) + \gamma_2 \left(\frac{u_{c,t}}{(1+\tau_t^c)^2} \frac{R}{\pi} \right) - \gamma_4 \left(\frac{u_h}{u_c} \frac{h\tau^h}{(1-\tau^h)} - c \right);$$

$$c \quad : \quad 0 = u_c + \gamma_1 \left[u_{cc} (\pi - 1) \pi - \frac{(1-\eta)}{\varphi} hau_{cc} \right] - \gamma_1 u_{cc} (\pi - 1) \pi + \tag{102}$$
$$+\gamma_2 \left(\frac{u_{cc}}{(1+\tau^c)} \right) - \gamma_2 \left(\frac{u_{cc}}{(1+\tau^c)} \frac{R}{\pi} \right) + \gamma_4 \tau^c - \gamma_3;$$

$$h : 0 = u_h + \gamma_1 \left(u_{hh} (1+\tau^c) \frac{h\eta}{\varphi} - \left((1-\eta) u_c - u_h (1+\tau^c) \frac{\eta}{a} \right) \frac{1}{\varphi} \right) + \gamma_3; \tag{103}$$

$$R \quad : \quad 0 = -\gamma_2 \frac{u_c}{(1+\tau^c)} \frac{\beta}{\pi} - \gamma_4 \frac{b}{R^2}; \tag{104}$$
$$g \quad : \quad 0 = u_g - \gamma_3 - \gamma_4; \tag{105}$$
$$b \quad : \quad 0 = -\frac{\gamma_4}{\pi} + \frac{\gamma_4}{\beta R}; \tag{106}$$
$$\pi \quad : \quad 0 = \gamma_1 (2\pi - 1) u_c - \gamma_1 (2\pi - 1) u_c + \gamma_2 \left(\frac{u_c}{(1+\tau^c)} \frac{R}{\pi^2} \right) - \gamma_3 \varphi (\pi - 1) + \gamma_4 \frac{b}{\pi^2}; \tag{107}$$

$$\gamma_1 \quad : \quad u_c (\pi - 1) \pi (1-\beta) - (1-\eta) u_c \frac{h}{\varphi} + u_h (1+\tau^c) \eta \frac{h}{\varphi} = 0; \tag{108}$$
$$\gamma_2 \quad : \quad \frac{u_c}{(1+\tau^c)} - \beta \left(\frac{u_c}{(1+\tau^c)} \frac{R}{\pi} \right) = 0; \tag{109}$$
$$\gamma_3 \quad : \quad h - c - g - \frac{\varphi}{2} (\pi - 1)^2 = 0; \tag{110}$$
$$\gamma_4 \quad : \quad \frac{b}{R} - \frac{b}{\pi} - g + c\tau^c = 0. \tag{111}$$

6 Appendix B-Proofs

6.1 The Social Planner's Program -Proof of Proposition 3

The Social Planner problem is characterized as:

$$\mathcal{L}^* = \max_{c_t, h_t, g_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, g_t) - \gamma_t \beta^t [a_t h_t - c_t - g_t]. \tag{112}$$

The optimal level of consumption, labor and public spending are given respectively by

$$u^*_{c,t} = \gamma_t, \tag{113}$$

$$u^*_{h,t} = -\gamma_t a_t, \tag{114}$$

and

$$u^*_{g,t} = \gamma_t. \tag{115}$$

Therefore, by combining these three expressions one can obtain

$$u^*_{c,t} = \frac{-u^*_{h,t}}{a_t} = u^*_{g,t}. \tag{116}$$

6.2 Proof of Proposition 5

Combining equation (52) and equation (55) eliminating γ_4 one can write

$$0 = \gamma_2 \left(\frac{u_c}{(1+\tau^c)} \frac{R}{\pi^2} \right) - \gamma_3 \varphi (\pi - 1) - \gamma_2 \frac{u_c}{(1+\tau^c)} \frac{\beta}{\pi} \frac{R^2}{b} \frac{b}{\pi^2}. \tag{117}$$

From (57) we have that

$$\frac{R}{\pi} = \frac{1}{\beta},$$

and therefore (117) becomes

$$0 = -\gamma_3 \varphi (\pi - 1).$$

Given that $\gamma_3 > 0$ represents the marginal utility of relaxing the resource constraint it follows that.

$$\pi = 1.$$

6.3 Proof of Proposition 6

The proof is done via a comparison between the decentralized equilibrium and the first best allocation. First of all, one can notice that in the decentralized equilibrium, part of the output is eroded by the inflation rate. Therefore any efficient steady state allocation need to forsake a zero inflation policy so that the decentralized market clearing condition is equivalent to the one in the efficient steady state, i.e.

$$y^* = h^* = c^* + g^*. \tag{118}$$

Second, in the efficient equilibrium

$$-\frac{u(h^*)}{u(c^*)} = 1. \tag{119}$$

In other words, the marginal utility of consumption equates the marginal utility of leisure. In the decentralized steady-state the marginal rate of substitution between consumption and hours worked is given by

$$-\frac{u(h^*)}{u(c^*)}=\frac{\eta-1}{\eta}\left(\frac{1-\tau^h}{1+\tau^c}\right). \tag{120}$$

Equating the last two expressions, efficiency requires

$$\frac{\eta-1}{\eta}\left(\frac{1-\tau^h}{1+\tau^c}\right)=1. \tag{121}$$

Finally, these efficiency conditions must hold together with the government budget constraint as

$$\frac{b_0^{opt}}{h^*}=\frac{\frac{c^*}{h^*}\tau^c+\frac{\eta-1}{\eta}\left(\tau^h\right)-\frac{g^*}{h^*}}{1-\beta}. \tag{122}$$

Combining the latter with the market clearing condition one can recover (26) presented in the main text, i.e.

$$\frac{b_0^{opt}}{h^*}=\frac{\frac{c^*}{h^*}(1+\tau^c)+\frac{\eta-1}{\eta}\left(\tau^h\right)-1}{1-\beta}. \tag{123}$$

Now, one can rewrite (121) as

$$\left(1-\tau^h\right)\frac{\eta-1}{\eta}=(1+\tau^c). \tag{124}$$

By plugging the latter into the government budget constraint, it yields

$$\frac{b_0^{opt}}{h^*}=\frac{\left(1-\tau^h\right)\frac{c^*}{h^*}\frac{\eta-1}{\eta}-\left(\frac{\eta}{\eta-1}-\tau^h\right)\frac{\eta-1}{\eta}}{1-\beta}. \tag{125}$$

Given that $\frac{c^*}{h^*}<1$ and $\eta>1$, it is easy to show that

$$\left(1-\tau^h\right)\frac{c^*}{h^*}\frac{\eta-1}{\eta}<\left(\frac{\eta}{\eta-1}-\tau^h\right)\frac{\eta-1}{\eta}. \tag{126}$$

Hence

$$\frac{b_0^{opt}}{h^*}<0 \text{ Q.E.D.} \tag{127}$$

In other words, efficiency requires the government to accumulate public assets, i.e. first best is not attainable for a generic level of public debt.

6.4 Proof of Corollary 7

Combining equations (25) and (26), it is straightforward to show that a generic level positive level of b_0 requires in an hypothetical first best decentralized equilibrium

$$\frac{\frac{c^*}{h^*}(1+\tau^c)+\left(\frac{1+\tau^c}{1-\tau^h}\right)\tau^h-1}{1-\beta}\geq 0 \tag{128}$$

or

$$\frac{c^*}{h^*}(1+\tau^c) + \left(\frac{1+\tau^c}{1-\tau^h}\right)\tau^h \geq 1, \tag{129}$$

which is possible if and only if either $\tau^h > 1$ or $\tau^c < -1$.

As a proof, let assume $b_0^{opt} = 0$. From (25), we can rewrite τ^c as

$$\tau^c = \left(\frac{\eta-1}{\eta}\left(1-\tau^h\right) - 1\right) \tag{130}$$

and we substitute it in (26) obtaining

$$\frac{c^*}{h^*}\left(\frac{\eta-1}{\eta}\left(1-\tau^h\right)\right) + \left(\frac{\eta-1}{\eta}\right)\tau^h \geq 1 \tag{131}$$

and therefore

$$\tau^h = \frac{1-\frac{\eta-1}{\eta}\frac{c^*}{h^*}}{\frac{\eta-1}{\eta}-\frac{\eta-1}{\eta}\frac{c^*}{h^*}} > 1 \tag{132}$$

On the other hand, if we rewrite τ^h from (25) as

$$\tau^h = \left(1-(1+\tau^c)\frac{\eta}{\eta-1}\right), \tag{133}$$

and we substitute it in (26), we obtain

$$\frac{\frac{c^*}{h^*}(1+\tau^c) + \frac{\eta-1}{\eta}\left(1-(1+\tau^c)\frac{\eta}{\eta-1}\right) - 1}{1-\beta} = 0, \tag{134}$$

which implies

$$(1+\tau^c) = \frac{1-\frac{\eta-1}{\eta}}{\left(\frac{c^*}{h^*}-1\right)} < 0, \tag{135}$$

$$\tau^c = \frac{1-\frac{\eta-1}{\eta}}{\left(\frac{c^*}{h^*}-1\right)} - 1. \tag{136}$$

Given that $\frac{c^*}{h^*} - 1 < 0$, (135) requires $\tau^c < -1$ Q.E.D.

If we allow for full profit taxation, i.e. $\tau^d = 1$, the steady state government budget constraint reads as

$$\frac{b_0^{opt}}{h^*} = \frac{\frac{c^*}{h^*}\tau^c + \frac{\eta-1}{\eta}\left(\tau^h\right) - \frac{g^*}{h^*} + \frac{1}{\eta}}{1-\beta}. \tag{137}$$

Efficiency requires

$$\frac{\eta-1}{\eta}\left(1-\tau^h\right) = (1+\tau^c). \tag{138}$$

Hence

$$\frac{b_0^{opt}}{h^*} = \frac{\frac{c^*}{h^*}\tau^c + \frac{\eta-1}{\eta}\left(\tau^h\right) - \frac{g^*}{h^*} + \frac{1}{\eta}}{1-\beta}, \tag{139}$$

$$\frac{b_0^{opt}}{h^*} = \frac{\left(1-\tau^h\right)\frac{\eta-1}{\eta}\frac{c^*}{h^*} - \left(1-\tau^h\right)\frac{\eta-1}{\eta}}{1-\beta}. \tag{140}$$

It is straightforward to show that $\left(1-\tau^h\right)\frac{\eta-1}{\eta}\frac{c^*}{h^*} - \left(1-\tau^h\right)\frac{\eta-1}{\eta} < 0$. Hence $\frac{b_0^{opt}}{h^*} < 0$ even in the presence of full profit taxation. Q.E.D.

6.5 Ramsey Steady State with only Labor Income Taxes (Proof of Proposition 8)

As discussed in the main text, in order to pin down the RSS, we fix an exogenous level of steady state public debt. By doing this, (67) becomes redundant, i.e. there is no need to take the foc wrt public debt. Therefore in finding the RSS one can substitute out for the government budget constraint and therefore eliminate from the Ramsey problem (60) γ_4. More precisely, given that γ_4 is indeterminate and therefore unconstrained, there always exists a value of γ_4, such that, for a given value of b_0 and the optimal monetary policy in place, i.e. $\pi = 1$, the RSS allocation in this modified problem is identical to the one in (60).

Since from (7)

$$-\frac{u_{h,t}}{u_{c,t}} = w_t\left(1-\tau_t^h\right) = w_t - \tau_t^h w_t,$$

and from the steady state version of the government budget constraint (86)

$$\tau_t^h w_t h_t = g_t + \tilde{x},$$

we can rewrite the real wage as $w_t = -\frac{u_{h,t}}{u_{c,t}} + \frac{g_t+\tilde{x}}{h_t}$ and transform the Lagrangian (60) as

$$\begin{aligned}
\mathcal{L} = & \max_{\left\{\begin{array}{c} c_t, h_t, \pi_t, g_t, \\ R_t \geq 1 \end{array}\right\}_{t=0}^{\infty}} E_0 \sum_{t=o}^{\infty} \beta^t \left[u\left(c_t, h_t, g_t\right)\right] + \\
& +\gamma_{1,t}\beta^t \left[\begin{array}{c} u_{c,t}\left(\pi_t - 1\right)\pi_t - \frac{h_t a_t u_{c,t}}{\varphi}\left(\left(1-\eta\right) - \eta\left(\frac{u_{h,t}}{u_{c,t}} - \frac{g_t+\tilde{x}}{h_t}\right)\right) - \\ \beta E_t\left[u_{c,t+1}\left(\pi_{t+1}-1\right)\left(\pi_{t+1}\right)\right] \end{array}\right] + \\
& +\gamma_{2,t}\beta^t\left[u_{c,t} - \beta E_t\left(u_{c,t+1}\frac{R_t}{\pi_{t+1}}\right)\right] + \\
& +\gamma_{3,t}\beta^t\left[a_t h_t - c_t - g_t - \frac{\varphi}{2}\left(\pi_t - 1\right)^2\right].
\end{aligned} \tag{141}$$

The first-order-conditions are given by the three constraints and

$$c_t \quad : \quad 0 = u_{c,t} + \gamma_{1,t}\left[u_{cc,t}\left(\pi_t - 1\right)\pi_t - \frac{h_t a_t u_{cc,t}}{\varphi}\left(1 - \eta + \eta\frac{g_t + \tilde{x}}{h_t}\right)\right] + \\ -\gamma_{1,t-1}\left(\frac{u_{cc,t}}{(1+\tau_t^c)}\left(\pi_t - 1\right)\left(\pi_t\right)\right) + \gamma_{2,t}\left(u_{cc,t}\right) - \gamma_{2,t-1}\left(u_{cc,t}\frac{R_{t-1}}{\pi_t}\right) - \gamma_{3,t}; \quad (142)$$

$$h_t \quad : \quad 0 = u_{h,t} - \gamma_{1,t}\frac{u_{c,t}}{\varphi}\left((1-\eta) - \eta\left(\frac{u_{h,t}}{u_{c,t}} - \frac{g_t + \tilde{x}}{h_t}\right) - h_t\eta\left(\frac{u_{h,t}}{u_{c,t}} + \frac{g_t + \tilde{x}}{h_t^2}\right)\right) + \\ +\gamma_{3,t}a_t; \quad (143)$$

$$\pi_t : 0 = \gamma_{1,t}\left(2\pi_t - 1\right)u_{c,t} - \gamma_{1,t-1}\left(2\pi_t - 1\right)u_{c,t} + \gamma_{2,t-1}\left(u_{c,t}\frac{R_{t-1}}{\pi_t^2}\right) - \gamma_{3,t}\varphi\left(\pi_t - 1\right); \quad (144)$$

$$R_t : 0 = -\gamma_{2,t}\beta\frac{u_{c,t+1}}{\pi_{t+1}}; \quad (145)$$

$$g_t : 0 = u_{g,t} - \gamma_{1,t}\frac{\eta}{\varphi}u_{c,t} - \gamma_{3,t}. \quad (146)$$

We can now impose the steady state and obtain $\gamma_2 = 0$ from (145). Using this results in (144), in the Euler equation, and in the New Keynesian Phillips curve, we obtain respectively:

$$\pi = 1,$$

$$R = \frac{1}{\beta},$$

and

$$\frac{-u_h}{u_c} = -\frac{(1-\eta)}{\eta} - \frac{g_t + \tilde{x}}{h_t}. \quad (147)$$

Note that since $u_h < 0$ and $u_c > 0$, it follows that

$$1 - \eta + \eta\frac{g_t + \tilde{x}}{h_t} < 0. \quad (148)$$

The remaining conditions simplify to

$$c : u_c + \gamma_1\left[-\frac{u_{cc}h}{\varphi}\left(1 - \eta + \eta\frac{g + \tilde{x}}{h}\right)\right] - \gamma_3 = 0; \quad (149)$$

$$h : u_h + \eta\gamma_1\frac{u_c}{\varphi}\left(\frac{u_{hh}h}{u_c} + \frac{g + \tilde{x}}{h}\right) + \gamma_3 = 0; \quad (150)$$

$$g : u_g - \gamma_1\frac{\eta}{\varphi}u_c - \gamma_3 = 0. \quad (151)$$

Note that γ_3 represents the marginal utility of relaxing the resource constraint, therefore $\gamma_3 > 0$. Moreover, the latter FOCs determine the possible alternative uses of the disposable resources such as consumption, labor and government spending. It must be the case that γ_3 is greater than all the marginal utilities, namely u_c, u_g, $-u_h$. Equation (151) therefore implies $\gamma_1 \leq 0$. Combining equation (149) with (151) to eliminate γ_3, we get

$$u_c = \frac{u_g + \gamma_1 \frac{u_{cc} h}{\varphi} \left(1 - \eta + \eta \frac{g + \tilde{x}}{h}\right)}{1 + \gamma_1 \frac{\eta}{\varphi}},$$

and given condition (148), $\gamma_1 \leq 0$ and $u_{cc} < 0$, it follows that

$$u_c = \frac{u_g + \gamma_1 \frac{u_{cc} h}{\varphi} \left(1 - \eta + \eta \frac{g + \tilde{x}}{h}\right)}{1 + \gamma_1 \frac{\eta}{\varphi}} \leq u_g + \gamma_1 \frac{u_{cc} h}{\varphi} \left(1 - \eta + \eta \frac{g + \tilde{x}}{h}\right) \leq u_g.$$

Therefore we have that $u_g \geq u_c \geq -u_h$. Q.E.D.

6.6 Ramsey Steady State with only Consumption Taxation (Proof of Proposition 9)

Here we follow the same approach as for Proposition (8). The Ramsey Planner under the consumption taxation regime solves the following problem

$$\begin{aligned}
\mathcal{L} = & \max_{\left\{ \begin{array}{c} c_t, h_t, \pi_t, g_t, \\ R_t \geq 1, \tau_t^c, b_t, \end{array} \right\}_{t=0}^{\infty}} E_0 \sum_{t=o}^{\infty} \beta^t u\left(c_t, h_t, g_t\right) + \qquad (152) \\
& + \gamma_t^1 \beta^t \left[\begin{array}{c} \left((1-\eta) a_t - \eta \frac{u_{h,t}}{u_{c,t}} \left(1 + \tau_t^c\right) \right) (h_t) \\ -\varphi \left(\pi_t - 1\right) \pi_t + \varphi \beta E_t \left[\frac{u_{c,t+1} \left(1 + \tau_t^c\right)}{u_{c,t} \left(1 + \tau_{t+1}^c\right)} \left(\pi_{t+1} - 1\right) \left(\pi_{t+1}\right) \right] \end{array} \right] + \\
& + \gamma_t^2 \beta^t \left[\frac{u_{c,t}}{\left(1 + \tau_t^c\right)} - \beta E_t \left(\frac{u_{c,t+1}}{\left(1 + \tau_{t+1}^c\right)} \frac{R_t}{\pi_{t+1}} \right) \right] + \\
& + \gamma_t^3 \beta^t \left[-g_t - \tilde{x} + c_t \tau_t^c \right] + \\
& + \gamma_t^4 \beta^t \left[a_t \left(h_t\right) - \left(c_t\right) - g_t - \frac{\varphi}{2} \left(\pi_t - 1\right)^2 \right].
\end{aligned}$$

To simplify matters, following Adam (2011) we substitute the tax instrument using the constant steady state version of the government budget constraint as

$$\tau_t^c = \frac{g_t + \tilde{x}}{c_t}. \qquad (153)$$

The first order conditions of the transformed problem are

$$c \quad : \quad 0 = u_{c,t} + \gamma_{1,t} \left[\begin{array}{c} -(\eta - 1)(u_{c,t} + u_{cc,t}c_t) h_t (c_{t+1} + g_{t+1} + \tilde{x}) \\ -\eta u_{h,t} h_t (c_{t+1} + g_{t+1} + \tilde{x}) + \\ -\varphi (\pi_t - 1) \pi_t (c_{t+1} + g_{t+1} + \tilde{x}) u_{c,t} \\ -\varphi (\pi_t - 1) \pi_t (c_{t+1} + g_{t+1} + \tilde{x}) u_{cc,t} c_t \\ +\varphi \beta E_t u_{c,t+1} c_{t+1} (\pi_{t+1} - 1)(\pi_{t+1}) \end{array} \right] +$$

$$+\gamma_{1,t-1} \frac{1}{\beta} \left[\begin{array}{c} -(\eta - 1) u_{c,t-1} c_{t-1} h_{t-1} - \\ \eta u_{h,t-1} (c_{t-1} + g_{t-1} + \tilde{x}) h_{t-1} - \varphi (\pi_{t-1} - 1) \pi_{t-1} u_{c,t-1} c_{t-1} + \\ -\varphi \beta (c_{t-1} + g_{t-1} + \tilde{x}) (\pi_t - 1)(\pi_t) u_{c,t} + \\ \varphi \beta (c_{t-1} + g_{t-1} + \tilde{x}) (\pi_t - 1)(\pi_t) u_{cc,t} c_t \end{array} \right] +$$

$$+\gamma_{2,t} \left[\frac{u_{cc,t} \left(1 + \frac{g_t + \tilde{x}}{c_t}\right) + u_{c,t} \frac{g_t + \tilde{x}}{c_t^2}}{\left(1 + \frac{g_t + \tilde{x}}{c_t}\right)^2} \right] - \gamma_{2,t-1} \frac{R_{t-1}}{\pi_t} \left[\frac{u_{cc,t} \left(1 + \frac{g_t + \tilde{x}}{c_t}\right) + u_{c,t} \frac{g_t + \tilde{x}}{c_t^2}}{\left(1 + \frac{g_t + \tilde{x}}{c_t}\right)^2} \right] +$$

$$-\gamma_{3,t}; \tag{154}$$

$$h \quad : \quad 0 = u_{h,t} + \gamma_{1,t} \left[\begin{array}{c} (1 - \eta) u_{c,t} c_t (c_{t+1} + g_{t+1} + \tilde{x}) - \\ \eta u_{h,t} (c_t + g_t + \tilde{x}) (c_{t+1} + g_{t+1} + \tilde{x}) + \\ -\eta h_t u_{hh,t} (c_t + g_t + \tilde{x}) (c_{t+1} + g_{t+1} + \tilde{x}) \end{array} \right] +$$

$$+\gamma_{3,t}; \tag{155}$$

$$g \quad : \quad u_{g,t} + \gamma_{1,t} \left[-\eta h_t u_{h,t} (c_{t+1} + g_{t+1} + \tilde{x}) + \varphi \beta E_t (\pi_{t+1} - 1)(\pi_{t+1}) u_{c,t+1} c_{t+1} \right] +$$

$$+\gamma_{1,t-1} \frac{1}{\beta} \left[\begin{array}{c} h_{t-1} (1 - \eta) u_{c,t-1} c_{t-1} - \eta h_{t-1} u_{h,t-1} (c_{t-1} + g_{t-1} + \tilde{x}) + \\ -\varphi (\pi_{t-1} - 1) \pi_{t-1} u_{c,t-1} c_t \end{array} \right] +$$

$$-\gamma_{2,t} \left[\frac{\frac{u_{c,t}}{c_t}}{\left(1 + \frac{g_t + \tilde{x}}{c_t}\right)^2} \right] + \gamma_{2,t-1} \left[\frac{R_{t-1}}{\pi_t} \frac{\frac{u_{c,t}}{c_t}}{\left(1 + \frac{g_t + \tilde{x}}{c_t}\right)^2} \right] - \gamma_{3,t}; \tag{156}$$

$$R : \gamma_{2,t} \beta E_t (c_t + g_t + \tilde{x}) (u_{c,t+1} c_{t+1}) = 0 \tag{157}$$

$$\pi \quad : \quad -\gamma_{1,t} \left[\varphi (2\pi_t - 1) u_{c,t} c_t (c_{t+1} + g_{t+1} + \tilde{x}) \right] +$$

$$+\gamma_{1,t-1} \left[\varphi (c_{t-1} + g_{t-1} + \tilde{x}) (2\pi_t - 1) u_{c,t} c_t \right] +$$

$$+\gamma_{2,t-1} \left[\frac{u_{c,t}}{\left(1 + \frac{g_t + \tilde{x}}{c_t}\right)} \frac{R_{t-1}}{\pi_t^2} \right] - \gamma_{3,t} \varphi (\pi_t - 1). \tag{158}$$

At steady-state we obtain

$$\gamma_2 = 0, \tag{159}$$

so that (158) gives

$$\pi = 1, \tag{160}$$

and therefore, from (8) we get

$$R = \frac{1}{\beta}. \tag{161}$$

The first order condition with respect to γ_1 becomes

$$-\frac{u_h}{u_c} = \frac{(\eta - 1)}{\eta} \frac{1}{\left(1 + \frac{g+\tilde{x}}{c}\right)}, \tag{162}$$

and since $u_h < 0$ and $u_c > 0$, the latter implies

$$0 < \frac{(\eta - 1)}{\eta} \frac{c}{(c + g + \tilde{x})} < 1, \tag{163}$$

therefore $-u_h < u_c$.

Using these results on (155), (156) and (**??**) it yields respectively

$$\gamma_1 (c + g + \tilde{x}) \eta \underbrace{\left[-\frac{(\eta - 1)}{\eta} u_c c + (c + g + \tilde{x}) (-u_h - h u_{hh})\right]}_{>0} = -u_h - \gamma_3 \leq 0 \tag{164}$$

This conditions implies that $\gamma_1 < 0$.

$$\gamma_1 h (\eta - 1) \left[\left(\frac{1}{\beta} + 1\right) \frac{\eta}{(\eta - 1)} (-u_h) (c + g + \tilde{x}) - \frac{1}{\beta} u_c c\right] = \gamma_3 - u_g \geq 0, \tag{165}$$

$$\gamma_1 h (\eta - 1) \left[\left(\frac{1}{\beta} + 1\right) \frac{\eta}{\eta - 1} (-u_h) (c + g + \tilde{x}) - (u_c + u_{cc} c) (c + g + \tilde{x}) - \frac{1}{\beta} u_c c\right] = \gamma_3 - u_c \geq 0. \tag{166}$$

Summing (164) and (165) we get

$$\gamma_1 (c + g + \tilde{x}) \eta \underbrace{\left[-\frac{(\eta - 1)}{\eta} u_c c + (c + g + \tilde{x}) (-u_h - h u_{hh})\right]}_{>0} + \gamma_1 h (\eta - 1) \underbrace{\left[\left(\frac{1}{\beta} + 1\right) \frac{\eta}{(\eta - 1)} (-u_h) (c + g + \tilde{x}) - \frac{1}{\beta} u_c c\right]}_{>0} = -u_h - u_g, \tag{167}$$

since

$$\gamma_1 (c + g + \tilde{x}) \eta \left[-\frac{(\eta - 1)}{\eta} u_c c + (c + g + \tilde{x}) (-u_h - h u_{hh})\right] < 0 \tag{168}$$

and

$$\gamma_1 h (\eta - 1) \left[\left(\frac{1}{\beta} + 1 \right) \frac{\eta}{(\eta - 1)} (-u_h) (c + g + \tilde{x}) - \frac{1}{\beta} u_c c \right] < 0, \tag{169}$$

it must be the case that

$$-u_h - u_g < 0, \tag{170}$$

and therefore

$$u_g > -u_h. \tag{171}$$

Combining (165) and (166) one obtains

$$u_g = u_c - \gamma_1 h (\eta - 1) (u_c + u_{cc} c) (c + g + \tilde{x}), \tag{172}$$

and therefore:
a) If $-\frac{c u_{cc}}{u_c} = 1$, then $(u_c + u_{cc} c) = 0$. It follows that

$$u_g = u_c. \tag{173}$$

b) If $-\frac{c u_{cc}}{u_c} > 1$, then $(u_c + u_{cc} c) < 0$. It follows that

$$u_g < u_c. \tag{174}$$

c) If $-\frac{c u_{cc}}{u_c} < 1$, then $(u_c + u_{cc} c) > 0$. It follows that

$$u_g > u_c. \text{ Q.E.D.} \tag{175}$$

6.7 Derivation of Conditions (29) and (32)

If the policy-maker can use only labour income taxation, we can rewrite the labour supply condition as

$$\omega_h h^{\eta+1} c^{\sigma} \left(\frac{w}{(w-1) h + c - \tilde{x}} \right) - w = 0. \tag{176}$$

Applying to the latter the implicit function theorem, it yields

$$\frac{\partial c}{\partial h} = - \frac{\left(\frac{w}{[(w-1)h+c-\tilde{x}]} \right) (-h u_h) \left(-\frac{u_{cc}}{(u_c)^2} \right) - \frac{1}{u_c} \left(\frac{w}{[(w-1)h+c-\tilde{x}]^2} \right) (-h u_h)}{\frac{1}{u_c} \left(\frac{w}{(w-1)h+c-\tilde{x}} \right) (-h u_{hh}) - \frac{1}{u_c} \left(\frac{w(w-1)}{[(w-1)h+c-\tilde{x}]^2} \right) (-h u_h)}, \tag{177}$$

$$\frac{\partial c}{\partial h} = - \frac{(-h u_h) \left(-\frac{u_{cc}}{(u_c)^2} \right) - \frac{1}{u_c} \left(\frac{1}{[(w-1)h+c-\tilde{x}]} \right) (-h u_h)}{\frac{1}{u_c} (-h u_{hh}) - \frac{1}{u_c} \left(\frac{(w-1)}{[(w-1)h+c-\tilde{x}]} \right) (-h u_h)}. \tag{178}$$

Collecting terms we rewrite it as

$$\frac{\partial c}{\partial h} = -\frac{\left[-\frac{cu_{cc}}{u_c}h - hc\left(\frac{1}{[(w-1)h+c-\tilde{x}]}\right)\right]}{\left[-\frac{hu_{hh}}{u_h}c - hc\left(\frac{(w-1)}{[(w-1)h+c-\tilde{x}]}\right)\right]}, \tag{179}$$

since $(w-1) = -\frac{1}{\eta}$, it yields

$$\frac{\partial c}{\partial h} = -\frac{h\left(-\frac{cu_{cc}}{u_c}\right) - ch\left(\frac{1}{\left[c-\frac{1}{\eta}h-\tilde{x}\right]}\right)}{c\left(-\frac{hu_{hh}}{u_h}\right) + ch\left(\frac{\frac{1}{\eta}}{\left(c-\frac{1}{\eta}h-\tilde{x}\right)}\right)}. \tag{180}$$

Assuming that $\left(c - \frac{1}{\eta}h - \tilde{x}\right) > 0$, then $c\left(-\frac{hu_{hh}}{u_h}\right) + ch\left(\frac{\frac{1}{\eta}}{\left(c-\frac{1}{\eta}h-\tilde{x}\right)}\right) > 0$. Therefore, the sign of (180) depends only on its numerator, i.e.

$$\left(\frac{\partial c}{\partial h}\right)_{sign} = \left(h\frac{cu_{cc}}{u_c} + ch\left(\frac{1}{\left[c - \frac{1}{\eta}h - \tilde{x}\right]}\right)\right)_{sign} \tag{181}$$

It follows that $\frac{\partial h}{\partial c} > 0$ requires

$$\left(\frac{c}{\left[c - \frac{1}{\eta}h - \tilde{x}\right]}\right) + \frac{cu_{cc}}{u_c} > 0.$$

The above expression collapses to condition (29)

$$\frac{\partial c}{\partial h} > 0 \text{ iff } -\frac{cu_{cc}}{u_c} < \left(\frac{c}{\left[c - \tilde{x} - \frac{1}{\eta}h\right]}\right), \tag{182}$$

hence for $\tilde{x} = 0$, $\eta \to \infty$, i.e. perfect competition case with no public debt,

$$\frac{\partial c}{\partial h} \geq 0 \text{ iff } -\frac{cu_{cc}}{u_c} \leq 1. \tag{183}$$

As $\tilde{x}$ turns positive, or *ceteris paribus*, $\eta < \infty$, the LHS of (182) becomes greater than one. Hence there are values of $-\frac{cu_{cc}}{u_c}$ greater that one for which $\frac{\partial h}{\partial c} \geq 0$.

Applying the implicit function theorem to the labour supply condition when only consumption taxation is available, it is easy to show that $\frac{\partial h}{\partial c}$ reads as

$$\frac{\partial h}{\partial c} = -\frac{\left(-\frac{cu_{cc}}{u_c} - 1\right)\left[-u_h u_c \left(\frac{h+\tilde{x}}{c}\right)\right]}{\frac{1}{c}\left[-\left(h\frac{u_{hh}}{u_h} + 1\right)u_h u_c - u_{hh} u_c \tilde{x}\right]}, \tag{184}$$

which again is negative (positive) for $-\frac{cu_{cc}}{u_c} > (<)\, 1$. Note that for $-\frac{cu_{cc}}{u_c} = 1$, $\frac{\partial h}{\partial c} = 0$.