Ultrahigh Energy Cosmic Rays, Cosmological Constant, and θ Vacua

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(Received 21 January 2003; revised manuscript received 24 March 2003; published 15 May 2003)

We propose that the origin of ultrahigh energy cosmic rays beyond the Greisen-Zatsepin-Kuzmin cutoff and the origin of small cosmological constant can both be explained by vacuum tunneling effects in a theory with degenerate vacua and fermionic doublets. By considering the possibility of tunneling from a particular winding number state, accompanied by violation of some global quantum number of fermions, the small value of the vacuum dark energy and the production of high energy cosmic rays are shown to be related. We predict that the energy of such cosmic rays should be at least 5×10^{14} GeV.

DOI: 10.1103/PhysRevLett.90.191301 PACS numbers: 98.80.Cq, 14.80.–j, 98.70.Sa

The two outstanding puzzles of modern astroparticle physics are the observed value of the small cosmological constant and the origin of ultrahigh energy cosmic rays. The latter with energies ranging from 10^5–$10^{20.5}$ eV, ever since it was observed in the first half of the last century, posed an open question which attracted many new ideas within conventional astrophysics, from the particle spectrum of the standard model (SM) to beyond the SM [1]. Undoubtedly such a vast range of energies could never be covered by a single source for the origin of the cosmic rays. The observed broken power law spectrum of cosmic rays gradually steepens as the energy increases from 4×10^{15} eV, known as a *knee,* to 5×10^{18} eV, known as an *ankle,* and subsequently flattens above 5×10^{18} eV. It is usually believed that the first transition in the observed spectrum reassigns the origin of cosmic rays from galactic to extra galactic sources. However, there are obvious constraints on a primary particle accelerated up to 5×10^{19} eV if they are either charged or a heavy nuclei which interacts with a cosmic microwave photon background $T \sim 3$ K, and thus they cannot traverse farther than a few Mpc without losing energy. This is known as the Greisen-Zatsepin-Kuzmin (GZK) effect [2,3]. Therefore distant astrophysical sources which might be able to generate such energetic particles might not be a suitable candidate for ultrahigh energy cosmic rays with energies beyond 5×10^{19} eV. A simple solution to this impasse is to look for a candidate which is not only capable of producing ultrahigh energy cosmic rays but also avoids the GZK cutoff.

Such candidates could be topological defects [4], or they could come from the decays of the primordial superheavy particles [5]. In the latter scenario the mass of the unknown X particles could be ranging from 10^{12} GeV and above. However, the longevity of the X particles (equivalent to the age of the Universe $\sim \tau_X \sim 10^{10}$ years) demands an extraordinary suppression in their interaction. Rather interesting solutions have been put forward [6,7]. In the latter reference it was assumed that the required lifetime can be obtained by imposing discrete gauge symmetries even if X is an elementary particle. Similar ideas in string theory can be found in [8], while in [6], the reason for the long lifetime of the X particles was explained via the instanton-mediated decay, which we shall explore here in some detail.

An interesting connection can be made between the abundance of the nonluminous cold dark matter, observed as 30% of the critical energy density of the Universe: $\rho_c \approx 4 \times 10^{-47}$ (GeV)^4, and the origin of the cosmic rays (above GZK cutoff) provided that the cold dark matter constituent is X particles with a mass $m_X \sim 10^{11}$–10^{15} GeV. Such heavy particles can be produced abundantly to match the correct cold dark matter abundance right after the end of inflation [9], or from the direct decay of the inflaton [10].

On the other hand, the majority of the energy density $\sim$70% is in the form of dark energy, whose constituent is largely unknown, but usually believed to be the *cosmological constant* [11]. The *bare* and the observed cosmological constant is a severe problem, especially why the observed cosmological constant is so small $\sim 2.8 \times 10^{-47}$ (GeV)^4, or, in other words, in Planck units $\sim 10^{-120}(M_{\rm p})^4$ (where we use the reduced Planck mass $M_{\rm P} \sim 2.4 \times 10^{18}$ GeV). One would naively expect that, even if the bare cosmological constant can be made to be vanishing, the quantum loop corrections would eventually lead to quadratic divergences $\sim M_{\rm p}^4$. In other words, what keeps the cosmological constant so small as we see today? Despite many attempts in a conventional big bang cosmological setup [12], a convincing solution is still elusive. Recently it was pointed out in [13] that the present observable cosmological constant can be obtained if the original vacuum can be associated with a nontrivial winding number. It was also assumed that the bare cosmological constant and the vacuum energy density vanished not in any one specific vacuum but in the superposed, or θ, vacuum at some high scale $\sim 4 \times 10^{16}$ GeV.

In this Letter we address some interesting cosmological consequences of the θ vacuum. We propose that if the X particles' decay can be explained via instanton mediation due to the transition from one nontrivial vacuum to another, then the superposed vacua can also be responsible for generating a nonvanishing but small cosmological constant as we observe now. As a result, we can relate the origin of the cosmological constant with the ultrahigh energy cosmic rays. Regarding the θ vacuum we also generalize the description of Ref. [13], where it was strictly assumed that the state of the Universe is solely given by the $|n = 0\rangle$ vacuum. In what follows, we assume that the quantum state of the Universe can be a superposition of the n vacua of the nontrivial SU(N). As we shall see we will have a concrete prediction for the mass of the fermion, which may act as a source for the ultrahigh energy cosmic rays. We begin by considering the origin of the θ vacuum.

Apart from the familiar invariance principle of gauge theories under small gauge transformations (those connected continuously to the identity), it is well known that most grand unified theories such as SU(N) or SO(N) where $N \geq 2$ are also invariant under large gauge transformations. Such transformations are not continuously connected to the identity; rather, they generate infinitely degenerate perturbative vacua separated by action barriers that prevent classical transitions between them. Quantum mechanical tunneling can lead to a superposed ground state of these perturbative vacua, exponentially smaller in energy than any of them, since the tunneling amplitude is itself suppressed exponentially by the height of the action barrier. A practical example is the QCD vacuum, where imaginary time (Euclidean space) solutions of minimum action (instantons) can be viewed as tunneling between adjacent vacua characterized by different winding numbers n [14]. Taking into account this quantum tunneling, one usually writes the true vacuum state as a weighted superposition of all identical $|n\rangle$ vacua

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta}|n\rangle, \tag{1}$$

where, for QCD, bounds from neutron electric dipole moment studies [15] suggest that $\theta_{\rm QCD} \leq 10^{-9}$. However, for the purpose of illustration, let us assume that θ is an unconstrained parameter of some SU(2)$_X$ gauge theory, which is broken at some high energy scale (we will generalize our main results for arbitrary even N_f, where N_f is the number of fermionic doublets). Then, there are SU(2)$_X$ instantons, which represent tunneling solutions between the vacua with different winding numbers.

Since this true ground state is lower in energy than any particular n vacuum by only an exponentially small amount, the observed small but finite vacuum energy density can be explained if the Universe has not yet settled down into the nonperturbative θ state, but is still in one of the perturbative vacuum states. If one calculates the vacuum energy density in a θ state or any n state for pure gauge theory, 't Hooft's formula [16] for the one-instanton weight shows that the contribution of large size instantons diverges. The vacuum energy density in any $|n\rangle$ vacua can be expressed as [13]

$$\rho_v = \langle n|H|n\rangle = 2Ke^{-S_0}, \tag{2}$$

where S_0 is the classical instanton action $8\pi^2/g^2$ and g is the coupling constant of the SU(2)$_X$ theory. Note that we assume the bare cosmological constant vanishes in the $\theta = 0$ vacuum.

In the dimensional regularization scheme [16],

$$K = 2^{10}\pi^6 g^{-8} \times \int \frac{dR}{R^5} \exp\left[-\frac{8\pi^2}{g^2(\mu_0)} + \frac{22}{3}\ln(\mu_0 R) + 6.998\,435\right]. \tag{3}$$

The integration over sizes R diverges, but this can be tamed by introducing a physical cutoff scale $\mu_0 \sim M$, set by an SU(2) scalar doublet Φ with a potential $V(\Phi) = \lambda(|\Phi|^2 - M^2/2)^2$, a la 't Hooft [16]. In this case, the pure gauge theory instanton should be replaced by the constrained instanton solution [17], and the vacuum energy density in the n vacuum is given by

$$\rho_v = \langle n|H|n\rangle \sim \left(\frac{8\pi}{g^2}\right)^4 M^4 e^{-8\pi^2/g^2}. \tag{4}$$

If M is larger than $H_{\rm inf}$ during inflation (for chaotic-type inflation model $H_{\rm inf} \sim 10^{13.5}$ GeV [18]), the inflation will naturally tend to wipe out inhomogeneities in Φ over a Hubble volume. We will show that this argument constrains the size of the instanton.

Let us assume, for the purpose of illustration, three degenerate vacua, $|n = 0\rangle$, $|n = \pm 1\rangle$. Note that we allow for the Universe to be in a state other than the nonperturbative $|\theta\rangle$ state. The ground state of the Universe would then be

$$|G\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |-1\rangle), \tag{5}$$

and this would have a larger energy than if all winding number states $|n = (-\infty, \infty)\rangle$ were included (the energy of the θ vacuum is the least). Since the $|\theta\rangle$ state is also an energy eigenstate unlike the $|n\rangle$ state, we can expand each of the three $|n\rangle$ states in terms of the $|\theta\rangle$ state. In the dilute gas approximation [19], corresponding to $MR \leq 1$, which is the regime we are working in, $\langle\theta|H|\theta\rangle = \rho_v(1 - \cos\theta)$. As a result, the probability distribution function of the vacuum energy has a peak at $2\rho_v$, corresponding to $\theta = \pi$, as shown in [13]. (In QCD, $\theta = \pi$ is a CP-conserving value, though here the θ is unrelated to strong interactions.) This peak can be interpreted as the value of the finite dark energy observed today.

The inclusion of massive spin-1/2 fermions in this SU(2)$_X$ gauge theory which admits instanton solutions has interesting physical consequences. 't Hooft has shown that for fermion mass $m \ll 1/R$, where R is the typical instanton size, N_f species of spin 1/2 fermions contribute a factor [16]

$$A_f = (Rm)^{N_f} \exp\left[-\frac{2}{3} N_f \ln(MR) + 0.291\,746 N_f \right], \quad (6)$$

to the tunneling amplitude. Using the constrained instanton solution, the constant K now reads

$$K = A_f 2^{10} \pi^6 g^{-8} \times \int \frac{dR}{R^5} \exp\left[-\frac{8\pi^2}{g^2(M)} - \pi^2 R^2 M^2 + \frac{43}{6} \ln\left(\frac{MR}{\sqrt{2}}\right) + 6.759\,189 \right]. \quad (7)$$

Let us suppose now that, as in [6], there are two fermionic doublets X and Y with different global quantum numbers, that they acquire this mass through the spontaneous breaking of the $SU(2)_X$ symmetry, and that the standard model quarks and leptons carry some quantum numbers under $SU(2)_X$. Mixing between X and Y can occur at the quantum level via the $SU(2)_X$ anomaly; therefore, the quantum transition between the adjacent vacua must be accompanied by the decay process [6]

$$X \to Y + \text{SM quarks and leptons}, \quad (8)$$

assuming that X fermions have a larger mass than Y fermions. The large mass of the X and Y (Kuzmin and Rubakov [6] assumed $m_X \geq 10^{13}$ GeV) particles implies that the decay products (quarks and leptons) are highly energetic and can lead to the production of highly energetic hadrons which can be the constituents of cosmic rays.

The large mass value in our case implies the hierarchy of scales $R^{-1} \geq M > H_{\rm inf}$. The first condition $MR \leq 1$ justifies the use of the $M \neq 0$ instanton solution, while the second condition $M > H_{\rm inf}$ assures us that gravitational effects on instantons are small during the inflationary phase and after inflation as well. Including the fermions, from Eq. (4), the vacuum energy in any perturbative $|n\rangle$ vacuum is

$$\rho_v \sim \left(\frac{8\pi}{g^2}\right)^4 m^2 M^2 e^{-8\pi^2/g^2}, \quad (9)$$

where we have used the fact that $MR \sim 1$. As we have allowed for a small probability of transition Γ in the horizon volume, the condition $\Gamma H_0^{-4} \sim 1$ holds, where the tunneling rate per unit volume is

$$\Gamma \simeq \left(\frac{8\pi}{g^2}\right)^4 m^2 M^2 e^{-16\pi^2/g^2}. \quad (10)$$

Using $\rho_v \sim 10^{-120} M_{\rm p}^4$ and $H_0^2 = \rho_v/(3M_{\rm p}^2)$ (where the subscript 0 denotes the present era), we can solve for the coupling

$$\alpha = \frac{g^2}{4\pi} \simeq \frac{1}{44.1}. \quad (11)$$

Using this value of the coupling, we find from either Eq. (9) or Eq. (10) that

$$m^2 M^2 \sim 10^{66}\ \text{GeV}^4. \quad (12)$$

From the condition $\Gamma H_0^{-4} \sim 1$, we also have $m^2 M^2 \sim \alpha^4 M_{\rm p}^4$. We are proposing to explain the ultrahigh energy cosmic rays which can have energies up to the mass of the decaying X fermion, so it follows that one can expect cosmic rays of energies up to $\sim 5 \times 10^{14}$ GeV (since M cannot be greater than $M_{\rm p}$). We also note that $M > H_{\rm inf}$ is satisfied. We may generalize our scenario to an arbitrary number of fermion doublets N_f [which is required to be even by the $SU(2)_X$ anomaly]. Accordingly, Eq. (12) is modified to

$$m^{N_f} M^{4-N_f} \sim 10^{66}\ \text{GeV}^4. \quad (13)$$

For example, for $N_f = 4$, $m \approx 3 \times 10^{16}$ GeV. For larger N_f, we find that the fermion mass scale is not considerably different. The fermion mass is indeed the prime result of this Letter, which clearly shows that the cascade decay of the fermions can give rise to ultrahigh energy cosmic rays with energies greater than the GZK cutoff.

Note that in the instanton-mediated decay of fermions, the predicted mass turns out to be heavier than 10^{13} GeV (for chaotic-type inflation, the inflaton mass is around 10^{13} GeV in order to produce the right amplitude for the density perturbations and the spectrum [18]), and also greater than the observed spectrum from HiRes (High Resolution Fly's eye experiment) [20] and AGASA (Akeno Giant Air Shower Array) [21]. Current analysis seems to be suggesting a relic fermion mass around 10^{12} GeV [22] to $10^{14.6}$ GeV [23]. Particularly in our case, in order to excite the superheavy fermions at the very first instance, one has to rely on nonthermal production mechanism for fermions after inflation [24] (see also the appendix of Ref. [25]). The upcoming experiment such as AUGER [26] will be able to see a considerable number of events above the GZK cutoff which will verify or falsify the energy scales which we predict here. If the inflaton coupling to the $SU(2)_X$ fermions is sufficiently large, then the adequate abundance of such fermions will be the right candidate for cold dark matter.

In summary, we argue that the problem of the cosmic dark energy, the small value of the cosmological constant, and ultrahigh energy cosmic rays can have a common origin. We have shown that if the longevity of the X particles is due to instanton-mediated decays, then the fermion mass, which sets the scale for the ultrahigh energy cosmic rays, must be larger than 5×10^{14} GeV. Note that this is in accordance with the observed small cosmological constant.

The authors are thankful to Robert Brandenberger and Guy Moore for discussion. P. J. acknowledges support from the Natural Sciences and Engineering Research Council of Canada. A. M. acknowledges partial support from CITA.

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