Coupled inflation and brane gases

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We study an effective four-dimensional theory with an action with two scalar fields minimally coupled to gravity, and with a matter action which couples to the two scalar fields via an overall field-dependent coefficient in the action. Such a theory could arise from a dimensional reduction of supergravity coupled to a gas of branes winding the compactified dimensions. We show the existence of solutions corresponding to power-law inflation. The graceful exit from inflation can be obtained by postulating the decay of the branes, as would occur if the branes are unstable in the vacuum and stabilized at high densities by plasma effects. This construction provides an avenue for connecting string gas cosmology and the late-time universe.

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I. INTRODUCTION

String gas cosmology, originally discussed in [1,2] (see also [3]) and later reconstructed in the context of branes as fundamental degrees of freedom in string theory [4,5], provides a mechanism for constructing a nonsingular cosmology and a mechanism which confines all but three spatial dimensions to be microscopic in size. String winding modes play a key role in this scenario: they are unable to annihilate in more than three spatial dimensions [1] because the probability that two string world sheets intersect is of measure zero (see [6] for some recent caveats regarding this scenario). A key assumption in string gas cosmology is that all spatial dimensions start out at string scale, and that the Universe is hot (and brane degrees of freedom excited). However, in order to connect this scenario to our current universe, a mechanism is required which expands the volume of our observed three spatial dimensions to be large enough to contain the currently observed Hubble volume. This is one aspect of the ''flatness problem'' of standard big bang cosmology (see e.g. [7] for a discussion), and a period of cosmological inflation after the winding modes in our three spatial dimensions have disappeared is the only currently known solution.

Thus, an outstanding challenge for string gas cosmology has been to provide a stringy realization of inflation [8]. One suggestion was recently made in [10] and made use of a gas of codimension one branes which provide a period of power-law inflation, with an equation of state $\omega = -2/3$ (where $\omega = p/\rho$, and p and ρ are the pressure and energy density, respectively), which is however incompatible with inflation being the source of the cosmic microwave anisotropies (see e.g. [11]). The graceful exit from inflation was achieved by considering the branes to be unstable in the vacuum, and stabilized by plasma effects in a hot gas, in analogy to how embedded defects in field theory can be stabilized in the plasma of the early universe [12,13].

In this paper we consider a more natural realization of inflation in the context of string gas cosmology. We study the period of cosmology after our three dimensions have shredded their winding modes (as discussed in detail in [5]), but the other dimensions are still confined by strings and branes which wind around them. We consider the four-dimensional effective action, and focus, in particular, on two scalar fields in this action, one corresponding to a modulus of the higher-dimensional theory, the other corresponding to the radion. We assume an exponential potential for the modulus field and consider brane/string matter (treated as a hydrodynamical fluid) whose overall action has a prefactor which depends exponentially on the two scalar fields. We then demonstrate that it is possible to obtain solutions of power-law inflation with an effective equation of state more negative than $\omega = -2/3$. The coupling of matter to the scalar fields required for acceleration arises automatically if we consider branes winding around the compact space as our matter. To obtain a graceful exit from inflation, we again require the branes to be unstable, as in [10]. Compared to [10], our mechanism may provide an inflationary phase with a smaller value of ω, and it does not require branes which are extended in our large spatial dimensions. Our mechanism relies on the analysis of [14] which we now review.

II. COUPLED INFLATION

In [14] a mechanism to realize an accelerated phase of expansion was proposed which relies on a rolling scalar

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field being coupled (roughly with gravitational strength) to some form of matter or radiation. Although the main motivation in [14] came from trying to explain the late-time acceleration phase that we seem to be entering into today, in this paper we show how one can obtain inflation with a gas of unstable branes via a similar mechanism. Since branes in general couple to both the dilaton and the radion (the volume of the extra dimensions), unless one invokes an additional mechanism to stabilize one of the scalars, one has to study the dynamics involving both the radion and the dilaton. Hence we first review and generalize the analysis done in [14].

A. Evolution equations

We start with an effective four-dimensional action with two scalar fields minimally coupled to gravity

$$\hat{S} = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\bigg[R - \partial_{\hat{m}}\phi\partial^{\hat{m}}\phi - \partial_{\hat{m}}\psi\partial^{\hat{m}}\psi - \frac{2V_0}{M_p^2}e^{-2(\alpha\phi+\beta\psi)}\bigg], \tag{1}$$

where M_p is the reduced Planck mass which we will set to 1 from now on. Such exponential potentials are common in dimensionally reduced supergravity/M theories in dilaton-radion systems [15], or in the context of large extra dimensions [16] and we will give some explicit examples later. In the above, α, β are constants which depend on the origin of the potential as well as on the dimensionality of the original model. Consider now that a form of matter or radiation also couples to the scalar fields:

$$S_{\rm mat} = \int d^4x\sqrt{-g}\tilde{\rho} = \int d^4x\sqrt{-g}\rho e^{2(\mu\psi+\nu\phi)} \tag{2}$$

where, as usual, $\tilde{\rho}$ is the observed energy density and we define a "bare density," $\rho \equiv \tilde{\rho}e^{-2(\mu\psi+\nu\phi)}$, which obeys the familiar evolution equations involving an equation of state parameter ω

$$\dot{\rho} + 3H(p+\rho) = 0 \tag{3}$$

with

$$p = \omega\rho \Rightarrow \rho = \rho_0\left(\frac{a}{a_0}\right)^{-3(1+\omega)}. \tag{4}$$

In the next section we will identify $\tilde{\rho}$ with the energy density of a gas of branes wrapping extra dimensions and will derive the coupling exponents μ and ν explicitly.

Because of this coupling, the brane action not only acts as a source term for gravity but also provides an effective potential term for the scalar fields. The field equations for the scalars now read

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V_{\rm eff}(\phi,\psi)}{\partial\phi} \tag{5}$$

and

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{\partial V_{\rm eff}(\phi,\psi)}{\partial\psi} \tag{6}$$

where

$$V_{\rm eff}(\phi) = V_0e^{-2(2\alpha\psi+\beta\phi)} + e^{2(\mu\psi+\nu\psi)}\rho. \tag{7}$$

Similar multifield exponential potential has been studied in the context of *assisted inflation*, see [17], the difference here being that ρ is not a constant but redshifts according to (4). Similar multifield potentials were also used to construct inflationary models in [18,19]. From here onwards it is useful to work in terms of rotated fields [20],

$$\begin{pmatrix}\psi' \\ \phi'\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\psi \\ \phi\end{pmatrix} \tag{8}$$

where we choose

$$\cos\theta = \frac{\mu}{\sqrt{\mu^2+\nu^2}} \tag{9}$$

such that

$$\tilde{\rho} = \rho e^{2\mu'\psi'} \quad \text{with} \quad \mu' = \sqrt{\mu^2+\nu^2} \tag{10}$$

depends only on a single field. The exponents α, β transform in the same way as the fields:

$$\begin{pmatrix}\alpha' \\ \beta'\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}. \tag{11}$$

Thus, the "rotated" field equations read as

$$\ddot{\phi}' + 3H\dot{\phi}' = 2\beta'V_0e^{-2(\alpha'\psi'+\beta'\phi')} \tag{12}$$

and

$$\ddot{\psi}' + 3H\dot{\psi}' = 2\alpha'V_0e^{-2(\alpha'\psi'+\beta'\phi')} - 2\mu'\rho e^{2\mu'\psi'}. \tag{13}$$

One can also write down the Friedmann equation for a Robertson-Walker metric:

$$H^2 = \frac{1}{3}\left(\frac{\dot{\phi}'^2}{2} + \frac{\dot{\psi}'^2}{2} + V_0e^{-2(\alpha'\psi'+\beta'\phi')} + \rho e^{2\mu'\psi'}\right). \tag{14}$$

B. Inflation from a single field

Before we solve (12)–(14) in its full generality, it is insightful to look at a special case when the dynamics reduces to that of a single field which has been studied in detail in [14]. This happens when

$$\frac{\beta}{\alpha} = \tan\theta = \frac{\nu}{\mu} \Rightarrow \beta' = 0. \tag{15}$$

In this case, ϕ' remains frozen (even if one starts with a nonzero $\dot{\phi}'$ it is quickly Hubble damped) and ψ' evolves under the influence of the effective potential (7) with exponents $\alpha' = \sqrt{\alpha^2+\beta^2}$ and μ'. As discussed in [14], provided the exponents are of $\mathcal{O}(1)$ and have the same sign, ψ' tracks the minimum formed between the two opposing exponentials. Since ρ redshifts, the minimum redshifts and one can solve for the evolution of ψ' and $a(t)$ to get

$$e^{2\psi'} = \left(\frac{\alpha' V_0}{\mu' \rho}\right)^{1/(\mu'+\alpha')} = \left(\frac{\alpha' V_0}{\mu' \rho_0}\right)^{1/(\mu'+\alpha')}\left(\frac{a}{a_0}\right)^{3(1+\omega)/(\mu'+\alpha')} \tag{16}$$

and

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{(2/3\alpha')[(\mu'+\alpha')/(1+\omega)]}. \tag{17}$$

Thus, the Universe accelerates if

$$\frac{\mu'}{\alpha'} > \frac{1}{2}(1 + 3\omega). \tag{18}$$

For nonrelativistic dust-type matter, like wrapped branes, we have $\omega = 0$, so that (18) becomes

$$\sqrt{\frac{\mu^2 + \nu^2}{\alpha^2 + \beta^2}} > \frac{1}{2}. \tag{19}$$

In the next section, when we discuss the dynamics involving a gas of branes, we will see that, although the system does not reduce to a single field dynamics, one can get power-law inflation in a manner similar to the case discussed above.

As a next step then, let us calculate the density fluctuations and spectral tilt in this model. From (16) and (17) one can compute $\dot{\psi}'$ and H and hence the amplitude of density fluctuations

$$\delta_H = \frac{H^2}{2\pi\dot{\psi}'} = \frac{(\mu' + \alpha')H}{3\pi M_p} = \frac{(\mu' + \alpha')}{3\pi M_p}\sqrt{\frac{V_0(\mu' + \alpha')e^{-2\alpha'\psi'}}{3\mu' M_p^2}} \tag{20}$$

or

$$\delta_H^2 \sim a^{-[3(1+\omega)\alpha'/(\mu'+\alpha')]}, \tag{21}$$

where the right-hand side is evaluated at the time when the length scale of the fluctuation being considered is exiting the Hubble radius (see e.g. [21,22] for reviews). Therefore, the spectral tilt η is given by

$$\eta \approx 1 + \frac{1}{a}\frac{d\ln\delta_H^2}{da} = 1 - \frac{3(1 + \omega)}{1 + \mu'/\alpha'}. \tag{22}$$

It is clear from (22) that in order to explain the observed spectral tilt of the CMB spectrum (see e.g. [11]), $\eta > 0.94$, one needs a very large ratio $\mu'/\alpha' \sim \mathcal{O}(10)$ which is difficult to achieve from string theory. Hence, it seems likely that one needs to supplement this inflationary mechanism with an additional mechanism to generate the observed almost scale-invariant spectrum of density perturbations. We will comment on this later.

C. Two field dynamics

To solve the evolution equations (12)–(14) for the two field case we choose the usual ansatz

$$a = a_0\left(\frac{t}{t_0}\right)^m; \qquad e^{\psi} = e^{\psi_0}\left(\frac{t}{t_0}\right)^n; \qquad e^{\phi} = e^{\phi_0}\left(\frac{t}{t_0}\right)^p. \tag{23}$$

In order that all the terms in (12)–(14) have the same t dependence, t^{-2} to be specific, we get by looking at the exponents

$$\alpha' n + \beta' p = 1 = \frac{3m}{2} - \mu' n. \tag{24}$$

Further, one can substitute the potentials associated with V_0 and ρ_0 from (12) and (13) in (14) to obtain another relation between the power-law exponents:

$$m^2 = \frac{1}{3}\left[\frac{1}{2}(n^2 + p^2) + (3m - 1)\left(\frac{p}{2\beta'}\left(1 + \frac{\alpha'}{\mu'}\right) - \frac{n}{2\mu'}\right)\right]. \tag{25}$$

One can thus solve (24) and (25) to obtain m, n and p in terms of the exponents α', β' and μ' (see the Appendix for details). In particular one finds

$$m = 2\frac{3\mu'\alpha' + \alpha'^2 + \beta'^2 + 2\mu'^2}{6\mu'\alpha' + 3\alpha'^2 + 3\beta'^2 + 8\mu'^2\beta'^2}. \tag{26}$$

In order to find out whether one gets inflation, one has to check whether $m > 1$ or not.

Intuitively, it is clear what needs to happen in order to have an accelerated expansion: Along the ψ' direction, due to the coupling, one has a slowly evolving minimum due to the two opposing exponential potentials as before. The field, though, can also roll along the ϕ' direction, since $\beta' \neq 0$. However, if β' is sufficiently small (in other words if the two exponents corresponding to the potential V_0 and ρ_0 are approximately collinear) then the field rolls slowly also along the ϕ' direction and thus we can have acceleration.

Finally, one has to check that the ansatz (23) gives meaningful solutions, i.e. solutions exist for positive V_0 and ρ_0. We show in the Appendix that this is indeed the case provided $m > 1/3$, which is obviously true for inflationary solutions.

III. SUPERGRAVITY AND BRANE GASES

A. Supergravity and effective potentials

Let us start with a typical bosonic sector of a supergravity theory:

$$\hat{S} = \frac{1}{16\pi\hat{G}}\int d^{\hat{D}}x\sqrt{-g}\left[\hat{R} - \partial_{\hat{m}}\phi\partial^{\hat{m}}\phi - \frac{1}{2}\sum_i e^{-2a_i\phi}\frac{1}{n_i!}F_{n_i}^2 - V(\phi)\right] \tag{27}$$

where hatted quantities denote the full higher ($\hat{D}$) dimensional objects, ϕ is the dilaton field and the field strengths F_{n_i}'s are n_i forms with $i = 1, \ldots, M$. For generality, we have also included a potential for the dilaton which may have both a classical [23] or a quantum [24] origin depending on the specific supergravity/string theory model. In general, the potential runs to either $\pm\infty$, leading to the much studied issue of dilaton stabilization. For the purposes of this paper, we do not concern ourselves with this issue and simply assume that either it is stabilized after inflation or it is coupled very weakly (if at all) to standard model particles and therefore even though the dilaton slowly runs towards infinity, constraints coming from fifth force experiments and variation of physical constants [25] are satisfied. Indeed a "least coupling mechanism" has been proposed [26] to explain why the dilaton may couple very weakly to the ordinary standard model particles. As a simple example of a running potential we choose

$$V(\phi) = V_0 e^{-2\delta\phi}. \tag{28}$$

We mention in passing that such exponential potentials are found in several supergravity theories [23].

To obtain an effective four-dimensional theory one has to perform a consistent dimensional reduction [27]. For a flux compactification the only consistent ansatz (without involving squashing) is given by [15,28]

$$\hat{g}_{\hat{m}\hat{n}} = \begin{pmatrix} g_{mn}(x) & 0 \\ 0 & e^{2\psi(x)} \overset{\circ}{g}_{\overset{\circ}{m}\overset{\circ}{n}}(y) \end{pmatrix} \tag{29}$$

and

$$F_{\overset{\circ}{m}_1 \cdots \overset{\circ}{m}_{\overset{\circ}{D}}} \sim \epsilon_{\overset{\circ}{m}_1 \cdots \overset{\circ}{m}_{\overset{\circ}{D}}} \tag{30}$$

where F has to be a $\overset{\circ}{D}$ form, $\overset{\circ}{D}$ being the number of extra dimensions and we use the symbol "$\circ$" to indicate extra dimensional quantities. Once one solves the Bianchi identity and the field equations for F, the F^2 term in the action (27) gives us a potential term for the scalars, ϕ and ψ. After performing the usual dimensional reduction by integrating the extra dimensions and applying conformal transformations

$$\hat{g} \rightarrow e^{-\overset{\circ}{D}\psi/(\overset{\circ}{D}+2)} \hat{g} \tag{31}$$

followed by

$$\psi \rightarrow \sqrt{\frac{\overset{\circ}{D}(\overset{\circ}{D}+2)}{2}}\psi \tag{32}$$

to go to the Einstein frame in four dimensions, one finds that

$$\int d^{\hat{D}}x\sqrt{-\hat{g}}\frac{e^{-2a_i\phi}}{n_i!}F_{n_i}^2 \tag{33}$$

becomes

$$\int d^4x\sqrt{-g}\overset{\circ}{c}{}^2 e^{-2(a\phi+\gamma'\psi)} \tag{34}$$

with

$$2\gamma' \equiv 3\sqrt{\frac{2\overset{\circ}{D}}{\overset{\circ}{D}+2}} \tag{35}$$

and a is the same exponent that appears in the dilaton coupling to the $\overset{\circ}{D}$ form in (27), its value depends on the specific supergravity model. Also, c° is a constant determining the strength of the flux background.

Let us now compute the four-dimensional effective potential coming from the dilaton potential $V(\phi)$. We observe that dimensional reduction followed by conformal transformation (32) gives a ψ dependence which is in fact the same as one gets from a higher-dimensional cosmological constant. The four-dimensional potential reads

$$V(\phi, \psi) = V_0 e^{-2(\delta\phi+\gamma\psi)} \tag{36}$$

with

$$2\gamma = \sqrt{\frac{2\overset{\circ}{D}}{\overset{\circ}{D}+2}}. \tag{37}$$

The full four-dimensional action then reads

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[R - \partial_m\phi\partial^m\phi - \partial_m\psi\partial^m\psi - 2(V_0' e^{-2(a\phi+\gamma'\psi)} + V_0 e^{-2(\delta\phi+\gamma\psi)})] \tag{38}$$

where as before we have set $M_p = 1$, and redefined c_0 in terms of V_0'.

As a specific example, let us consider type IIA string theory. In the string frame the relevant bosonic part of the action reads

$$\hat{S}_{II} = \int d^{10}x\sqrt{-g}e^{-2\phi}[\hat{R} + 4\partial_{\hat{m}}\phi\partial^{\hat{m}}\phi - V(\phi)] + \frac{1}{2}\frac{1}{4!}F_4^2 + \cdots \tag{39}$$

where we have only included the 4-form whose dual is a 6-form field strength and hence can be consistently turned on as there are 6 extra dimensions. The ellipsis involves other form and fermionic fields which are set to zero as usual. To be general we have also added a potential for the dilaton which could result from quantum corrections. Performing the well-known conformal transformation

$$\hat{g}_{\hat{m}\hat{n}} \rightarrow e^{-4\phi/(\overset{\circ}{D}+2)}\hat{g}_{\hat{m}\hat{n}} \tag{40}$$

followed by

$$\phi \rightarrow \sqrt{\frac{4}{\overset{\circ}{D}+2}}\phi \tag{41}$$

one recovers an action of the form (27) in the 10-dimensional Einstein frame with $2a_4 = -1/\sqrt{2}$ corresponding to the 4-form coupling exponent, or in terms of the 6-form dual, $2a_6 = 1/\sqrt{2}$:

$$\frac{1}{4!}e^{\phi}F_4^2 \leftrightarrow \frac{1}{6!}e^{-\phi}F_6^2. \tag{42}$$

The dimensionally reduced four-dimensional action then looks like (38) with

$$2a_6 = \frac{1}{\sqrt{2}} \quad \text{and} \quad 2\gamma' = 3\sqrt{\frac{3}{2}} \tag{43}$$

where γ' and a_6 can be identified with α and β of Sec. II respectively.

Instead of looking at a flux background one may also study a potential of the form (36) coming from a higher-dimensional dilatonic potential where substituting $\mathring{D} = 6$ we get

$$2\gamma = \sqrt{\frac{3}{2}} \tag{44}$$

and δ remains a free parameter.

B. Brane stress energy

Let us now consider a gas of branes wrapping all the compact internal dimensions and hence these are $\mathring{D}$ branes. The action for such a gas is given by

$$S_{\rm brane} = \int d^{\hat{D}}x\sqrt{-\hat{g}}\rho_0 e^{2\nu\phi}e^{-3\alpha} = \int d^{\hat{D}}x\sqrt{-\hat{g}}\hat{\rho} \tag{45}$$

where $e^{\alpha} = a/a_0$ denotes the usual scale factor of our observable universe. The exponential involving the dilaton in the last term originates from the dilaton coupling present in the brane action in the string frame which depends on several factors like the nature of the brane/string (whether it is fundamental or solitonic etc. [29]), the dimensionality of the supergravity model, etc. The second exponential corresponds to the well-known fact that the brane energy density redshifts as nonrelativistic dust along the transverse directions, which in this case are the three large spatial dimensions.

In order to obtain the brane energy density in the four-dimensional Einstein frame one has to perform the conformal rescalings (32), so that in terms of the rescaled ψ one has

$$S_{\rm brane} = \int d^4x\sqrt{-g}\rho_0 e^{2(\mu\psi+\nu\phi)}\left(\frac{a}{a_0}\right)^{-3} \tag{46}$$

with

$$2\mu = \sqrt{\frac{\mathring{D}}{2(\mathring{D}+2)}}, \tag{47}$$

or

$$S_{\rm brane} = \int d^4x\sqrt{-g}\rho_0 e^{2\mu'\psi'}\left(\frac{a}{a_0}\right)^{-3} = \int d^4x\sqrt{-g}\tilde{\rho} \tag{48}$$

where ψ' is now the linear combination of the dilaton and the radion as defined earlier in (8). $\tilde{\rho}$ corresponds to the observed four-dimensional energy density of the wrapped branes.

Let us focus our attention now onto the 10-dimensional superstring theories. The Dirac-Born-Infeld (DBI) action for a p brane in a string frame is given by [30]

$$S_{\rm DBI} = T_p\int d^{p+1}\sigma e^{-2\phi}\sqrt{-\gamma} \tag{49}$$

where γ is the induced metric on the world volume of the brane parametrized by σ's. Equation (49) leads to an action for a gas of 6-branes wrapping all the compact extra six directions, which looks like

$$S_{\rm brane} = \int d^{10}x\sqrt{-\hat{g}}\rho_0 e^{-2\phi}e^{-3\alpha} \tag{50}$$

in the string frame. Conformal transformation of $\hat{g}$ (41) and subsequent rescaling of ϕ then gives us the action in the Einstein frame (46) with

$$2\nu = \frac{\mathring{D}}{2}\sqrt{\frac{1}{\mathring{D}+2}} = \frac{3}{\sqrt{8}} \tag{51}$$

while substituting $\mathring{D} = 6$ in (47) gives us the exponent μ

$$2\mu = \sqrt{\frac{3}{8}}. \tag{52}$$

Moreover, one finds in this case

$$\psi' = \cos(\pi/3)\psi + \sin(\pi/3)\phi \quad \text{and} \quad \mu' = \sqrt{\frac{3}{8}}. \tag{53}$$

C. Inflation from 10-dimensional universe

We have finally accumulated all the objects necessary to understand whether or not one can indeed obtain a phase of acceleration in the early universe with branes in conjunction with string theory potentials for modulii fields and the dilaton. Let us first look at the case when no flux is turned on [$V_0' = 0$ in (38)] but rather we have a dilatonic potential of the form (36). In this case we have one free parameter, namely δ, and whether one has acceleration or not depends on it. First, realize that in order to end inflation we want

$$\alpha_{\rm eff} \equiv \sqrt{\alpha'^2 + \beta'^2} > 1. \tag{54}$$

This is because, once the branes have decayed, the scalars evolve as if under the influence of an exponential potential with an effective exponent $\alpha_{\rm eff}$ [28]. Therefore, in order for the scalars to track radiation [31] after the end of inflation we require (54). Equation (54) implies a bound $\delta > \sqrt{5/8}$. Now, plugging in all the relevant exponents in the expression for m in (26) one finds that it is possible to have an inflationary paradigm provided

$$\sqrt{\frac{5}{8}} < \delta < \sqrt{\frac{9}{8}}, \tag{55}$$

in other words, when δ is close to 1, and this seems reasonable from the string theory point of view. For these values of δ one finds

$$1 < m < 1.1. \tag{56}$$

Next, let us look at the potential coming from the 4-form flux. Substituting the exponents a_6 and γ' in (26) one finds

$$m < 1. \tag{57}$$

Thus, with just a 4-form flux and branes one cannot get acceleration. This situation however can change if a separate mechanism is available to stabilize a particular linear combination of the dilaton and the radion.

IV. GRACEFUL EXIT AND REHEATING

Branes in string theory can also be interpreted as solitonic solutions in the corresponding low energy supergravity theory. One subclass of such branes are the unstable branes of string theory (e.g. even-dimensional branes in type IIB string theory). Provided that the tachyon which describes the decay of these branes interacts with fields which are in thermal equilibrium in the early universe, these branes could be stabilized at early times—like the embedded Z string in the standard electroweak theory [12]—and thus trigger inflation as described above. The phase of inflation would last until the fields which mediate the interaction, e.g. the photon in the case of the standard model Z string, fall out of equilibrium. At that time, the tachyonic instability could set in. The energy density stored in the unstable branes would lead to reheating. In the example below, the role of the bulk modes is played by the Kaluza-Klein modes.

To be specific, we consider a subclass of branelike solutions of the supergravity equations known as black branes [32] which appear to us (i.e. in the effective four-dimensional world) as black holes. Stability of such black branes (both charged and uncharged) has been discussed in detail in [33], and, in particular, it was realized that when the Kaluza-Klein modes of the gravitational supermultiplet are lighter than the tension of the branes, these modes become unstable (Gregory-Laflamme instability) [33,34]. Since in our model the volume of the internal manifold slowly grows, the Gregory-Laflamme instability thus provides us with a natural mechanism to end inflation. Assume that the mass scale associated with the tension of the branes is given by

$$T = 10^{-B} M_p. \tag{58}$$

The masses of the Kaluza-Klein modes are, on the other hand, given by

$$M_K = e^{-\zeta\psi} M_p \quad \text{with} \quad \zeta = \sqrt{\frac{\mathring{D}+2}{2\mathring{D}}}. \tag{59}$$

The branes become unstable when $M_K \sim T$. In terms of the rotated basis this happens when

$$\begin{aligned} 10^B &= e^{\zeta(\psi' \cos\theta - \phi' \sin\theta)} \\ &= e^{\zeta(\psi'_0 \cos\theta - \phi'_0 \sin\theta)} \left(\frac{t}{t_0}\right)^{\zeta(n\cos\theta - p\sin\theta)} \\ &= e^{-\zeta\psi_0} \left(\frac{a}{a_0}\right)^{\zeta(n\cos\theta - p\sin\theta)/m} \\ &\equiv N_0 e^{\mathcal{N}\zeta(n\cos\theta - p\sin\theta)/m} \end{aligned} \tag{60}$$

where $\mathcal{N}$ is the number of e foldings. In the spirit of string gas cosmology [1] we assume that, initially, the internal volume is of Planck size, i.e. $\psi \approx 0$. This implies $N_0 \sim \mathcal{O}(1)$. One can then estimate the number $\mathcal{N}$ of e foldings as

$$\mathcal{N} = \frac{2.3Bm}{\zeta(n\cos\theta - p\sin\theta)}. \tag{61}$$

For a range of the relevant parameters one can indeed obtain a sufficient number of e foldings. For example, for a ten-dimensional model, substituting $\mathring{D} = 6$ and $\theta = \pi/3$ one finds

$$\mathcal{N} = \frac{2.3Bm}{0.41n - 0.71p}. \tag{62}$$

For the exponent calculated for a gas of six branes (53) with a dilatonic potential [(36) and (44)], δ within the range (55), one finds that for a moderate hierarchy, $B \sim$ 5–7 we get around 50–60 e foldings which is necessary to solve the flatness and horizon problems.

After the black branes have decayed, the field starts rolling along the exponential potential of the dilaton and the radion. At late times, there are two possible scenarios. If the potential is indeed a pure exponential as we considered in our model, then the fields would continue to roll tracking first the radiation and later on the matter energy density (since $\sqrt{\alpha^2 + \beta^2} > 1$). One can try to connect this model to a late-time coupled quintessence regime as discussed in [14]. However, the phenomenological viability of this scenario requires a mechanism which ensures that the standard model particles are only very weakly coupled to radion and the dilaton—otherwise one has conflicts with fifth force experiments. Indeed, in [26] such a least coupling principle was proposed for the coupling of the dilaton to standard model particles. We here do not further speculate on the details of such mechanisms. The other possibility could be if the radion-dilaton potential has a minimum where the fields could be stabilized after the end of inflation. This possibility might be realized if, for example, the potential is really a sum of exponentials. This occurs quite commonly in supergravity reductions [15,28]. In this case, of course, the moduli are stabilized and one does not need to worry about the fifth force constraints (as long as the potential is sufficiently curved at the minimum).

V. DISCUSSION AND CONCLUSIONS

We have presented a mechanism for obtaining inflation in the context of string gas cosmology. We start with the conventional scenario of string gas cosmology: the Universe starts out hot and small (string scale), with all spatial dimensions of comparable scale. As discussed in [1,4,5], the fundamental string winding modes can disappear in at most three spatial dimensions, thus leading to an explanation of why the spatial dimensions predicted by string theory but not seen experimentally are confined. Radion stabilization is a natural result of the string winding and momentum modes about the extra spatial dimensions [9].

At the later times studied in this paper, we have focused on the effective four-dimensional field theory coming from dimensionally reducing the Lagrangian of string gas cosmology. Assuming the existence of an exponential potential for the dilaton and the radion at these later times, we have shown that the coupling of branes winding the extra dimensions couple to the radion and dilaton and can lead to a period of power-law inflation. This inflationary period is generated by the combined radion-dilaton system tracking its time-dependent potential minimum position. The time dependence of the effective potential is induced by the coupling of the brane gas to the two fields. A graceful exit from inflation is obtained by making use of a gas of unstable branes, branes which are stabilized at early times via their interactions with the Kaluza-Klein modes of the fields of the bulk gravitational supermultiplet. The decay of these branes then leads to reheating after inflation.

For the specific branes we considered we obtained a power-law exponent m which is too small to be consistent with the observational limits on the tilt in the power spectrum of density fluctuations. In addition, a hierarchy between the Planck scale and the tension of the branes is required in order to obtain a sufficient number of e foldings of inflation. Both of these problems can be solved provided the brane coupling exponents are larger than what we obtained for 6-branes in the context of 10-dimensional supergravity/string theory. This can happen in several ways. As noted in [24], quantum stingy loop corrections can change several coefficients in the effective dilatonic action which in turn will change the exponents. (For example, if the coefficient in front of the kinetic term for the dilaton changes, one has to rescale the dilaton field which in turn changes its coupling exponent to the brane.)

One could also consider various types of branes with different dimensionalities, perhaps in the context of lower-dimensional supergravity models. The various exponents again are expected to be different, as is evident from the general analysis we performed. Finally, it is not necessary to restrict oneself only to radion-dilaton systems. For example, one could consider squashed configurations where the branes couple to the volume as well as the shape moduli (the dilaton would be stabilized by some other mechanism). As shown in [28], for such configurations it is possible to consistently turn on other lower form fluxes (not just the 6-form as considered here) which may come with the right exponents to realize an accelerated regime in conjunction with the branes. We leave an exploration of all these possibilities for the future.

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APPENDIX: TWO FIELD SOLUTION

We want to solve cosmological evolution equations of the form (13) and (14):

$$\ddot{\phi} + 3H\dot{\phi} = 2\beta V_0 e^{-2(\alpha\psi+\beta\phi)}, \tag{A1}$$

$$\ddot{\psi} + 3H\dot{\psi} = 2\alpha V_0 e^{-2(\alpha\psi+\beta\phi)} - 2\mu\rho e^{2\mu\psi}, \tag{A2}$$

$$H^2 = \frac{1}{3}\left(\frac{\dot{\phi}^2}{2} + \frac{\dot{\psi}^2}{2} + V_0 e^{-2(\alpha\psi+\beta\phi)} + \rho e^{2\mu\psi}\right). \tag{A3}$$

We choose the ansatz as in (23)

$$a = a_0\left(\frac{t}{t_0}\right)^m; \qquad e^{\psi} = e^{\psi_0}\left(\frac{t}{t_0}\right)^n; \qquad e^{\phi} = e^{\phi_0}\left(\frac{t}{t_0}\right)^p. \tag{A4}$$

In order that all the terms in (A2) and (A3) have the same t dependence, t^{-2}, to be specific, we get by looking at the exponents

$$\alpha n + \beta p = 1 = \frac{3m}{2} - \mu n. \tag{A5}$$

Further, one can substitute the potentials associated with V_0 and ρ_0 from (A2) in terms of the field derivatives in (A3) to obtain

$$H^2 = \frac{1}{3}\left(\frac{\dot{\phi}^2}{2} + \frac{\dot{\psi}^2}{2} - \frac{1}{2\mu}(\ddot{\psi} + 3H\dot{\psi}) - \frac{1}{2\beta}(1 + \alpha/\mu)(\ddot{\phi} + 3H\dot{\phi})\right). \tag{A6}$$

Substituting the ansatz (A4) in the above equation one finds another relation between the power-law exponents

$$m^2 = \frac{1}{3}\left[\frac{1}{2}(n^2 + p^2) + (3m - 1)\left(\frac{p}{2\beta}\left(1 + \frac{\alpha}{\mu}\right) - \frac{n}{2\mu}\right)\right]. \tag{A7}$$

One can thus solve (A7) and (A8) to obtain m, n and p in terms of the exponents α, β and μ:

$$m = 2\frac{3\mu\alpha + \alpha^2 + \beta^2 + 2\mu^2}{6\mu\alpha + 3\alpha^2 + 3\beta^2 + 8\mu^2\beta^2}, \tag{A8}$$

$$n = \frac{3\alpha + 6\mu - 8\beta^2\mu}{6\mu\alpha + 3\alpha^2 + 3\beta^2 + 8\mu^2\beta^2}, \tag{A9}$$

$$p = \frac{\beta(8\mu^2 + 3 + 8\mu\alpha)}{6\mu\alpha + 3\alpha^2 + 3\beta^2 + 8\mu^2\beta^2}. \tag{A10}$$

Although the t dependence now cancels in all the evolution equations by virtue of (A8)–(A10), we are still left with matching the coefficients in the two equations (A1) and (A2). These equations essentially determine the other unknown parameters ϕ_0 and ψ_0 in terms of V_0 and ρ_0 or vice versa. In fact to have a consistent solution one should check whether these equations can be satisfied for positive values of V_0 and ρ_0 as those are the physical scenarios we are interested in. After some algebra one finds

$$V_0 = e^{-2(\alpha\psi_0 + \beta\phi_0)}\frac{p(3m-1)}{2\beta} \tag{A11}$$

and

$$\rho_0 = e^{-2\mu\psi}\left(\frac{3m-1}{2\mu}\right)\left(\frac{\alpha}{\beta}p - n\right) = e^{-2\mu\psi}(3m-1)\left(\frac{4(\alpha^2+\beta^2) - 3 + 4\mu\alpha}{6\mu\alpha + 3\alpha^2 + 3\beta^2 + 8\mu^2\beta^2}\right). \tag{A12}$$

From (A10) it is clear that the ratio p/β is positive. Also, for the solutions we are looking at, $m > 1 > 1/3$, and thus the right-hand side of (A11) is positive implying $V_0 > 0$. Next let us look at the right-hand side of (A12).

In order for the potential to track radiation after inflation we demanded $\alpha^2 + \beta^2 > 1$ and thus it is clear that ρ_0 is positive too. We indeed have consistent attractor solutions.

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