3-form induced potentials, dilaton stabilization, and running moduli

Andrew R. Frey*
Department of Physics, University of California, Santa Barbara, California 93106

Anupam Mazumdar†
CHEP, McGill University, 3600 University Road, Montréal, Quebec, Canada H3A 2T8

(Received 8 November 2002; published 27 February 2003)

We study the potential induced by imaginary self-dual 3-forms in compactifications of string theory and the cosmological evolution associated with it. The potential contains exponentials of the volume moduli of the compactification, and we demonstrate that the exponential form of the potential leads to a power law for the scale factor of the universe. This power law does not support accelerated expansion. We explain this result in terms of supersymmetry and comment on corrections to the potential that could lead to inflation or quintessence.

DOI: 10.1103/PhysRevD.67.046006 PACS number(s): 11.25.Mj, 04.65.+e, 98.80.Cq

I. INTRODUCTION

If we believe that string theory (or M theory) is the fundamental description of interactions in our Universe, then we are obviously forced to place the basic processes of cosmology into a string theoretic framework. Important steps have been made in this direction by examining four dimensional supergravity models for potentials that could support the early phase of the accelerated expansion of the Universe, known as inflation, which solves some of the outstanding problems of the hot big bang cosmology [1]. See, for recent examples, [2,3]. Other work has identified string theory models in which D-brane physics leads to inflation [3–6].[1] At the same time, however, it has proven challenging to incorporate cosmological acceleration into string theory backgrounds because they tend to relax to supersymmetric vacua [7,8]. In this paper, we ask whether a stringy potential generated by higher dimensional magnetic fields can give rise to accelerated expansion. We restrict our analysis to the classical potential of supergravity.

We study a class of exact solutions to type IIB supergravity that have a vacuum state [denoted by superscript (0)] with 3-form magnetic fluxes that satisfy a self-duality relation

$$\star_6^{(0)}(F-C^{(0)}H)=e^{-\Phi^{(0)}}H \tag{1}$$

on the compact space, which should be Calabi-Yau (CY) space [9,10]. These vacua were described in some detail in [11] and in dual versions in [12–16]. The metric is of ''warped product'' form,

$$ds^{(0)2}=e^{A}\eta_{\mu\nu}dx^{\mu}dx^{\nu}+e^{-A}g_{mn}dy^{m}dy^{n}, \tag{2}$$

so these models have the phenomenology of the Randall-Sundrum models [17–19]. The warp factor depends on the position of D3-branes (and orientifold planes) on the compact space and also determines the 5-form field strength. The condition (1) gives rise to a potential for many of the light scalars, including the dilaton generically, which vanishes at the classical minimum and furthermore has no preferred compactification volume. We will be interested in the behavior of these systems above the minimum, and the 4D metric will generalize $\eta_{\mu\nu}\to g_{\mu\nu}$.

For simplicity, we will mainly consider the case where the internal manifold is a $T^6/\mathbf{Z}_2$ orientifold, as described in [20–22] (or in dual forms in [16,23]). We take the torus coordinates to have square periodicities, $x^m \simeq x^m + 2\pi l_s$, so that the geometric structure is encoded in the metric. On this torus, the 3-form components must satisfy the Dirac quantization conditions

$$H_{mnp}=\frac{1}{2\pi l_s}h_{mnp}, \quad F_{mnp}=\frac{1}{2\pi l_s}f_{mnp}, \quad h_{mnp},f_{mnp}\in\mathbf{Z}. \tag{3}$$

Boundary conditions at the orientifold planes give large Kaluza-Klein masses to many fields (including the metric components $g_{\mu m}$, for example), and the remaining theory is described by an effective 4D gauged $\mathcal{N}=4$ supergravity with completely or partially broken supersymmetry via the super-Higgs effect [24–29].

In the following section, we discuss the dimensional reduction of the type IIB superstring in toroidal compactifications with self-dual 3-form flux, ignoring the warp factor, paying particular attention to the potential for a subset of the light scalars. Next, in Sec. III, we find the cosmological evolution driven by our potential based on known inflationary models; we find that our potentials do not lead to an accelerating universe. Finally, in Sec. IV, we comment on the generalization of our results to more complicated models, compare our results to other models that do lead to inflation, and discuss corrections to our potential that might or might not lead to inflation.

*Electronic address: frey@vulcan.physics.ucsb.edu
†Electronic address: anupamm@hep.physics.mcgill.ca

[1] As did [47], which found inflation in a small region of moduli space.

II. DIMENSIONAL REDUCTION AND POTENTIAL

Here we will review the dimensional reduction of 10D type IIB supergravity in compactifications with imaginary self-dual 3-form flux on the internal manifold. For simplicity and specificity, we will concentrate on the toroidal compactifications of [21,22], extending our analysis to more general cases in Sec. IV. We will ignore the warp factor, which assumes that the compactification radius is large compared to the string scale.[2]

A. Kinetic terms

We will start with the kinetic terms, mostly following the analysis of [22], using the $\mathcal{N}=4$ $SO(6,22)\times SU(1,1)/SO(6)\times SO(22)\times U(1)$ language because we are studying configurations away from the moduli space at the bottom of the potential. Our main purpose is to identify the physical interpretation of the canonically normalized scalars, so we will skip the algebraic details.

As was shown in [22], the moduli must be tensor densities in order to avoid double trace terms in the action,

$$\gamma^{mn}=\frac{\Delta}{2}e^{-\Phi}g^{mn},\quad \beta^{mn}=\frac{\Delta}{2\times 4!}\epsilon^{mnpqrs}C_{pqrs},$$

$$\Delta\equiv\sqrt{\det g_{mn}}, \tag{4}$$

along with the D-brane positions[3] $\alpha_I^m=X_I^m/2\pi l_s$ and the 10D dilaton-axion. For the purpose of cosmology, we want to work in the 4D Einstein frame (note that this is different than in [22] because we are allowing the dilaton to vary)

$$g^E_{\mu\nu}=\frac{\Delta}{2}e^{-2\Phi}g_{\mu\nu}. \tag{5}$$

From stringy dualities, it can be seen that the moduli definitions (4) correspond to the geometric moduli g_{mn}, B_{mn} in a toroidal heterotic compactification, and the metric (5) is the 4D "canonical metric" [30,31] in the heterotic description [22].[4]

The kinetic action obtained from dimensional reduction of type IIB supergravity (SUGRA) and the D3-brane action is then

$$S_{\rm kin}=\frac{M_P^2}{16\pi}\int d^4x\sqrt{-g_E}\Big[R_E+\frac{1}{4}\partial_\mu\gamma_{mn}\partial^\mu\gamma^{mn}+\frac{1}{4}\gamma_{mp}\gamma_{nq}D_\mu\beta^{mn}D^\mu\beta^{qp}-\frac{1}{2}\gamma_{mn}\partial_\mu\alpha_I^m\partial^\mu\alpha_I^n-\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}e^{2\Phi}\partial_\mu C\partial^\mu C\Big],$$

$$M_P^2=\frac{1}{8\pi^2 l_s^2}. \tag{6}$$

Here, M_P is the Planck mass, and we are using a coset space covariant derivative

$$D_\mu\beta^{mn}\equiv\partial_\mu\beta^{mn}+\frac{1}{2}(\alpha_I^m\partial_\mu\alpha_I^n-\alpha_I^n\partial_\mu\alpha_I^m) \tag{7}$$

which arises from the magnetic coupling of the D3-branes to β; this is the dimensionally reduced action for the heterotic theory of [30,32], as one might expect. In deriving the action, one needs the identity

$$\gamma^{mn}\partial_\mu\gamma_{mn}=-\gamma_{mn}\partial_\mu\gamma^{mn}=6\partial_\mu\Phi-4\partial_\mu\ln\Delta. \tag{8}$$

It is easiest to study the cosmology of canonically normalized scalars; so we will break down the geometric moduli. For simplicity we will consider only the factorized case $T^6=(T^2)^3$. We can then parametrize the metric on an individual 2-torus [say, the $(4-7)$ torus] as

$$\gamma^{mn}=e^{2\sigma}\begin{bmatrix}e^{-\zeta}+e^{\zeta}d^2 & -e^{\zeta}d\\ -e^{\zeta}d & e^{\zeta}\end{bmatrix}. \tag{9}$$

Here, σ gives the overall size of the T^2, ζ gives the relative length of the two sides, and d controls the angle between the two directions of periodicity. Then the γ kinetic term becomes

$$S_{\rm kin}=-\frac{M_P^2}{16\pi}\int d^4x\sqrt{-g_E}\sum_{i=1}^{3}\Big[2\partial_\mu\sigma_i\partial^\mu\sigma_i+\frac{1}{2}\partial_\mu\zeta_i\partial^\mu\zeta_i+\frac{1}{2}\partial_\mu d_i\partial^\mu d_i\Big]. \tag{10}$$

For canonical normalization, the coefficient of the kinetic terms should simply be $-1/2$, so a further rescaling is necessary.

B. Potential

The scalar potential comes from dimensional reduction of the background 3-form terms in the type IIB action. After converting to our variables, the potential for the bulk modes is, in generality,

[2]Actually, because the warp factor A scales like R^{-4} [14,22], the radius need only be a few times the string scale for our approximation to be reasonable.

[3]If the D-branes are coincident, the index I labels the adjoint representation of $U(N)$; the kinetic terms remain the same [29].

[4]Strictly speaking, these are only the heterotic dual variables with vanishing fluxes; see [16].

$$V=\frac{M_P^2}{4!\times 32\pi}(\det\gamma_{mn})\gamma^{mq}\gamma^{nr}\gamma^{ps}[e^{\Phi}(F-CH)_{mnp}(F-CH)_{qrs}+e^{-\Phi}H_{mnp}H_{qrs}] \tag{11}$$

along with an additional term that subtracts off the vacuum energy.[5] This potential was derived from dimensional reduction in [11,21], from gauged supergravity in [24,28], and from the superpotential of [12]. One feature to note in this potential is that it always has (at least) three flat directions at the minimum, corresponding to the radii of factorization $T^6=T^2\times T^2\times T^2$. Also, the β moduli do not enter into the potential, although some become Goldstone bosons via the super Higgs effect [22,24–26].

For cosmological purposes, we will need to have a more explicit form of the potential in hand. Since there are 23 scalars γ^{mn},Φ,C, writing the full potential for a given set of 3-form fluxes would be prohibitively complicated, but we can write down a few simple examples and focus on the universal aspects.

The simplest case is to take the three T^2 to be square, so that the geometric moduli are $\gamma^{44}=\gamma^{77}=e^{2\sigma_1}$, etc., with all others vanishing. Then, above a vacuum that satisfies Eq. (1), we can calculate the potential

$$V_{\rm dil}=\frac{M_P^4}{4(8\pi)^3}h^2e^{-2\Sigma_i\sigma_i}\left[e^{-\Phi^{(0)}}\cosh(\Phi-\Phi^{(0)})+\frac{1}{2}e^{\Phi}(C-C^{(0)})^2-1\right], \tag{12}$$

$$h^2=\frac{1}{6}h_{mnp}h_{qrs}\delta^{mq}\delta^{nr}\delta^{ps}. \tag{13}$$

This potential was written explicitly in $SU(1,1)$ notation in [28] and is valid for any 3-form background. The most important feature of this potential is that there is a vanishing vacuum energy, and, further, the radial moduli σ feels a potential only when the dilaton-axion system is excited. Since this is the simplest potential to write down, it will be our primary focus in Sec. III. It is very interesting to note that the cosmology of this potential for the dilaton-axion has been discussed earlier in [3,33,34] from SUGRA. Importantly, though, their models did not include the radial moduli or the negative term that subtracts off the cosmological constant.

Adding the complex structure is more complicated and more model-dependent. The simplest possible case, for example, $f_{456}=-h_{789}$, is nongeneric in that Eq. (1) is satisfied at $\Phi-\Sigma_i\zeta_i=C=d_i=0$, so the ζ_i give extra moduli compared to other background fluxes (at the classical level). However, we still have $\Phi-\Sigma_i\zeta_i$ fixed by a cosh potential with a polynomial in C,d_i:

$$V_0=\frac{M_P^4}{4(8\pi)^3}h^2e^{-2\Sigma_i\sigma_i}\left\{\cosh\left(\Phi-\sum_i\zeta_i\right)+\frac{1}{2}e^{\Phi+\Sigma_i\zeta_i}[C^2-2Cd_1d_2d_3+d_1^2d_2^2d_3^2+e^{-2\zeta_3}d_1^2d_2^2+e^{-2\zeta_2}d_1^2d_3^2+e^{-2\zeta_1}d_2^2d_3^2+e^{-2\zeta_2-2\zeta_3}d_1^2+e^{-2\zeta_1-2\zeta_3}d_2^2+e^{-2\zeta_1-2\zeta_2}d_3^2]-1\right\} \tag{14}$$

using again Eq. (13). It is straightforward but tedious to show that this potential is positive definite, and the only extremum is at $\Phi-\Sigma_i\zeta_i=C=d_i=0$. As this case is nonsupersymmetric, quantum mechanical corrections should lift the flat directions.

On the other end of the supersymmetry spectrum are the $\mathcal{N}=3$ models of [22], which fix the dilaton as well as all the complex structure. If we ignore C,d_i (set them to a vanishing vacuum value), we find a potential

$$V_3=\frac{M_P^4}{(8\pi)^3}h^2e^{-2\Sigma_i\sigma_i}[\cosh(\Phi-\zeta_1-\zeta_2-\zeta_3)+\cosh(\Phi-\zeta_1+\zeta_2+\zeta_3)+\cosh(\Phi+\zeta_1-\zeta_2+\zeta_3)+\cosh(\Phi+\zeta_1+\zeta_2-\zeta_3)-4]. \tag{15}$$

This again has the same cosh structure for the dilaton; the only difference is a factor of 4 due to the number of components of flux in the background.

Including the non-Abelian coupling for the D3-brane scalars α_I^m introduces new terms in the potential (see [29] for a supersymmetry based approach). In the absence of fluxes and even in the ground state, this potential is monotonic and simply forces the α_I^m to commute. Otherwise, the branes pick up a 5-brane dipole moment and become noncommuting, as discussed in [35]. Writing the brane positions as $U(N)$ matrices, the potential is

$$V_{\rm b}=2\pi M_P^4\left[2\pi e^{\Phi}\gamma_{mp}\gamma_{nq}{\rm tr}([\alpha^m,\alpha^n][\alpha^q,\alpha^p])+\frac{i}{12}(\det\gamma_{mn})^{1/2}e^{\Phi}(e^{-\Phi}h-\star_6(f-Ch))_{mnp}{\rm tr}(\alpha^m\alpha^n\alpha^p)\right]. \tag{16}$$

To illustrate this potential, we take $f_{456}=-h_{789}$ as before, set $C=d_i=\zeta_i=0$, and consider $\alpha^{4,5,6}\propto I_N$ and $\alpha^{7,8,9}=\rho t^{1,2,3}$ with t^i a representation of $SU(2)$. Then

$$V_{\rm b}=2\pi M_P^4\left[16\pi e^{\Phi}(e^{-2\sigma_1-2\sigma_2}+e^{-2\sigma_1-2\sigma_3}+e^{-2\sigma_2-2\sigma_3})\rho^4+\frac{h_{789}}{2}e^{-2\sum_i\sigma_i}e^{\Phi}(e^{-\Phi}-1)\rho^3\right]. \tag{17}$$

[5]This comes from the D3/O3 tension, which must cancel the vacuum potential for string tadpole conditions to be satisfied to leading order in l_s.

There are actually more terms in this potential as required by supersymmetry; these are just the lowest order terms that appear in the D-brane action given by [35]. For example, the underlying $\mathcal{N}=4$ supersymmetry gives a ρ^6 term,[6] and there is also a ρ^2 term from gravitational back reaction that has been calculated using supersymmetry in one case (see [36]); in any event, there is a local maximum in the α_I^m direction. Like the bulk potential, this potential has exponential prefactors from the σ moduli, and if the bulk scalars are away from their minimum, there is the same $\exp[-2\Sigma_i\sigma_i]$ factor.

The key point to take from this discussion of the potential is the exponential prefactor that appears in all terms, whether bulk or brane modes.

III. COSMOLOGICAL EVOLUTION

In this section we seek the cosmological evolution of the dilaton and the moduli fields in a flat $d=4$ dimensional space time background. However, for the purpose of illustration it is prudent that we consider a toy model which illustrates the behavior of the potentials V_{dil}, V_0 and V_3 described in the earlier section.

$$V \approx e^{-\Sigma_i \alpha_i \sigma_i} V(\Phi). \tag{18}$$

Let us also assume that the above potential has a global minimum Φ_0 determined by $V(\Phi)$. At Φ_0 the potential vanishes. In the above, Φ mimics the dilaton and σ_i play the role of moduli with various coefficients α_i determines the slope of the potential. For generality we have assumed that there are i number of moduli. In our original potential all the slopes are fixed at $\alpha_i = 4\sqrt{\pi}/M_P$ (with normalized scalars), see Eq. (14). We will model V_b by slightly different potential.

For the sake of simplicity and generality in Eq. (14), we do not assume any form for d_i and ζ_i at the moment. It is interesting to note that the potential Eq. (18) is quite adequate to determine the cosmological evolution if they dominate the energy density, which is fixed by the value $V(\Phi)$ in our case. Further note that $V(\Phi) \propto (M_P)^4$.[7] Therefore, given generic initial conditions for all the moduli $\sigma_i \sim M_P$ in the dimensionally reduced action, we hope that the rolling moduli could lead to the expansion of the universe. In order to see this clearly, one must obtain the equations of motion for both dilaton and moduli if coupled to the gravity in a Robertson-Walker space-time metric with an expansion factor $a(t)$, where t represents the physical time. The equations of motion are in the Einstein frame

$$\ddot{\Phi} + 3H\dot{\Phi} + e^{-\Sigma_i \alpha_i \sigma_i} V'(\Phi) = 0, \tag{19}$$

$$\ddot{\sigma}_i + 3H\dot{\sigma}_i - \alpha_i e^{-\Sigma_i \alpha_i \sigma_i} V(\Phi) = 0, \tag{20}$$

$$H^2 = \frac{8\pi}{3M_{\rm P}^2}\left[\frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}\sum_i \dot{\sigma}_i^2 + e^{-\Sigma_i \alpha_i \sigma_i} V(\Phi)\right]. \tag{21}$$

The Hubble expansion is given by $\dot{a}/a$; an overdot denotes derivative with respect to physical time and prime denotes differentiation with respect to Φ.

Note that depending upon the slopes of the fields along their classical trajectories the dilaton can roll slowly compared to the moduli, in which case we might be able to solve the moduli equations exactly.[8] With this simple assumption we first consider Eqs. (20),(21) with $\dot{\Phi} \ll \dot{\sigma}_i$, and $V(\Phi) \sim V_0$, the latter condition is true if the dilaton time varying VEV changes slowly. Much stronger condition can be laid on the kinetic terms for the moduli and dilaton if we assume

$$\dot{\sigma}_i \gg \frac{M_P V'(\Phi)}{2\sqrt{2\pi}\alpha_i V(\Phi)}\dot{\Phi}. \tag{22}$$

The above equation can be derived from Eqs. (19),(20) by assuming $\ddot{\Phi} \ll 3H\dot{\Phi}$, $\ddot{\sigma}_i \ll 3H\dot{\sigma}_i$ and $\dot{\Phi} \ll \dot{\sigma}$, which is equivalent to slow-roll conditions.

Now we are interested in solving the moduli field evolution without imposing slow roll conditions on them. We argue that there exists an attractor region with a power law solution $a(t) \propto t^p$, which from Eq. (21), dimensionally satisfies $H^2 \propto t^{-2} \propto e^{-\Sigma_i \alpha_i \sigma_i} V(\Phi)$. Hence we write

$$e^{\alpha_i \sigma_i} = \frac{k_i}{t^{c_i}}, \tag{23}$$

$$\sum_{i=1}^{n} c_i = 2, \tag{24}$$

where k_i are dimensional and c_i are dimensionless constants respectively. Equation (23), coupled with the equations of motion Eq. (20) results in

$$(3p-1)c_i = \alpha_i^2 V(\Phi)\prod_{k=1}^{n} k_k, \tag{25}$$

from which we find, using Eq. (24) and Eq. (20),

$$V(\Phi)\prod_{k=1}^{n} k_k = \frac{2(3p-1)}{\sum_{i=1}^{n} \alpha_i^2},$$

$$\left(\frac{c_i}{\alpha_i}\right)^2 = \frac{4\alpha_i^2}{\left(\sum_{k=1}^{n} \alpha_k^2\right)^2}. \tag{26}$$

[6] We thank S. Ferrara for discussions on this point.

[7] Strictly speaking potential energy ought to be less than $(M_P)^4$ in order to make sense of field theoretic description of the expanding Universe.

[8] We are obviously assuming *a priori* that the dilaton is moving very slowly which may or may not be the case. Nevertheless, our scenario shall be able to discern some of the aspects of the actual dynamics, such as inflationary or noninflationary.

When substituted into Eq. (21) with $\dot{\Phi} \ll \dot{\sigma}_i$, we obtain the key result without using any slow roll condition for the moduli where the exponent of the scale factor $a(t) \propto t^p$ goes as

$$p = \frac{16\pi}{M_P^2} \frac{1}{\sum_{j=1}^{n} \alpha_j^2}. \tag{27}$$

We also note that the scaling solution for the moduli fields can be found quickly as follows for any two moduli, σ_i and σ_k:

$$\left(\frac{\dot{\sigma}_i}{\dot{\sigma}_k}\right)^2 = \left(\frac{\alpha_i}{\alpha_k}\right)^2. \tag{28}$$

The above equation ensures the late time attractor behavior for all the moduli in our case, which has a similarity to the assisted inflation discussed in [37,38]. From Eqs. (23),(24), we can also write

$$\sigma_i = \sigma_i(0) - \frac{c_i}{\alpha_i} \ln t, \tag{29}$$

where $\sigma_i(0)$ is a constant depending on the initial conditions.

Inflationary solutions exist provided $p > 1$, which can be attained in our case only when the slopes α_i are small enough, or in other words the moduli should have sufficiently shallower slope. The power law solution also applies to any p in the range $0 < p < 1$, where the expansion is noninflationary.

Note that so far we have neglected the dynamics of the dilaton. In spite of rolling down slowly, Φ eventually comes down to the bottom of the potential. So, the prime question is how fast does it roll down to its minimum Φ_0. This will again depend on the exact slope of the potential for $V(\Phi)$. Nevertheless, if we demand that the dilaton is indeed rolling down slowly such as $\ddot{\Phi} \ll 3H\dot{\Phi}$, then we can mimic the slow-roll regime for the dilaton, and the situation mimics that of soft inflation studied in Refs. [39–41],

$$f(\Phi) = f(\Phi_0) - p \ln t, \tag{30}$$

where

$$f(\Phi) \equiv \frac{8\pi}{M_P^2} \int d\Phi \frac{V(\Phi)}{V'(\Phi)}. \tag{31}$$

Here the subscript 0 indicates the initial value.

With $a \propto t^p$ and $e^{-\alpha_i \sigma_i} V(\Phi) \propto H^2$, we can then parametrize the dilaton equation of motion by

$$\ddot{\Phi} + 3H\dot{\Phi} = -cH^2\Phi, \tag{32}$$

where c is a constant factor which determines the unknown shape parameter of $V(\Phi)$, which ought to be smaller than one in order to be consistent with the Hubble equation (21). In this case, we can find the exact solution for the dilaton

$$\Phi(t) \propto a^{-\eta}; \quad \eta = \frac{1}{2}\left[\left(3 - \frac{1}{p}\right) - \sqrt{\left(3 - \frac{1}{p}\right)^2 - 4c}\right]. \tag{33}$$

Unlike the dilaton, the moduli have no minimum, and they face the usual run away moduli problem. Note that once dilaton reaches its minimum the potential Eq. (18) vanishes, and so the effective potential for the moduli. However, once the expansion of the universe driven by the dynamics for the moduli comes to an end, the dilaton settles down at Φ_0, then the moduli still continue to evolve accordingly

$$\frac{d}{dt}(\dot{\sigma}_i a(t)^3) = 0, \tag{34}$$

provided there is some source of energy-momentum tensor supporting the expansion of the universe. The moduli can indeed come to rest at some finite value.

So far we have been concentrating upon the toy model with the potential Eq. (18). Nevertheless, the situation remains unchanged for the type of potentials we are interested in; see Eqs. (12),(14),(15). Note that the dynamical behavior of the moduli will remain unchanged, but the dilaton may roll slow or fast depending upon the actual slope of the dilaton potential. By inspecting the potentials we find the corresponding slope of the moduli, i.e. $\alpha_i = 4\sqrt{\pi}/M_P$, and $n = 3$. Therefore, the moduli driven expansion of the universe leads to

$$p = \frac{1}{3} < 1; \quad a(t) \propto t^{1/3}. \tag{35}$$

The expansion is noninflationary and will not solve any of the outstanding problems of the big bang cosmology. Nevertheless, this expansion which is slower than either radiation dominated or matter dominated epoch could be the precursor or end stage of inflation in this particular model.

Now, we briefly comment on bulk potential derived in Eq. (17). Note, even if the dilaton is settled down the minimum with $e^{-\Phi} = 1$, the moduli fields still contribute to the potential. It would then be interesting to note whether we get any expansion of the universe from the moduli driven potential. Further note that the structure of the potential is quite different from Eq. (18). The potential rather follows (taking ρ to be slowly rolling and $\rho \ll 1$)

$$V_b = 32\pi^2 M_P^4 \rho^4 \sum_{s=1}^{n} \exp\left(\sum_{j=1}^{m} \alpha_{sj} \sigma_j\right). \tag{36}$$

This kind of potential has also been solved exactly without using slow-roll conditions [38]. Of course with the possibility of some of $\alpha_{sj} = 0$ for some combination of s, j. Our case Eq. (17) exemplifies with $s, j = 1,2,3$. For Eq. (36), again we demand that $\exp(\Sigma_{j=1}^{m} \alpha_{sj} \sigma_j) \propto 1/t^2$. The late time attractor solution for the moduli fields can be established with [38]

$$\left(\frac{\dot{\sigma}_j}{\dot{\sigma}_l}\right)^2 = \left(\frac{\sum_{q=1}^{n} \alpha_{qj} B^q}{\sum_{r=1}^{n} \alpha_{rl} B^r}\right)^2 . \tag{37}$$

In the above equation $B \equiv (\Sigma_{j=1}^{m} \alpha_{sj} \alpha_{qj})^{T}_{COF}$, where T stands for transpose and COF stands for the cofactor, and $B^s \equiv \Sigma_{q=1}^{n} B_{sq}$ is the sum of elements in row s. The power law solution $a(t) \propto t^p$ can be found to be [38]

$$p = \frac{16\pi}{M_P^2} \frac{\sum_{s}^{n} \sum_{q}^{n} B_{sq}}{\det \mathrm{A}}, \tag{38}$$

where $A_{sq} = \Sigma_{j=1}^{m} \alpha_{sj} \alpha_{qj}$.

Now, we can read α_{sj} from Eq. (17). After little calculation with the normalized α_{sj}, we obtain the value of p from Eq. (38)

$$p = \frac{3}{16} \ll 1. \tag{39}$$

Again we find that there is no accelerated expansion. The assisted inflation in all these cases provides expansion but could not be used to solve inflation or even late time acceleration during the matter dominated era. In all our examples we found that the moduli trajectories follow the late time attractor towards the supersymmetric vacuum. Finally, a word upon supersymmetry breaking in the observable sector, which will induce mass ~ 1 TeV to the moduli and dilaton in gravity mediation. Unless the moduli amplitude is damped considerably, the large amplitude oscillations of the moduli field will eventually be a cause for worry (through particle production). The late time moduli domination may lead to the infamous moduli problem [42].

IV. DISCUSSION

In this section, we would like to comment on the conclusion that we cannot get power-law inflation (or quintessence) from the 3-form induced potential. The reason seems related to comments in [7,8]; exponential potentials consistent with the constraints of supersymmetry are generically too steep. Our results, then, are consistent with a generalization to many fields of the work of [7,8] that a system cannot simultaneously relax to a supersymmetric minimum and cause cosmological acceleration. Even though the models considered here do not necessarily preserve supersymmetry, they are all classically of ''no-scale'' structure, meaning that they all have vanishing cosmological constant and no potential for the radial moduli. So even the nonsupersymmetric vacua have characteristics of supersymmetric cases. Furthermore, the potential arises from the supergravity Ward identity [24,28], which means it suffers from the same kind of constraints imposed by the arguments of [7,8]. Heuristically, the vacua of our system give Minkowski spacetime, which is static, and there is no way to accelerate into a static state.

This sort of argument based on supersymmetry is readily generalized to the Calabi-Yau models with 3-form fluxes that were studied in [11]. Indeed, the form of the bulk mode potential (11) is identical, although the complex structure decomposition of the metric will differ from case to case. The key thing to note is that the overall scale of the internal manifold is always a modulus, as if we set $\sigma_{1,2,3} = \sigma$. In fact, it works out so that the exponential prefactor gives the same $a \sim t^{1/3}$ evolution. The potential for brane modes should also be similar, at least for small non-Abelian parts of the brane coordinates. Considering a more complicated CY compactification is not the route to an accelerating universe. Again, this seems to be a feature of the broken supersymmetry.

We should contrast this case to other work that does find inflationary physics in supergravity. In the 1980s, Refs. [43,44] found no-scale supergravities with inflation, but they specified the potential to give slow-roll inflation. The freedom to insist on inflation does not exist here. More recently, other gauged supergravities have been found that can give at least a give few e-foldings of inflation [2,3,33,34], but these do not yet have a known embedding in string theory. These gauged supergravities are not of the no-scale type and have a cosmological constant. Also, [3,4,6] describe inflation based on the motion of branes in a warp factor. In fact, [3,6] use a background very similar to the one considered here but include the warp factor.

There is clearly, then, some hope for finding acceleration in compactifications with 3-form magnetic fields, and it is possible to think of other methods than D3-brane motion. For example, the warp factor can modify the potential, although it does not seem likely to change the basic features. Another possibility is that the small volume region of moduli space, where supergravity breaks down, has a different form of the potential. It has been argued that some type IIB compactifications with flux with one T^2 shrinking are dual to heterotic compactifications with intrinsically stringy monodromies [16,45], so it is conceivable that inflation could occur in such a compactification with a decelerating end stage described by our model.

Finally, there are many possible corrections associated with supersymmetry breaking. It is known that there should be stringy corrections to the potential in nonsupersymmetric cases and that these would break the no-scale structure, giving the radial modulus mass (at least in the CY case) [46], and there should also be supergravity loop corrections. It would be very difficult to compute this potential, but it seems likely that the potential could have a local maximum for the compactification radius, allowing for inflation. There are also potentials from instanton corrections, given by wrapped Euclidean D3-branes [22]. Since the instanton action is proportional to the volume of the cycle it wraps, it would actually generate a potential like the exponential of an exponential. This type of potential could very possibly be shallow enough to support inflation, although we have not investigated this point.

In summary, we have examined the cosmology induced by 3-form fluxes in type IIB superstring compactifications

and concluded that the classical bulk action does not lead to inflation or quintessence because the potential contains exponential factors that are too steep, much as in [7,8]. However, we have noted loopholes in our analysis which could allow accelerating cosmologies. We leave the exploration of those loopholes for future work.

ACKNOWLEDGMENTS

The authors are thankful to Cliff Burgess, Ed Copeland, Sergio Ferrara, and Joseph Polchinski for helpful discussions and feedback. The work of A.F. is supported by National Science Foundation grant PHY97-22022.

[1] A.D. Linde, *Particle Physics and Inflationary Cosmology*, Contemporary Concepts in Physics Vol. 5 (Harwood Academic, Chur, Switzerland, 1990), p. 362.
[2] R. Kallosh, A.D. Linde, S. Prokushkin, and M. Shmakova, Phys. Rev. D **65**, 105016 (2002).
[3] R. Kallosh, hep-th/0205315.
[4] C. Herdeiro, S. Hirano, and R. Kallosh, J. High Energy Phys. **12**, 027 (2001).
[5] A. Mazumdar, S. Panda, and A. Perez-Lorenzana, Nucl. Phys. **B614**, 101 (2001).
[6] K. Dasgupta, C. Herdeiro, S. Hirano, and R. Kallosh, Phys. Rev. D **65**, 126002 (2002).
[7] S. Hellerman, N. Kaloper, and L. Susskind, J. High Energy Phys. **06**, 003 (2001).
[8] W. Fischler, A. Kashani-Poor, R. McNees, and S. Paban, J. High Energy Phys. **07**, 003 (2001).
[9] M. Graña and J. Polchinski, Phys. Rev. D **63**, 026001 (2001).
[10] S.S. Gubser, hep-th/0010010.
[11] S.B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev. D **66**, 106006 (2002).
[12] S. Gukov, C. Vafa, and E. Witten, Nucl. Phys. **B584**, 69 (2000).
[13] K. Dasgupta, G. Rajesh, and S. Sethi, J. High Energy Phys. **08**, 023 (1999).
[14] B.R. Greene, K. Schalm, and G. Shiu, Nucl. Phys. **B584**, 480 (2000).
[15] K. Becker and M. Becker, Nucl. Phys. **B477**, 155 (1996).
[16] K. Becker and K. Dasgupta, J. High Energy Phys. **11**, 006 (2002).
[17] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[18] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[19] P. Mayr, J. High Energy Phys. **11**, 013 (2000).
[20] H. Verlinde, Nucl. Phys. **B580**, 264 (2000).
[21] S. Kachru, M. Schulz, and S. Trivedi, hep-th/0201028.
[22] A.R. Frey and J. Polchinski, Phys. Rev. D **65**, 126009 (2002).
[23] R. Argurio, V.L. Campos, G. Ferretti, and R. Heise, Nucl. Phys. **B640**, 351 (2002).
[24] V.A. Tsokur and Y.M. Zinoviev, Phys. At. Nucl. **59**, 2192 (1996).
[25] L. Andrianopoli, R. D'Auria, S. Ferrara, and M.A. Lledo, Nucl. Phys. **B640**, 46 (2002).
[26] L. Andrianopoli, R. D'Auria, S. Ferrara, and M.A. Lledo, J. High Energy Phys. **07**, 010 (2002).
[27] L. Andrianopoli, R. D'Auria, S. Ferrara, and M.A. Lledo, Nucl. Phys. **B640**, 63 (2002).
[28] R. D'Auria, S. Ferrara, and S. Vaula, New J. Phys. **4**, 71 (2002).
[29] S. Ferrara and M. Porrati, Phys. Lett. B **545**, 411 (2002).
[30] A. Sen, Int. J. Mod. Phys. A **9**, 3707 (1994).
[31] J.H. Schwarz and A. Sen, Phys. Lett. B **312**, 105 (1993).
[32] J. Maharana and J.H. Schwarz, Nucl. Phys. **B390**, 3 (1993).
[33] P. Fre, M. Trigiante, and A. Van Proeyen, Class. Quantum Grav. **19**, 4167 (2002).
[34] R. Kallosh, A. Linde, S. Prokushkin, and M. Shmakova, Phys. Rev. D **66**, 123503 (2002).
[35] R.C. Myers, J. High Energy Phys. **12**, 022 (1999).
[36] J. Polchinski and M.J. Strassler, hep-th/0003136.
[37] A.R. Liddle, A. Mazumdar, and F.E. Schunck, Phys. Rev. D **58**, 061301 (1998).
[38] E.J. Copeland, A. Mazumdar, and N.J. Nunes, Phys. Rev. D **60**, 083506 (1999).
[39] A.L. Berkin, K.-I. Maeda, and J. Yokoyama, Phys. Rev. Lett. **65**, 141 (1990).
[40] A.L. Berkin and K.-I. Maeda, Phys. Rev. D **44**, 1691 (1991).
[41] A. Mazumdar, Phys. Lett. B **469**, 55 (1999).
[42] B. de Carlos, J.A. Casas, F. Quevedo, and E. Roulet, Phys. Lett. B **318**, 447 (1993).
[43] C. Kounnas and M. Quiros, Phys. Lett. **151B**, 189 (1985).
[44] J.M. Molera and M. Quiros, Phys. Rev. D **36**, 375 (1987).
[45] S. Hellerman, J. McGreevy, and B. Williams, hep-th/0208174.
[46] K. Becker, M. Becker, M. Haack, and J. Louis, J. High Energy Phys. **06**, 060 (2002).
[47] C.P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh, and R.-J. Zhang, J. High Energy Phys. **07**, 047 (2001).