Q-ball formation in the wake of Hubble-induced radiative corrections

Rouzbeh Allahverdi
Physik Department, TU München, James Frank Strasse, D-85748, Garching, Germany

Anupam Mazumdar and Altug Ozpineci
The Abdus Salam International Centre for Theoretical Physics, I-34100 Trieste, Italy
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We discuss some interesting aspects of Q-ball formation during the early oscillations of the flat directions. These oscillations are triggered by the running of a soft $(\text{mass})^2$ stemming from the nonzero energy density of the Universe. However, this is quite different from standard Q-ball formation. The running in the presence of gauge and Yukawa couplings becomes strong if $m_{1/2}/m_0$ is sufficiently large. Moreover, Q balls which are formed during the early oscillations constantly evolve, due to the redshift of the Hubble-induced soft mass, until the low-energy supersymmtery breaking becomes dominant. For smaller $m_{1/2}/m_0$, Q balls are not formed during early oscillations because of the shrinking of the instability band due to the Hubble expansion. In this case, Q balls are formed only at the weak scale, but typically carry smaller charges, as a result of their amplitude redshift. Therefore, the Hubble-induced corrections to the flat directions give rise to a successful Q-ball cosmology.

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I. INTRODUCTION

The presence of *flat directions*, generally denoted by ϕ, in the field space along which the classical potential vanishes is quite generic in the minimal supersymmetric standard model (MSSM) [1]. This has interesting cosmological implications. In particular flat directions which are built on squarks and sleptons, and carry a nonzero $B-L$, can generate the observed baryon asymmetry in the context of the Affleck-Dine baryogenesis mechanism [2–4]. As a second offshoot, the formation of Q balls [5,6] in supersymmetric (SUSY) theories can come from the presence of flat directions carrying a $U(1)$ charge [7–12]. A Q ball is a nontopological soliton whose stability is confirmed due to the presence of a nonvanishing charge Q it carries associated with a $U(1)$ symmetry [5,6].

It has been shown [9–11] that homogeneous oscillations of flat directions can be fragmented into Q balls if the flat direction potential grows more slowly than $|\phi|^2$, or, equivalently if $V(\phi)/|\phi|^2$ has a minimum at $|\phi|\neq 0$. The numerical simulations also support this idea [13]. This can be understood by noting that flat direction oscillations for such potentials behave as a fluid with a negative pressure. Therefore, spatial inhomogeneities around the zero-mode condensate along the flat direction, set by inflationary fluctuations [10], can exponentially grow. When the modes go nonlinear, a lump of Q matter forms with a physical size set by the wavelength of the fastest growing mode. The Q-ball formation is usually studied in this context when the low-energy SUSY breaking triggers the flat direction oscillations.

However, all the flat directions of the MSSM are subject to modification at scales well above the low-energy SUSY breaking scale. In fact, nonrenormalizable superpotential terms induced by the new physics remove the flatness for large $|\phi|$ [3,4]. Furthermore, in supergravity (SUGRA) models, SUSY breaking terms by the inflation sector generally induce soft mass terms for scalar fields [3,4,14] (unless forbidden by some symmetry). It is also possible to obtain Hubble-induced gaugino soft mass $m_{1/2}$, and Hubble-induced A term unless prohibited by an R symmetry. This has a very important consequence for Affleck-Dine baryogenesis as we will explain shortly.

Very recently, the running of the flat direction soft masses when the Hubble-induced SUSY breaking is dominant, called the Hubble-induced radiative corrections, has been studied [15]. There, the main focus was on the viability of the Affleck-Dine baryogenesis in the wake of Hubble-induced radiative corrections. The main conclusion was that the $H_{\rm u}L$ flat direction is the most promising one. A similar case study can be made for the Q-ball formation in the early Universe, which is the topic of this paper. We show that flat directions made up of squarks and sleptons may undergo early oscillations, as a result of the Hubble-induced radiative corrections. The Q balls can be formed during these early oscillations if $m_{1/2}/m_0$ is sufficiently large. For smaller values of $m_{1/2}/m_0$, Q balls are formed when the low-energy SUSY breaking becomes dominant. The main point here is that during early oscillations the soft masses are determined by the Hubble parameter. Therefore, the width of the instability band is redshifted $\sim H$ which is quicker than the redshift of the unstable modes $\propto a^{-1}$, where a is the scale factor of the Universe. If perturbations do not have enough time to grow then Q balls are not formed. However, the expansion of the Universe reduces the amplitude of oscillations by the time the low-energy SUSY breaking takes over. This has interesting consequences for the subsequent formation of Q balls.

We begin by discussing the dynamics of the flat direction. Then we briefly describe the Hubble-induced radiative corrections to the scalar potential. In the subsequent section we discuss Q-ball formation and consequences of a phase of early oscillations. Finally, we conclude our paper.

II. THE FLAT DIRECTION DYNAMICS

In the early Universe the energy density stored in the inflaton field [16] is the dominant source of SUSY breaking, and induces a $(\text{mass})^2 \propto H^2$ for all the MSSM flat directions, where H is the expansion rate of the Universe [3,4,14]. The effect of such a mass term crucially depends on the size and the sign of the constant of proportionality. For a positive $(\text{mass})^2 \ll H^2$, SUSY breaking by the inflation sector has no significant consequences. On the other hand, if $(\text{mass})^2 \gg H^2$, the flat direction is heavy enough to settle down at the bottom of the potential during inflation. A stable scalar field might even act as cold dark matter [17].

Perhaps, the most interesting case occurs for a $(\text{mass})^2 \sim -H^2$, since it naturally leads to a nonzero vacuum expectation value (VEV) for the flat direction at the onset of its oscillations. This can be realized at the tree level in simple extensions of minimal supergravity models [3,4], and from one-loop corrections to the Kähler potential in no-scale supergravity models [18]. A detailed examination of the scenario with $(\text{mass})^2 \sim -H^2$, including a systematic treatment of nonrenormalizable superpotential terms which lift the flat direction has been performed in Ref. [4]. The SUSY breaking by the inflaton energy density and by the hidden sector results in terms

$$-C_I H^2|\phi|^2 + \left(a\lambda_n H \frac{\phi^n}{nM^{n-3}} + \text{H.c.}\right) + m_{\phi,0}^2|\phi|^2 + \left(A_{\phi,0}\lambda_n \frac{\phi^n}{nM^{n-3}} + \text{H.c.}\right) \tag{1}$$

in the scalar potential.[1] The first and the third terms are the Hubble-induced and low-energy soft mass terms, respectively, while the second and the fourth terms are the Hubble-induced and the low-energy A terms, respectively. The Hubble-induced soft terms typically dominate the low-energy ones for $H > m_{3/2}$, with $m_{3/2}$ being the gravitino mass. If $C_I > 0$, the absolute value of the flat direction during inflation settles at the minimum given by

$$|\phi| \simeq \left(\frac{C_I}{(n-1)\lambda_n} H_I M^{n-3}\right)^{1/n-2}, \tag{2}$$

with H_I being the Hubble constant during the inflationary epoch. After the end of inflation, $\langle\phi\rangle$ initially continues to track the instantaneous local minimum of the scalar potential, which can be estimated by replacing H_I by $H(t)$ in Eq. (2). Once $H \simeq m_{3/2}$, the low-energy soft SUSY breaking terms take over and ϕ starts oscillating. It has recently been noticed that various thermal effects can trigger oscillations at $H \gg m_{3/2}$ [20,21]. Here we show that it is also possible to obtain a phase of early oscillations from the Hubble-induced radiative corrections to the scalar potential.

[1] For our purpose we consider nonrenormalizable superpotential terms $\lambda_n \Phi^n / nM^{n-3}$, where Φ is the superfield comprising ϕ and its fermionic partner, and M is the scale of new physics which induces such a term. All of the MSSM flat directions are lifted at $n \leq 9$ level, if there is no other symmetry except the standard model gauge group [19].

III. HUBBLE-INDUCED RADIATIVE CORRECTIONS

It has been shown in a recent study [15], that the Hubble-induced radiative corrections can significantly change the shape of the flat direction potential. This is especially important during and right after the end of inflation if the inflationary scale is above the weak scale, and if thermal equilibrium is achieved at sufficiently late times. Otherwise, thermal corrections to the scalar masses [20,21] always dominate the radiative corrections. We therefore consider models with a low reheat temperature which can be naturally realized in models where the inflation sector is gravitationally coupled to the matter sector.

All fields which have gauge or Yukawa couplings to the flat direction contribute to the logarithmic running of its $(\text{mass})^2$. Therefore, one should study the evolution of the flat direction $(\text{mass})^2$ from some higher scale such as M_{GUT} down to low scales in order to determine the location of the true minimum of the potential. The running of low-energy soft breaking masses has been studied in great detail in the context of MSSM phenomenology [22]. The one-loop beta functions for the $(\text{mass})^2$ of the MSSM scalars receive opposite contributions from the scalar and the gaugino loops [1]. If the top Yukawa coupling is the only large one (i.e., as long as $\tan\beta$ is not very large), the beta functions for the $(\text{mass})^2$ of squarks of the first and the second generations, then the sleptons only receive significant contributions from the gaugino loops. A review of these effects can be found in Ref. [22]. Here we only mention the main results for the universal boundary conditions, where at $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV the soft $(\text{mass})^2$ of all the scalars is m_0^2, while the gauginos have a common soft mass $m_{1/2}$.

For the first and second generations of squarks

$$m^2 \simeq m_0^2 + (5-7)m_{1/2}^2, \tag{3}$$

at the weak scale, while for the right-handed and the left-handed sleptons

$$m^2 \simeq m_0^2 + 0.1 m_{1/2}^2, \quad m^2 \simeq m_0^2 + 0.5 m_{1/2}^2, \tag{4}$$

respectively. These results are independent of $\tan\beta$, while the soft breaking $(\text{mass})^2$ of the third generation of squarks has some dependence on $\tan\beta$. On the other hand, the $(\text{mass})^2$ of H_u becomes negative at low scales, e.g.,

$$m^2 \simeq -\frac{1}{2}m_0^2 - 2m_{1/2}^2, \tag{5}$$

for the choice of $\tan\beta = 1.65$ [22]. The important point is that the sum $m_{H_u}^2 + m_L^2$, which describes the mass of the $H_u L$ flat direction, is driven to negative values at the weak scale only for $m_{1/2} \gtrsim m_0$.

Similarly, one could follow the evolution of the soft breaking terms when the Hubble-induced supersymmetry breaking terms are dominant.[2] However, the boundary conditions are quite different here, since m_0^2 and $m_{1/2}$ are determined by the scale of inflation, and the form of the Kähler potential. They become completely negligible at low scales, and, they have no bearing on present phenomenology, e.g., at present the Hubble parameter is $H(0)\sim\mathcal{O}(10^{-33})$ eV. For more details in this regard we refer the reader to Ref. [15]. Here we draw the main conclusions which play important roles in our subsequent discussion on the formation of Q balls.

A typical flat direction ϕ is a linear combination $\phi=\Sigma_{i=1}^{N}a_i\varphi_i$ of the MSSM scalars φ_i, implying that $m_\phi^2=\Sigma_{i=1}^{N}|a_i|^2m_\varphi^2$. As mentioned before, for a given m_0^2, the running of m_ϕ^2 crucially depends on $m_{1/2}$. A Hubble-induced gaugino mass can be produced from a non-minimal dependence of the gauge superfield kinetic terms on the inflaton field. Generically, the gauge superfield kinetic terms must depend on the field(s) of the hidden, or, secluded sector in order to obtain gaugino masses of roughly the same order, or larger than the scalar masses, as required by phenomenology. Having $m_{1/2}\sim H$ thus appears to be quite natural unless an R symmetry forbids terms which are linear in the inflaton superfield [4]. The same also holds for the Hubble-induced A terms. The μ term is a bit different. Since, it does not break supersymmetry, there is *a priori* no reason to assume that μ of order H will be created. As noticed in Ref. [15], the viability of Affleck-Dine baryogenesis only requires $\mu(M_{\rm GUT})\lesssim H/4$.

For $C_I\approx-1$, only the H_uL flat direction acquires a negative (mass)^2 at low scales. The flipping in the sign occurs at a scale q_c, which is above $\sim\mathcal{O}(1)$ TeV, unless $m_{1/2}<H$. However, the exact value of q_c also depends on a number of other model parameters, e.g., $\tan\beta$.

For $C_I\approx+1$, the H_uL flat direction always has a negative (mass)^2 at small scales, but for $m_{1/2}\gtrsim 3H$ the (mass)^2 flips its sign twice. The slepton masses only receive positive contributions from the electroweak gaugino loops. Therefore, the LLE flat direction (mass)^2 remains negative down to small scales unless $m_{1/2}\gtrsim 2H$, in particular $q_c\gtrsim 10^9$ GeV for $m_{1/2}\gtrsim 3H$. The squared masses of all the squarks (but the right-handed top squark), and with a fair approximation the $U_3D_iD_j$ and the LQD flat directions, change sign unless $m_{1/2}\lesssim H/3$. In particular, $q_c\simeq 10^{10}$ (10^{15}) GeV, for $m_{1/2}/H=1$ (3). This is due to the large positive contribution from gluino loops to the squared squark masses below $M_{\rm GUT}$. The exact value of q_c is almost independent of other model parameters in this case, unlike the H_uL case for $C_I\approx-1$. This is simply because the top Yukawa coupling has almost no effect on the flat directions other than H_uL. This also suggests that the same conclusions essentially hold for flat directions built on sleptons and particularly squarks when $C_I\approx-1$, which we numerically verified.

The (mass)^2 of the flat directions made up of squarks and sleptons increases towards the lower scales and can be written as

$$m^2\sim m_0^2\left(1+K\ln\left(\frac{|\phi|^2}{M_{\rm GUT}^2}\right)\right),\quad |\phi|\leq M_{\rm GUT},\qquad(6)$$

where K is a negative constant approximately given by [10,11]

$$K\approx-\frac{\alpha}{8\pi}\frac{m_{1/2}^2}{m_0^2}.\qquad(7)$$

Here α represents the gauge fine structure constant at the GUT scale, and $m_{1/2}$, m_0^2 are the universal gaugino soft mass and scalar soft (mass)^2 at the GUT scale, respectively. When the low-energy SUSY breaking is dominant one typically finds, by taking phenomenological constraints at the weak scale into account, $|K|\simeq 0.01-0.1$ [10,11]. However, our case at hand is different. In the early Universe the Hubble-induced SUSY breaking determines the ratio $m_{1/2}^2/m_0^2$, which can be $\gg 1$ without affecting the low energy phenomenology. This is an important point, to which we shall come back shortly.

IV. EARLY *Q*-BALL FORMATION

A. Cosmological setup

During inflation all scalar fields, including the flat directions, with a mass $m<H$ have quantum fluctuations [16]

$$\delta\phi\sim\frac{H_I}{2\pi},\qquad(8)$$

which become classical when a particular mode leaves the horizon. These fluctuations, upon reentering the horizon, act as initial seed perturbations which will get amplified when the flat direction starts oscillating after the end of inflation [10]. Note, for our purpose we strictly assume that the Universe remains matter-dominated for a sufficiently long time after the end of inflation. This will be the case for models with a low reheat temperature.

B. The flat direction oscillations

As mentioned earlier, the flat directions built on squarks and/or sleptons receive a positive contribution from the gaugino loops which increases their (mass)^2 towards lower scales. For $|m_0^2|\approx H^2$, the rate at which m_ϕ^2 increases only

[2]Apart from the gauge and the Yukawa interactions of the flat directions there also exist nonrenormalizable interactions coming from nonminimal kinetic terms in the Kähler potential and nonminimal coupling of the flat directions to the gravity. The flat direction (mass)^2 receives radiative corrections from all these interactions. However, at scales well below the Planck scale 2×10^{18} GeV, the running from nonrenormalizable interactions are taken over by that from the gauge and the Yukawa interactions. We therefore assume that well below the grand unified theory (GUT) scale it is sufficient to consider the running due to gauge and Yukawa couplings, while including the running from nonrenormalizable interactions between $M_{\rm Planck}$ and $M_{\rm GUT}$ in the boundary condition for C_I at $M_{\rm GUT}$. Roughly speaking, this is equivalent to integrating out the heavier modes above the GUT scale.

depends on $m_{1/2}/H$. For a large enough $m_{1/2}/H$ the flat direction (mass)^2 exceeds H^2 at a large scale $\sim q_c$, irrespective of the sign of C_I in Eq. (1), leading to early oscillations of the flat direction. In order to examine such a possibility we should compare q_c with the instantaneous VEV of the flat direction $(HM^{n-3})^{1/n-2}$; see Eq. (2). We note that as long as $q_c<(HM^{n-3})^{1/n-2}$, the flat direction tracks the instantaneous VEV. Also note that the value of q_c is fixed by the ratio $m_{1/2}/H$ while the Hubble parameter is constantly decreasing in time. Therefore, an overlap eventually occurs when

$$H_O \sim \frac{q_c^{n-2}}{M^{n-3}}. \tag{9}$$

If $m_{3/2}<H_O<H_I$, then the flat direction starts early oscillations with an initial amplitude $\sim q_c$. Note that any overlap during inflation has no interesting consequences, because oscillations exponentially die down during inflation. A phase of early oscillations is ensured in gravity-mediated SUSY breaking models if $H_O>10^3$ GeV, while for gauge-mediated models $H_O>10^{-4}$ GeV can be sufficient, since the mass of the gravitino is generically much smaller in these models.

In order to study the possibility of Q-ball formation during early oscillations, we briefly recall the analysis of the amplification of the instabilities [9–11]. Let us consider an oscillating homogeneous background $\phi=(Re^{i\theta})/\sqrt{2}$ with a perturbation $\delta R \propto e^{(\alpha(t)+ikx)}$, and $\delta\theta \propto e^{(\alpha(t)+ikx)}$. The equations of motion for the perturbations read [9–11]

$$\left(\ddot{\alpha}+\dot{\alpha}^2+3H\dot{\alpha}+\frac{k^2}{a^2}+V''-\dot{\theta}^2\right)\times\left(\ddot{\alpha}+\dot{\alpha}^2+3H\dot{\alpha}+\frac{k^2}{a^2}+\frac{2\dot{R}}{R}\dot{\alpha}\right)+4\dot{\theta}^2\dot{\alpha}^2=0, \tag{10}$$

where V is the flat direction potential, dot denotes time derivative, and prime denotes derivative with respect to R. If $\dot{\theta}^2-V''>0$, then there is an instability band within the momentum range $0<k<k_{\max}$, where

$$k_{\max}=a\sqrt{\dot{\theta}^2-V''}. \tag{11}$$

In the above expression $k_{\max}$ can either be a constant, or increasing. In these cases the modes grow. Otherwise, the modes are removed constantly from the instability band due to the momentum redshift.

For quantitative estimates it is sufficient to focus on the case of gravity-mediated models [10,11], as this is clearly the situation of interest regarding the Hubble-induced SUSY breaking. We first analyze the situation when $|K|$ in Eq. (6) is smaller than one. We have verified numerically that $|K|<1$, if $m_{1/2}<5H$. The behavior of the potential, which is governed by the running mass of the flat direction, is given by

$$V\sim H^2\left(1+K\ \ln\left(\frac{|\phi|^2}{q_c^2}\right)\right)|\phi|^2, \quad |\phi|\leqslant q_c, \tag{12}$$

where we have taken into account the Hubble-induced corrections to the flat direction. The flat direction ϕ starts oscillating about the origin, with an amplitude $\sim q_c$, when $H\sim H_O$. Upon averaging over oscillations, one obtains an equation of state with a negative pressure

$$p\approx\frac{K}{2}\rho, \tag{13}$$

which results in $k_{\max}\approx 2a|K|^{1/2}H$ [10,11]. It takes a finite amount of time for a certain mode to become nonlinear. A fairly good estimate of this time scale has been given in Refs. [10,11]:

$$\Delta t\approx\frac{10}{|K|}H^{-1}. \tag{14}$$

This is also a good estimate of the time scale for forming a Q ball from the oscillations. For $|K|<1$, this time scale exceeds one Hubble time, therefore several oscillations will be needed to form Q balls. However, this is not the end of the story, because the flat direction (mass)^2 is also redshifted. As a result physical modes within the instability band $k_{\text{phys}}<|K|^{1/2}H$ are either outside or, at best, marginally inside the horizon. Furthermore, the width of the instability band shrinks more quickly $\propto a^{-3/2}$, compared to the redshift of momentum modes $\propto a^{-1}$ in a matter-dominated Universe. This implies that no subhorizon mode can be made unstable before the low-energy SUSY breaking takes over. Therefore, we shall not be able to form Q balls for $|K|<1$.

The situation is quite different if $|K|\geqslant 1$, which is the case if $m_{1/2}\gtrsim 5H$. Now, we cannot use the expression in Eq. (14), which was derived when $|K|<1$, reliably. On the other hand, if we continue doing so, the time required for perturbations to become nonlinear is seen to be less than the period of oscillations $2\pi/m$. It may sound contradictory to the whole essence of forming Q balls from negative pressure behavior of an oscillating condensate. However, in this particular case it is a rapid change in the shape of the potential which is responsible for the growth of the instabilities. This happens due to the strong running of the flat direction (mass)^2. As a consequence, the fragmentation of the zero-mode condensate occurs very rapidly within one oscillation due to nonadiabatic time variation in m_ϕ^2. This we call here an *instant* Q-ball formation. This is an interesting possibility, which has not been investigated so far. This phenomena cannot happen when the low-energy SUSY breaking terms are dominant, because one typically encounters $|K|\lesssim 0.1$ [10–12], based on phenomenological constraints at the weak scale.

Finally, we note that for $H>m_{3/2}$, the shape of the potential is slowly varying in time due to the redshift of m_0^2 by the Hubble expansion. This implies that any Q balls which are formed during early oscillations must evolve too. It is conceivable though that these Q balls are eventually stabilized with a size set by the final shape of the potential. Nevertheless, their charge and number density can be quite different if they are formed at early times. A more precise estimate requires a better knowledge of the detailed dynamics of the growth of the perturbations when $|K|>1$. This we are lacking at the moment and it goes beyond the scope of the present paper.

V. LATE-TIME FORMATION OF SMALL *Q* BALLS

In this section, we study two consequences when $|K|<1$, occurring for $3H \leqslant m_{1/2} \leqslant 5H$. As mentioned earlier, this can lead to early oscillations, albeit no early formation of Q balls.

A. Baryogenesis

The scale q_c at which the flat direction $(\text{mass})^2$ becomes $\approx H^2$ is mainly determined by the ratio $m_{1/2}/H$; note that the dependence is practically exponential. For example, for $m_{1/2} \approx 4H$, we find $q_c \geqslant 3 \times 10^{15}$ GeV for all squarks and sleptons, except the right-handed top squark. With such a value for $m_{1/2}$, for a typical A term, we find $A_{q_c} \sim \mathcal{O}(H)$, almost independent of its boundary value. This is a consequence of the gaugino loop contribution to the A-term beta function. Moreover, a relative phase between $m_{1/2}$ and $A(M_{\rm GUT})$ implies that the phase of the A term also runs along with the VEV. This is interesting since one can obtain a phase mismatch required for the torque, which induces a spiral motion to the flat direction and consequently generates a baryon asymmetry. We have checked that for $m_{1/2} \gtrsim 3H$ a good yield of baryon asymmetry with $n_B/n_\phi \sim 10^{-2}$ can be comfortably accommodated. Once the Hubble-induced A term is effectively switched off, the comoving baryon asymmetry becomes frozen. Our conclusion is that baryogenesis during early oscillations only depends on the Hubble-induced SUSY breaking parameters, and it is independent of the low-energy soft mass and A term.

B. Damping the amplitude of oscillations

From the onset of oscillations at H_O, given by Eq. (9), the flat direction simply oscillates until the low-energy SUSY breaking effects take over. The amplitude of the oscillations decreases as $\propto t^{-1/2}$, while the redshift of the flat direction mass is $\propto t^{-1}$. As we have described, no Q balls are formed during this interval for $|K|<1$. The situation rapidly changes once the low-energy SUSY breaking terms take over, because the width of the instability band becomes constant thereon. Then there is a chance for the subhorizon modes to grow, and eventually collapse into lumps of Q balls. However, the amplitude of flat direction oscillations at that time has decreased to a new value

$$|\phi| \sim q_c \left(\frac{H_0}{H_O} \right)^{1/2}, \qquad (15)$$

due to the redshift during the matter-dominated era, where H_0 determines the Hubble rate at the time of low energy SUSY breaking scale. By replacing H_O from Eq. (9), we obtain

$$|\phi| \sim q_c \left(\frac{H_0 M^{n-3}}{q_c^{n-2}} \right)^{2/3}. \qquad (16)$$

Now, in order to illustrate the significance of dampening of the amplitude, let us compare it with the amplitude when the oscillations had started at $H \simeq H_0$. In the latter case the flat direction would have tracked the instantaneous VEV, which at $H \simeq H_0$ is given by $\phi_0 \sim (H_0 M^{n-3})^{1/(n-2)}$. The ratio of the two amplitudes is then given by

$$\frac{|\phi|}{|\phi_0|} \sim \left[\frac{(H_0 M^{n-3})^{1/(n-2)}}{q_c} \right]^{(n-4)/2}. \qquad (17)$$

Note that this is less than one if $q_c > (H_0 M^{n-3})^{1/(n-2)}$. Therefore, early oscillations in general damp the amplitude of the oscillations at H_0, leading to the formation of smaller Q balls.

This has important consequences for Q-ball cosmology. Recent detailed analysis by lattice simulations show that almost all of the generated baryon-lepton asymmetry is absorbed into the Q balls [13]. This can be problematic if large Q balls are formed. In gravity-mediated SUSY breaking models the late decay of large B balls below the freeze-out temperature for the lightest supersymmetric particle (LSP) annihilation, while generating $n_B/n_\gamma \sim \mathcal{O}(10^{-10})$, results in LSP overproduction [11].[3] Note that the charge of the Q balls in gravity-mediated models is proportional to the square of the amplitude; $Q \propto |\phi|^2$ [13]. Therefore, early oscillations of the flat direction indeed help in reducing the Q-ball charge. As an example, consider a flat direction which is lifted at the $n=8$ superpotential level with $M = M_{\rm Planck}$. Then it turns out that $m_{1/2} \approx 4H$ will be sufficient to trigger early oscillations, resulting in the formation of Q balls which are 6 orders of magnitude smaller. The situation is clearly better for smaller n and/or $M = M_{\rm GUT}$, since a smaller $m_{1/2}/H$ will also help in decreasing the amplitude of the oscillations.

In the gauge-mediated models the SUGRA contribution dominates the flat direction potential at large VEVs, though with a smaller gravitino mass $m_{3/2}$. For example, with $m_{3/2} \sim 10^{-4}$ GeV, the potential reads

$$V \sim m_{3/2}^2 \left(1 + K \ln \left(\frac{|\phi|^2}{M_{\rm GUT}^2} \right) \right) |\phi|^2, \qquad (18)$$

for $|\phi| > 10^{10}$ GeV. On the other hand, for $|\phi| \leqslant 10^{10}$ GeV, the gauge-mediated contribution

$$V \sim m_{\rm SUSY}^4 \ln \left(\frac{|\phi|^2}{m_{\rm SUSY}^2} \right), \quad |\phi| \geqslant m_{\rm SUSY}, \qquad (19)$$

dominates the potential, with $m_{\rm SUSY} \sim \mathcal{O}(\text{TeV})$ [24]. In this case B balls with a charge $B \geqslant 10^{12}$ act as a stable dark matter candidate in the Universe [9]. However, stable B balls overclose the Universe in almost all the regions of the parameter space, if they ought to yield an acceptable baryon asymmetry [25].

[3]This problem can be alleviated for the Higgsino- and *W*-ino-like dark matter [23].

Our earlier estimates can be repeated to show that the Hubble-induced radiative corrections indeed lead to early oscillations which eventually decrease the charge of the Q balls. However, the reduction in the Q-ball charge now depends on where $|\phi|$ starts; see Eq. (16). If $|\phi| \geqslant 10^{10}$ GeV, the SUGRA corrections dominate and $Q \propto |\phi|^2$; thus our earlier arguments hold even for the gauge-mediated models. On the other hand, for $|\phi| < 10^{10}$ GeV, we have $Q \propto |\phi|^4$ [8]. Therefore, early oscillations reduce the Q-ball charge even more significantly. As an example, consider a flat direction lifted at the $n=9$ superpotential level with $M = M_{\rm Planck}$. Then for $m_{1/2} \approx 4H$, the early oscillations result in the formation of gravity-mediated type Q balls which are 10^7 times smaller.[4] For smaller n, and/or $M = M_{\rm GUT}$, we find $|\phi| < 10^{10}$ GeV from Eq. (16), leading to the formation of even smaller Q balls.

We conclude that for $m_{1/2} \gtrsim 3H$, the flat directions built on squarks and/or sleptons in general undergo early oscillations. Then the amplitude of the oscillations is redshifted by the expansion of the Universe. This results in the formation of smaller Q balls in both gravity and gauge-mediated models of SUSY breaking. In Ref. [27] a gauged $U(1)_{B-L}$, which is broken at a scale $v \sim 120^{14}$ GeV, has been invoked to reduce the charge of a Q ball. In the gravity-mediated models the D term from $U(1)_{B-L}$ helps form smaller B balls from the oscillations of the flat directions with a nonzero $B-L$ at the weak scale. However, as noticed by the authors, this mechanism does not improve the situation for the gauge-mediated models. The scenario mentioned here works well for both gravity- and gauge-mediated models, thus helping the emergence of a successful Q-ball cosmology.

VI. CONCLUSION

We have considered possible effects of radiative corrections to the Hubble-induced soft (mass)^2 of the flat directions on Q-ball formation. The key observation is that for a Hubble-induced soft gaugino mass $m_{1/2} \gtrsim 3H$, the (mass)^2 of all squarks and sleptons exceeds H^2 at a scale $q_c \gtrsim 10^{15}$ GeV and, for $m_{1/2} \gtrsim 4H$, it turns out that $q_c \sim M_{\rm GUT}$. Note, that the ratio $m_{1/2}/H$ is not constrained by the weak scale physics since the Hubble-induced SUSY breaking has no bearing on present phenomenology. Our conclusion is that all MSSM flat directions start oscillating shortly after the end of inflation, independently from the mechanism of low-energy SUSY breaking mediation. The formation of Q balls during early oscillations is different from the standard case. For $m_{1/2} > 5H$, the logarithmic running of the flat direction (mass)^2 is so strong that its nonadiabatic time variation may result in the fragmentation of the homogeneous condensate during just one oscillation, dubbed as *instant* Q-ball formation. However, Q balls which may be formed in this fashion constantly evolve. This is because of the change in the shape of the potential caused by redshift of the flat direction (mass)^2. It is conceivable that Q balls are eventually stabilized with a size set by the final shape of the potential, though with a different charge and number density. A more precise estimate requires quantitative analysis of Q-ball formation in this regime.

For $3H \lesssim m_{1/2} \lesssim 5H$, the early oscillations in general start, but no Q balls can be formed due to the continuous shrinking of the instability band caused by the Hubble expansion. The expansion also redshifts the amplitude of the oscillations leading to the formation of considerably smaller Q balls at low scales. We noted that for $m_{1/2} \gtrsim 3H$, the generated baryon asymmetry is determined by the Hubble-induced A term as it is also running, which can provide a phase mismatch required for generating baryogenesis. We presented examples of the flat directions which are lifted by nonrenormalizable superpotential terms of higher order and found a substantial reduction in the charge of the Q balls. Therefore, early oscillations can lead to a successful Q-ball cosmology in both gravity- and gauge-mediated SUSY breaking models.

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