Phase and Amplitude Control of Dipole Crabbing Modes in Multi-Cell Cavities

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ABSTRACT

This report sets out the established theory of driving a mode in a beam loaded, multi-cell cavity via a waveguide coupling and derives associated envelope equations from which time domain solutions are developed. Time domain solution allows accurate simulation with realistic microphonic spectra, beamloading, drive amplifier characteristics, controller and measurement errors to be encompassed. A computer code that uses the model is used to compute limits on phase and amplitude stability of a dipole mode as a function of disturbance and control system latency for a superconducting cavity being developed for use as a crab cavity for the ILC. A crab cavity is a deflection cavity operated with a 90^{o} phase shift with respect to the beam so that the front of a bunch gets kicked one way whilst the back gets kicked the other way.

CONTENTS

1. Background

The work described in this report has arisen from a requirement to predict the phase and amplitude control performance of superconducting crab cavities as required for the ILC [1].

The major output from the work is the computer code given in appendix III. It was unclear at the outset as to whether a satisfactory estimation of the phase and amplitude control performance could be made from an analytical analysis and hence we proceeded with the development of a code. The report contains derivations of the equations used and solved by code. It should be noted that many of the equations derived in the report are well known and can be found in similar but not necessarily identical forms elsewhere.

The code models the control of a multi-cell crab cavity with microphonics and beamloading, driven by an amplifier of limited bandwidth and in response to filtered measurements from an output coupler. Microphonics is included by specifying the natural frequency of the cavity as a function of time. The code does not include any coupling where electromagnetic forces drive mechanical vibrations [2]. Power consumption is not a key issue for the ILC crab cavities hence the cavity can be filled slowly well before the bunch arrives. This removes the requirement for the code to give an accurate representation of Lorentz detuning and potential electromechanical resonances. The code does not include noise from the electronic circuits of the LLRF system.

The code anticipates that a digital controller will be utilised and typical control loop delays assumed in simulations are consistent available digital processing technology [3]. The code shows how far controller gain must be reduced from the stability limit associated with controller time delays in the presence of unpredictable disturbances. The important disturbances are microphonics and beamloading. Beamloading for a crab cavity is likely to be unpredictable as it depends on the lateral offset of a bunch [4]. Microphonics is likely to be predicable however the analysis determines the limit of control when microphonics is managed by a PI controller that does not look forward or predict response.

2. Report Organisation

A large part of the report relates to the equations used by the code and simulations results demonstrating its outputs for a range of simplified or exaggerated situations which demonstrate qualitative validity. Sections 1 to 15 set up the equations.

Key equations with respect to the code are:-

- the enveloped equations (11.3a) and (11.3b) which are solved by the code,
- the equivalent circuit beamloading equations (7.3) and (7.3) and the consequential cavity beamloading equations (15.26) and (15.27),
- the PI controller with delayed action defined by equations (18.1) and (18.2),

Other useful results which are not needed by the code but can be used for design and code validation are:-

- the equations which determine the cavity power requirement from beamloading as a function of bunch offset and cavity parameters given in section 16,
- the stability limit for proportional control with fixed delay and periodic update (21.13),
- the stability limit for proportional control with fixed delay and rapid update (21.27),
- the stability limit for PI control with fixed delay and rapid update (21.41)
- an empirical estimate for (21.41) is given in (21.42)
- an empirical estimate for PI control with fixed delay and periodic update (21.45)

Distributed through the report are examples using ILC crab cavity proposed, parameters.

Shunt impedance, R/Q, bunch parameters, gradient, stored energies and power requirements are first given at the end of section 14. Discussion of the choice of external Q is given in section 16.

Input to the code detailing likely control system parameters and Microphonic spectra are first given in the table on figure 12 in section 19.

Section 8 is included as an illustration of beamloading and computes its results from the full time dependent differential equations for the cavity rather than from the envelope equations. An important part of our code validation procedure was the checking the co-incidence of results from integration of the full time dependent equations and integration of the envelope equations. Section 17 is provided as an illustration showing that beamloading has been correctly implemented.

The code went through various stages of development. A version of the code which assumed that the cavity only had excitation of one mode only was developed first. Including results for the analysis of a single mode cavity allows one to see more clearly the effect of adjacent modes when the multi-cell cavity is analysed. The single mode analysis is relevant to the testing of the phase control system performed with single cell cavities [3]. (*During the course of the project insufficient resources were available for the manufacture of a full nine cell cavity and its cryomodule.*) The simulations in section 19 for single mode cavity excitation have been chosen to illustrate how phase and amplitude control depends independently on beam-loading, microphonics and external Q. The results also illustrate the control action being taken by the power amplifiers.

Section 20 illustrates how control performance depends on gain in preparation for a more comprehensive study in sections 21 and 22. Section 21 derives analytic formulae for the stability limit of proportional control and proportional integral control.

Section 22 gives the first results from systematic application of the code for cases which are relevant to the ILC crab cavity and similar systems, albeit cavities where the operating mode is well separated from other modes. The first important result determined in section 22.1 and section 22.2 is that the phase control performance is likely to be independent of the external Q (determined by input coupler). This is not a result we had anticipated and is only true for certain parameter ranges. The following sub-sections of section 22 indicate the position of optimum performance with respect to the stability limit as a function of measurement errors, latency, microphonics and beam-loading. Due to project time constraints the coverage of the parameter space involved and interpretation of the result is sparse.

Section 23 presents some analyses for the ILC crab cavity where adjacent modes to the operating mode have a significant effect on the control performance. The simulations in this report make assumptions on the level and frequencies of of microphonics. Real microphonic data will not be available until the the cavities have been tested in a horizontal crymodule.

3. Modelling Cavity Filling

A general layout for a driven superconducting cavity might be drawn as

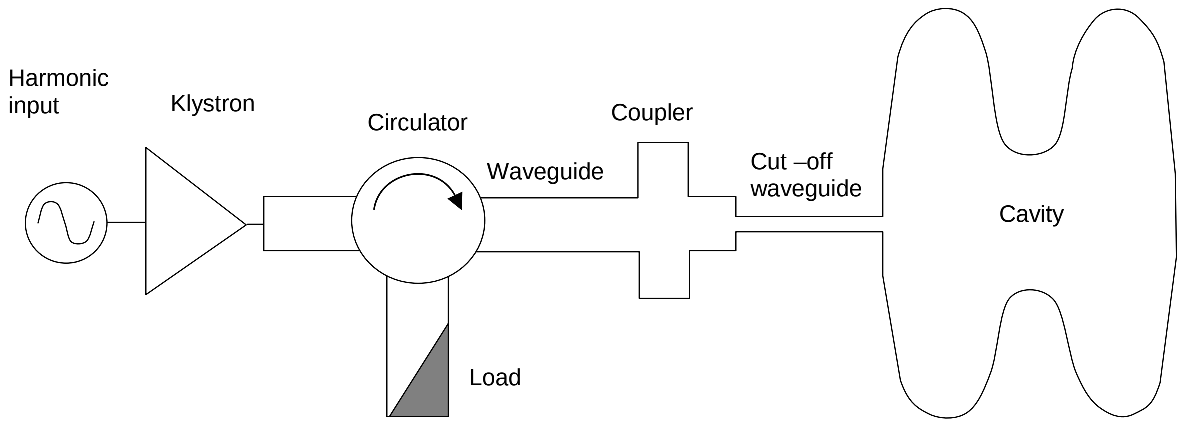

Figure 1 Cavity Drive Schematic

At each step in the waveguide the impedance changes, i.e. the ratio of the voltage to current changes and there could be some reflection of power. Where the reflected power from successive transitions cancel, all the power travels forward. This means that if the new section has higher impedance than the previous section, the voltage is higher and the current is smaller but the power flow stays the same. When reflected power cancels, the system is matched and the sequence of transitions has a pure transformer action.

To get large voltage transformations it is sometimes useful to use a short waveguide section that is beyond cut-off. The cut off section still transmits and reflects power without energy storage or modifying the frequency hence still acts as part of the transformer.

Often all this complication is reduced to the equivalent circuit

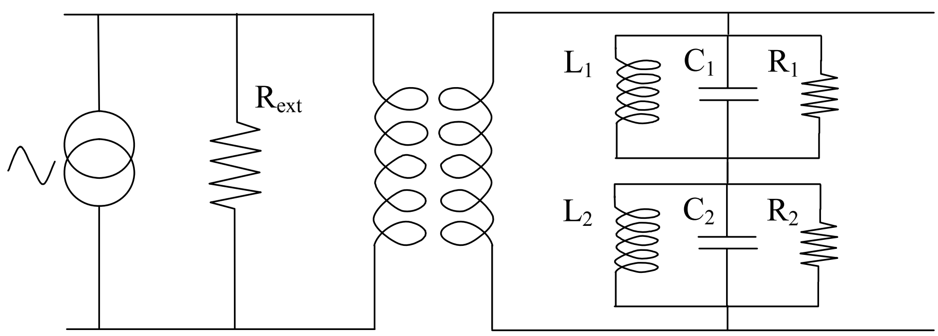

Figure 2 Equivalent circuit for a cavity with two mode

Modelling the cavity modes as lumped parallel resonant circuits in series is valid provided the cavity is only excited at frequencies that don't excite modes that are not included in the model. A cavity can do one of three things, it can reflect energy and it can store energy and it can re-emit energy as a function of the phase and amplitude of the incoming wave. These properties can all be represented by a parallel lumped circuit.

The difficulty with simplification to this level is that one cannot explicitly see forward and reflected power in the waveguide. Here we will use a modified formulation where the waveguide is explicit. Explicit representation of transformer is omitted as an appropriate voltage transformation is achieved with the correct coupler design.

4. Waveguide Driven Circuit Model

The time evolution of a cavity mode obeys the same differential equation as a parallel lumped circuit [5]. Where a cavity has the potential to resonate at a number of frequencies each mode adds a voltage contribution at the coupler and hence the modes are modelled as parallel resonators in series as shown in figure 3.

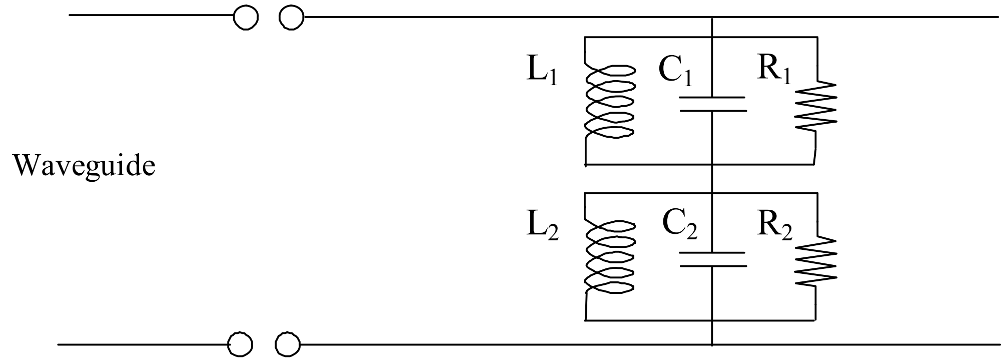

Figure 3 Simplified equivalent circuit

At the terminal the voltage in the waveguide must equal the voltage in the lumped circuit.

Along the waveguide the voltage and current satisfies the equations

$$\frac{\partial V}{\partial z} = -L_{wg}\frac{\partial I}{\partial t} \quad \text{and} \quad \frac{\partial I}{\partial z} = -C_{wg}\frac{\partial V}{\partial t}$$

hence
$$\frac{\partial^2 V}{\partial z^2} = L_{wg}C_{wg}\frac{\partial^2 V}{\partial t^2}$$

where C_{wg} is the capacitance per unit length and L_{wg} is the inductance per unit length.

For a fixed frequency source of angular frequency ω the voltage along the waveguide is given as

$$V(z,t) = \mathcal{F}\exp\{\, j(kz-\omega t)\} + \mathcal{R}\exp\{-j(kz+\omega t)\} \tag{4.1}$$

where $k = \omega\sqrt{L_{wg}C_{wg}}$

and $\mathcal{F}$ is the amplitude of the forward wave and $\mathcal{R}$ is the amplitude of the reflected wave.

The current on the waveguide is therefore given as

$$I(z,t) = \mathcal{F}\frac{\omega C_{wg}}{k}\exp\{\, j(kz-\omega t)\} - \mathcal{R}\frac{\omega C_{wg}}{k}\exp\{-j(kz+\omega t)\}$$

this can be written as

$$I(z,t) = \frac{1}{Z_{wg}}\left[\mathcal{F}\exp\{\, j(kz-\omega t)\} - \mathcal{R}\exp\{-j(kz+\omega t)\}\right] \tag{4.2}$$

$$\text{where } Z_{wg} = \sqrt{\frac{L_{wg}}{C_{wg}}} \tag{4.3}$$

If the terminal between the cavity and the waveguide is at z = 0 then the current in the waveguide equals the sum of the currents through the equivalent circuit components of each series resonator (i.e. we get an equation for each resonator / mode) hence

$$\frac{1}{L_i}\int V_i dt + C_i\frac{dV_i}{dt} + \frac{V_i}{R_i} = \frac{1}{Z_{wg}}\{\mathcal{F} - \mathcal{R}\}\exp(-j\omega t) \tag{4.4}$$

where V_i is the voltage for the ith mode and with respect to the model in figure 3 is the voltage across one of the parallel resonators.

From (4.1) and adding series voltages for each mode the voltage at z = 0 is given as

$$V = \sum_{modes} V_i = (\mathcal{F} + \mathcal{R})\exp(-j\omega t) \tag{4.5}$$

Eliminating the reflected power between (4.4) and (4.5) gives

$$\frac{1}{L_i}\int V_i dt + C_i\frac{dV_i}{dt} + \frac{V_i}{R_i} + \frac{1}{Z_{wg}}\sum_{j=1}^{N} V_j = \frac{2\mathcal{F}}{Z_{wg}}\exp(-j\omega t) \tag{4.6}$$

If the coupling to different modes is dissimilar then Z_{wg} takes a different value for each mode. This equation determines the modal voltages in the cavity as a function of the amplitude of the forward wave in the waveguide.

Now define the natural frequency of the i^{th} mode as

$$\omega_i = \frac{1}{\sqrt{L_i C_i}} \tag{4.7}$$

To evaluate Z_{wg} we write

$$Q_{ei} = \frac{\omega_i U_{stored}}{P_{emitted}} = \frac{\frac{1}{2}\omega_i C_i V_i^2}{\frac{1}{2}\left(V_i^2 / Z_{wg}\right)} = \omega_i Z_{wgi} C_i \tag{4.8}$$

$$Q_{oi} = \frac{\omega_i U_{stored}}{P_{diss}} = \frac{\frac{1}{2}\omega_i C_i V_i^2}{\frac{1}{2}\left(V_i^2 / R_i\right)} = \omega_i R_i C_i \tag{4.9}$$

where U is an energy, P is a power and diss is an abbreviation for dissipation in the cavity).

Hence dividing (4.8) and (4.9) we have that

$$\frac{Q_{ei}}{Q_{oi}} = \frac{Z_{wgi}}{R_i} \tag{4.10}$$

which can be re-arranged as

$$Z_{wgi} = \left(\frac{R}{Q_{oi}}\right)_C Q_{ei} \tag{4.11}$$

i.e. Z_{wgi} is the product of the external Q with the R/Q of the bare cavity. The suffix C is used to denote the circuit definition of R/Q. Note that Z_{wgi} is not that of the physical waveguide from the RF generator as represented in figure 2 by the transmission line from the current source to the transformer. The transformer models the coupler which transforms the voltage.

Differentiation of (4.6) and division by C_i gives

$$\frac{d^2V_i}{dt^2} + \frac{\omega_i}{\omega_i R_i C_i}\frac{dV_i}{dt} + \frac{\omega_i}{\omega_i Z_{wg} C_i}\sum_{j=1}^{N}\frac{dV_j}{dt} + \frac{1}{L_i C_i}V_i = \frac{2\omega_i}{\omega_i Z_{wgi} C_i}\frac{d}{dt}\{\mathcal{F}\exp(-j\omega t)\} \tag{4.12}$$

and using (4.7) to (4.11) in (4.12) gives

$$\frac{d^2V_i}{dt^2}+\frac{\omega_i}{Q_{oi}}\frac{dV_i}{dt}+\frac{1}{Q_{ei}}\omega_i\sum_{j=1}^{N}\frac{dV_j}{dt}+\omega_i^2\,V_i=\frac{2\,\omega_i}{Q_{ei}}\frac{d}{dt}\{\mathcal{F}\exp(-j\omega t)\} \tag{4.13}$$

defining

$$\frac{1}{Q_{Li}}=\frac{1}{Q_{oi}}+\frac{1}{Q_{ei}} \tag{4.14}$$

equation (4.12) becomes

$$\frac{d^2V_i}{dt^2}+\frac{\omega_o}{Q_{Li}}\frac{dV_i}{dt}+\frac{1}{Q_{ei}}\omega_i\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{dV_j}{dt}+\omega_o^2\,V=\frac{2\,\omega_o}{Q_e}\frac{d}{dt}\{\mathcal{F}\exp(-j\omega t)\} \tag{4.15}$$

5. Steady State for a Single Cavity Mode

The steady state solution of (4.6) for a single mode (N = 1) has to be of the form

$$V(t)=\mathcal{V}\exp(-j\,\omega\,t) \tag{5.1}$$

where $\mathcal{V}$ is complex (i.e. has a phase shift from the forward power).

Solving (4.6) for the single mode case and omitting the suffix gives

$$\left\{-\frac{1}{j\omega L}-j\omega C+\left(\frac{1}{R}+\frac{1}{Z_{wg}}\right)\right\}\mathcal{V}=\frac{2\mathcal{F}}{Z_{wg}} \tag{5.2}$$

Power dissipated in the cavity P_c is given by

$$P_c=\frac{\mathcal{V}\mathcal{V}^{*}}{2R} \tag{5.3}$$

Power carried by the forward wave P_f is given as

$$P_f=\frac{\mathcal{F}\mathcal{F}^{*}}{2Z_{wg}} \tag{5.4}$$

Hence multiplying each side of (5.2) by its complex conjugate and substituting from (5.3) and (5.4) gives

$$\left\{\left(\frac{1}{\omega L}-\omega C\right)^2+\left(\frac{1}{R}+\frac{1}{Z_{wg}}\right)^2\right\}RP_c=4\frac{P_f}{Z_{wg}} \tag{5.5}$$

hence as expected

$$P_c=\frac{4P_f}{Z_{wg}R\left\{\left(\frac{1}{\omega L}-\omega C\right)^2+\left(\frac{1}{R}+\frac{1}{Z_{wg}}\right)^2\right\}} \tag{5.5}$$

6. Single Mode Resonant Loss Free Operation

At resonance when

$$\omega = \omega_o = \frac{1}{\sqrt{LC}}$$

then

$$P_c = \frac{4 Z_{wg} R\ P_f}{\left(R + Z_{wg}\right)^2} \tag{6.1}$$

and from (5.2)

$$\mathcal{V} = \frac{2R\mathcal{F}}{Z_{wg} + R} \tag{6.2}$$

For a loss free cavity $R \to \infty$ so

$$\mathcal{V} = 2\mathcal{F} \tag{6.3}$$

High voltages are achieved in the cavity by an impedance change not shown in figure 3.

From (4.5) and (5.1)

$$\mathcal{V} = \mathcal{F} + \mathcal{R} \tag{6.4}$$

hence for a lossless cavity (6.3) and (6.4) give

$$\mathcal{F} = \mathcal{R} \tag{6.5}$$

as expected.

Away from steady state we must solve the differential equation (4.6) or equivalently (4.15).

7. Beamloading

If a charged bunch moves though a cavity it leaves its image charge q_m where it enters the cavity and collects some more where it leaves, see figure 4. Importantly the bunch then carries the new image charge with it as it moves on leaving an opposite charge behind.

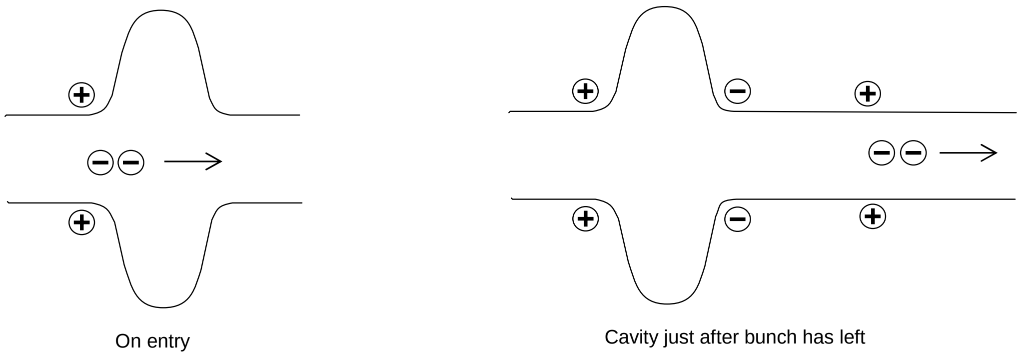

Figure 4 Image charge illustrating beamloading

Beam loading can be simulated by allowing a current impulse to flow into the equivalent circuit. If this impulse is very short compared to the period of the cavity then its only action is to change the charge across the capacitance C. For this case the voltage V jumps instantaneously to a new value. Consequently as an alternative to modelling beamloading with an additional current source in the equivalent circuit we can also model beamloading by letting the voltage jump each time a bunch passes through the cavity. With respect to a numerical solution of equation (4.6) over millions of RF cycles it is easier to let the voltage jump rather than to input short current pulses as a source term. If the time over which the voltage is allowed to jump is zero, then in equation (4.6) the integral of the voltage is unchanged allowing the differential of V immediately after the discontinuity to be determined and hence the numerical solution of (4.6) through the discontinuity to be determined. If the cavity is already excited, the charge that moves across adds to the charge that is already there hence the energy is increased as

$$U + \delta U = \frac{1}{2}\frac{\left(q_m + \delta q_m\right)^2}{C} \qquad \text{so that} \qquad \delta U \approx \frac{1}{2}\frac{2 q_m \, \delta q_m}{C} = V\delta Q \tag{7.1}$$

The new voltage in the cavity after the charge has passed must be determined by phasor addition i.e.

$$\underline{V}_{\text{final}} = \frac{1}{C}\left(\underline{q_m} + \underline{\delta q_m}\right) \tag{7.2}$$

(Note that at any instant the charge is always in phase with the voltage i.e. $\underline{q_m} = C\underline{V}$.)

Suppose that the phase of the cavity charge with respect to some reference is θ and the phase of the image charge added by beamloading is α then from figure 4a we see that

$$\mathrm{Re}\left(V_{\text{final}}\right) = \frac{1}{C}\left(q_m \cos\theta + \delta q_m \cos\alpha\right) \tag{7.3}$$

$$\mathrm{Im}\left(V_{\text{final}}\right) = \frac{1}{C}\left(q_m \sin\theta + \delta q_m \sin\alpha\right) \tag{7.4}$$

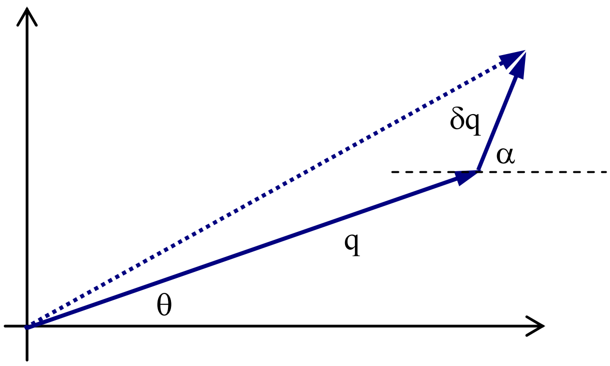

Figure 4a Phasor addition of image charge

The phase α for the incremental addition of charge is determined by the difference between the peak of the reference phase and the reference phase when the bunch is at the centre of the cavity.

A proof can be constructed from the time reversal symmetry of electrodynamics when there are no system losses. The argument goes as follows,

1) Let the bunch travel through the cavity and induce a field.

2) At some time in the future when the bunch is a long way down the beam pipe let time run backwards.

3) After the bunch has gone back through the cavity and into the beam pipe again time reversal symmetry gives us zero field in the cavity.

4) The bunch extracts maximum energy from the cavity when it is at the centre of the cavity when the field is maximum (one might question whether this is exactly true for all energies?)

5) For the bunch to extract all the energy from the cavity this must be the same as extracting the maximum energy hence the bunch must return to see a field maximum.

6) By time reversal symmetry, the return position and the cavity phase is perfectly synchronised with the forward position and the cavity phase hence the bunch must have been at the centre of the cavity to correspond to a field maximum on the forward path.

Note that time reversal symmetry works for both the relativistic and non relativistic cases.[1]

The quantity $\underline{\delta q_m}$ is the charge added to the effective capacitance of the mode. It is of course related to the charge that passes through the cavity. This relationship will be determined for a dipole mode in section 15.

8. Cavity Response to Beamloading

Equation (6.4) is valid away from steady state hence if the cavity voltage $\mathcal{V}$ jumps and the forward wave amplitude $\mathcal{F}$ stays the same then the amplitude of the reflected wave $\mathcal{R}$ increases so that the system starts returning towards steady state. From (6.4) we have that

$$\mathcal{R} = \mathcal{V} - \mathcal{F} \tag{8.1}$$

The cavity can start moving back towards steady state faster if the amplitude of the reflected wave is increased. Turning the forward power off will increase the reflected power. Indeed phase shifting the forward power by 180^o gives an even greater reflected power and hence the rate at which the system starts returning to steady state is even higher. Effectively phase shifting the forward power by 180^o sucks power out of the cavity at an enhanced rate.

Superconducting accelerator, cavity cells are typically designed so that the charged bunches traverse the cavity in half an RF cycle. In this instance the bunch can be simulated by supplying current to the effective capacitance of the cavity cell for half a cycle so that the integrated current is equal to the bunch charge. If the time dependence of the electric field at the centre of a cavity cell is taken to be $\sin \omega t$ and the bunch is phased to receive maximum acceleration or de-acceleration then the beam load current will be supplied between $\omega t = 0$ and $\omega t = \pi$. This means that the peak of the average beam current coincides with the peak voltage. At resonance the integral term in (4.6) cancels the differential term implying that the peak voltage is in phase with the drive current. Taken together one sees that the peak beam load current is in phase with the drive when the accelerator is providing maximum acceleration.

Equation (4.15) is easily integrated numerically for the initial fill and after the passage of a bunch which kicks the voltage for the three cases described above. These cases where (i) when the drive is constant, (ii) when the drive is switched off as the bunch kicks the cavity and (iii) when the phase of the forward power is shifted by 180^o as the bunch kicks the cavity. These cases are shown in figure 5.

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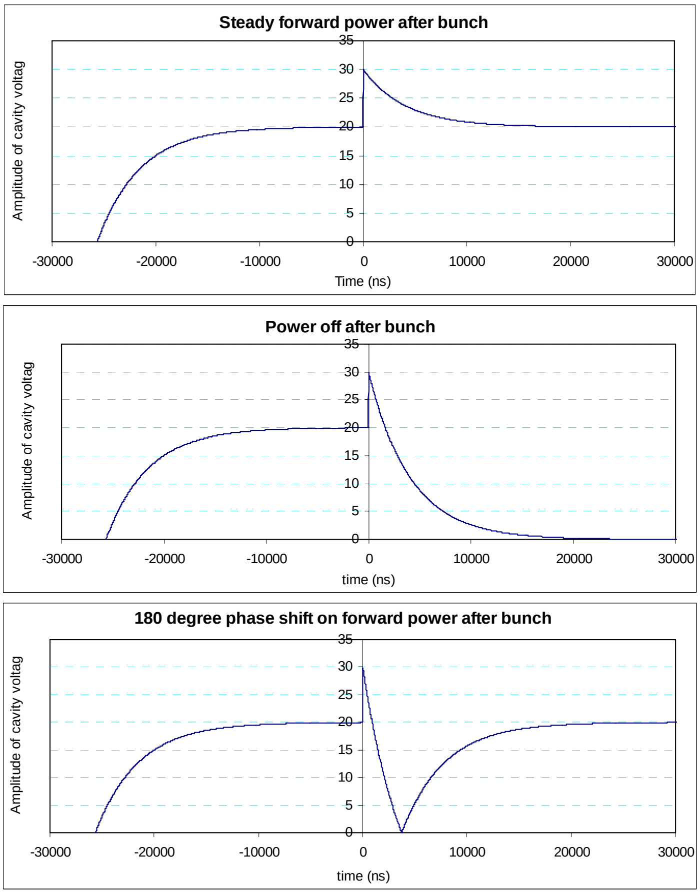
[1] (We have checked the result using a MAGIC simulation of a symmetric pill box cavity for a mildly relativistic bunch.)



Figures 5a, 5b & 5c Response to beam-load impulse at t = 0. Frequency=3.9 GHz, Q=5×10^4

The charts were produced with 180 iterations per period. The phase shift remained close to zero to better than 0.1 degrees. Increasing the number of iterations per period reduced phase errors towards zero. The program used to compute figure 5 is given in Appendix 1.

In a second calculation the forward power was controlled by a PI (Proportional Integral) controller, the response is shown in figure 6. For the case shown the amplitude of the peak forward power was limited to a maximum of 110% of that used for figures 5.

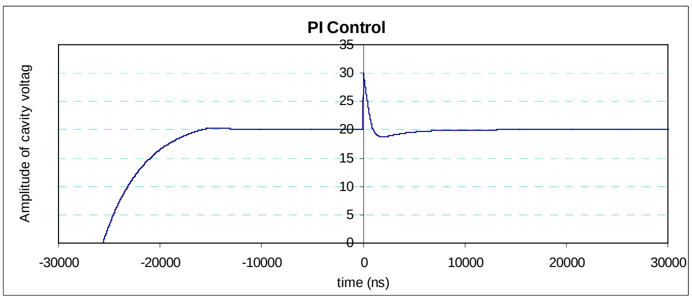


Figure 6 Response to beam-load impulse at t = 0 with PI control. Frequency=3.9 GHz, Q=5×10^4 max amplitude of input = 11

Note that the initial recovery is at the rate obtained with 180^o phase shift in figure 5c.

For a multi-cell cavity beamloading can be modelled by applying a periodic current waveform to an appropriate equivalent circuit. The duration will be determined by the time it takes for the bunch to pass through N cells and the period by twice the time it takes the bunch to pass through one cell. The precise waveform can only be determined by a full electromagnetic simulation. If in the solution one is only interested in the behaviour of a single mode then one extracts the Fourier component of the current waveform that excites that mode. The net effect of the current will be to change the voltage phasor of the mode. Simulation of beamloading can therefore still be modelled by discrete changes in the cavity voltage phasor.

9. The Envelope Equations

The solution of the differential equation (4.15) is only fast with a modern PC. For high Q calculations it is difficult to obtain phase prediction with an accuracy of milli-degrees. A faster solution technique is to assume a solution of the form

$$V_m(t) = \{A_{mr}(t) + jA_{mi}(t)\}\exp\{-j\omega t\} \tag{9.1}$$

where m is the index that runs over modes, $A_{mr}(t)$ and $A_{mi}(t)$ are slowly varying functions of time and ω is the drive frequency [6]. A similar form is assumed for $\mathcal{F}$. It will be seen that that when the assumed solution is substituted into the differential equation, second derivatives can be neglected. Differentiating the RHS of (4.15) gives

$$\frac{d^2V_m}{dt^2} + \frac{\omega_m}{Q_{om}}\frac{dV_m}{dt} + \omega_m^2 V_m + \frac{\omega_m}{Q_{em}}\sum_{j=1}^{N}\frac{dV_j}{dt} = \frac{2\omega_m}{Q_{em}}\left(\dot{\mathcal{F}} - j\omega\mathcal{F}\right)\exp(-j\omega t) \tag{9.2}$$

Differentiation of (9.1) gives

$$\frac{dV_m}{dt} = \left(\dot{A}_{mr} - j\omega A_{mr} + j\dot{A}_{mi} + \omega A_{mi}\right)\exp\{-j\omega t\} \tag{9.3}$$

second differentiation gives

$$\frac{d^2V_m}{dt^2} = \left(\ddot{A}_{mr} - 2j\omega\dot{A}_{mr} - \omega^2 A_r + j\ddot{A}_{mi} + 2\omega\dot{A}_{mi} - j\omega^2 A_{mi}\right)\exp\{-j\omega t\} \tag{9.4}$$

Substituting (9.3) and (9.4) in (9.2), cancelling a factor of $\exp(-j\omega t)$ and separating into real and imaginary parts gives the envelope equations as

$$\left(\ddot{A}_{mr} + 2\omega\dot{A}_{mi} - \omega^2 A_{mr}\right) + \frac{\omega_m}{Q_{om}}\left(\dot{A}_{mr} + \omega A_{mi}\right) + \frac{\omega_m}{Q_{em}}\sum_{j=1}^{N}\left(\dot{A}_{jr} + \omega A_{ji}\right) + \omega_m^2 A_{mr} = \frac{2\omega_m}{Q_{em}}\left(\dot{\mathcal{F}}_r + \omega\mathcal{F}_i\right) \tag{9.5a}$$

$$\left(\ddot{A}_{mi} - 2\omega\dot{A}_{mr} - \omega^2 A_{mi}\right) + \frac{\omega_m}{Q_{om}}\left(\dot{A}_{mi} - \omega A_{mr}\right) + \frac{\omega_m}{Q_{em}}\sum_{j=1}^{N}\left(\dot{A}_{ji} - \omega A_{jr}\right) + \omega_m^2 A_{mi} = \frac{2\omega_m}{Q_{em}}\left(\dot{\mathcal{F}}_i - \omega\mathcal{F}_r\right) \tag{9.6a}$$

Now ω_o and ω are big numbers whilst $\dot{A}$ and $\ddot{A}$ are not so big as A is a slowly varying envelope function. This means that in (9.5a) and (9.6a) the second derivative terms i.e. $\ddot{A}$ are an order of magnitude smaller than all the other terms, i.e. we retain order ω and ω^2 though we shall see that order ω^2 cancels. Later we will also take Q_L as large but not yet.

Eliminating 2nd derivatives in (9.5a) and (9.6a) they become

$$2\omega\,\dot{A}_{mi} + \frac{\omega_m}{Q_{om}}\dot{A}_{mr} + \frac{\omega_m}{Q_{em}}\sum_{j=1}^{N}\dot{A}_{jr} + \frac{\omega\omega_m}{Q_{om}}A_{mi} + \frac{\omega\omega_m}{Q_{em}}\sum_{j=1}^{N}A_{ji} + \left(\omega_m^2 - \omega^2\right)A_{mr} = \frac{2\omega_m}{Q_{em}}\left(\dot{\mathcal{F}}_r + \omega\mathcal{F}_i\right) \tag{9.5b}$$

$$-2\omega\,\dot{A}_{mr} + \frac{\omega_m}{Q_{om}}\dot{A}_{mi} + \frac{\omega_m}{Q_{em}}\sum_{j=1}^{N}\dot{A}_{ji} - \frac{\omega\omega_m}{Q_{om}}A_{mr} - \frac{\omega\omega_m}{Q_{em}}\sum_{j=1}^{N}A_{jr} + \left(\omega_m^2 - \omega^2\right)A_{mi} = \frac{2\omega_m}{Q_{em}}\left(\dot{\mathcal{F}}_i - \omega\mathcal{F}_r\right) \tag{9.6b}$$

10. Single Mode Diagonalisation

For the case of a single mode equation (9.5b) and (9.6b) can be diagonalised exactly. In this section we drop the suffix m.

Eliminating $\dot{A}_i$ by $\frac{\omega_o}{Q_L}\times(7.5b) - 2\omega\times(7.6b)$ gives

$$\left\{4\omega^2 + \left(\frac{\omega_o}{Q_L}\right)^2\right\}\dot{A}_r + \left(\omega^2 + \omega_o^2\right)\frac{\omega_o}{Q_L}A_r + \left\{\left(\frac{\omega_o}{Q_L}\right)^2 - 2\left(\omega_o^2 - \omega^2\right)\right\}\omega A_i = \frac{2\omega_o^2}{Q_e Q_L}\left(\dot{\mathcal{F}}_r + \omega\mathcal{F}_i\right) - \frac{4\omega\omega_o}{Q_e}\left(\dot{\mathcal{F}}_i - \omega\mathcal{F}_r\right) \tag{10.1a}$$

Eliminating $\dot{A}_r$ by $2\omega\times(7.5b) + \frac{\omega_o}{Q_L}\times(7.6b)$ gives

$$\left\{4\omega^2 + \left(\frac{\omega_o}{Q_L}\right)^2\right\}\dot{A}_i + \left(\omega^2 + \omega_o^2\right)\frac{\omega_o}{Q_L}A_i - \left\{\left(\frac{\omega_o}{Q_L}\right)^2 - 2\left(\omega_o^2 - \omega^2\right)\right\}\omega A_r = \frac{2\omega_o^2}{Q_e Q_L}\left(\dot{\mathcal{F}}_i - \omega\mathcal{F}_r\right) + \frac{4\omega_o\omega}{Q_e}\left(\dot{\mathcal{F}}_r + \omega\mathcal{F}_i\right) \tag{10.1b}$$

Dividing (10.1a) and (10.1b) by ω_o^3 they can be written as

$$\left\{\left(\frac{2\omega}{\omega_o}\right)^2 + \left(\frac{1}{Q_L}\right)^2\right\}\frac{1}{\omega_o}\dot{A}_r + \left(\frac{\omega^2}{\omega_o^2} + 1\right)\frac{1}{Q_L}A_r + \left\{\left(\frac{1}{Q_L}\right)^2 - \frac{2\omega}{\omega_o}\left(\frac{\omega_o}{\omega} - \frac{\omega}{\omega_o}\right)\right\}\frac{\omega}{\omega_o}A_i = \frac{2}{Q_e Q_L}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_r + \frac{\omega}{\omega_o}\mathcal{F}_i\right) - \frac{4}{Q_e}\frac{\omega}{\omega_o}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_i - \frac{\omega}{\omega_o}\mathcal{F}_r\right) \tag{10.2a}$$

$$\left\{\left(\frac{2\omega}{\omega_o}\right)^2+\left(\frac{1}{Q_L}\right)^2\right\}\frac{1}{\omega_o}\dot{A}_i+\left(\frac{\omega^2}{\omega_o^2}+1\right)\frac{1}{Q_L}A_i-\left\{\left(\frac{1}{Q_L}\right)^2-\frac{2\omega}{\omega_o}\left(\frac{\omega_o}{\omega}-\frac{\omega}{\omega_o}\right)\right\}\frac{\omega}{\omega_o}A_r$$
$$=\frac{2}{Q_eQ_L}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_i-\frac{\omega}{\omega_o}\mathcal{F}_r\right)+\frac{4}{Q_e}\frac{\omega}{\omega_o}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_r+\frac{\omega}{\omega_o}\mathcal{F}_i\right) \tag{10.2b}$$

For superconducting cavities Q_L and Q_e are invariably made very large (to benefit from its superconducting properties) hence the equations after multiplication by $\omega_o^2/4\omega^2$ approximate to

$$\frac{1}{\omega_o}\dot{A}_r+\frac{1}{4}\left(1+\frac{\omega_o^2}{\omega^2}\right)\frac{1}{Q_L}A_r-\frac{1}{2}\left(\frac{\omega_o}{\omega}-\frac{\omega}{\omega_o}\right)A_i=-\frac{1}{Q_e}\frac{\omega_o}{\omega}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_i-\frac{\omega}{\omega_o}\mathcal{F}_r\right) \tag{10.3a}$$

$$\frac{1}{\omega_o}\dot{A}_i+\frac{1}{4}\left(1+\frac{\omega_o^2}{\omega^2}\right)\frac{1}{Q_L}A_i+\frac{1}{2}\left(\frac{\omega_o}{\omega}-\frac{\omega}{\omega_o}\right)A_r=\frac{1}{Q_e}\frac{\omega_o}{\omega}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_r+\frac{\omega}{\omega_o}\mathcal{F}_i\right) \tag{10.3b}$$

These equations have been integrated for the same parameters as figure 6, results are given in figure 7. The program is given in appendix 2.

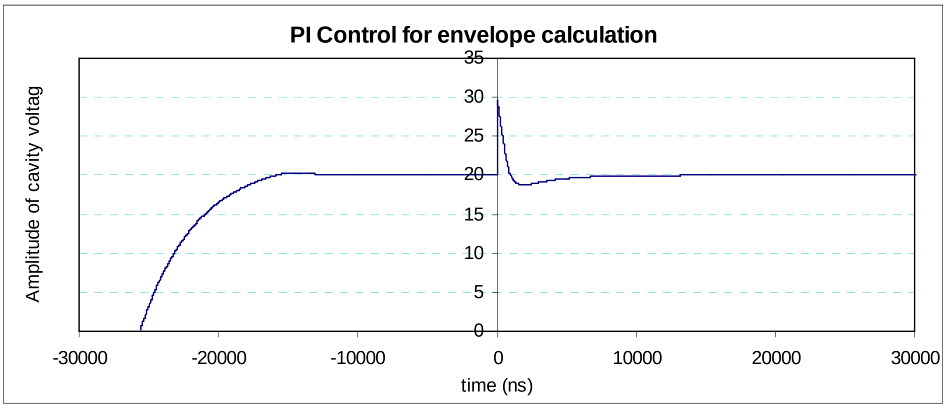


Figure 7 Response to beam-load impulse at t = 0 with PI control by integrating the envelope equations. Frequency=3.9 GHz, Q=5×10^4 max amplitude of input = 11

11. Approximate Envelope Equations for the Multimode Case

For the multi-mode case we go back to considering equations (9.5b) and (9.6b). In superconducting cavities Q_{Lm} and Q_{em} are always going to be large (~10^6) hence terms in $1/Q^2$ can always be neglected where they are multiplied by the same power of ω. This observation allows us to diagonalise the derivative terms of (9.5b) and (9.6b) by substituting (9.6b) into (9.5a) and (9.5a) into (9.6b) neglecting powers $1/Q^2$.

$$2\omega\dot{A}_{mi}+\frac{1}{2Q_{om}}\frac{\omega_m}{\omega}\left(\omega_m^2-\omega^2\right)A_{mi}+\frac{1}{2Q_{em}}\frac{\omega_m}{\omega}\sum_{j=1}^{N}\left(\omega_j^2-\omega^2\right)A_{ji}+\frac{\omega\omega_m}{Q_{om}}A_{mi}$$
$$+\frac{\omega\omega_m}{Q_{em}}\sum_{j=1}^{N}A_{ji}+\left(\omega_m^2-\omega^2\right)A_{mr}=\frac{2\omega_m}{Q_{em}}\left(\dot{\mathcal{F}}_r+\omega\mathcal{F}_i\right)$$

(11.1a)

$$-2\omega\dot{A}_{mr}-\frac{1}{2Q_{om}}\frac{\omega_m}{\omega}\left(\omega_m^2-\omega^2\right)A_{mr}-\frac{1}{2Q_{em}}\frac{\omega_m}{\omega}\sum_{j=1}^{N}\left(\omega_j^2-\omega^2\right)A_{jr}-\frac{\omega\omega_m}{Q_{om}}A_{mr}$$
$$-\frac{\omega\omega_m}{Q_{em}}\sum_{j=1}^{N}A_{jr}+\left(\omega_m^2-\omega^2\right)A_{mi}=\frac{2\omega_m}{Q_{em}}\left(\dot{\mathcal{F}}_i-\omega\mathcal{F}_r\right)$$

(11.1b)

Combing terms and normalising the differentials in (11.1a) and (11.1b) and swapping the order of (11.1a) and (11.1b) gives

$$\frac{\dot{A}_{mr}}{\omega_m}+\frac{1}{4Q_{om}}\left(\frac{\omega_m^2}{\omega^2}+1\right)A_{mr}+\frac{1}{4Q_{em}}\sum_{j=1}^{N}\left(\frac{\omega_j^2}{\omega^2}+1\right)A_{jr}-\left(\frac{\omega_m}{\omega}-\frac{\omega}{\omega_m}\right)\frac{A_{mi}}{2}=-\frac{1}{Q_{em}}\left(\frac{\dot{\mathcal{F}}_i}{\omega}-\mathcal{F}_r\right)$$

(11.2a)

$$\frac{\dot{A}_{mi}}{\omega_m}+\frac{1}{4Q_{om}}\left(\frac{\omega_m^2}{\omega^2}+1\right)A_{mi}+\frac{1}{4Q_{em}}\sum_{j=1}^{N}\left(\frac{\omega_j^2}{\omega^2}+1\right)A_{ji}+\left(\frac{\omega_m}{\omega}-\frac{\omega}{\omega_m}\right)\frac{A_{mr}}{2}=\frac{1}{Q_{em}}\left(\frac{\dot{\mathcal{F}}_r}{\omega}+\mathcal{F}_i\right)$$

(11.2b)

For Runge Kutta Solution we write these equations in the form

$$\dot{A}_{mr}=-\frac{\omega_m}{4Q_{om}}\left(\frac{\omega_m^2}{\omega^2}+1\right)A_{mr}-\frac{\omega_m}{4Q_{em}}\sum_{j=1}^{N}\left(\frac{\omega_j^2}{\omega^2}+1\right)A_{jr}+\left(\omega_m^2-\omega^2\right)\frac{A_{mi}}{2\omega}-\frac{\omega_m}{\omega Q_{em}}\left(\dot{\mathcal{F}}_i-\omega\mathcal{F}_r\right)$$

(11.3a)

$$\dot{A}_{mi}=-\frac{\omega_m}{4Q_{om}}\left(\frac{\omega_m^2}{\omega^2}+1\right)A_{mi}-\frac{\omega_m}{4Q_{em}}\sum_{j=1}^{N}\left(\frac{\omega_j^2}{\omega^2}+1\right)A_{ji}-\left(\omega_m^2-\omega^2\right)\frac{A_{mr}}{2\omega}+\frac{\omega_m}{\omega Q_{em}}\left(\dot{\mathcal{F}}_r+\omega\mathcal{F}_i\right)$$

(11.3b)

which is of the form

$$\dot{A}_r(m) = -f_o(m)g_1(m)A_r(m) - f_e(m)\sum_{j=1}^{N} g_1(j)A_r(j) + g_2(m)A_i(m) - g_3(m)\left(\dot{\mathcal{F}}_i - \omega\mathcal{F}_r\right) \tag{11.4a}$$

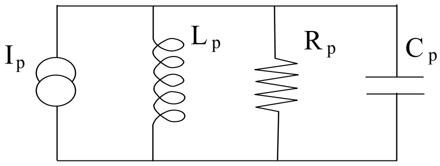
$$\dot{A}_i(m) = -f_o(m)g_1(m)A_i(m) - f_e(m)\sum_{j=1}^{N} g_1(j)A_i(j) - g_2(m)A_i(m) + g_3(m)\left(\dot{\mathcal{F}}_r + \omega\mathcal{F}_i\right) \tag{11.4b}$$

where

$$f_o(m) = \frac{\omega_m}{4Q_{om}} \quad f_e(m) = \frac{\omega_m}{4Q_{em}} \quad g_1(m) = \left(\frac{\omega_m^2}{\omega^2} + 1\right) \quad g_2(m) = \frac{\left(\omega_m^2 - \omega^2\right)}{2\omega} \quad g_4(m) = \frac{\omega_m}{\omega Q_{em}}$$

12. Definition of shunt impedance for the accelerating mode

For a parallel resonant circuit where R_p is the shunt impedance (resistance),



the stored energy U_{stored} is given as

$$U_{stored} = \frac{C V_{peak}^2}{2} \tag{12.1}$$

and the dissipated energy P_{diss} is given as

$$P_{diss} = \frac{V_{peak}^2}{2R_p} \tag{12.2}$$

hence the Q factor is given as

$$Q = \frac{\omega U_{stored}}{P_{diss}} = R_p\sqrt{\frac{C_p}{L_p}} \tag{12.3}$$

Note also that

$$\frac{R_p}{Q} = \sqrt{\frac{L_p}{C_p}} = \frac{1}{\omega\, C_p} \tag{12.4}$$

The R over Q written as (R/Q) applies to a parallel resonant circuit and not to a series resonant circuit as for a series resonant circuit one has that

$$Q = \frac{\omega U_{stored}}{P_{diss}} = \frac{\frac{1}{2}\omega L_s I_{peak}^2}{\frac{1}{2}R_s I_{peak}^2} = \frac{1}{R_s}\sqrt{\frac{L_s}{C_s}} \tag{12.5}$$

The R/Q takes different values for different modes. It is an important figure of merit as it determines how much energy a charged bunch passing through a cavity will deliver to its associated mode. This result will be derived in a later section.

For accelerator cavities the shunt impedance R_c is often defined in a different way. It is defined such that

$$P_{diss} = \frac{V_c^2}{R_c} \tag{12.6}$$

where V_c is the voltage that accelerates the beam. For a cavity mode with a uniform electric field along the beam axis and for a charged bunch travelling at the velocity of light and at a phase such that the electric field is maximum when the bunch is at the centre of the cavity

$$V_c = \frac{V_{peak}}{d}\int_{-d/2c}^{d/2c}\cos(\omega t)c\,dt = V_{peak}\frac{\sin\left(\frac{\omega d}{2c}\right)}{\frac{\omega d}{2c}} \le \frac{2}{\pi}V_{peak} \tag{12.7}$$

where the cavity length is d and V_{peak} is the end to end voltage on the axis. The maximum acceleration voltage is achieved when

$$\frac{\omega d}{2c} = \frac{\pi}{2} \tag{12.8}$$

Equating (12.2) and (12.6)

$$\frac{V_c^2}{R_c} = \frac{V_{peak}^2}{2R_p} \tag{12.9}$$

For a cavity of optimum length (maximum acceleration voltage) equations (12.7) and (12.9) give

$$\frac{R_c}{R_p} = 2\left(\frac{2}{\pi}\right)^2 = 0.8106 \tag{12.10}$$

i.e. the accelerator definition of shunt impedance is close to the external circuit shunt impedance.

Eliminating U_{diss} between (12.3) and (12.6) we can also write

$$Q = \frac{\omega U_{stored}}{P_{diss}} = \frac{\omega R_c U_{stored}}{V_c^2} \tag{12.11}$$

hence

$$\frac{R_c}{Q} = \frac{V_c^2}{\omega U_{stored}} \tag{12.12}$$

13. Definition of shunt impedance for dipole TM_{1np} modes

For dipole and higher order modes the maximum voltage between two points on the cavity walls no longer occurs on the axis, indeed there is no electric field on the axis hence the end to end voltage on the axis is zero. The concept of a peak longitudinal voltage V_{peak} still exists hence equation (12.2) can still be used to define a shunt impedance. Supposing that V_{peak} for occurs at radius a (and in a direction appropriate to the dipole) then equation (12.2) could be used to define a shunt impedance R_{px} as

$$P_{diss} = \frac{V_{peak}^2(a)}{2\,R_{px}} \tag{13.1}$$

Note that a is often taken as the beam pipe radius.

Taking m = 0 for an accelerating mode, m = 1 for a dipole mode, m = 2 for a quadrupole mode etc., the variation of the axial electric field E_z out from the centre varies with radius as r^m hence one has that[2]

$$V_{peak}(r) = \frac{r^m}{a^m} V_{peak}(a) \tag{13.2}$$

Using (12.3) and (13.1) the Q factor might be expressed as

$$Q = \frac{\omega\,U_{stored}}{P_{diss}} = \frac{2\,\omega\,R_{px}\,U_{stored}}{V_{peak}^2(a)} \tag{13.3}$$

Hence the R upon Q might be written as

$$\frac{R_{px}}{Q} = \frac{V_{peak}^2(a)}{2\,\omega\,U_{stored}} \tag{13.4}$$

This definition of the shunt impedance and its associated R/Q is inconvenient as it relates to a potential in the cavity far from beam axis and which does not immediately relate to the deflecting properties of the mode. The definition of the shunt voltage for a dipole cavity is instead defined by the equation

$$P_{diss} = \frac{V_{cz}^2(a)}{\left(\frac{a\,\omega}{c}\right)^2 R_d} \tag{13.5}$$

where $V_{cz}(a)$ is the voltage that a bunch moving parallel to the axis at radius a experiences including transit time effects. From (13.5) and the definition of Q given in (4.9) one gets that

$$\frac{R_d}{Q} = \frac{V_{cz}^2(a)}{\omega\left(\frac{a\,\omega}{c}\right)^2 U_{stored}} \tag{13.6}$$

The usefulness of this definition comes from a relationship between the on axis transverse kick $V_\perp$ to the off axis longitudinal kick $V_z(a)$ derived from the Panofsky Wenzel theorem i.e.

$$V_{cz}(a) = j\,\frac{a\,\omega}{c}\,V_\perp \tag{13.7}$$

Using (13.7) equation (13.6) becomes

$$\frac{R_d}{Q} = \frac{|V_\perp|^2}{\omega\,U_{stored}} \tag{13.8}$$

[2] This only strictly applies if the cavity is azimuthally symmetric.

This equation conveniently has the same form as (12.12). Note that for quadrupole and higher order modes there is no transverse kick for on axis particles hence (13.8) only applies to dipole modes.

For higher modes one normally defines the shunt impedance as

$$R_m = \frac{V_{cz}^2(a)}{\left(\frac{a\,\omega}{c}\right)^{2m} U_{diss}} = \frac{V_{cz}^2(a)}{\left(\frac{2\pi a}{\lambda}\right)^{2m} U_{diss}}$$

where λ is the free space wavelength at the frequency of the mode. If a is at a radius where the field is maximum, then the factor $\left(\frac{2\pi a}{\lambda}\right)$ is going to be close and probably slightly more than one. This estimation is based on the simplistic assumption that the peak field will be a quarter of a wavelength from the field null occurring on the axis.

To derive equation (13.7) from the Panofsky Wenzel theorem [7] one writes

$$V_z(a) = \int_0^{d/c} E_z(a)\exp(j\omega t)\,c\,dt \approx a\int_0^{d/c} \nabla_\perp E_z(0)\exp(j\omega t)\,c\,dt = a\int_0^{d} \nabla_\perp E_z(0)\exp(j\,\omega z/c)\,dz \tag{13.9}$$

where the first approximation comes from a Taylor expansion for small radius a. Note that this approximation is not useful for quadrupole and higher modes. The Panofsky Wenzel theorem can be stated as

$$\int_0^d F_\perp \exp(j\omega z/v)\,dz = -j\,q\frac{v}{\omega}\left[E_\perp \exp(j\omega z/v)\right]_0^d + j\,q\,\frac{v}{\omega}\int_0^d \nabla_\perp E_z \exp(j\omega z/v)\,dz \tag{13.10}$$

where v is the velocity of the bunch which will be taken to be the velocity of light c. The first term on the RHS vanishes assuming the electric field in the beam pipe vanishes, i.e. the mode is cut-off.

Combining (13.9) and (13.10) gives

$$V_z(a) = -j\frac{a\,\omega}{q\,v}\int_0^d F_\perp \exp(j\omega z/v)\,dz \tag{13.11}$$

The transverse momentum kick is given as force × time hence the deflection energy kick is the velocity of light × the momentum kick so that the deflection voltage kick $V_\perp$ is the energy kick divided by the charge hence

$$V_\perp = \frac{1}{q}\int_0^{d/c} F_\perp \exp(j\omega t)\,c\,dt = \frac{1}{q}\int_0^d F_\perp \exp(j\,\omega z/c)\,dz \tag{13.12}$$

Definition (13.12) in result (13.11) gives the earlier result (13.7).

Note that the use of a voltage kick or energy rather than a momentum kick is very dangerous as the new energy of the bunch is <u>not the initial energy plus the energy kick</u>, voltage kicks must be combined as momenta.

Re-arranging equation (13.6) to give the stored energy in terms of the R/Q on gets

$$U_{stored} = \frac{V_{cz}^2(a)}{\omega\left(\frac{a\,\omega}{c}\right)^2 \frac{R_d}{Q}} \tag{13.13}$$

Later this expression will be needed for an arbitrary radius hence utilising the relationship (13.2) for the case of a dipole (m = 1) equation (13.13) becomes

$$U_{stored} = \frac{V_{cz}^2(r)}{\omega\left(\frac{r\,\omega}{c}\right)^2 \frac{R_d}{Q}} \tag{13.14}$$

i.e. we are not restricted to radius a.

14. Crab Cavity Beam-loading from offset Bunches

A crab cavity is a deflection cavity operated with a 90° phase shift [8]. The fields that acts on an offset bunch in a pillbox dipole cavity radius excited in a TM_{110} mode are given by

$$E_z = E_o J_1\left(\frac{u_{11} r}{a}\right)\cos\phi\ \exp(j\omega t) \tag{14.1a}$$

$$H_r = j\,E_o\sqrt{\frac{\varepsilon_o}{\mu_o}}\,\frac{\omega_{110}\,a^2}{c\ r\ u_{11}^2}\,J_1\left(\frac{u_{11} r}{a}\right)\sin\phi\ \exp(j\omega t) \tag{14.1b}$$

$$H_\phi = j\,E_o\sqrt{\frac{\varepsilon_o}{\mu_o}}\,\frac{\omega_{110}\,a}{c\ \ u_{11}^2}\,J_1'\left(\frac{u_{11} r}{a}\right)\cos\phi\ \exp(j\omega t) \tag{14.1c}$$

$$E_r = E_\phi = H_z = 0 \tag{14.1d}$$

where

$$\omega_{110} = \frac{c u_{11}}{a} \tag{14.1e}$$

and where u_{11} is the first root of $J_1(x)$

Importantly the longitudinal electric field is 90° out of phase with the magnetic field. For a crab cavity the charged bunch is at the centre of the cavity when the magnetic field is zero. This means that the longitudinal electric field is maximized and hence equation (12.7) applies. The electric field that bunches see is therefore determined by (13.14) i.e.

$$V_{cz}(r) = \frac{r\omega}{c}\sqrt{\omega\frac{R_d}{Q}U_{stored}} \tag{14.2}$$

It should be noted that for a deflection cavity as opposed to a crab cavity the cosine in (12.7) is replaced with a sine and hence the integrated voltage and hence the beam-loading is zero.

Equation (14.2) is easy to understand as from (12.4)

$$\omega\frac{R_d}{Q} \sim \frac{1}{C_d}$$

hence (14.2) just re-states the relationship between the charge, the peak voltage at $r = c/\omega$ that an electron sees for optimum cavity length and stored energy of a capacitor.

For the FNAL CKM 3.9 GHz cavity the R_d/Q of a single cell has been determined by McAshan & Wanzenberg [9]. They use a slightly different definition for R_d/Q that is given here. Our definition is based on that used in Padamsee's book [10]. From equation 14 in [9]

$$\left(\frac{R}{Q}\right)'_{FNAL} = \frac{|V_L(r)|^2}{2\omega U\left(\frac{r\omega}{c}\right)^2}$$

where $V_L(r)$ includes transit time effects hence

$$|V_L(r)| = \frac{r\omega}{c}\sqrt{2\,\omega\left(\frac{R}{Q}\right)'_{FNAL} U_{stored}} \qquad (14.2\ _{FNAL})$$

comparison with (14.2) gives

$$2\left(\frac{R}{Q}\right)'_{FNAL} = \frac{R_d}{Q} \qquad (14.3)$$

Henceforth we will write

$$\left(\frac{R}{Q}\right)'_{FNAL} = \left(\frac{R}{Q}\right)_F$$

where the suffix F implies a factor of one half as in the parallel equivalent circuit definition and retain the transit time factor unlike the equivalent circuit definition.

The paper http://accelconf.web.cern.ch/AccelConf/p01/papers/MPPH129.pdf gives

$$\left(\frac{R}{Q}\right)_F = 702 \quad \Omega\,m^{-1} \qquad \text{(26 cells)}$$

For 1 cell of a multi-cell cavity[3] $\left(\frac{R}{Q}\right)_F = 27 \quad \Omega$ and with the definition here

$$\frac{R_d}{Q} = 54\ \Omega \ \text{ per cell}$$

for a transverse kick of 5 MV m^{-1} and operation at 3.9 GHz the paper gives

$$U_{stored} = 0.73 \quad J\,m^{-1}$$

hence per cell

$$U_{stored} = 0.0281\ J \ \text{ per cell}$$

From (14.2 $_{FNAL}$) one calculates that for a metre length of cavities

$$|V_L(r)| = 4.096\times 10^8\ r \qquad V\,m^{-1}$$

hence for beam offsets of 0.6 mm and 1 mm the kick per metre length of cavity system is 2.458×10^5 V m^{-1} and 4.096×10^5 V m^{-1} respectively.

For the ILC [11] the nominal bunch charge is 3.2×10^{-9} C hence the energy delivered for 0.6 mm and 1 mm offsets are $qV = 0.787\times 10^{-3}$ $J\,m^{-1}$ per bunch and

[3] Because the endcells of a multi-cell cavity have a slightly different shape, for one cell on its own i.e. a cavity made of a single cell of 2 pieces 18.6mm long will have R/Q nearer to 40.2 Ω and it will need to be run up to 34.8 mJ to give 5 MV/m deflection.

$qV = 1.311\times10^{-3} \quad \mathrm{J\,m^{-1}}$ per bunch respectively which give for 3300 bunches 2.6 J and 4.3 J respectively which are bigger than the required energy for the cavity. The high power amplifier has to deliver or extract this amount of energy in 1 ms hence amplifier power requirements are 2600 $\mathrm{Wm^{-1}}$ and 4300 $\mathrm{Wm^{-1}}$ respectively plus cavity losses which can be neglected, plus reflection losses associated with any miss-match between Q_{beam} and Q_{ext} , noting that the impedance of the external circuit can only be matched to the impedance of the beam at one power level. Note that 4300 $\mathrm{Wm^{-1}}$ corresponds to 165 Watts per cell.

Calculations for a 9 cell CKM cavity planned for the ILC [1] give

$\left(\frac{R}{Q}\right)_F = 664 \quad \Omega\,\mathrm{m}^{-1}$ and the stored energy is 0.795 J $\mathrm{m^{-1}}$ hence we obtain

$\left|V_L(r)\right| = 4.157\times10^{8}\ r \quad \mathrm{V\,m^{-1}}$ which is practically what is obtained for the original design.

For the purpose of understanding the control of phase in a cavity, precise values for (R/Q) and U_{stored} are not necessary. For the computations in subsequent section we use $(R/Q)_F = 26.5\ \Omega$ and $U_{stored} = 0.0284$ Joules per cell.

15. Voltage Increment needed for Envelope Equations

From (14.2)

$$U_{stored} = \left(\frac{c}{r\omega}\right)^2 \frac{1}{\omega\left(\frac{R_d}{Q}\right)} V_{cz}^2(r) \tag{15.1}$$

where $V_{cz}(r)$ includes transit time effects for ideal crab cavity phasing. Differentiation gives

$$\delta U_{stored} = \left(\frac{c}{r\omega}\right)^2 \frac{1}{\omega\left(\frac{R_d}{Q}\right)} 2\,V_{cz}(r)\,\delta V_{cz}(r) \tag{15.2}$$

Equation (7.1) gives

$$\delta U_{stored} = qV_{seen} \tag{15.3}$$

where q is the bunch charge and V_{seen} is the voltage experience by the bunch for non-ideal crab cavity phasing. Combining (15.2) and (15.3) gives

$$\delta V_{cz}(r) = \frac{1}{2}\left(\frac{r\omega}{c}\right)^2 \omega\left(\frac{R_d}{Q}\right)\frac{V_{seen}}{V_{cz}(r)}\,q \tag{15.4}$$

The voltage that the bunch sees depends on the phase ϕ with which it traverses the cavity with respect to the peak field in the cavity. If the cavity has a uniform longitudinal electric field[4] E_z along any offset path taken by the bunch then the maximum voltage seen by the bunch is

[4] the analysis can be more general with the use of a transit factor that encompasses non-constant E_Z

$$V_{cz} = E_z \int_{-\frac{d}{2}}^{\frac{d}{2}} \cos(\omega t)\, dz \qquad (15.5)$$

In this integral t = 0 when z = 0 so that a particle in the crabbing phase gets maximum longitudinal acceleration/retardation depending upon which side of the cavity's axis the beam enters.

If the bunch travels at the velocity of light then z – ct and if the length of the cavity cell is $d = \frac{\pi c}{\omega}$ then (15.5) becomes

$$V_{cz} = E_z \int_{-\frac{d}{2}}^{\frac{d}{2}} \cos(\omega t)\, dz = c\, E_z \int_{-\frac{d}{2c}}^{\frac{d}{2c}} \cos(\omega t)\, dt = \frac{\omega d}{\pi} E_z \int_{-\frac{\pi}{2\omega}}^{\frac{\pi}{2\omega}} \cos(\omega t)\, dt = \frac{2}{\pi} E_z\, d \qquad (15.6)$$

For a bunch that traverses the cavity with phase error ϕ then

$$V_{seen} = E_z \int_{-\frac{d}{2}}^{\frac{d}{2}} \cos(\omega t - \phi)\, dz = \frac{\omega d}{\pi} E_z \int_{-\frac{\pi}{2\omega}}^{\frac{\pi}{2\omega}} \cos(\omega t - \phi)\, dt = \frac{d}{\pi} E_z \left\{ \sin\left(\frac{\pi}{2} - \phi\right) - \sin\left(-\frac{\pi}{2} - \phi\right) \right\}$$

$$= \frac{2}{\pi} E_z\, d \cos\phi \qquad (15.7)$$

hence from (15.6) and (15.7)

$$V_{seen} = V_{cz}(r_b) \cos\phi \qquad (15.8)$$

Where r_b is the lateral displacement of the beam.

hence (15.4) can be written

$$\delta V_{cz}(r) = \frac{1}{2}\left(\frac{r\omega}{c}\right)^2 \omega \left(\frac{R_d}{Q}\right) \frac{V_{cz}(r_b)}{V_{cz}(r)} q \cos\phi \qquad (15.9)$$

From (12.2) with m = 1 we can write

$$\frac{V_{cz}(a)}{a} = \frac{V_{cz}(r)}{r} = \frac{V_{cz}(r_b)}{r_b}$$

Hence (15.9) can be written

$$\delta V_{cz}(r) = \frac{1}{2}\left(\frac{r\omega}{c}\right)^2 \omega \left(\frac{R_d}{Q}\right) \frac{r_b}{r} q \cos\phi$$

Evaluating the voltage increment at radius r = a for a bunch with an offset r_b then the cavity kick is therefore

$$\delta V_{cz}(a) = \frac{1}{2}\left(\frac{a\omega}{c}\right)^2 \omega \left(\frac{R_d}{Q}\right) \frac{r_b}{a} q \cos\phi = \frac{1}{2} \frac{a\omega}{c} \frac{r_b \omega}{c} \omega \left(\frac{R_d}{Q}\right) q \cos\phi \qquad (15.10)$$

The radius a is to be determined by ensuring that the stored energy for the equivalent circuit of figure 3 is the same as the stored energy given by (15.1). For the equivalent circuit and using (4.9) gives

$$U_{stored} = \tfrac{1}{2} C V_i^2 = \frac{V_i^2}{2\omega \left(\frac{R}{Q_{oi}}\right)_C} \qquad (15.11)$$

equating this to the stored energy given for the dipole mode by (15.1) evaluated at radius a and reverting to the FNAL definition of R/Q given in (14.3) gives

$$\frac{V_i^2}{2\omega\left(\dfrac{R}{Q_{oi}}\right)_C} = \left(\frac{c}{a\omega}\right)^2 \frac{1}{2\omega\left(\dfrac{R}{Q}\right)_F}\left(\frac{2}{\pi}\right)^2 V_z^2(a) \tag{15.12}$$

where we have replaced the $V_{cz}(a)$ in (15.1) by $\frac{2}{\pi}V_z(a)$ so that the voltages on each side do not contain the transit time factor.

It is apparent from (15.12) that

$$\text{if} \quad \frac{a\omega}{c} = \frac{2}{\pi} \quad \text{then} \quad \left(\frac{R}{Q_{oi}}\right)_C = \left(\frac{R}{Q}\right)_F \tag{15.13}$$

creating meaning for the circuit voltage V_i .

Equation (15.10) can now be used to determine the beam loading voltage kick that must be applied to the cavity response equation (4.15) each time a bunch passes through the cavity at phase ϕ. Using (15.13) in (15.10) we evaluate the cavity voltage increment from the initially existing field acting on the bunch δV_i (without the transit time factor) as

$$\delta V_i = \frac{\pi}{2}\delta V_c = \frac{1}{2}\frac{r_b\omega}{c}\omega\left(\frac{R_d}{Q}\right) q\cos\phi = \frac{r_b\omega}{c}\omega\left(\frac{R}{Q}\right)_F q\cos\phi \tag{15.14}$$

Side Note

P*adamsee [10] gives the beam induced voltage on pg 386 eqn. (19.12) as*

$\tilde{V}_{br} = \frac{\tilde{i}_b R_a}{2(1+\beta)} = -\frac{I_o R_a}{(1+\beta)}$ *where* $\beta = \frac{Q_o}{Q_e}$, $\tilde{i}_b$ *is the Fourier component of the actual beam current that excites the cavity,* I_o *is the time average beam current and* R_a *is the shunt impedance of the cavity in the equivalent circuit. This is the steady state voltage attained after many bunches when the external circuit is connected and dissipating the beam current. The minus sign represents the fact that the beam in an accelerator cavity is acting to reduce the voltage with respect to the generator. This equation is not comparable with (15.14).*

The derivation of (15.14) using the integral (15.7) only considered the action of the cavity field on the bunch and not the action of the bunch back on the cavity. Effectively equation (15.14) is a limiting case of equations (7.2) – (7.4) as illustrated in the approximate form of (7.1). The δV_i is not the full change in the voltage but just the change in its component in the direction of the existing field. Note that in equations (7.2) to (7.4) the quantity q_m was charge added to the cavity mode whereas in (15.3) the quantity q is the bunch charge. We also note that $\phi = \alpha - \theta$ i.e. ϕ is the phase of the bunch with respect to the field in the cavity and not the reference.

If we apply the kick (15.14) to the envelope equations we must take account of the fact that the voltage kick is being applied at phase ϕ .

From (7.3) and (7.4) and (9.1) we identify the real and imaginary parts of the field phasor $\underline{V}_{initial}$ before the bunch passes as

$$A_r = \frac{q_m}{C}\cos\theta \tag{15.15}$$

$$A_i = \frac{q_m}{C}\sin\theta \tag{15.16}$$

and we have that

$$\underline{V}^2_{\text{initial}} = A_r^2 + A_i^2 \tag{15.17}$$

hence

$$\left|\underline{V}_{\text{initial}}\right| = V_{\text{initial}} = \frac{q_m}{C} \tag{15.18}$$

Equations (7.3) and (7.4) can be therefore be written as

$$\text{Re}(V_{\text{final}}) = A_r + \frac{\delta q_m}{C}\cos\alpha \tag{15.19}$$

$$\text{Im}(V_{\text{final}}) = A_i + \frac{\delta q_m}{C}\sin\alpha \tag{15.20}$$

The square of the final voltage phasor is now given as

$$\underline{V}^2_{\text{final}} = \left(\underline{V}_{\text{initial}} + \underline{\delta V}\right)^2 = \left(A_r + \frac{\delta q_m}{C}\cos\alpha\right)^2 + \left(A_i + \frac{\delta q_m}{C}\sin\alpha\right)^2 \tag{15.21}$$

Hence expanding and subtracting (15.17) gives

$$\underline{V}_{\text{initial}} \cdot \underline{\delta V} + \tfrac{1}{2}\delta V^2 = \frac{\delta q_m}{C}\left(A_r\cos\alpha + A_i\sin\alpha\right) + \frac{1}{2}\left(\frac{\delta q_m}{C}\right)^2 \tag{15.22}$$

Using (15.18) in (15.15) and (15.16) so that A_r and A_i can be replaced with $\underline{V}_{\text{initial}}$ and the phase angle in (15.22) gives

$$\underline{V}_{\text{initial}} \cdot \underline{\delta V} + \tfrac{1}{2}\delta V^2 = \frac{\delta q_m}{C} V_{\text{initial}}\left(\cos\theta\cos\alpha + \sin\theta\sin\alpha\right) + \frac{1}{2}\left(\frac{\delta q_m}{C}\right)^2 \tag{15.23}$$

Hence as $\phi = \alpha - \theta$ we have

$$\underline{V}_{\text{initial}} \cdot \underline{\delta V} + \tfrac{1}{2}\delta V^2 = \frac{\delta q_m}{C} V_{\text{initial}}\cos\phi + \frac{1}{2}\left(\frac{\delta q_m}{C}\right)^2 \tag{15.24}$$

Comparison of this equation with its approximate form given in (15.15) allows us to identify

$$\frac{\delta q_m}{C} = \frac{r_b\omega}{c}\omega\left(\frac{R}{Q}\right)_F q \tag{15.25}$$

Hence from (15.19) and (15.20) beamloading in the envelope equations is determined as

$$A_r(\text{final}) = A_r(\text{initial}) + \frac{r_b\omega}{c}\omega\left(\frac{R}{Q}\right)_F q\cos\alpha \tag{15.26}$$

$$A_i(\text{final}) = A_i(\text{initial}) + \frac{r_b\omega}{c}\omega\left(\frac{R}{Q}\right)_F q\sin\alpha \tag{15.27}$$

By consideration of figure 5 we note that:-

$$\delta V_i = \delta\left|\,\underline{V_i}\,\right| = \delta A_r\cos\theta + \delta A_i\sin\theta \tag{15.28}$$

where $\delta A_r = A_r(\text{final}) - A_r(\text{initial})$ and $\delta A_i = A_i(\text{final}) - A_i(\text{initial})$

Putting our preferred cavity parameters and the ILC bunch charge into (15.14) where $(R_d/Q) = 53\ \Omega$ per cell, $q = 3.2\times10^{-9}$ C, $f = 3.9\times10^9$ gives

$$\delta V_i = 1.697 \times 10^5 \, r_b \cos\phi \tag{15.29}$$

hence for r = 1 mm and ϕ = 1 degree then

$$\delta V_i = 169.7 \text{ Volts per cell} \tag{15.30}$$

From (15.12) and (15.14)

$$V_i = \sqrt{2\omega_o \left(\frac{R}{Q}\right)_F U_{stored}} \tag{15.31}$$

hence

$$V_i = \sqrt{2 \times 2\pi \times 3.9 \times 10^9 \times 26.5 \times 0.0284} = 1.92 \times 10^5 \text{ Volts per cell} \tag{15.32}$$

hence

$$\frac{\delta V_i}{V_i} \approx 0.88 \times 10^{-3} \tag{15.33}$$

i.e. individual bunches have a negligible effect on the amplitude.

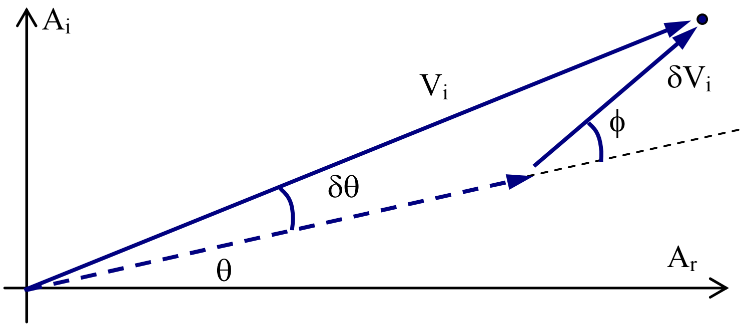


Figure 4b Phase addition of image charge giving a voltage increment

By consideration of figure 4b the change in the phase caused by a single bunch is given as

$$\delta\theta = \sin^{-1}\left(\frac{\delta V_i}{V_i} \tan\phi\right) \tag{15.34}$$

where $\phi = \alpha - \theta$ is the phase error on the bunch with respect to the cavity field. The phase on the cavity phase θ should not exceed 0.1 degrees at 3.9 GHz when properly controlled and α is unlikely to exceed 1 degree (at 3.9 GHz), hence

$$\delta\phi_{cavity} = \sin^{-1}\left(0.88 \times 10^{-3} \tan(1)\right) = 0.88 \times 10^{-3} \text{ degrees} \tag{15.35}$$

Phase synchronisation of the ILC crab cavity systems on the positron and electron beam lines needs to be better than 0.124 degrees for a 14 mad crossing angle [12]. If the synchronisation error is composed of three equal uncorrelated components, namely cavity 1 to reference 1, reference 1 to reference 2 and reference 2 to cavity 2 then the synchronisation requirement for each component is $0.124/\sqrt{3} = 0.072$ degrees. From (15.35) one concludes that it needs about 80 offset bunches at 1 mm and with a beam to cavity phase error of 1 degree before a change in the cavity phase reaches the limit of acceptability. The ILC bunches are 330 ns apart hence the control system needs to complete its correction on a time scale of 26 μs.

16. Power Requirement for Beam loading

From (6.1) and (4.10) one has for an unloaded cavity, excited at resonance and in steady state that

$$P_f = \frac{\left(1+\frac{Z_{wg}}{R}\right)^2}{4\frac{Z_{wg}}{R}} P_c = \frac{\left(1+\frac{Q_e}{Q_o}\right)^2}{4\frac{Q_e}{Q_o}} P_c \tag{16.1}$$

This equation expresses the condition for impedance matching between the waveguide and the cavity.

When a cavity is excited with a beam, the beam can be modelled as an additional shunt resistance R_b in parallel to the cavity resistance R. The resistance R is positive if power is transferred from the cavity to the beam and negative if power is transferred from the beam to the cavity. If beam-loading is high then we can neglect R with respect to R_b hence (16.1) can be written

$$P_f = \frac{\left(1+\frac{Q_e}{Q_b}\right)^2}{4\frac{Q_e}{Q_b}} P_b \tag{16.2}$$

where

$$Q_b = \frac{\omega U}{P_b} \tag{16.3}$$

The maximum power that the beam can extract is given by

$$P_b = qV_{seen} f_{rep} = q\,V_z(r)\cos\phi\; f_{rep} \tag{16.4}$$

where f_{rep} is the bunch repetition frequency, q is the bunch charge and $V_z(r)$ is given by (14.2) i.e.

$$V_z(r) = \frac{r\omega}{c}\sqrt{\omega\frac{R_d}{Q}U_{stored}} \tag{14.2}$$

If U_{stored} is specified, R/Q is known and the offset r is known then the power P_b can be determined and hence the forward power requirement.

The current ILC design spaces the bunches by 330 ns hence f_{rep} = 3.03 MHz. Putting in values appropriate to the ILC crab cavity one gets

$q = 3.2\times10^{-9}$ C, $f_{rep} = 3.03\times10^{6}$, $U_{stored} = 0.0284$ J per cell $r = 1\times10^{-3}$ m ,

$\frac{R_d}{Q_o} = 53\,\Omega$ and $\phi = 0$ gives

$V_z(1\text{mm}) = 15700$ V, $P_b = 152$ W, $Q_b = 4.57\times10^{6}$

Figure 8 plots equation (14.2) with these parameters.

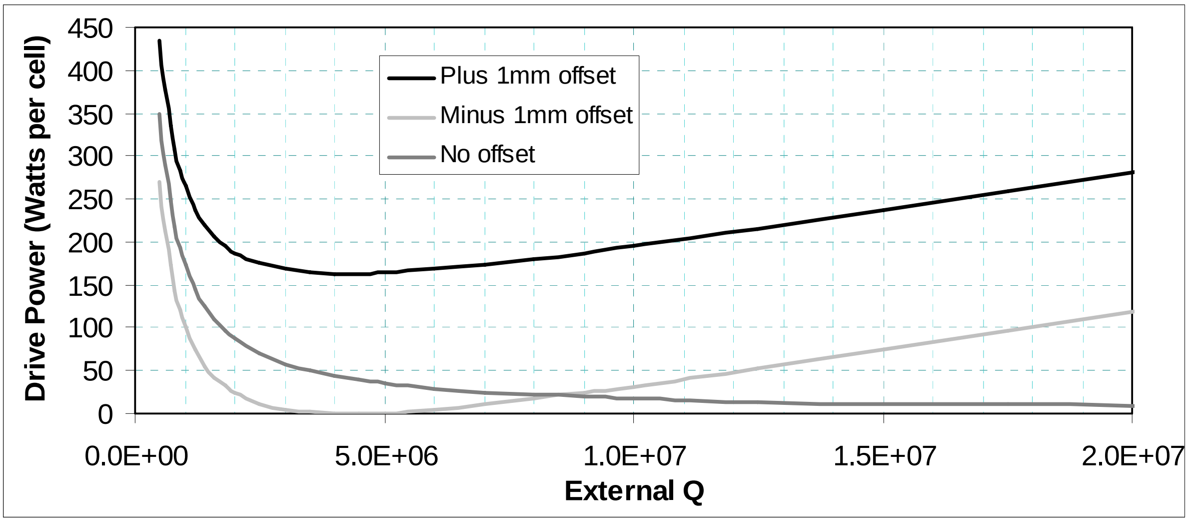


Figure 8 Drive power as function of Q_{ext} for ±1 mm beam offset (no microphonics).

For cavity sizes up to nine cells and for drive powers of a few hundred Watts per cell the total power requirement at 3.9 GHz for the proposed ILC crab cavity can be met with either a Klystron, or a TWT or a solid state amplifier. Control of microphonics might be expected to be better the lower the external Q and this will be analysed in later sections. On this basis and provided the amplifier has adequate flexibility one would probably want to aim for an external Q near to 3×10^6 hence many of the later calculations use this value as a reference case.

17. Beam Loading Simulation – an illustration

Beam loading can be simulated by incrementing the voltage by an appropriate amplitude and phase each time a bunch passes through the cavity. Appendix 3 gives the code for integration of the envelope equations with repetitive beam loading and a controller on the phase and amplitude of the forward wave from the generator. Figures 8 to 11 show illustrative results for highly exaggerated dipole beam loading using an unrealistic 40 mm beam offset, a massive 5 degree timing error and a 15.38 μs repetition rate (chosen so that the effect of the beam can be clearly seen in the figures). The external Q has been taken as 10^6 and the beam is turned on after 2.0×10^6 cycles and off 1 ms later. The controller uses a simple PI algorithm and the control action is applied with a delay of 3900 cycles = 1 μs.

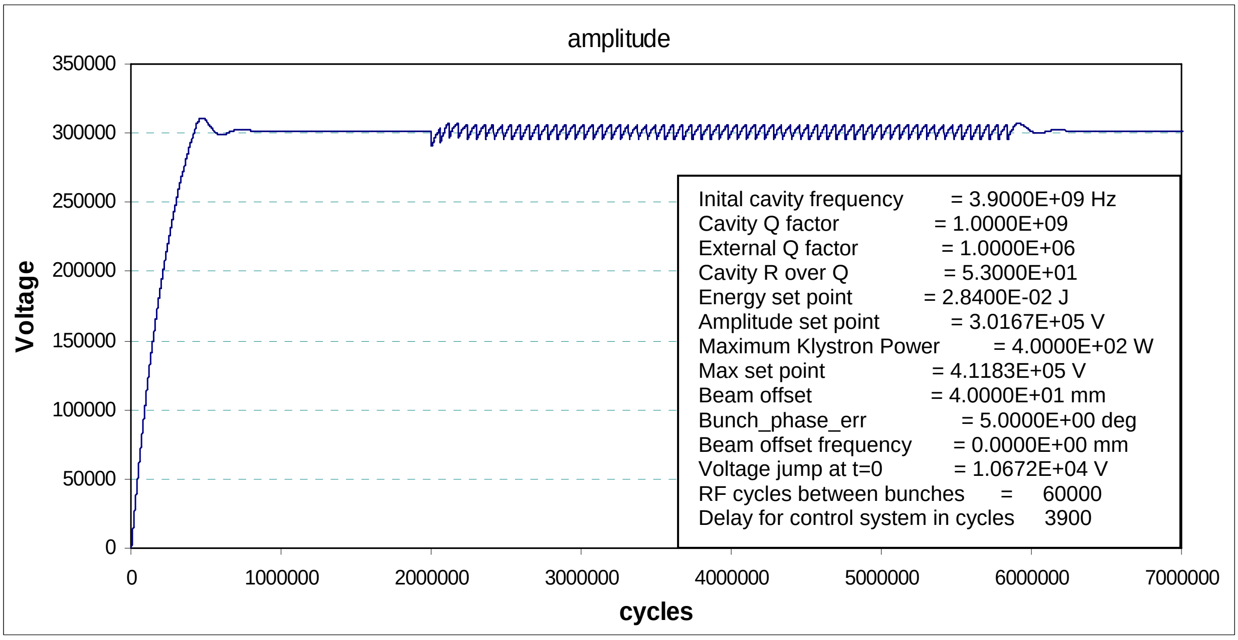


Figure 8 Cavity voltage after switch on and after arrival of beam

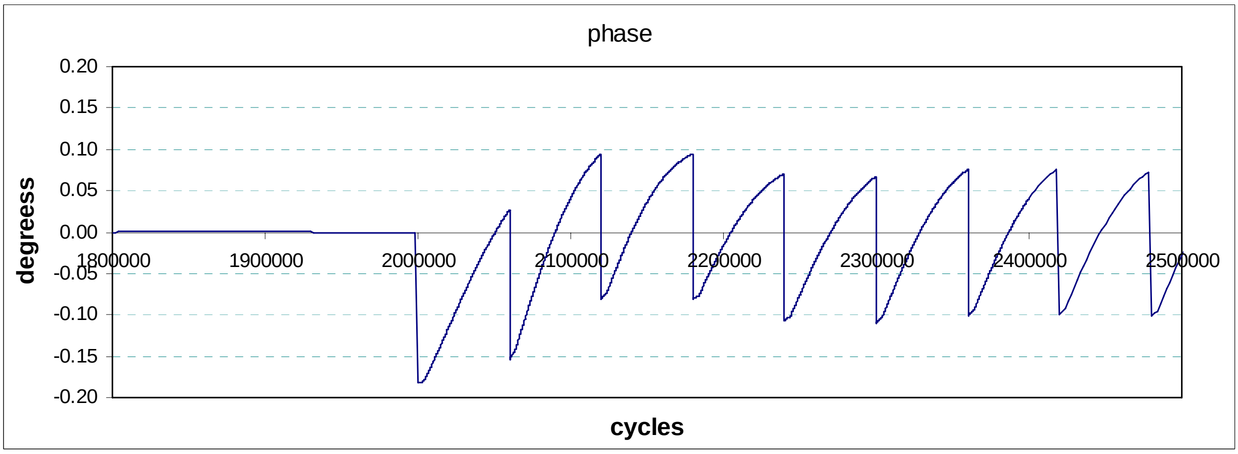


Figure 9 Cavity phase showing arrival of beam

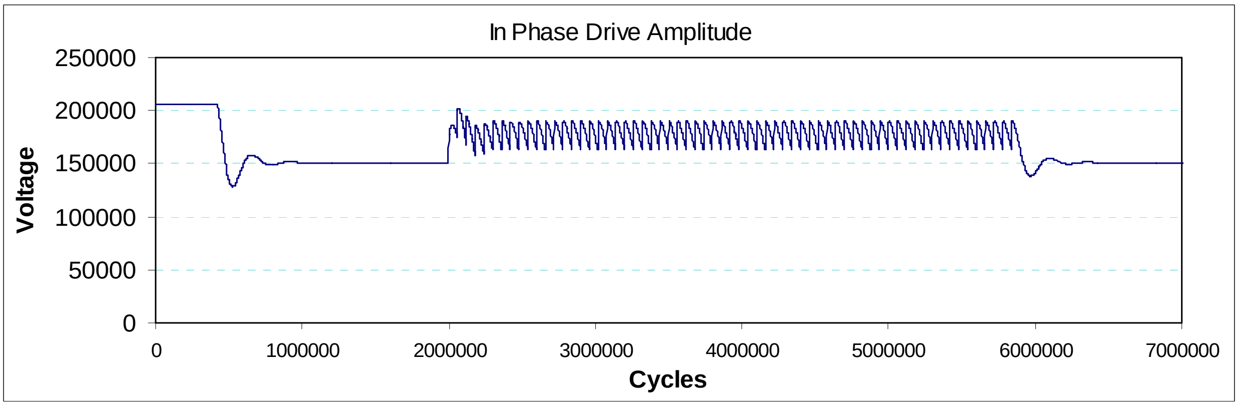


Figure 10 Amplitude of drive voltage (1:1 transformer ratio) after switch on and after arrival of beam

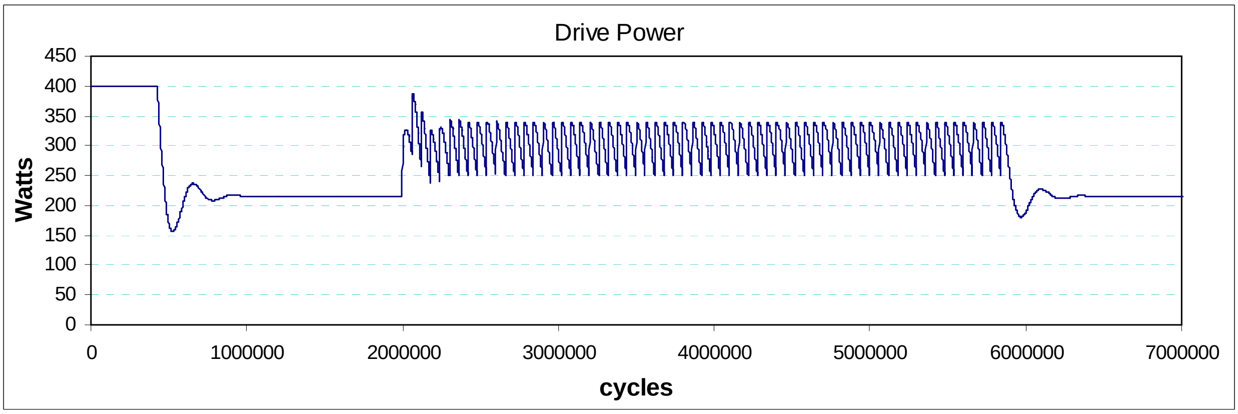


Figure 11 Input power from drive after switch on and after arrival of beam

Note that the amplifier has been limited to 400 Watts. The control system attempts to correct the phase and amplitude after every bunch.

18. The Controller

The controller employed for these and subsequent simulations is a simple PI controller with a delayed action. A differential term is not employed as it is anticipated that noise on measurements of actual cavity fields cannot be adequately suppressed for the differential term to be useful whilst retaining system response. A PI algorithm responds well to random and unpredictable system disturbances such as beam offsets. In high Q superconducting cavities, tiny mechanical vibrations referred to as Microphonics [2] can give significant phase and frequency shifts. Microphonics are typically predictable on the time scale of sub milli-seconds and hence predictive, feed-forward algorithms have the potential to out perform the simple PI algorithm. Advanced control algorithms are not considered in this report as microphonic data for the real system would not be available until a full system is tested. Performance with a PI algorithm should be regarded as a baseline performance.

For the multi-mode cavity an issue with the controller is whether by clever filtering one can determine the amplitude and phase of the operating mode. If one can and with reference to the envelope equations (11.2) one determines the drive as

$$\mathcal{F}_{\mathrm{r}}\left(t + t_{\mathrm{delay}}\right) = c_{\mathrm{pr}}\left(V_{\mathrm{sp}} - A_{1\mathrm{r}}\right) + c_{\mathrm{ir}}\left(\frac{\omega}{2\pi}\right)\int_{-\infty}^{t} dt\,\left(V_{\mathrm{sp}} - A_{1\mathrm{r}}\right) \tag{18.1}$$

$$\mathcal{F}_{\mathrm{i}}\left(t + t_{\mathrm{delay}}\right) = -c_{\mathrm{pi}}\,A_{1\mathrm{i}} - c_{\mathrm{ii}}\left(\frac{\omega}{2\pi}\right)\int_{-\infty}^{t} dt\,A_{1\mathrm{i}} \tag{18.2}$$

where t_{delay} is the time it takes to measure the error and adjust the amplifier output and V_{sp} is the set point voltage required in the cavity. The set point for the phase is zero hence the set point for the out of phase part of the voltage A_{1i} is also zero.

In the multi-mode software model we assume that the operating mode cannot be measured directly and instead one measures a time average of amplitude and phase of all the modes in the cavity where differing modes have a differing weighting according to their coupling to the output coupler.

The ratio of the integral coefficients c_i to the proportional coefficients c_p have been chosen such that the response immediately after rapid cavity filling at full power is slightly under-damped. This choice generally gave a slightly better control of amplitude and phase during the bunch than critical damping.

In some simulations such as those in sections 19.4, 20 and 22 the coefficients for the imaginary part were taken to be several times larger than those for the real part. The idea behind this followed from the fact that the crab cavity would operate at zero phase and better control of phase than amplitude is required. It was later realised that there is no real penalty for having coefficients for the real part equal to those for the imaginary part and the benefit is better control of the amplitude. Equal coefficients are used in sections 19.1, 19.2, 19.3, 21 and 23. Where larger control coefficients have been used for the imaginary part, these coefficients set the stability limit (*see section 21*).

19. Single mode simulations for ILC bunch parameters

19.1 $Q_e = 10^6$ (no microphonics, no measurement error, severe random beamloading)

This section considers the performance of single mode crab cavities with ILC bunch parameters [11]. Multi-mode cavities are considered in section 23. The first simulations of sections 19.1 and 19.2 focus on the effect of beam loading. The simulation shown in figures 12 to 16 illustrates beam loading response for a far less extreme case than was shown in section 17. Bunch charges and repetition rate are now nominal ILC values. In this simulation the beam is given a random offset with a maximum value of 1 mm and a maximum phase error of 1 degree; ILC actual values at the Crab Cavity location are more likely to be a slowly varying offset to 0.6 mm and beam arrival phase error of up to 0.3 degrees (at 3.9 GHz). The external Q has been set again as 10^6 so that results can be compared with calculations in section 16. The offset and phase errors are randomized with a flat distribution to the maximum offsets stated above. The controller has a relatively low gain with respect to the stability limit. The bunch arrives at cycle 500000.

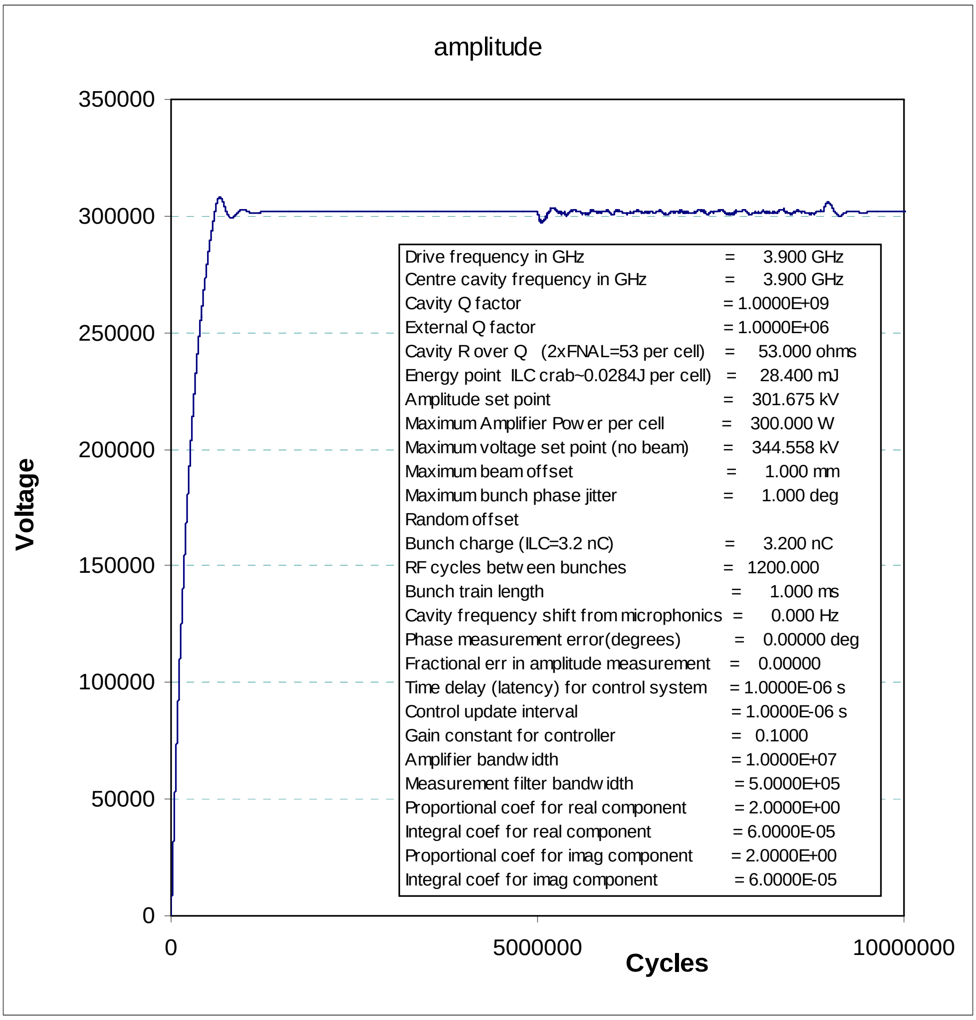


Figure 12 Cavity voltage after switch on and after arrival of beam

After initial settling, the RMS amplitude fluctuation during the bunch (cycle 5200000 to 8800000) in figure 12 with beam are about 0.2%

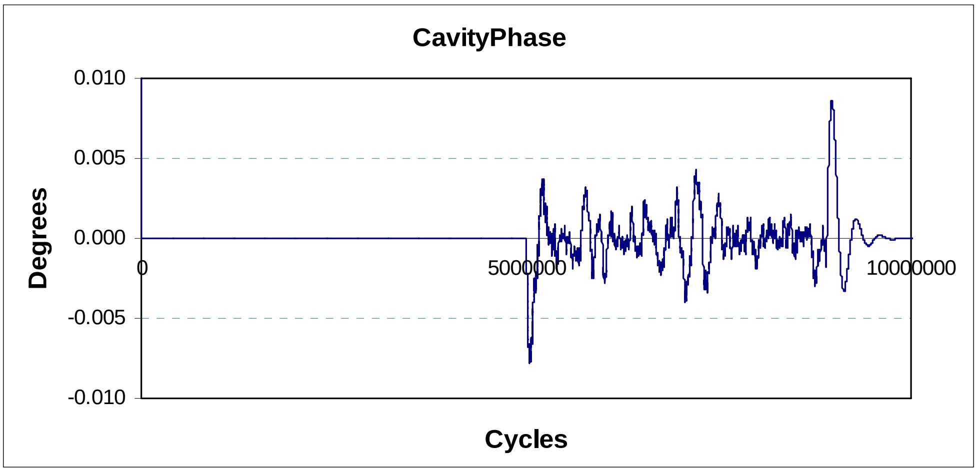


Figure 13 Cavity phase before and after bunch arrival at 5000000 cycles

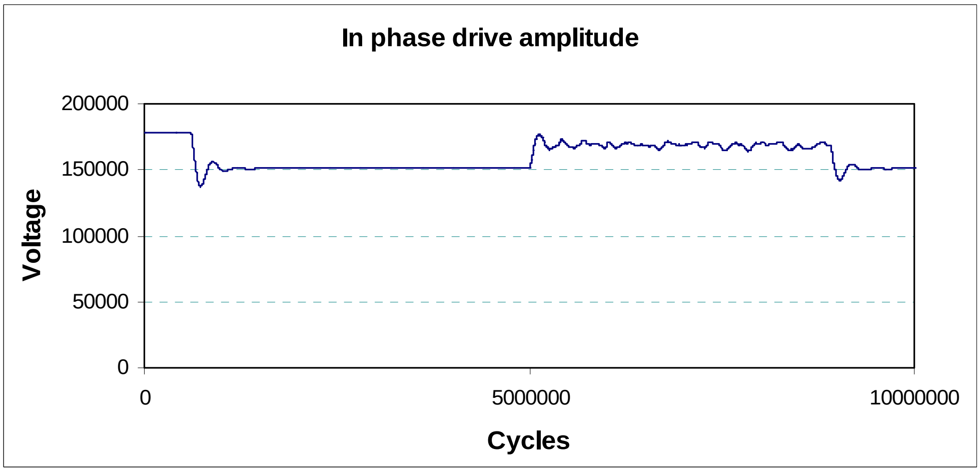


Figure 14 Amplitude of drive voltage (1:1 transformer ratio). Note that the peak value is limited as the drive amplifier has a maximum output of 300 W.

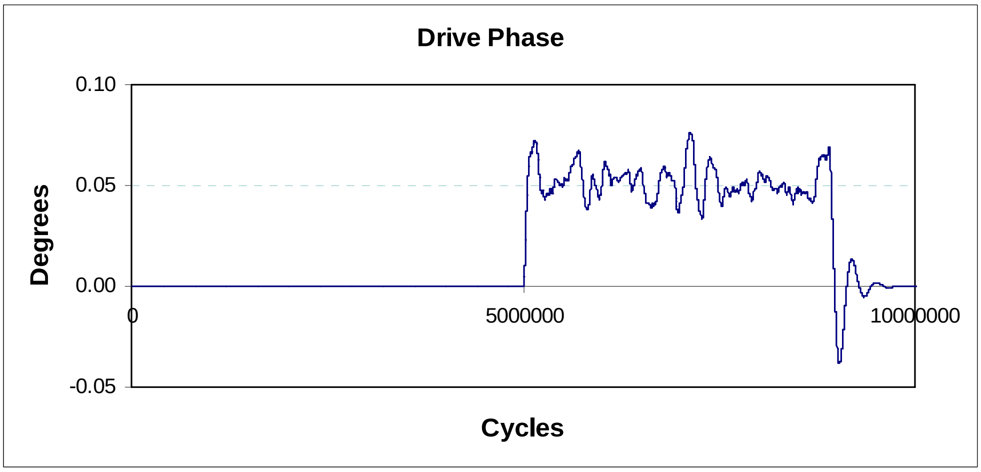


Figure 15 Phase of drive voltage (1:1 transformer ratio)

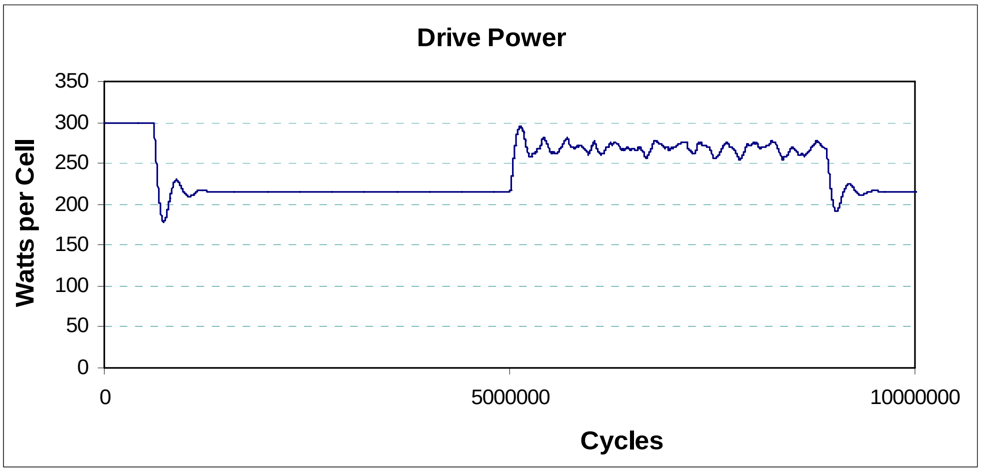


Figure 16 Input power from drive after switch on and after arrival of beam

Inspection of figure 16 suggests that the drive amplifier needs to respond in a time period of about 50000 RF cycles = 13 μm hence the bandwidth required to compensate for random beam loading is about 100 kHz. The actual bandwidth used for the simulation was 10 MHz.

19.2 $Q_e = 3 \times 10^6$ (no microphonics, no meas. err., severe random beamloading)

One might guess that increasing the external Q factor of the cavity increases the phase error. The simulations in this section show that this is not the necessarily the case if one increases the gain in the same proportion to any increase in Q. The simulation in this section shows that increasing the cavity external Q with respect to the previous calculation reduces power consumption with no other detrimental effect provided the gain is increased. When the external Q is increased from 10^6 to 3×10^6 the amplitude response as shown in figure 17 is practically unchanged except where the set point is just attained. If gain is not increased then control deteriorates.

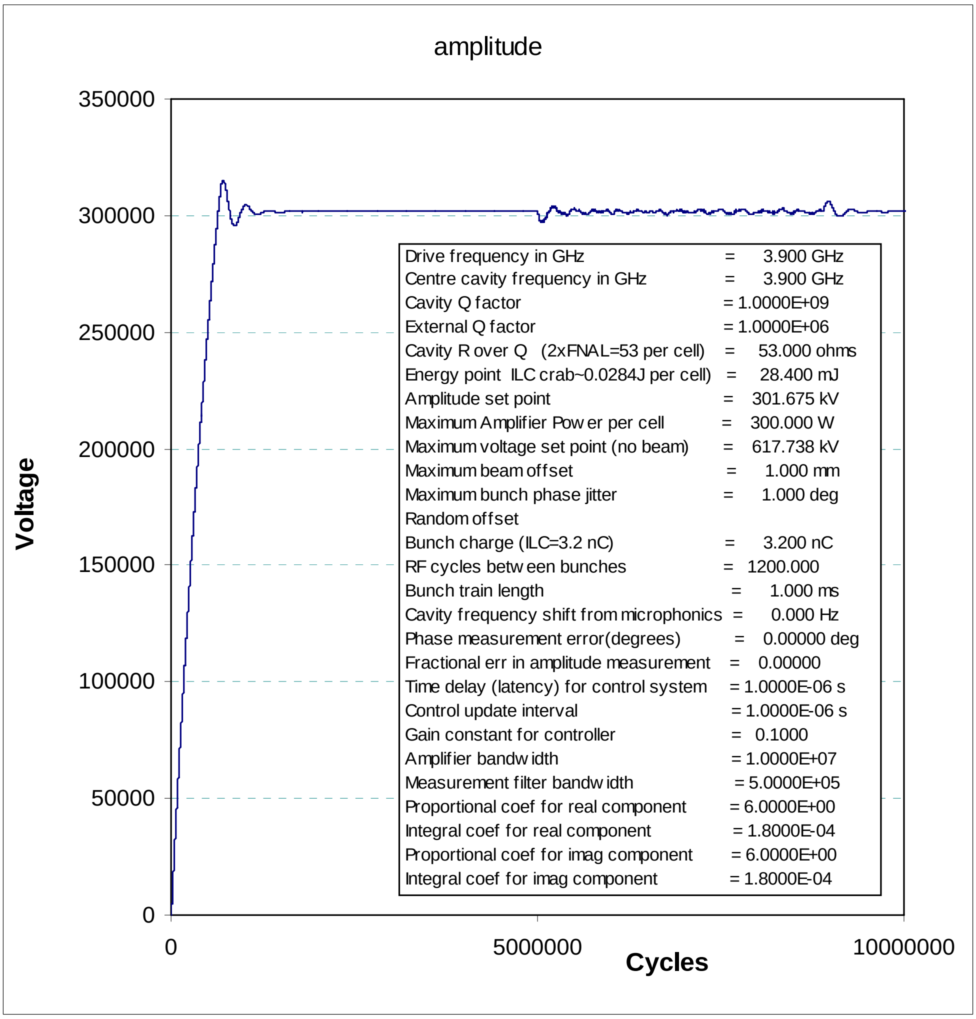


Figure 17 Cavity amplitude showing fill and full bunch, $Q_e = 3 \times 10^6$

The phase response of the cavity as shown in figure 18 is also almost identical to the low external Q case of figure 13.

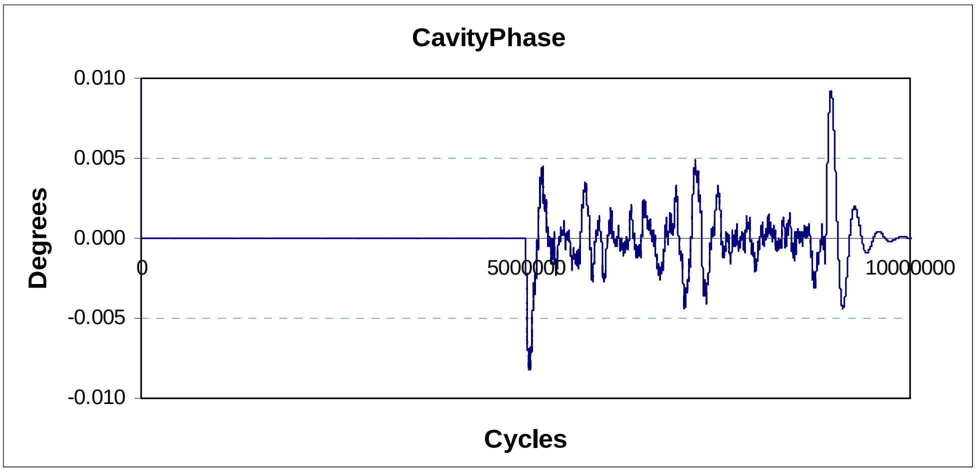


Figure 18 Cavity phase showing fill and whole bunch, $Q_e = 3 \times 10^6$

Figure 19 shows the power requirement before and after the beam arrives and should be compared with figure 16. In figure 16 a power requirement of 215 Watts is required before the beam arrives. After the beam arrives this rises to an average value of 270 Watts. In figure 19 there is a power requirement of 72 Watts before the beam arrives and this rises to an average of 132 Watts once the beam arrives. The power in each case rises by 55 Watts and 60 Watts respectively which is roughly the same as expected. The rise in power is required on a time scale of about 13 μs. There would appear to be no disadvantage in having a high external Q of 3×10^6 provided the amplifier has the required flexibility of output. In section 14 we determined that the minimum power requirement for a beam offset of 1 mm occurs at Q external = 4.51×10^6. For this external Q and for a random offset the power requirement falls to an average of 115 W.

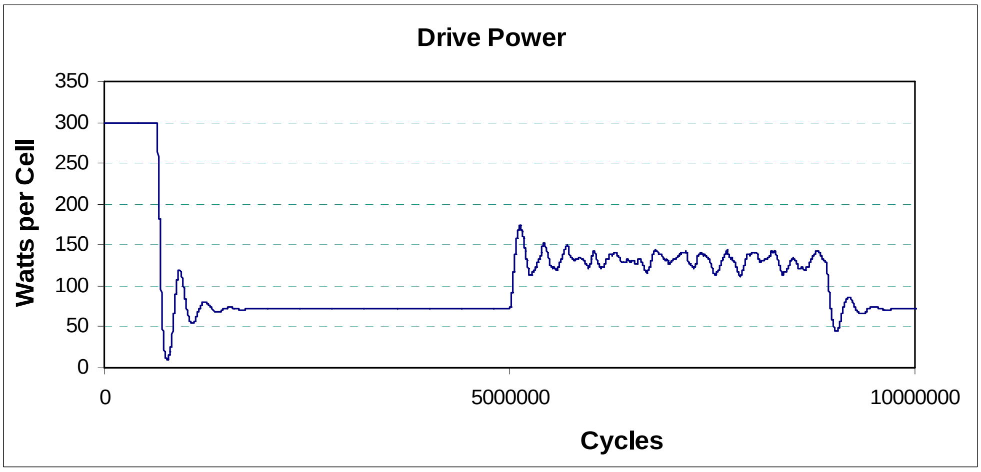


Figure 19 Input power for fill and whole bunch, $Q_e = 3 \times 10^6$

Figure 20 plots the phase of the drive, note that it does not average to zero.

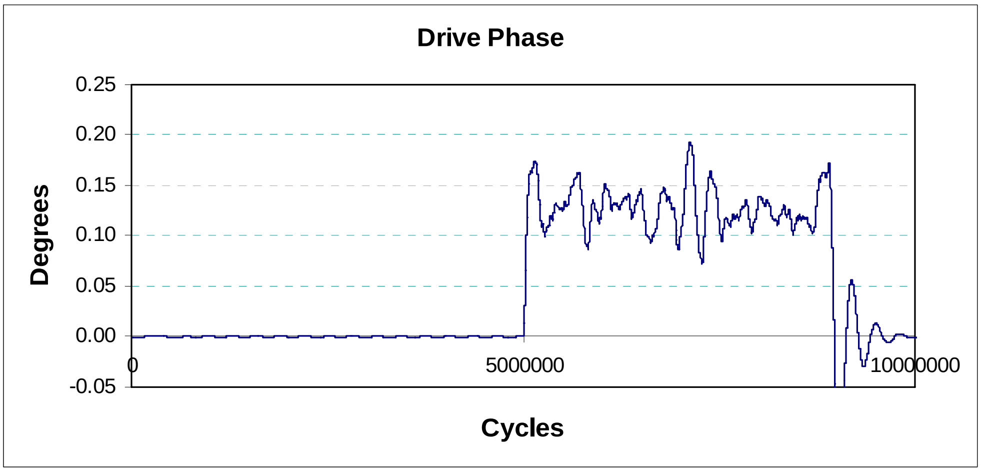


Figure 20 Phase of drive voltage (1:1 transformer ratio) after switch on and after arrival of beam, $Q_e = 3 \times 10^6$

19.3 Oscillatory offset (no microphonics)

Maximum power requirement occurs when the beam offset stays at its maximum offset. The most demanding power requirement is when the beam oscillates slowly from its extreme offset where it is taking power to its extreme offset where it is delivering power. For this case the power supply either has to turn itself on and off or switch its phase. Figures 21 to 23 illustrate the situation where the control algorithm turns the power supply on and off rather than switching phase and moving around zero output. The offset is taken to be sinusoidal with a frequency of 2 kHz so that there are two periods during the bunch. Figure 21 shows the amplitude on an expanded scale. Note that the control coefficients as defined in section 18 are given in a separate box and are written in terms of Q_e in accordance with the identified scaling.

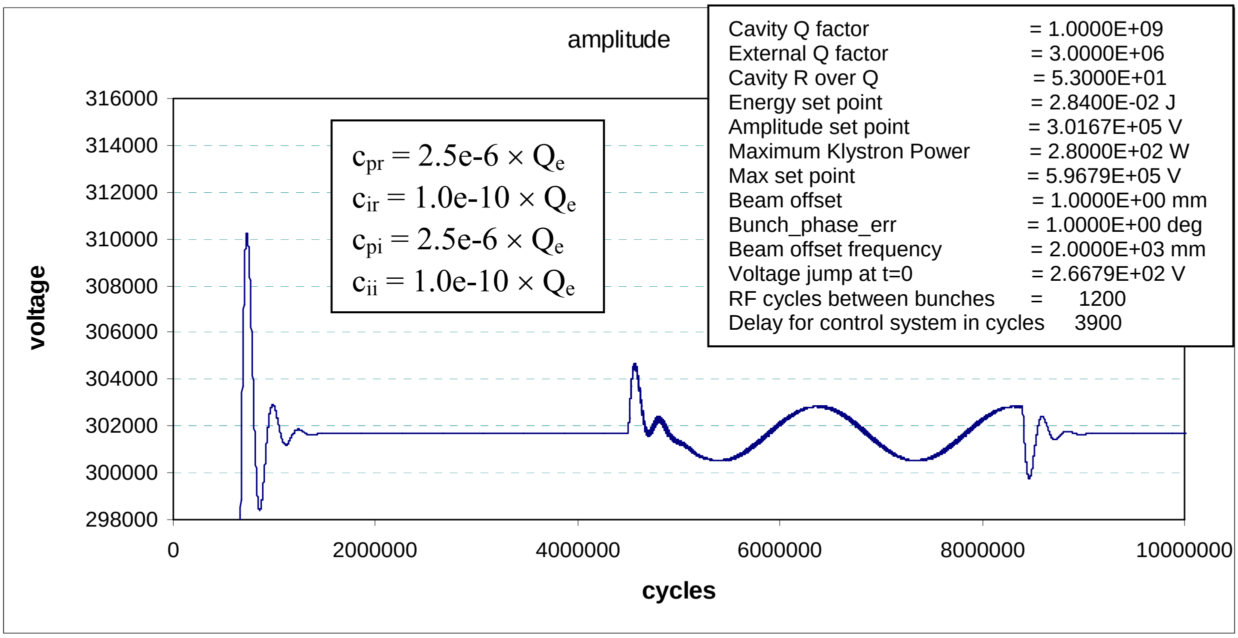


Figure 21 Cavity amplitude for oscillating beam offset, $Q_e = 3 \times 10^6$

The simple PI controller keeps amplitude fluctuations to less than 0.8% . Figure 22 shows the power supply response that is needed to achieve this.

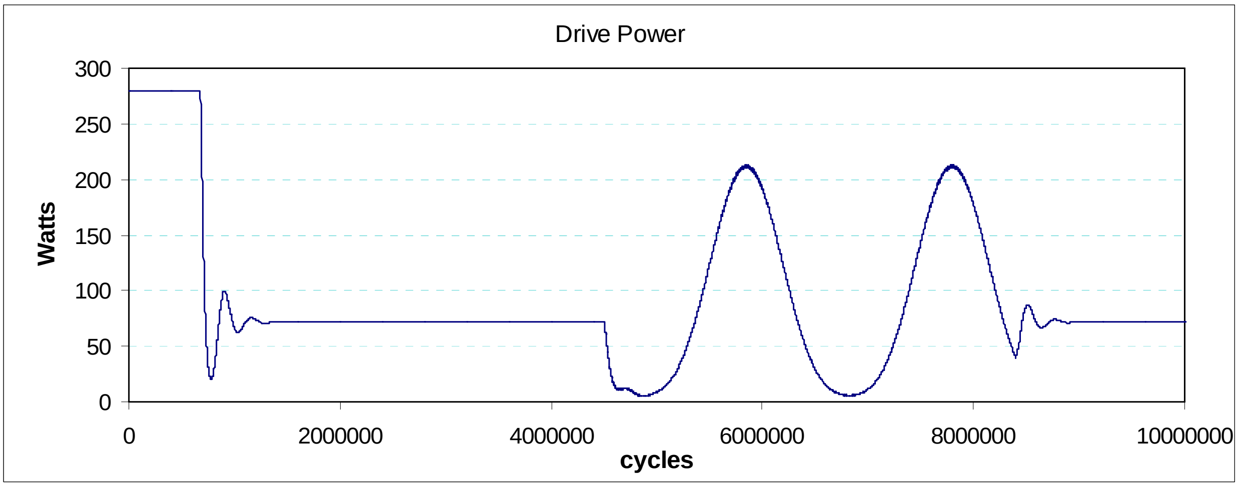


Figure 22 Input power for oscillating beam offset, $Q_e = 3 \times 10^6$

The power supply has to turn off and back on again at the oscillation frequency of the beam.

As shown in figure 23 cavity phase can still be maintained within the required tolerance.

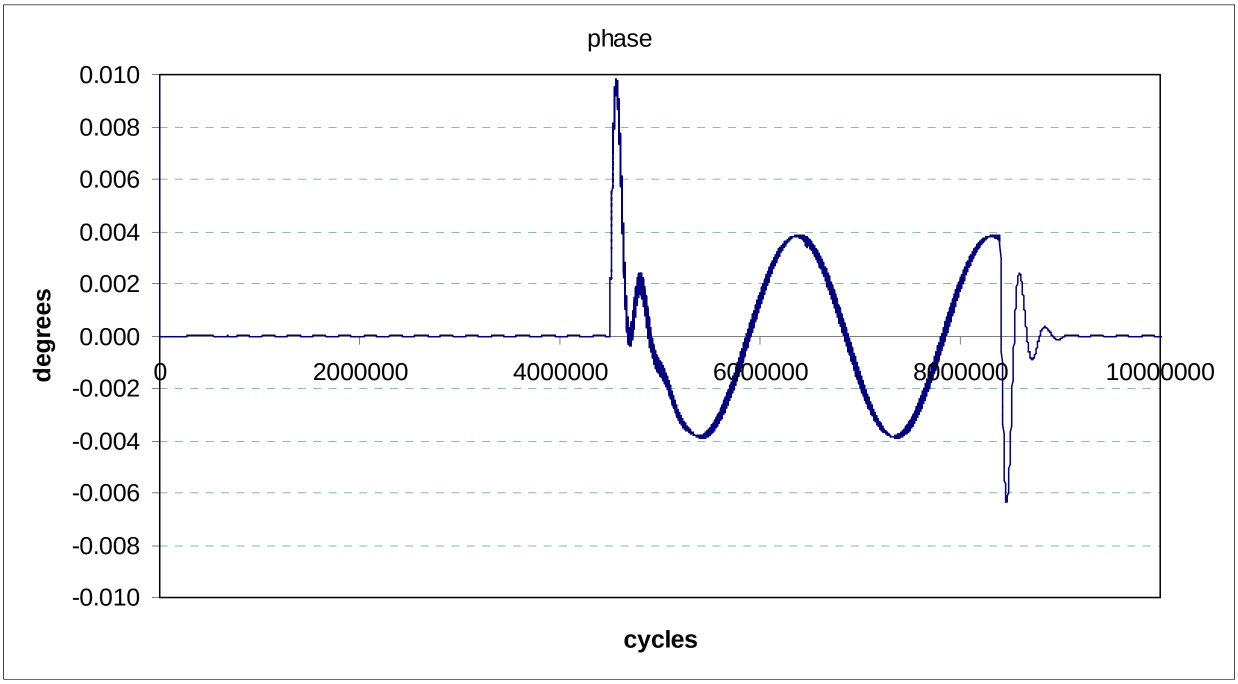


Figure 23 Cavity phase for oscillating beam offset, $Q_e = 3 \times 10^6$

If the cavity external Q is further increased the power supply must reverse its phase. Figures 24 to 28 illustrate this situation for the case of $Q_{external} = 1.75 \times 10^7$. As this external Q is much higher than the required external Q for matching the beam load at an offset of 1 mm the amplifier power must be increased. Here it is increase to 400 W per cell. Figure 24 shows the amplitude during cavity filling a period of settling and then during the pulse.

As the power has been increased the initial fill is rapid. Amplitude control is satisfactory. Figures 25, 26 and 27 show the amplifier power output, its drive amplitude and its drive phase respectively.

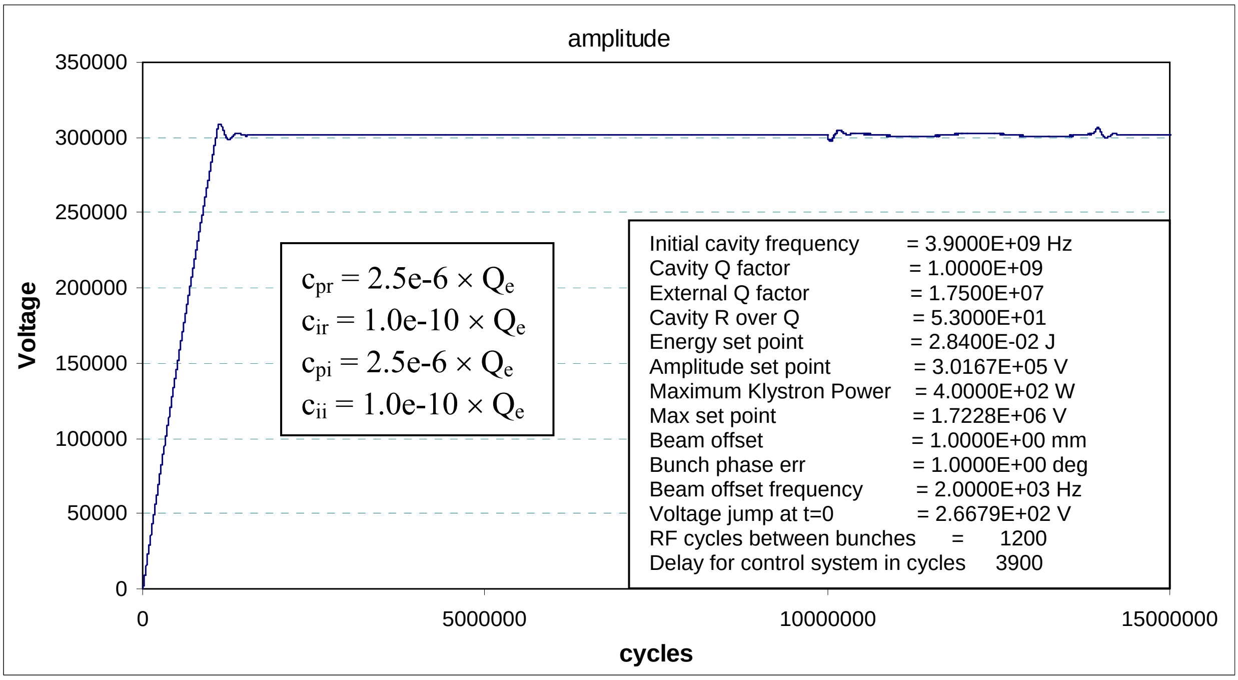


Figure 24 Cavity amplitude for oscillating beam offset, $Q_e = 1.75 \times 10^7$

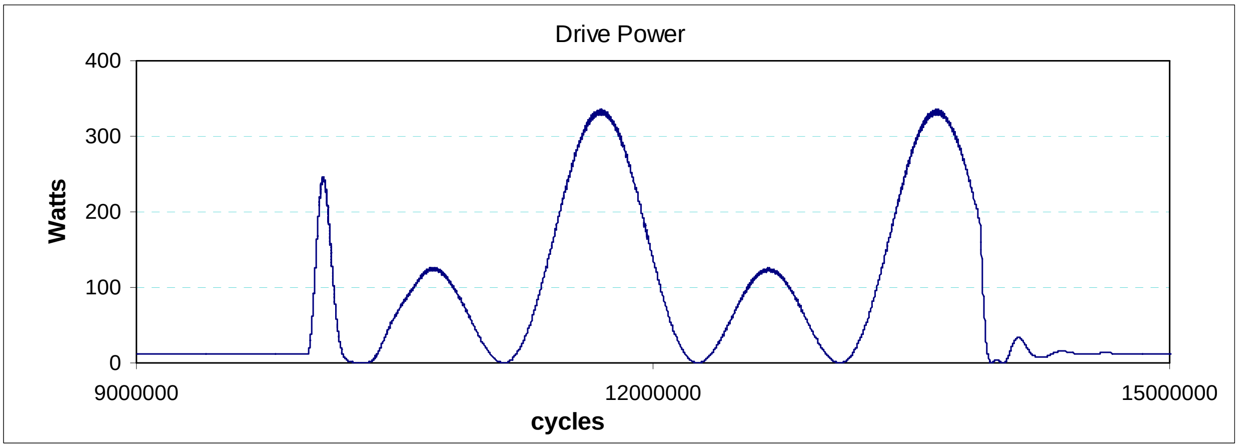


Figure 25 Drive power during bunch for oscillating beam offset, $Q_e = 1.75 \times 10^7$

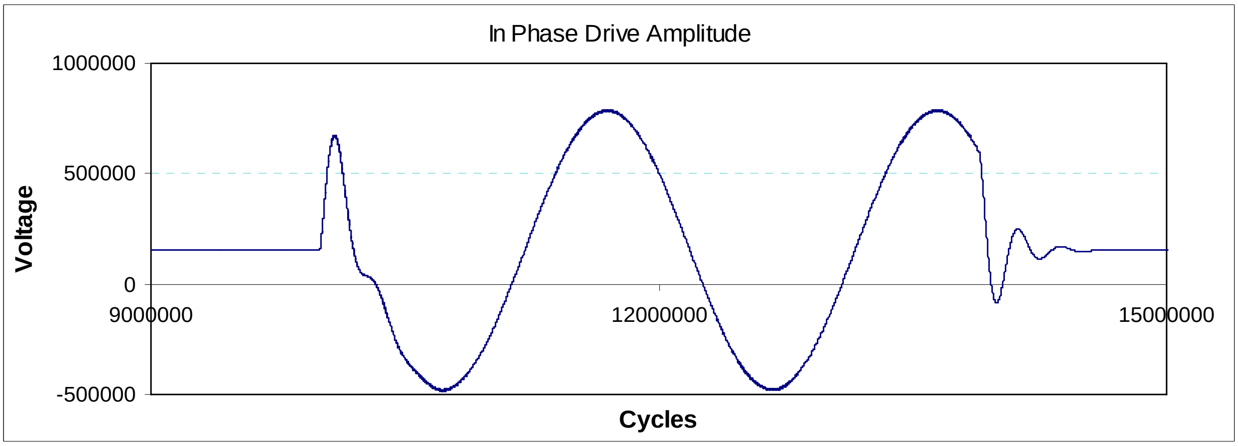


Figure 26 In phase drive amp. during bunch for oscillating beam offset, $Q_e = 1.75 \times 10^7$

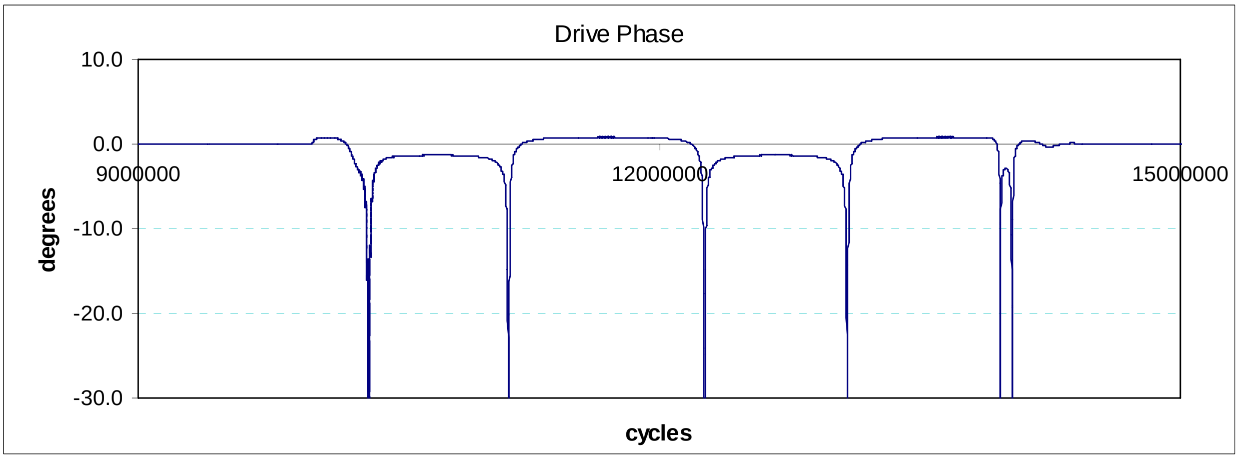


Figure 27 Drive phase during bunch for oscillating beam offset, $Q_e = 1.75 \times 10^7$

Figure 28 shows that for the higher external Q, control of the cavity phase and using the same controller parameters which include a scaling factor of Q_e is nominally the same. Accuracy of phase control depends primarily on the gain of the controller. When the controller has a delayed action, control becomes unstable when the gain (or indeed the integral term in the controller) rises above a certain level. It will be seen later that the point of instability in almost independent of Q_e when the control parameters have the scaling factor of Q_e .

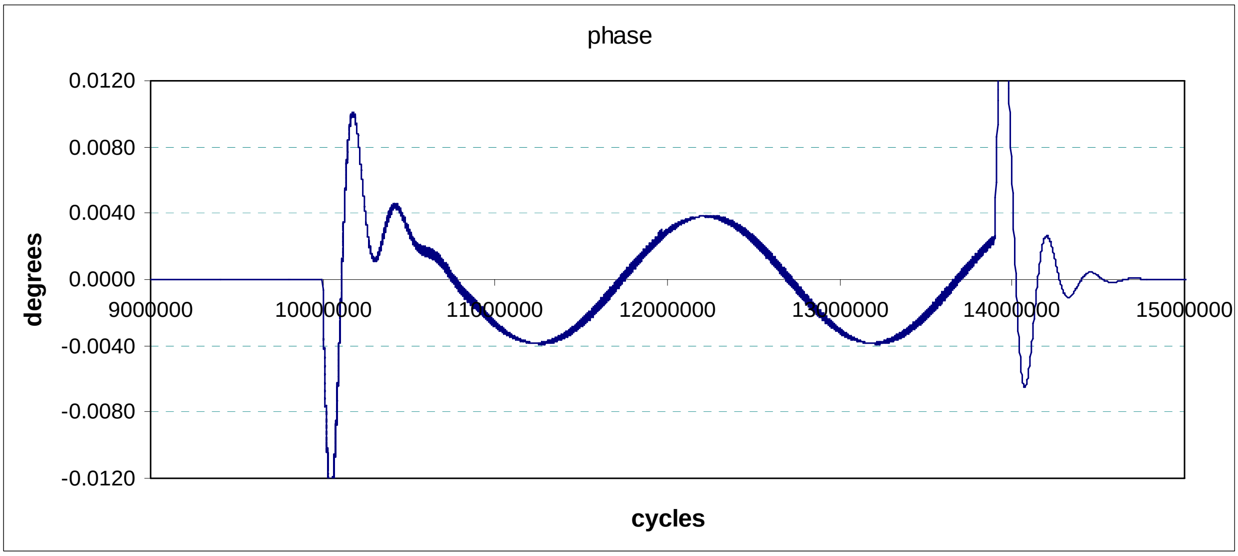


Figure 28 Cavity phase during bunch for oscillating beam offset, $Q_e = 1.75 \times 10^7$

19.4 Oscillatory offset and microphonics

In this section micro-phonics are simulated by giving the cavity frequency a sinusoidal variation.

The control action on the out of phase drive has been increased to keep the phase errors within the previous margin of 0.04^{o}. Figures 29 to 34 illustrate the effect of a mechanical resonance at 230 Hz that causes a maximum cavity frequency shift of 600 Hz. (See for instance "Microphonics detuning compensation in 3.9 GHz Superconducting Cavities", Carcagno, Bellantoni et al. FERMILAB-Conf-03/315-E). Note that the control parameters for the quadrature component have bee increased by a factor of four with respect to the in phase component for these simulations. The motivation for this was that a higher degree of phase control is required than amplitude control. (*Later we find no disadvantage in equal coefficients*)

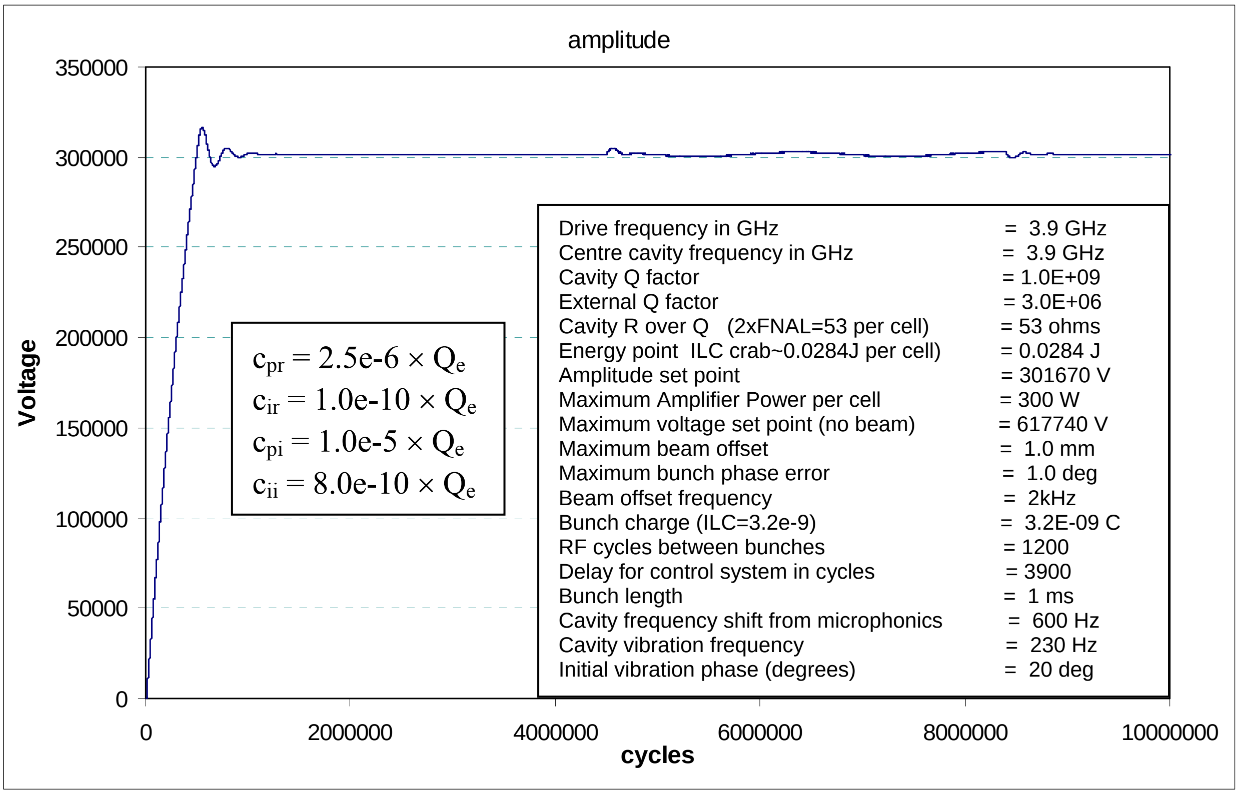


Figure 29 Cavity amplitude for oscillating beam offset and micrphonics, $Q_e = 3 \times 10^6$

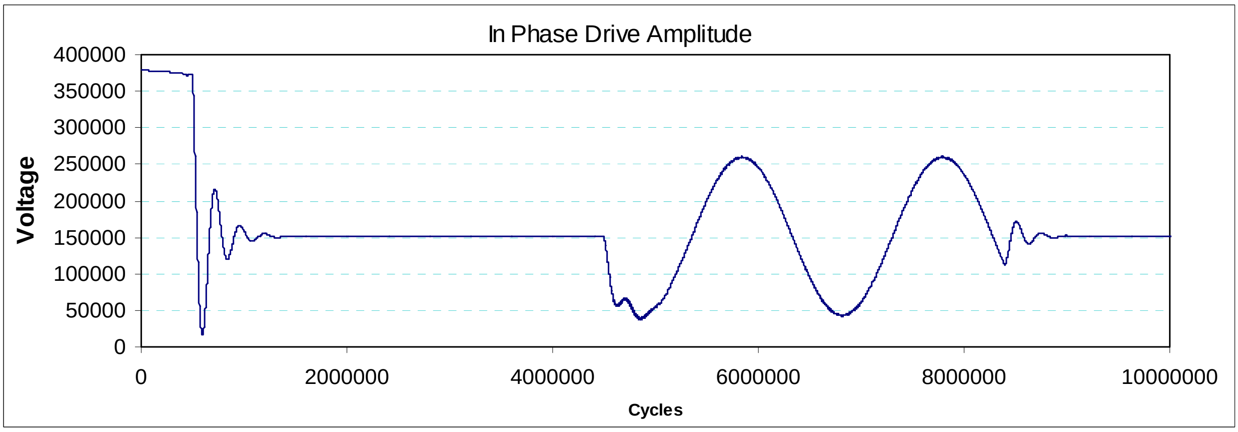


Figure 30 In phase drive amp. for oscillating beam offset & microphonics, $Q_e = 3 \times 10^6$

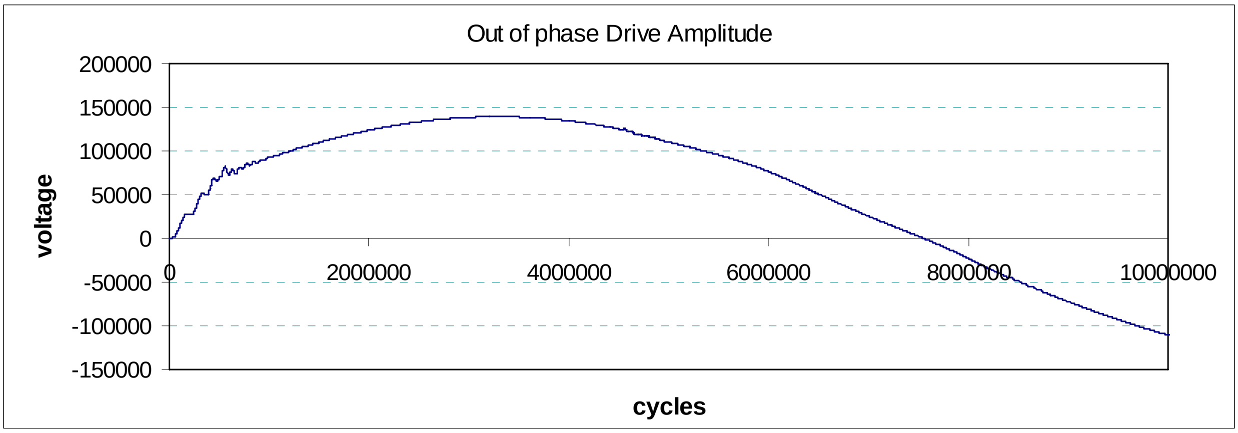


Figure 31 Out of phase drive ampl. oscillating beam offset & microphonics $Q_e = 3 \times 10^6$

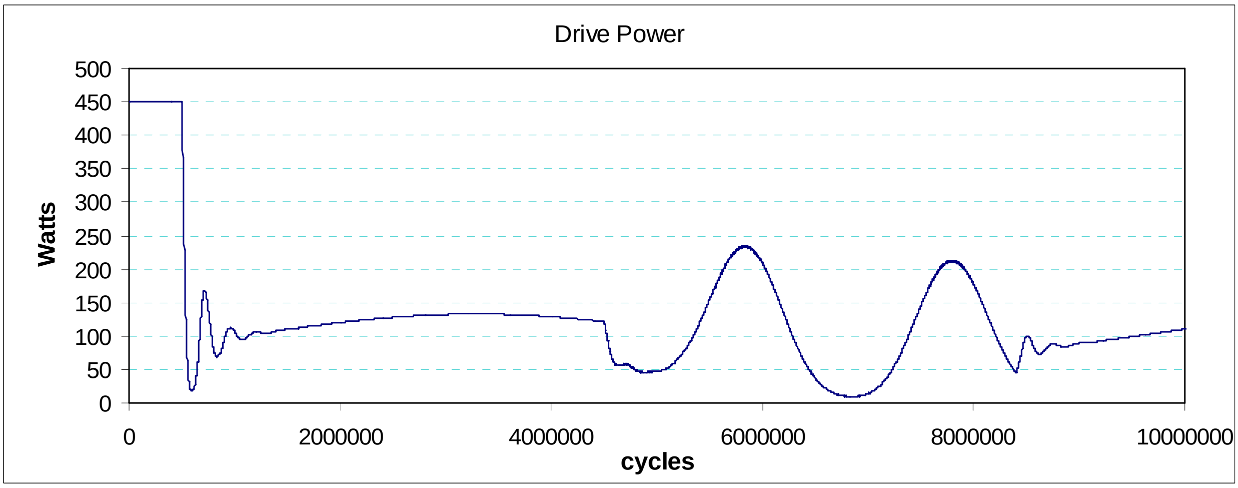


Figure 32 Out of phase drive power, oscillating beam offset & microphonics $Q_e = 3 \times 10^6$

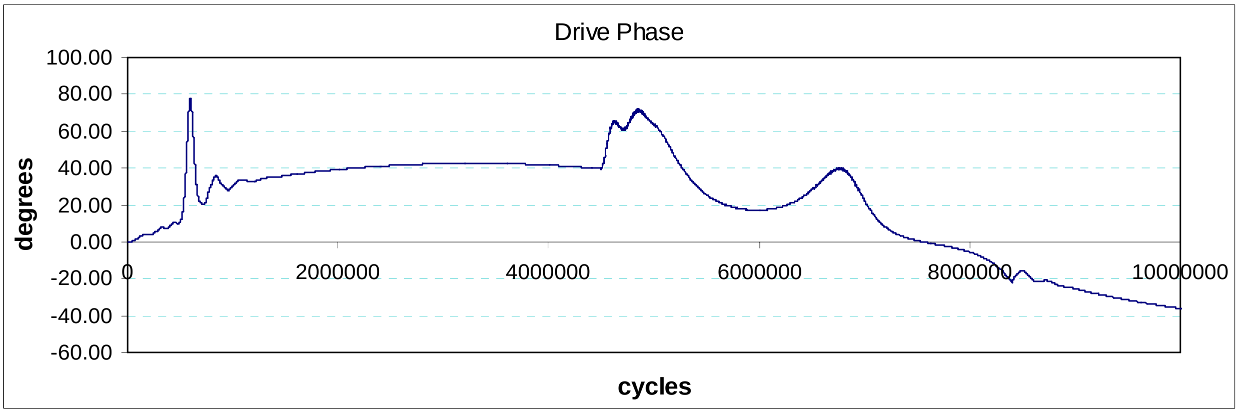


Figure 33 Out of phase drive phase oscillating beam offset & microphonics $Q_e = 3 \times 10^6$

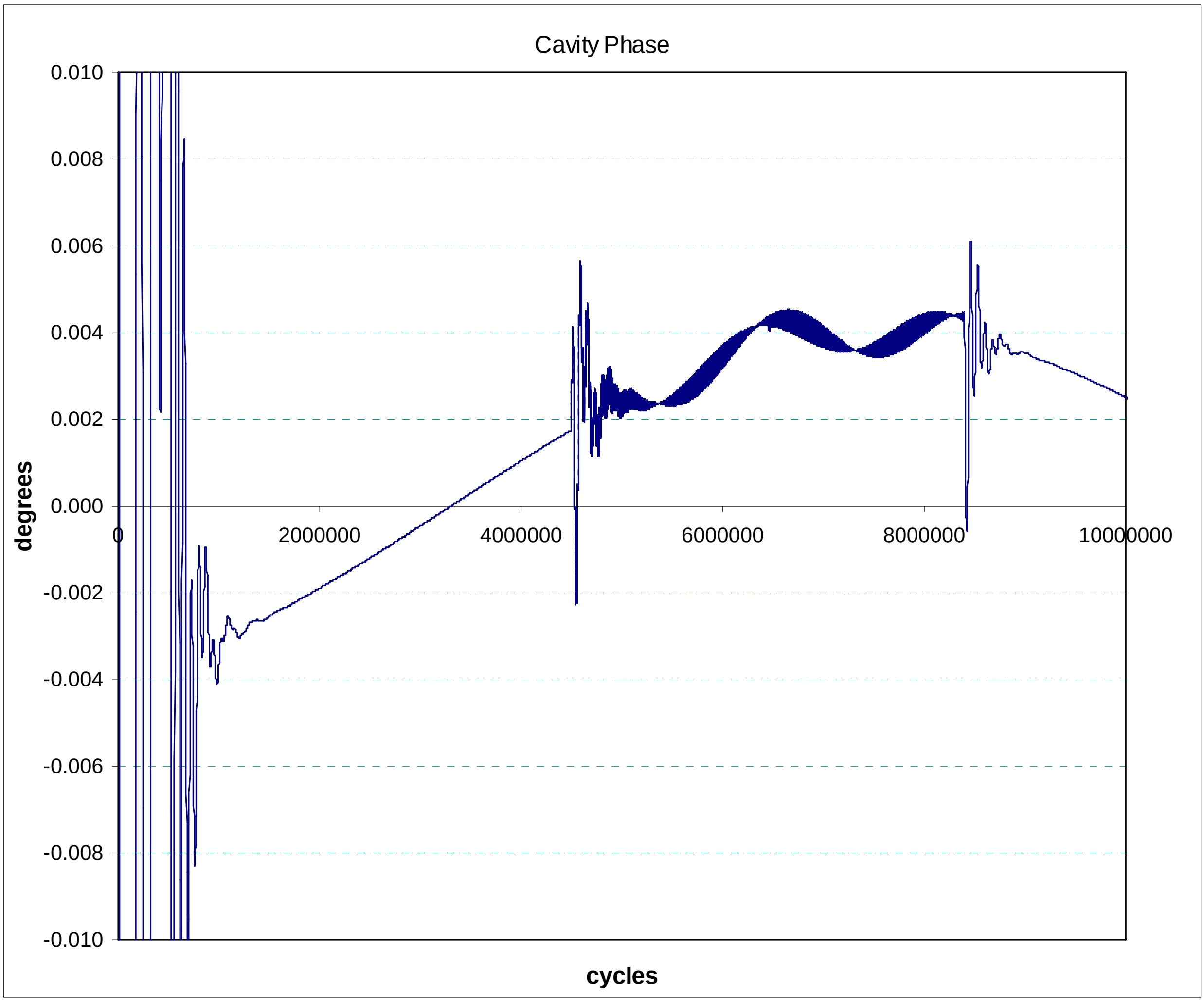


Figure 34 Cavity phase for oscillating beam offset & microphonics $Q_e = 3 \times 10^6$

Figures 30 and 31 show that the in phase component of the drive power predominantly corrects beamloading from beam offset and the out of phase component of the drive power corrects microphonics.

Figure 32 shows how microphonics will push up the power supply requirement. The minimum additional power requirement is given as $P = U\delta\omega$ where U is the energy stored in the cavity and $\delta\omega$ is the cavity detuning [2]. For this simulation U = 0.0284 J and $\delta\omega = 2\pi \times 600\,\mathrm{Hz}$ hence P = 107 W. Comparing figures 22 and 32 for the period before the bunch arrives the peak power rises from 70 W to 140 W. This is a little less than the prediction of 107 W, however one has to bear in mind that the phase and amplitude control here is not perfect.

Figure 34 suggests that with a sufficiently high gain, phase control to the required stability specification for the ILC crab cavity of 0.05 degrees rms can be achieved. Whether the gain used in the calculation gives a stable control system in practice will depend on measurement jitter at an appropriate sampling rate, delays in the controller and delays in the power supply responding to the new requirement.

20. Gain Optimisation

Figure 35 shows the effect of winding up the gain when the overall delay is fixed at 3900 RF cycles and there are no measurement errors. In this figure a gain G = 0.5 corresponds to the simulation presented in figures 29 - 34 hence the parameters given in the large table on figure 29 apply. Figure 35 shows that by reducing the gain with respect to the previous calculation by a factor of two gives an RMS error that is increased from 0.0036 degrees to 0.0072 degrees. An increase in the gain to 0.63 results in oscillations that degrade the phase. This point is the stability limit for the control algorithm.

Figure 36 shows the effect of including some measurement error. The measurement model used here supposes that the cavity only supports one mode and the output from the probe is taken through a low pass filter whose time constant has been taken equal to the time it takes for the controller to re-compute the required control.

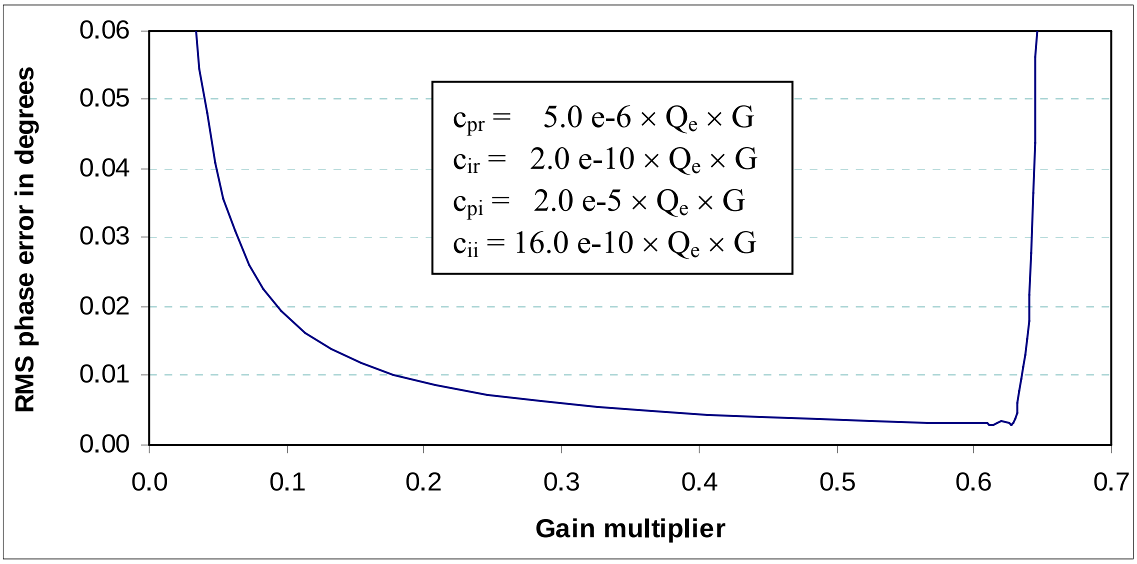


Figure 35 RMS phase error as a function of controller gain with an oscillating beam offset and microphonics but no measurement errors. $Q_e = 3 \times 10^6$

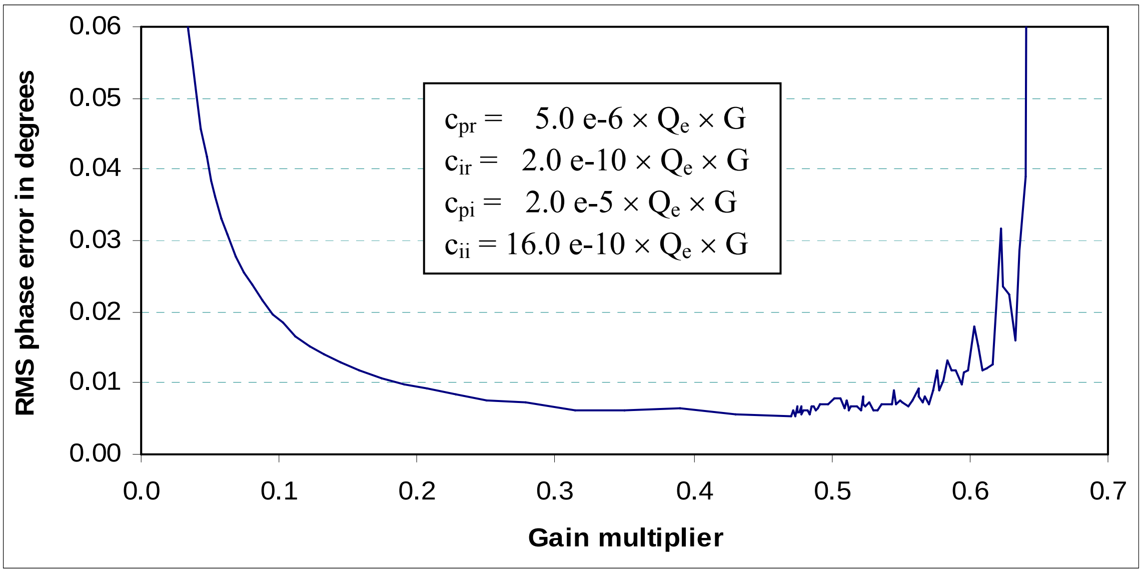


Figure 36 RMS phase error as a function of controller gain with an oscillating beam offset and microphonics and 0.005 degrees and 0.002 phase and amplitude measurement jitter respectively for a 1 MHz bandwidth. $Q_e = 3 \times 10^6$

21. The Stability Limit

Later, figure 40 in section 22 illustrates how the stability limit on gain goes to infinity as the time delay in the control system goes to zero. Section 22.5 shows that the stability limit still exists in the absence of beamloading and microphonic disturbances. This means that the stability limit can be analysed from equations (10.2) with $\omega = \omega_o$ which give

$$\left\{4+\left(\frac{1}{Q_L}\right)^2\right\}\frac{1}{\omega_o}\dot{A}_r+\frac{2}{Q_L}A_r+\left(\frac{1}{Q_L}\right)^2 A_i=\frac{2}{Q_eQ_L}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_r+\mathcal{F}_i\right)-\frac{4}{Q_e}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_i-\mathcal{F}_r\right) \quad (21.1a)$$

$$\left\{4+\left(\frac{1}{Q_L}\right)^2\right\}\frac{1}{\omega_o}\dot{A}_i+\frac{2}{Q_L}A_i-\left(\frac{1}{Q_L}\right)^2 A_r=\frac{2}{Q_eQ_L}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_i-\mathcal{F}_r\right)+\frac{4}{Q_e}\left(\frac{1}{\omega_o}\dot{\mathcal{F}}_r+\mathcal{F}_i\right) \quad (21.1b)$$

For typical accelerator cavities we can invariably neglect terms of order $1/Q^2$. The drive is also expected to be a slowly varying function with respect to the cavity frequency ω_o hence the terms $\dot{\mathcal{F}}/\omega_o$ can also be neglected. This means that for the stability analysis we consider the equations

$$\frac{1}{\omega_o}\dot{A}_r+\frac{1}{2Q_L}A_r=\frac{\mathcal{F}_r}{Q_e} \quad (21.2a)$$

$$\frac{1}{\omega_o}\dot{A}_i+\frac{1}{2Q_L}A_i=\frac{\mathcal{F}_i}{Q_e} \quad (21.2b)$$

where the force terms are delayed. As equations (21.2) are identical we need only consider one of them hence we will consider the first equation.

21.1 Analysis for Proportional Control

Formal methods for the determination of the stability limit of time-delay systems when the input is sampled and the output is updated in step changes are very complicated, they involve Z transforms and the solution of equations with infinite numbers of roots. Here we do not attempt such a complicated analysis. Firstly we make an estimate from the simple rule that positive feed back must be avoided for the system with periodic update. This estimate turns out to be in exact correspondence with the full model when the measurement filter is turned off, the amplifier has instantaneous response and the update interval equals the delay time (latency). Secondly we will make an estimate using Laplace transform methods assuming continuous update and hence should allow a larger delay for the same gain. In this section we confine ourselves to proportional control and in the next second we consider proportional integral control.

21.1.1 Simple method

A minimum requirement for stability is that the error $\varepsilon = A_r - V_{sp}$ does not change its sign in the time it takes before the force $\dot{\mathcal{F}}$ finishes acting and gets updated, note that V_{sp} is the set point defined in (18.1). If the error did change sign before the force finishes acting, the next force to be applied will be the value at the start of the previous delay interval and will reinforce the error for the whole delay period plus the overshoot period. In this case the error grows rather than oscillating about zero as shown in figure 37.

Suppose initially the control system is off and there is an error $\varepsilon = A_r - V_{sp}$. At time t_o the control system is switched on and a force is applied that starts reducing the error to zero, see figure 37.

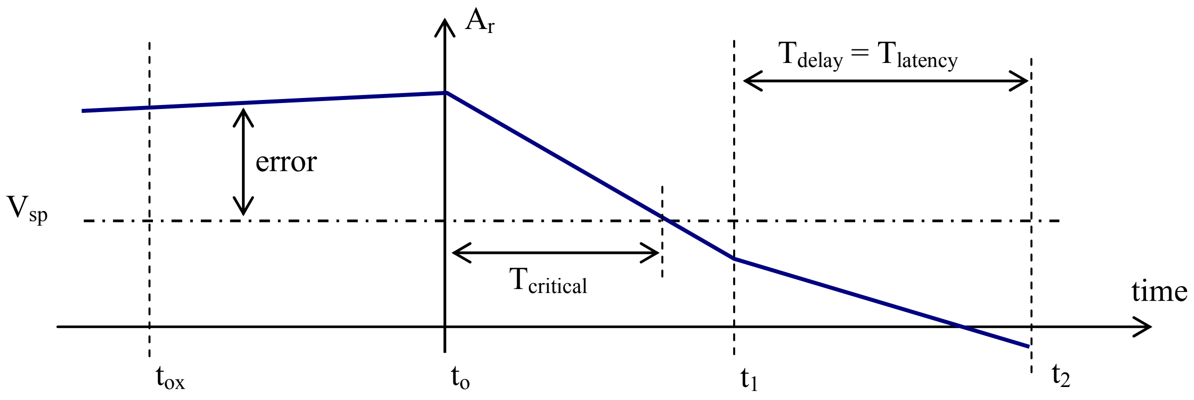


Figure 37 Control delay timing

Define the time $T_{critical}$ as the time it would take this force to bring the error to zero so that

$$T_{critical} = -\frac{A_r(t_o) - V_{sp}}{\dot{A}_r(t_o)} \tag{21.3}$$

For (21.2a) the minimum stability condition is therefore determined as

$$T_{latency} < -\frac{A_r(t_o) - V_{sp}}{\dot{A}_r(t_o)} \qquad \text{for } A_r(t_o) - V_{sp} \neq 0 \tag{21.4}$$

which is the same as

$$-\frac{\dot{A}_r(t_o)}{A_r(t_o) - V_{sp}} < \frac{1}{T_{latency}} \qquad \text{for } A_r(t_o) - V_{sp} \neq 0 \tag{21.5}$$

Equation (21.2a) re-arranges to give

$$\frac{\dot{A}_r}{A_r} = \frac{\omega_o}{Q_e}\left(\frac{\mathcal{F}_r}{A_r} - \frac{Q_e}{2Q_L}\right) \tag{21.6}$$

Substituting (21.6) in (21.5) to remove $\dot{A}_r$ gives

$$-\frac{\omega_o}{Q_e}\left(\frac{\mathcal{F}_r}{A_r} - \frac{Q_e}{2Q_L}\right)\left(\frac{A_r}{A_r - V_{sp}}\right) < \frac{1}{T_{latency}}$$

where all values are evaluated at t_o . Hence

$$-\frac{\mathcal{F}_r}{A_r - V_{sp}} < \frac{Q_e}{\omega_o T_{latency}} - \frac{Q_e}{2Q_L}\frac{A_r}{A_r - V_{sp}} \tag{21.7}$$

For the case of proportional control i.e. when $c_{ir} = 0$, putting the force from (18.1) in equation (21.7) gives

$$\frac{c_{pr}}{Q_e}\left\{\frac{A_r(t_o - T_{delay}) - V_{sp}}{A_r(t_o) - V_{sp}}\right\} < \frac{1}{\omega_o T_{latency}} - \frac{A_r(t_o)}{2Q_L\{A_r(t_o) - V_{sp}\}} \tag{21.8}$$

From (21.2) and (18.1) the steady state error is determined from

$$\frac{A_r}{2Q_L} = \frac{c_{pr}(V_{sp} - A_r)}{Q_e} \tag{21.9}$$

which re-arranges to give $\frac{c_{pr}}{Q_e} = \frac{A_r}{2Q_L(A_r - V_{sp})}$ providing an estimate of the third term in (21.8). Using this estimate, the stability limit for proportional control becomes

$$\frac{c_{pr}}{Q_e}\left\{1+\frac{A_r(t_o - T_{delay}) - V_{sp}}{A_r(t_o) - V_{sp}}\right\} < \frac{1}{\omega_o T_{latency}} \qquad \text{for } A_r(t_o) - V_{sp} \neq 0 \qquad (21.10)$$

From the diagram one can see that at the stability limit the field oscillates about the set point with a period of $4T_{critical}$ and when

$$A_r(t_o) - V_{sp} \neq 0 \qquad \text{then} \qquad A_r(t_o - T_{delay}) - V_{sp} \approx 0 \qquad (21.11)$$

hence (21.10) gives

$$\frac{c_{pr}}{Q_e} < \frac{1}{\omega_o T_{latency}} \qquad \text{(for proportional control)} \qquad (21.12)$$

For 3.9 GHz and a latency of 1 μs then $\frac{c_{pr}}{Q_e} < 4.08\times10^{-5}$

Formulae (21.12) can also be expressed in the more familiar format often given with reference to phase lock loops. Writing $Q_e = \omega_o/\delta\omega$ where $\delta\omega$ is the cavity bandwidth, equation (21.12) becomes

$$c_{pr} < \frac{1}{\text{bandwidth} \times T_{latency}} \qquad \text{(for PI control)} \qquad (21.13)$$

The stability limit for the ILC Tesla cavities and performance with adaptive feed-forward control algorithm has been recently discussed by Elmar Vogel [13].

21.1.2 Numerical result for periodic update

Figure 38 shows the numerical computation where only proportional control is used, the measurement filter is off and the amplifier has infinite bandwidth. The stability limit appears at G = 2.04 hence $\frac{c_p}{Q_e} = 4.08\times10^{-5}$.

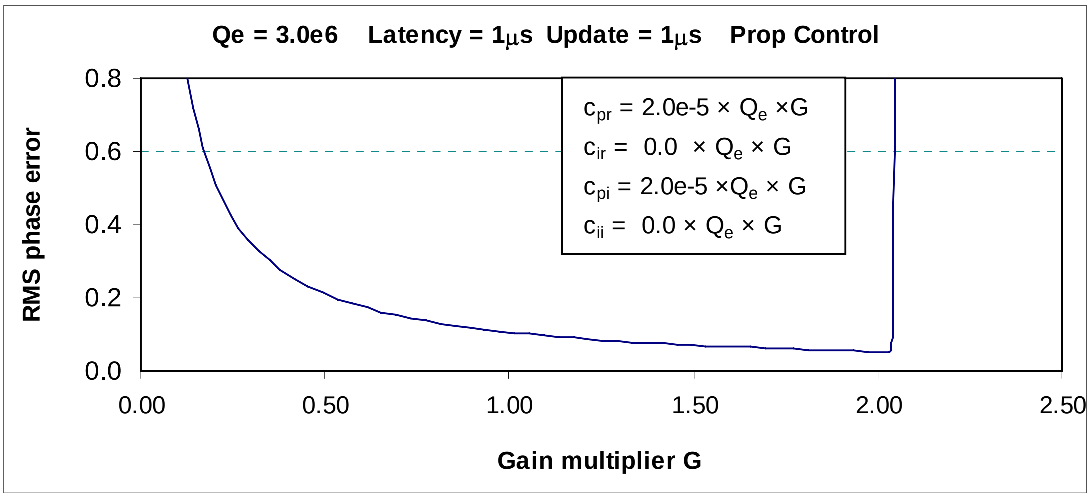


Figure 38 Stability limit for proportional control only

21.1.3 Laplace transform method for proportional control

From the system equation (21.2) and the controller (18.1) with $c_i = 0$

$$\frac{Q_e}{\omega_o}\dot{A}_r + \frac{Q_e}{2Q_L}A_r = -c_p\left\{A_r\left(t - T_{delay}\right) - V_{sp}\right\} \tag{21.14}$$

The Laplace transform method gives the stability limit for this differential equation as written but takes no account of the periodic update associated with a digital control system, i.e. whilst the action is delayed a new force is determined at every instant from the delayed data. Taking the Laplace transform of (21.14) gives

$$\frac{Q_e}{\omega_o}s\,\tilde{A} + \frac{Q_e}{2Q_L}\tilde{A} = -c_p\,\tilde{A}\,e^{-sT_{delay}} + c_p\frac{V_{sp}}{s} \tag{21.15}$$

Hence $$\tilde{A} = \frac{\dfrac{c_p}{Q_e}\dfrac{V_{sp}}{s}}{\dfrac{s}{\omega_o} + \dfrac{1}{2Q_L} + \dfrac{c_p}{Q_e}e^{-sT_{delay}}} \tag{21.16}$$

For stability we require that the roots of the denominator lie in the left hand plane. Given that we already know that the equation moves from a region of stability to a region of instability as c_p is increased it is reasonable to assume that the point of instability is given by one of the roots of the denominator, i.e. of

$$\frac{s}{\omega_o} + \frac{1}{2Q_L} + \frac{c_p}{Q_e}e^{-sT_{delay}} = 0 \tag{21.17}$$

As the roots are complex roots it is convenient to set

$$s = \sigma + j\lambda \tag{21.18}$$

where σ and λ are real numbers. Putting (21.18) in (21.17) and setting real and imaginary parts to zero gives

$$\frac{\sigma}{\omega_o} + \frac{1}{2Q_L} + \frac{c_p}{Q_e}e^{-\sigma T_{delay}}\cos\lambda T_{delay} = 0 \tag{21.19}$$

$$\frac{\lambda}{\omega_o} - \frac{c_p}{Q_e}e^{-\sigma T_{delay}}\sin\lambda T_{delay} = 0 \tag{21.20}$$

It is convenient to eliminate the sine and cosine terms in (21.19) and (21.20), doing this gives

$$\left(\frac{\sigma}{\omega_o} + \frac{1}{2Q_L}\right)^2 + \left(\frac{\lambda}{\omega_o}\right)^2 = \left(\frac{c_p}{Q_e}\right)^2 e^{-2\sigma T_{delay}} \tag{21.21}$$

The roots move into the LHP when $\sigma = 0$ hence from (21.21) the value of λ at the point of instability is given as

$$\lambda = \pm\,\omega_o\sqrt{\left(\frac{c_p}{Q_e}\right)^2 - \left(\frac{1}{2Q_L}\right)^2} \quad \text{when } \sigma = 0 \tag{21.22}$$

From (21.20)

$$T_{delay} = \frac{1}{\lambda}\sin^{-1}\left(\frac{\lambda Q_e}{\omega_o c_p}\right) + 2\pi n \qquad \text{when } \sigma = 0 \tag{21.23}$$

or from (21.19)

$$T_{delay} = \frac{1}{\lambda}\cos^{-1}\left(\frac{Q_e}{2Q_L c_p}\right) + 2\pi n \qquad \text{when } \sigma = 0 \tag{21.24}$$

and where n is a positive or negative integer. As expected there are an infinite number of solutions and each one represents a pole crossing the axis. Stability is for the smallest value of T_{delay} as this corresponds to the last pole crossing into the left hand plane.

The maximum proportional control coefficient is determined by solving (21.22) and (21.23) for c_p when $n = 0$.

From the approximate analysis we expect that the maximum $\frac{c_{pr}}{Q_e} \approx 4.0\times10^{-5}$ which is much bigger than $\frac{1}{2Q_L} \approx 1.667\times10^{-7}$ for the planned cavity hence

$$\lambda \approx \pm\omega_o \frac{c_p}{Q_e} \qquad \text{for last root with typical values} \tag{21.25}$$

$$\omega_o T_{delay} \approx \frac{Q_e}{c_p}\sin^{-1}(1) = \frac{\pi}{2}\frac{Q_e}{c_p} \tag{21.26}$$

$$\text{or} \quad \frac{c_p}{Q_e} < \frac{\pi}{2\omega_o T_{delay}} \qquad \text{(for proportional control)} \tag{21.27}$$

For 3.9 GHz and a latency of 1 μs then $\frac{c_{pr}}{Q_e} < 6.41\times10^{-5}$

which is a factor 0.5π better than the value for periodic update after each latency period.

21.1.4 Numerical result for proportional control with rapid update

Figure 39 shows the numerical computation where only proportional control is used and update is rapid. The stability limit appears at G = 3.19 hence $\frac{c_p}{Q_e} = 6.38\times10^{-5}$.

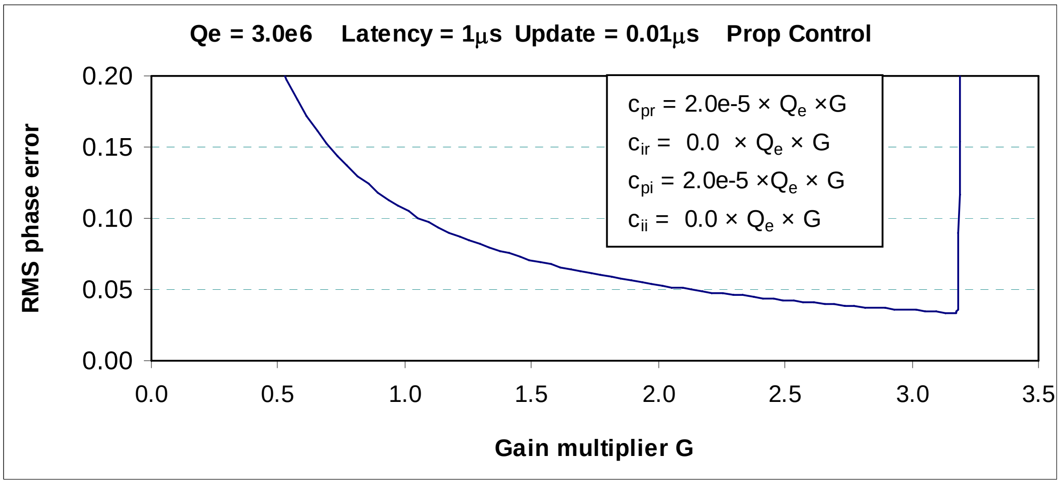


Figure 38 Stability limit for proportional control only

21.2 Analysis for Proportional Integral Control

21.2.1 Laplace transform method for PI control

From the system equation (21.2) and the controller (18.1)

$$\frac{Q_e}{\omega_o}\dot{A}_r + \frac{Q_e}{2Q_L}A_r = -c_p\left\{A_r\left(t - T_{delay}\right) - V_{sp}\right\} - c_i\frac{\omega}{2\pi}\int_{-\infty}^{t}\left\{A_r\left(t' - T_{delay}\right) - V_{sp}\right\}dt' \qquad (21.28)$$

Differentiation gives

$$\frac{Q_e}{\omega_o}\ddot{A}_r + \frac{Q_e}{2Q_L}\dot{A}_r = -c_p\dot{A}_r\left(t - T_{delay}\right) - c_i\left(\frac{\omega}{2\pi}\right)\left\{A_r\left(t - T_{delay}\right) - V_{sp}\right\} \qquad (21.29)$$

Laplace transform of (21.29) gives

$$\frac{Q_e}{\omega_o}s^2\,\tilde{A} + \frac{Q_e}{2Q_L}s\,\tilde{A} = -\left(c_p\,s + c_i f\right)\tilde{A}\,e^{-sT_{delay}} + c_i f\frac{V_{sp}}{s} \qquad (21.30)$$

where $f = \dfrac{\omega}{2\pi}$ is the drive frequency, hence

$$\tilde{A} = \frac{c_i f\dfrac{V_{sp}}{s}}{\dfrac{Q_e}{\omega_o}s^2 + \dfrac{Q_e}{2Q_L}s + \left(c_p\,s + c_i f\right)e^{-sT_{delay}}} \qquad (21.31)$$

For stability we require that the roots of the denominator lie in the left hand plane. Given that we already know that the equation moves from a region of stability to a region of instability as c_p and c_i are increased it is reasonable to assume that the point of instability is given by one of the roots of the denominator, i.e. of

$$\frac{s^2}{\omega_o} + \frac{s}{2Q_L} + \frac{\left(c_p s + c_i f\right)}{Q_e}e^{-sT_{delay}} = 0 \qquad (21.32)$$

As the roots are complex roots it is convenient to set

$$s = \sigma + j\lambda \qquad (21.33)$$

where σ and λ are real numbers. Putting (21.33) in (21.32) and setting real and imaginary parts to zero gives

$$\frac{\sigma^2 - \lambda^2}{\omega_o} + \frac{\sigma}{2Q_L} + \frac{\left(c_p\sigma + c_i f\right)}{Q_e}e^{-\sigma T_{delay}}\cos\lambda T_{delay} + \frac{c_p\lambda}{Q_e}e^{-\sigma T_{delay}}\sin\lambda T_{delay} = 0 \qquad (21.34)$$

$$\frac{2\lambda\sigma}{\omega_o} + \frac{\lambda}{2Q_L} + \frac{c_p\lambda}{Q_e}e^{-\sigma T_{delay}}\cos\lambda T_{delay} - \frac{c_p\sigma + c_i f}{Q_e}e^{-\sigma T_{delay}}\sin\lambda T_{delay} = 0 \qquad (21.35)$$

The roots move into the LHP when $\sigma = 0$ hence from (21.34) and (21.35) the value of λ at the point of instability is determined from

$$\frac{-\lambda^2}{\omega_o} + \frac{c_i f}{Q_e}\cos\lambda T_{delay} + \frac{c_p\lambda}{Q_e}\sin\lambda T_{delay} = 0 \qquad (21.36)$$

and

$$\frac{\lambda}{2Q_L} + \frac{c_p\lambda}{Q_e}\cos\lambda T_{delay} - \frac{c_i f}{Q_e}\sin\lambda T_{delay} = 0 \qquad (21.37)$$

Eliminating $\sin\lambda T_{delay} = 0$ between (21.36) and (21.37) gives

$$-\lambda^2\left(\frac{1}{\omega_o}-\frac{c_p}{2Q_L c_i f}\right)+\frac{c_i f}{Q_e}\left(1+\lambda^2\frac{c_p^2}{f^2 c_i^2}\right)\cos\lambda T_{delay} = 0 \qquad (21.38)$$

Eliminating $\cos\lambda T_{delay} = 0$ between (21.36) and (21.37) gives

$$-\left(\frac{\lambda^3}{\omega_o}\frac{c_p}{c_i f}+\frac{\lambda}{2Q_L}\right)+\frac{c_i f}{Q_e}\left(1+\lambda^2\frac{c_p^2}{f^2 c_i^2}\right)\sin\lambda T_{delay} = 0 \qquad (21.39)$$

Elimination of the cosine and sine terms in (21.38) and (21.39) gives

$$\lambda^4\left(\frac{1}{\omega_o}-\frac{c_p}{2Q_L c_i f}\right)^2+\lambda^2\left(\frac{\lambda^2}{\omega_o}\frac{c_p}{c_i f}+\frac{1}{2Q_L}\right)^2=\left(\frac{c_i f}{Q_e}\right)^2\left(1+\lambda^2\frac{c_p^2}{c_i^2 f^2}\right)^2$$

hence

$$\lambda^6\left(\frac{c_p}{\omega_o c_i f}\right)^2+\lambda^4\left\{\left(\frac{1}{\omega_o}+\frac{c_p}{2Q_L c_i f}\right)^2-\frac{c_p}{\omega_o c_i f Q_L}-\left(\frac{c_p^4}{Q_e^2\, c_i^2 f^2}\right)\right\}+\lambda^2\left\{\frac{1}{4Q_L^2}-\frac{2c_p^2}{Q_e^2}\right\}-\left(\frac{c_i f}{Q_e}\right)^2=0$$

which simplifies to

$$\lambda^6\left(\frac{c_p}{\omega_o c_i f}\right)^2+\lambda^4\left\{\left(\frac{1}{\omega_o}\right)^2+\left(\frac{c_p}{2Q_L c_i f}\right)^2-\left(\frac{c_p^4}{Q_e^2\, c_i^2 f^2}\right)\right\}+\lambda^2\left\{\frac{1}{4Q_L^2}-\frac{2c_p^2}{Q_e^2}\right\}-\left(\frac{c_i f}{Q_e}\right)^2=0$$

or

$$\lambda^6+\lambda^4\left\{\left(\frac{c_i f}{c_p}\right)^2+\left(\frac{\omega_o}{2Q_L}\right)^2-\left(\frac{\omega_o c_p}{Q_e}\right)^2\right\}+\lambda^2\left\{\left(\frac{\omega_o}{2Q_L}\frac{c_i f}{c_p}\right)^2-2\left(\frac{\omega_o c_i f}{Q_e}\right)^2\right\}-\left(\frac{\omega_o c_i^2 f^2}{Q_e c_p}\right)^2=0 \qquad (21.40)$$

This cubic equation can be solved exactly though the analytic expression is long. For typical values the equation only has one real root for λ^2. Table 19.1 gives coefficients in (21.40) and the positive solution for λ for a typical choice of control parameters known to give good control.

f	3.900×10^{9}
ω_o	24.50442×10^{9}
Q_e	3.0×10^{6}
Q_L	3.0×10^{6}
c_p / Q_e	2.0×10^{-5}
$c_i f / Q_e$	6.240
$c_i f / c_p$	3.120×10^{5}
$\omega_o / (2Q_L)$	4.084×10^{3}
$\omega_o c_p / Q_e$	4.901×10^{5}
$\omega_o c_i f / (2Q_L c_p)$	2.548×10^{4}
$\omega_o c_i f / Q_e$	1.529×10^{11}
$\omega_o c_i^2 f^2 / (Q_e c_p)$	4.771×10^{16}
λ	7.074×10^{5}

Table 19.1 Parameters for (21.40) known to give good control

It is apparent from the table that all the terms in (21.40) except one have similar magnitudes hence one does not expect a simple approximate formula for its solution.

Once λ has been computed for specific control parameters one can then use (21.38) to determine T_{delay}. Re-arranging (21.38) gives

$$T_{delay} = \frac{1}{\lambda}\cos^{-1}\left\{\frac{\lambda^2 Q_e}{\omega_o c_i f}\left(\frac{1-\dfrac{\omega_o c_p f}{2Q_L c_i}}{1+\lambda^2\dfrac{c_p^2}{c_i^2 f^2}}\right)\right\} \tag{21.41}$$

Putting values from the table above gives

$$T_{delay} = \frac{1}{7.074 \times 10^{5}}\cos^{-1}\left\{3.273\left(\frac{1-0.013}{1+5.141}\right)\right\} = 1.438 \times 10^{-6} \quad \text{s}$$

Figure 38 gives results for repeated calculations to determine delay times for four values of the proportional control term and for a range of values for the integral term until the stability limit is reached. For this calculation $Q_e = Q_L = 3 \times 10^{6}$.

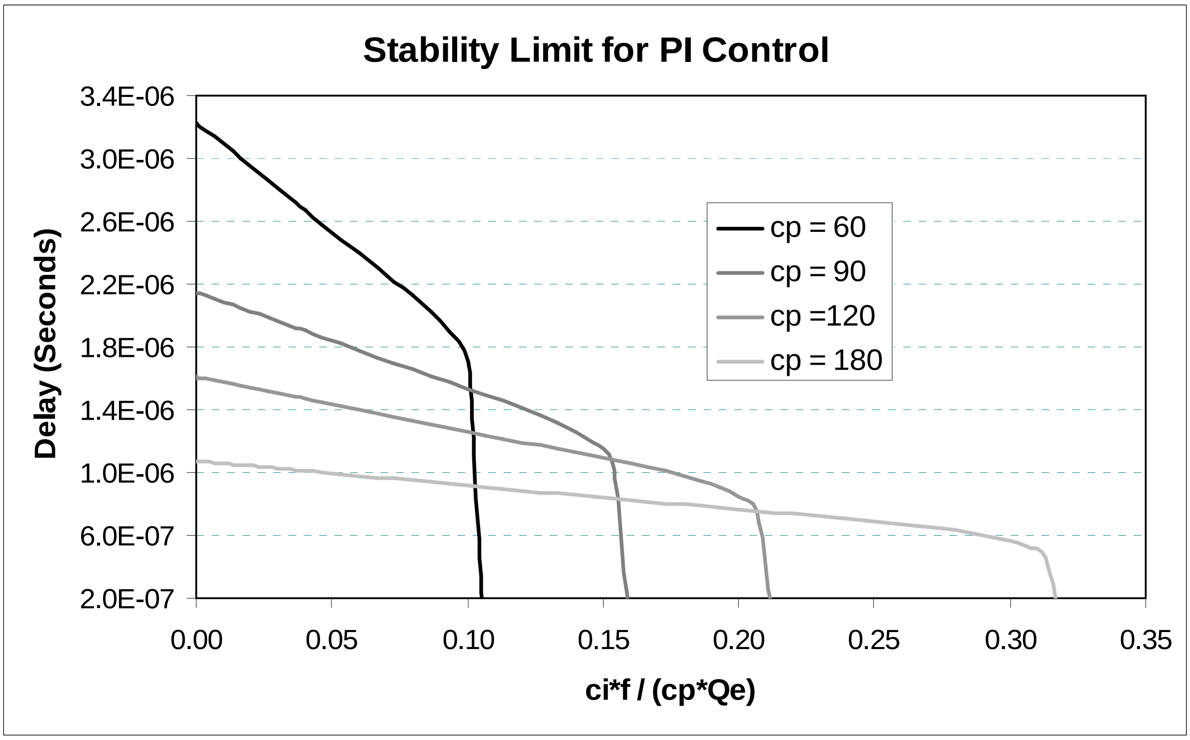


Figure 39 Stability limit for proportional integral control with rapid update

The intercept on the y axis is determined from (21.27) i.e.

$$T_{delay}(c_i = 0) = \frac{Q_e}{4 f c_p}$$

Since the gradient of the curves in figure 39 are constant to the point where the maximum permissible delay suddenly drops they can be approximated by a simple empirical formula which works out to be

$$T_{delay} = \frac{Q_e}{4f\,c_p}\left(1 - 250\ \frac{c_i f}{c_p^2 Q_e}\right) \tag{21.42}$$

and where the drop comes when

$$\frac{c_i f}{c_p^2 Q_e} = \frac{1}{500} \tag{21.43}$$

Equation (21.43) can be expressed as

$$\mathrm{Max}\left(\frac{c_i}{c_p}\right) = \frac{c_p Q_e}{500 f} \tag{21.44}$$

21.2.2 Formula and numerical analysis for periodic update with PI control

For periodic update as opposed to regular update and from the analysis for proportional control one expects the delay time to be reduced by a factor $2/\pi$. For periodic update with interval T_{delay} one expects from (21.42) that the delay time is given as

$$T_{delay} = \frac{Q_e}{\omega_o\, c_p}\left(1 - 250\ \frac{c_i f}{c_p^2 Q_e}\right) \qquad \text{(periodic update)} \qquad (21.45)$$

Putting typical numbers into (21.44) for the ILC crab cavity and computing the delay we get

frequency	=	3.9×10^9
Q_e	=	3.0×10^6
c_p	=	37.5
c_i	=	0.003
T_{delay}	=	1.0012×10^{-6}

Figure 39 shows the computation for this case where PI control is used, the stability limit appears at G = 0.625 hence at the point of instability $c_{pr} = 37.5$ and $c_{ir} = 0.003$. Remarkably this is just about exact. The remark to be made is that there is almost certainly a Z transform analysis similar to the Laplace transform analysis that yields the extra factor of $2/\pi$.

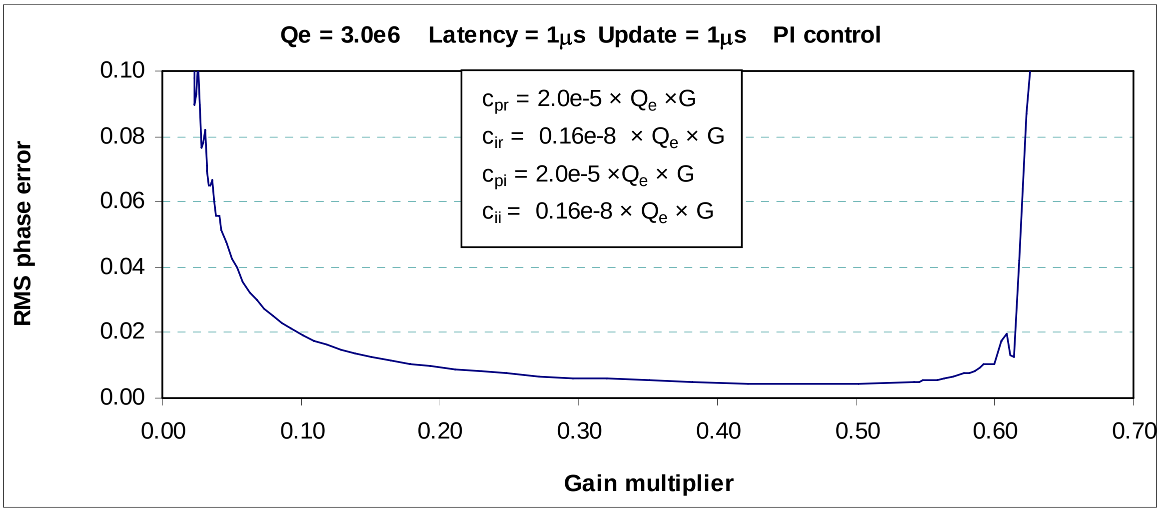


Figure 39 Calculated stability limit for proportional integral control with periodic update

21.2.3 Attempted approximate analysis for PI control

It is of interest to see if this formula can be derived in a simple way. For the case of PI control the condition for stability derived from (21.7) becomes

$$\frac{c_{pr}\left(A_r\left(t_o - T_{delay}\right) - V_{sp}\right) + c_{ir}\int_{-\infty}^{t} dt\left(A_r\left(t - T_{delay}\right) - V_{sp}\right)}{Q_e\left(A_r\left(t_o\right) - V_{sp}\right)} < \frac{1}{\omega_o T_{latency}} - \frac{A_r\left(t_o\right)}{2Q_L\left\{A_r\left(t_o\right) - V_{sp}\right\}} \tag{21.46}$$

For a PI controller the steady state error can be zero as the integral provides whatever steady force is necessary to maintain the set point. In steady state we have $\varepsilon = A_r - V_{sp} = 0$ and $\dot{A}_r = 0$, hence from (21.2a) and (18.1) we have the steady state value of the integral determined as

$$\frac{A_r}{2Q_L} = -\frac{c_{ir}}{Q_e}\int_{-\infty}^{t} dt\left(A_r\left(t - T_{delay}\right) - V_{sp}\right) \qquad \text{(Steady state constraint)} \tag{21.47}$$

Putting (21.47) in (21.46) gives

$$\frac{c_{pr}\left(A_r\left(t_o - T_{delay}\right) - V_{sp}\right) - \frac{Q_e}{2Q_L}A_r\left(t_o\right)}{Q_e\left(A_r\left(t_o\right) - V_{sp}\right)} < \frac{1}{\omega_o T_{latency}} - \frac{A_r\left(t_o\right)}{2Q_L\left\{A_r\left(t_o\right) - V_{sp}\right\}} \tag{21.48}$$

Cancelling terms gives

$$\frac{c_{pr}\left(A_r\left(t_o - T_{delay}\right) - V_{sp}\right)}{Q_e\left(A_r\left(t_o\right) - V_{sp}\right)} < \frac{1}{\omega_o T_{delay}} \tag{21.49}$$

Comparison with (21.45) for update at the delay frequency suggests that

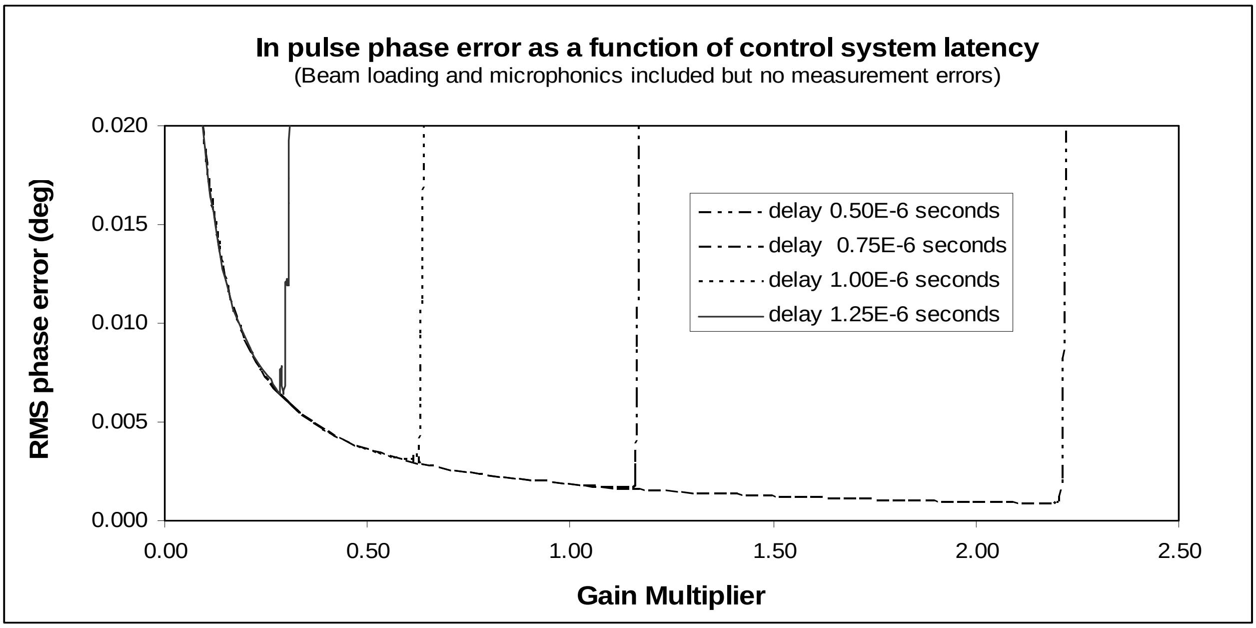
$$\frac{\left(A_r\left(t_o\right) - V_{sp}\right)}{\left(A_r\left(t_o - T_{delay}\right) - V_{sp}\right)} = 1 - \frac{250\, c_i\, f}{c_p^2\, Q_L} \tag{21.50}$$

As yet we do not have a simple argument to get this formula.

22. Ultimate Phase Performance for the Single Mode Cavity

One expects the ultimate phase stabilisation performance of the cavity to depend on a number of factors. These include control system delay (latency), cavity Q factor, measurement error and the level and rate of change of disturbance from beamloading and microphonics. The analysis in this section and section 23 which use parameters consistent with available digital technology will eventually show that the most significant constraint on phase control is likely to be measurement accuracy. The accuracy to which one can determine the phase and amplitude of the operating mode can be seriously limited by the presence of modes with frequencies close to the operating mode and which are simultaneously detected by the output couplers. This multi-mode case will be considered in section 23.

In this section the effect of the latency, Q factor, disturbance and measurement error on control phase performance is studied for the single mode cavity. The ILC requirement for phase stability of an individual crab cavity is nominally better than 0.05 degrees. Figure 40 shows how RMS phase error depends on control system time delay. Here the control update is set to match the delay, i.e. a measurement is requested, the value is returned to the controller, the controller computes and applies a new phase and amplitude correction for the drive and the update is made at end of the delay period associated with measurement and control. Parameters used for figure 40 are the same as those used in section 19.4 see figure 29.



Drive frequency in GHz	= 3.900 GHz	Bunch charge (ILC=3.2 nC)	= 3.200 nC
Centre cavity frequency in GHz	= 3.900 GHz	RF cycles between bunches	= 1200
Cavity Q factor	= 1.0000E+09	Bunch train length	= 1.000 ms
External Q factor	= 3.0000E+06	Cavity frequency shift from microphonics	= 600.000 Hz
Cavity R over Q (2xFNAL=53 per cell)	= 53.000 ohms	Cavity vibration frequency	= 230.000 Hz
Energy point ILC crab~0.0284J per cell)	= 28.400 mJ	Initial vibration phase (degrees)	= 20.000 deg
Amplitude set point	= 301.675 kV	Phase measurement error(degrees)	= 0.00000 deg
Maximum Amplifier Power per cell	= 1200.000 W	Fractional err in amplitude measurement	= 0.00000
Maximum voltage set point (no beam)	= 1235.476 kV	Time delay (latency) for control system	= see figure 38
Maximum beam offset	= 1.000 mm	Control update interval	= time delay
Maximum bunch phase error	= 1.000 deg	Initial gain constant for controller	= 0.0020
Beam offset frequency	= 2000.000 Hz	Amplifier bandwidth	= 1.0000E+07

Figure 40 In pulse RMS phase error as a function of control system latency for cavity and beam parameters as defined in list and where the update is made after the given time delay.

It is seen straight away from figure 40 that latency is a major constraint on the maximum gain. The curve has a knee such that phase performance deteriorates very quickly when the latency rises above 1.25 μs. The curves in figure 40 were produced for an external Q of 3.0×10^6. When minimal power consumption is not the key issue one has some flexibility in the choice of external Q. It is therefore of interest to investigate whether phase performance can be enhanced by changing the external Q. In sections 19.1 and 19.2 it was seen for a specific case that similar control performance is obtained by increasing gain in proportion to Q_e as Q_e is increased. As ultimate performance depends on the gain at the stability limit it is of interest to see how the stability limit depends on Q_e. The results of section 22.1 show this dependency. The three figures are for Q externals of 0.3×10^6 , 1×10^6 and 3×10^6. The phase performance is plotted against gain multiplier. The actual gain is obtained as the gain multiplier times the external Q, times some fixed coefficients used for all cases as illustrated in the inset box of figures 35 and 36. The control coefficients given in the tables under each figure are the values where RMS phase error is minimised. Where there is no measurement error the RMS phase error is minimised at the stability limit. The three plots of section 22.1 are almost identical. This means that the stability limit is a function of the ratio of the gain to the external Q and that choosing a different external Q does not necessarily allow an improved phase control performance. Note that the third figure of section 22.1 was given as one of the curves in figure 40.

Section 22.2 repeats the calculations of 22.1 with phase and amplitude measurement errors. Again the three curves are almost identical indicating that the scaling law with Q_e and gain still applies. With measurement errors the minimum phase error no longer occurs at the stability limit.

Section 22.3 repeats the calculations of 22.1 with an increased latency. The third figure has already been shown in figure 38. The fourth figure in this section has beamloading switched off. As the figure is almost identical to the third figure with beamloading included one might guess that phase errors are driven by microphonics rather than beamloading. Section 22.4 follows this up by switching off microphonics but including beamloading. According phase control is greatly enhanced. Section 22.5 turns off beamloading and microphonics. In the absence of disturbance, phase control is perfect up to the stability limit.

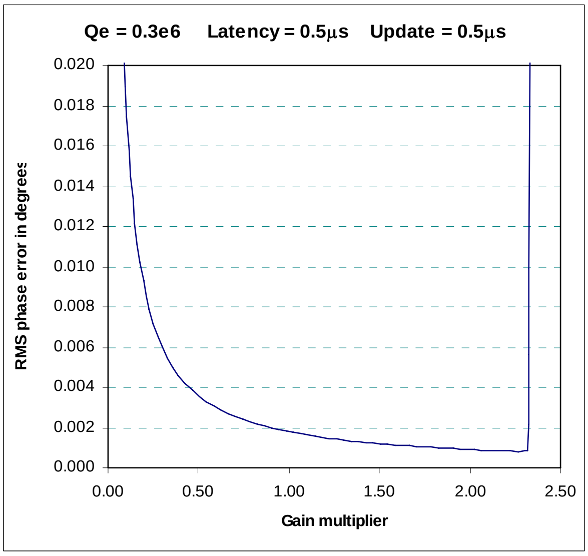
Section 22.6 repeats the calculations of 22.2 with an increased latency.

Section 22.7 shows that phase control performance is improved if the frequency of the disturbance is reduced.

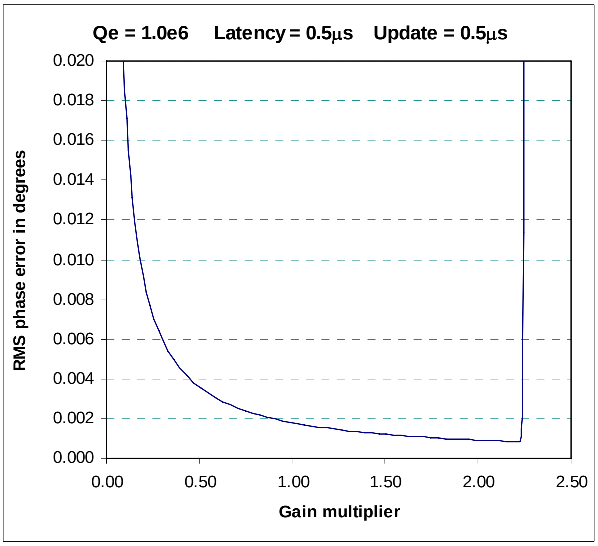
Section 22.8 again considers phase control performance as a function Q_e as in section 22.2 and 22.6 but with further increased latency going below the knee in figure 38. In this case one loses the result of phase control performance being independent of Q_e when gain is increased in proportion to the Q_e .

Section 22.9 goes to an even higher latency but allows update to be more frequent than the latency. This might be achieved by using parallel processors with staggered operation.

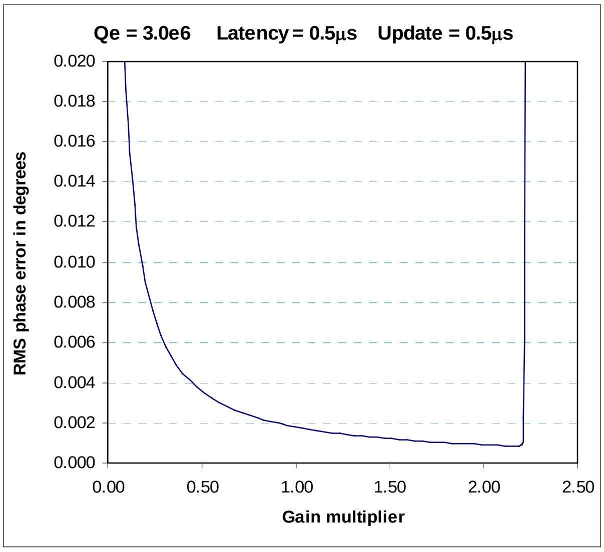
22.1 No measurement error, Latency = 0.5 μs, Update = 0.5 μs, vary Q



Drive frequency in GHz = 3.900 GHz
Centre cavity frequency in GHz = 3.900 GHz
Cavity Q factor = 1.0000E+09
External Q factor = 0.3000E+06
Cavity R over Q (2xFNAL=53 per cell) = 53.000 ohms
Energy point ILC crab~0.0284J per cell) = 28.400 mJ
Amplitude set point = 301.675 kV
Maximum Amplifier Power per cell = 1200.000 W
Maximum voltage set point (no beam) = 390.692 kV
Maximum beam offset = 1.000 mm
Maximum bunch phase error = 1.000 deg
Beam offset frequency = 2000.000 Hz
Bunch charge (ILC=3.2 nC) = 3.200 nC
RF cycles between bunches = 1200
Bunch train length = 1.000 ms
Cavity frequency shift from microphonics= 600.000 Hz
Cavity vibration frequency = 23.000 Hz
Initial vibration phase (degrees) = 20.000 deg
Phase measurement error(degrees) = 0.00000 deg
Fractional err in amplitude measurement = 0.00000
Time delay (latency) for control system = 0.5000E-06 s
Control update interval = 0.5000E-06 s
Amplifier bandwidth = 1.0000E+07
Optimal gain constant for controller = 2.2643
Minimum rms phase error = 0.00082
Maximum power delivered = 822.8670
Proportional coef for real component = 3.3965E+00
Integral coef for real component = 1.3586E-04
Proportional coef for imag component = 1.3586E+01
Integral coef for imag component = 1.0869E-03

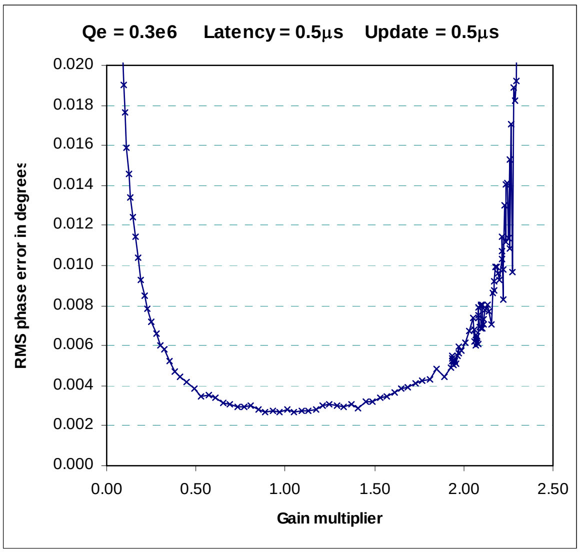


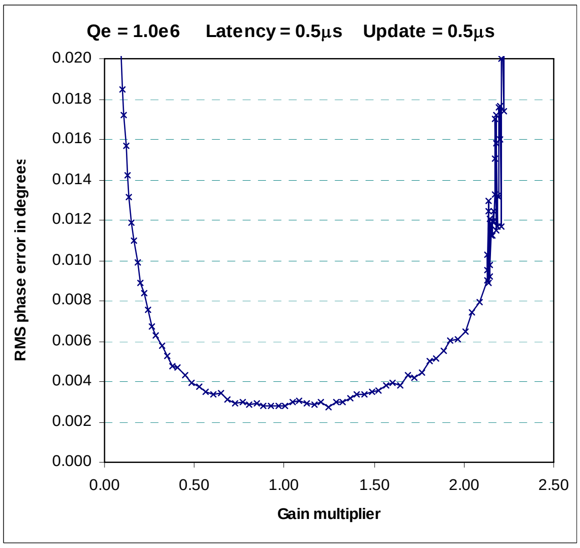
Drive frequency in GHz = 3.900 GHz
Centre cavity frequency in GHz = 3.900 GHz
Cavity Q factor = 1.0000E+09
External Q factor = 1.0000E+06
Cavity R over Q (2xFNAL=53 per cell) = 53.000 ohms
Energy point ILC crab~0.0284J per cell) = 28.400 mJ
Amplitude set point = 301.675 kV
Maximum Amplifier Power per cell = 1200.000 W
Maximum voltage set point (no beam) = 713.302 kV
Maximum beam offset = 1.000 mm
Maximum bunch phase error = 1.000 deg
Beam offset frequency = 2000.000 Hz
Bunch charge (ILC=3.2 nC) = 3.200 nC
RF cycles between bunches = 1200
Bunch train length = 1.000 ms
Cavity frequency shift from microphonics= 600.000 Hz
Cavity vibration frequency = 230.000 Hz
Initial vibration phase (degrees) = 20.000 deg
Phase measurement error(degrees) = 0.00000 deg
Fractional err in amplitude measurement = 0.00000
Time delay (latency) for control system = 0.5000E-06 s
Control update interval = 0.5000E-06 s
Initial gain constant for controller = 0.0020
Amplifier bandwidth = 1.0000E+07
Minimum rms phase error = 0.00085
Maximum power delivered = 335.6143
Proportional coef for real component = 1.0938E+01
Integral coef for real component = 4.3750E-04
Proportional coef for imag component = 4.3750E+01
Integral coef for imag component = 3.5000E-03



Drive frequency in GHz = 3.900 GHz
Centre cavity frequency in GHz = 3.900 GHz
Cavity Q factor = 1.0000E+09
External Q factor = 3.0000E+06
Cavity R over Q (2xFNAL=53 per cell) = 53.000 ohms
Energy point ILC crab~0.0284J per cell)= 28.400 mJ
Amplitude set point = 301.675 kV
Maximum Amplifier Power per cell = 1200.000 W
Maximum voltage set point (no beam) = 1235.476 kV
Maximum beam offset = 1.000 mm
Maximum bunch phase error = 1.000 deg
Beam offset frequency = 2000.000 Hz
Bunch charge (ILC=3.2 nC) = 3.200 nC
RF cycles between bunches = 1200
Bunch train length = 1.000 ms
Cavity frequency shift from microphonics= 600.000 Hz
Cavity vibration frequency = 230.000 Hz
Initial vibration phase (degrees) = 20.000 deg
Phase measurement error(degrees) = 0.00000 deg
Fractional err in amplitude measurement = 0.00000
Time delay (latency) for control system = 5.0000E-07 s
Control update interval = 5.0000E-07 s
Amplifier bandwidth = 1.0000E+07
Optimal gain constant for controller = 2.1539
Minimum rms phase error = 0.00086
Maximum power delivered = 231.3614
Proportional coef for real component = 3.2309E+01
Integral coef for real component = 1.2924E-03
Proportional coef for imag component = 1.2924E+02
Integral coef for imag component = 1.0339E-02

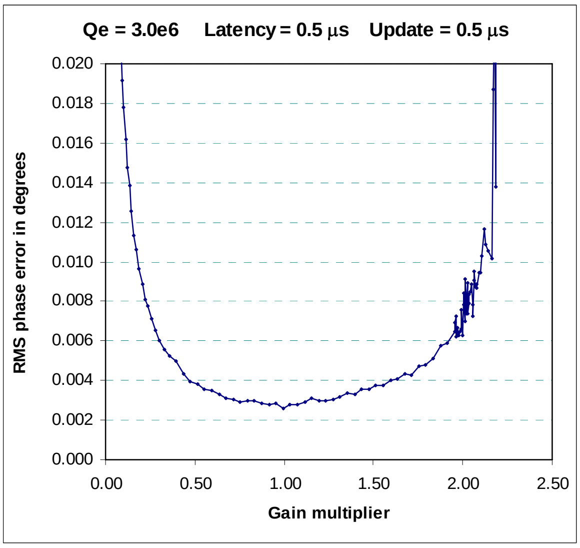
22.2 Measurement error = 0.005 degrees, Latency = 0.5 μs, Update = 0.5 μs, vary Q





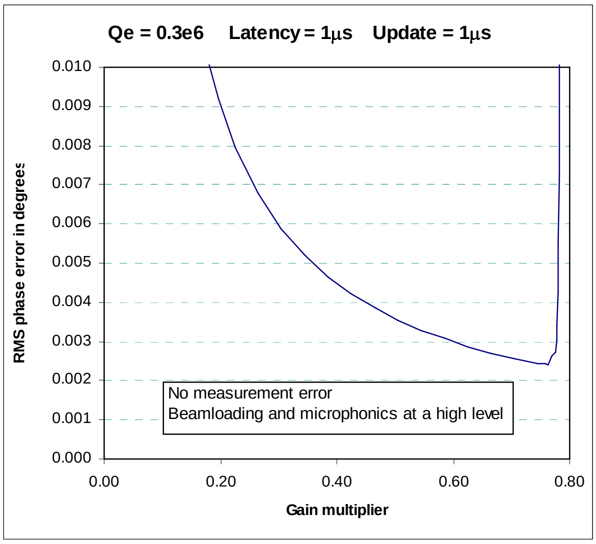
Drive frequency in GHz	= 3.900 GHz
Centre cavity frequency in GHz	= 3.900 GHz
Cavity Q factor	= 1.0000E+09
External Q factor	= 0.3000E+06
Cavity R over Q (2xFNAL=53 per cell)	= 53.000 ohms
Energy point ILC crab~0.0284J per cell)	= 28.400 mJ
Amplitude set point	= 301.675 kV
Maximum Amplifier Power per cell	= 1200.000 W
Maximum voltage set point (no beam)	= 390.692 kV
Maximum beam offset	= 1.000 mm
Maximum bunch phase error	= 1.000 deg
Beam offset frequency	= 2000.000 Hz
Bunch charge (ILC=3.2 nC)	= 3.200 nC
RF cycles between bunches	= 1200
Bunch train length	= 1.000 ms
Cavity frequency shift from microphonics	= 600.000 Hz
Cavity vibration frequency	= 230.000 Hz
Initial vibration phase (degrees)	= 20.000 deg
Phase measurement error(degrees)	= 0.00500 deg
Fractional err in amplitude measurement	= 0.00200
Time delay (latency) for control system	= 2.0000E-06 s
Control update interval	= 1.0000E-06 s
Amplifier bandwidth	= 1.0000E+07
Optimal gain constant for controller	= 0.8897
Minimum rms phase error	= 0.00271
Maximum power delivered	= 832.3693
Proportional coef for real component	= 1.3345E+00
Integral coef for real component	= 5.3380E-05
Proportional coef for imag component	= 5.3380E+00
Integral coef for imag component	= 4.2704E-04

Drive frequency in GHz	= 3.900 GHz
Centre cavity frequency in GHz	= 3.900 GHz
Cavity Q factor	= 1.0000E+09
External Q factor	= 1.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	= 53.000 ohms
Energy point ILC crab~0.0284J per cell)	= 28.400 mJ
Amplitude set point	= 301.675 kV
Maximum Amplifier Power per cell	= 1200.000 W
Maximum voltage set point (no beam)	= 713.302 kV
Maximum beam offset	= 1.000 mm
Maximum bunch phase error	= 1.000 deg
Beam offset frequency	= 2000.000 Hz
Bunch charge (ILC=3.2 nC)	= 3.200 nC
RF cycles between bunches	= 1200
Bunch train length	= 1.000 ms
Cavity frequency shift from microphonics	= 600.000 Hz
Cavity vibration frequency	= 230.000 Hz
Initial vibration phase (degrees)	= 20.000 deg
Phase measurement error(degrees)	= 0.00500 deg
Fractional err in amplitude measurement	= 0.00200
Time delay (latency) for control system	= 0.5000E-06 s
Control update interval	= 0.5000E-06 s
Amplifier bandwidth	= 1.0000E+07
Optimal gain constant for controller	= 1.2461
Minimum rms phase error	= 0.00317
Maximum power delivered	= 350.8633
Proportional coef for real component	= 6.2305E+00
Integral coef for real component	= 2.4922E-04
Proportional coef for imag component	= 2.4922E+01
Integral coef for imag component	= 1.9938E-03

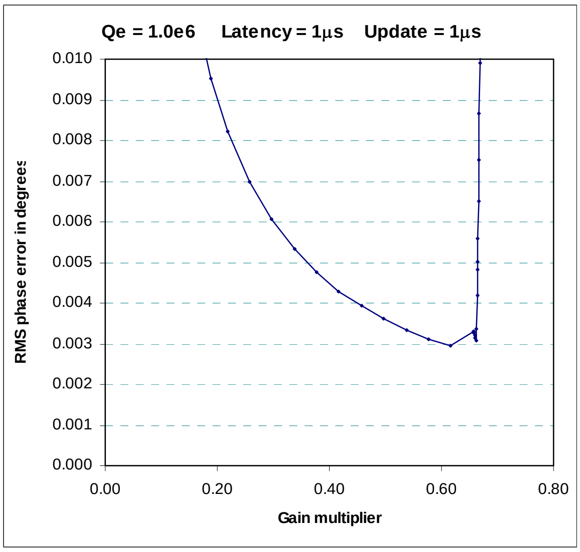


Drive frequency in GHz = 3.900 GHz
Centre cavity frequency in GHz = 3.900 GHz
Cavity Q factor = 1.0000E+09
External Q factor = 3.0000E+06
Cavity R over Q (2xFNAL=53 per cell) = 53.000 ohms
Energy point ILC crab~0.0284J per cell) = 28.400 mJ
Amplitude set point = 301.675 kV
Maximum Amplifier Power per cell = 1200.000 W
Maximum voltage set point (no beam) = 1235.476 kV
Maximum beam offset = 1.000 mm
Maximum bunch phase error = 1.000 deg
Beam offset frequency = 2000.000 Hz
Bunch charge (ILC=3.2 nC) = 3.200 nC
RF cycles between bunches = 1200
Bunch train length = 1.000 ms
Cavity frequency shift from microphonics= 600.000 Hz
Cavity vibration frequency = 23.000 Hz
Initial vibration phase (degrees) = 20.000 deg
Phase measurement error(degrees) = 0.00500 deg
Fractional err in amplitude measurement = 0.00200
Time delay (latency) for control system = 0.5000E-06 s
Control update interval = 0.5000E-06 s
Amplifier bandwidth = 1.0000E+07
Optimal gain constant for controller = 0.9942
Minimum rms phase error = 0.00297
Maximum power delivered = 248.0229
Proportional coef for real component = 1.4913E+01
Integral coef for real component = 5.9654E-04
Proportional coef for imag component = 5.9654E+01
Integral coef for imag component = 4.7723E-03

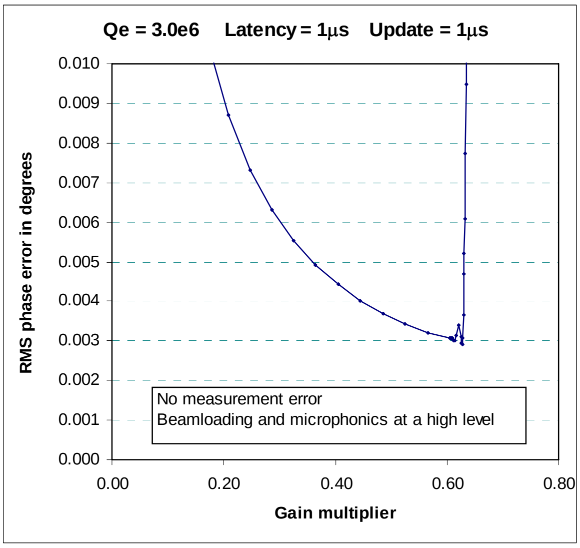
22.3 No measurement error, Latency = 1 μs, Update = 1 μs, vary Q



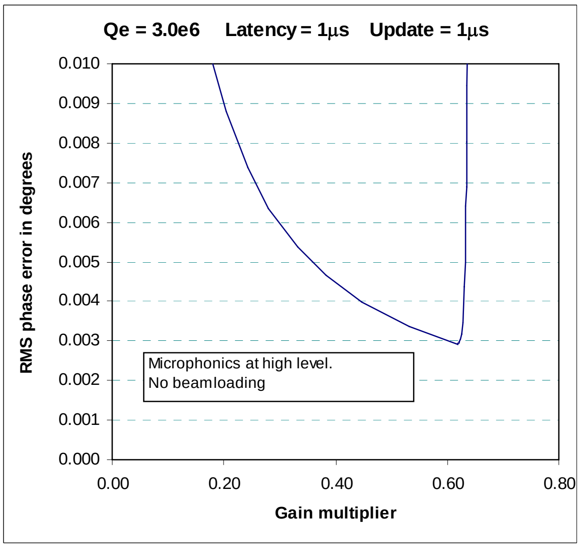
Drive frequency in GHz	= 3.900 GHz
Centre cavity frequency in GHz	= 3.900 GHz
Cavity Q factor	= 1.0000E+09
External Q factor	= 0.3000E+06
Cavity R over Q (2xFNAL=53 per cell)	= 53.000 ohms
Energy point ILC crab~0.0284J per cell)	= 28.400 mJ
Amplitude set point	= 301.675 kV
Maximum Amplifier Power per cell	= 1200.000 W
Maximum voltage set point (no beam)	= 390.692 kV
Maximum beam offset	= 1.000 mm
Maximum bunch phase error	= 1.000 deg
Beam offset frequency	= 2000.000 Hz
Bunch charge (ILC=3.2 nC)	= 3.200 nC
RF cycles between bunches	= 1200
Bunch train length	= 1.000 ms
Cavity frequency shift from microphonics	= 600.000 Hz
Cavity vibration frequency	= 23.000 Hz
Initial vibration phase (degrees)	= 20.000 deg
Phase measurement error(degrees)	= 0.00000 deg
Fractional err in amplitude measurement	= 0.00000
Time delay (latency) for control system	= 2.0000E-06 s
Control update interval	= 1.0000E-06 s
Amplifier bandwidth	= 1.0000E+07
Optimal gain constant for controller	= 0.7633
Minimum rms phase error	= 0.00242
Maximum power delivered	= 823.2286
Proportional coef for real component	= 1.1450E+00
Integral coef for real component	= 4.5800E-05
Proportional coef for imag component	= 4.5800E+00
Integral coef for imag component	= 3.6640E-04



Drive frequency in GHz	= 3.900 GHz
Centre cavity frequency in GHz	= 3.900 GHz
Cavity Q factor	= 1.0000E+09
External Q factor	= 1.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	= 53.000 ohms
Energy point ILC crab~0.0284J per cell)	= 28.400 mJ
Amplitude set point	= 301.675 kV
Maximum Amplifier Power per cell	= 1200.000 W
Maximum voltage set point (no beam)	= 713.302 kV
Maximum beam offset	= 1.000 mm
Maximum bunch phase error	= 1.000 deg
Beam offset frequency	= 2000.000 Hz
Bunch charge (ILC=3.2 nC)	= 3.200 nC
RF cycles between bunches	= 1200
Bunch train length	= 1.000 ms
Cavity frequency shift from microphonics	= 600.000 Hz
Cavity vibration frequency	= 23.000 Hz
Initial vibration phase (degrees)	= 20.000 deg
Phase measurement error(degrees)	= 0.00000 deg
Fractional err in amplitude measurement	= 0.00000
Time delay (latency) for control system	= 1.0000E-06 s
Control update interval	= 1.0000E-06 s
Amplifier bandwidth	= 1.0000E+07
Optimal gain constant for controller	= 0.6169
Minimum rms phase error	= 0.00295
Maximum power delivered	= 336.5188
Proportional coef for real component	= 3.0843E+00
Integral coef for real component	= 1.2337E-04
Proportional coef for imag component	= 1.2337E+01
Integral coef for imag component	= 9.8697E-04

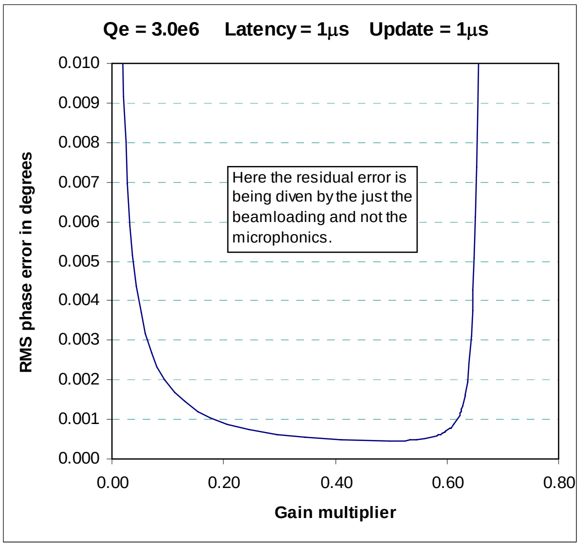


Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	3.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	1235.476 kV
Maximum beam offset	=	1.000 mm
Maximum bunch phase error	=	1.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	600.000 Hz
Cavity vibration frequency	=	23.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00000 deg
Fractional err in amplitude measurement	=	0.00000
Time delay (latency) for control system	=	1.0000E-06 s
Control update interval	=	1.0000E-06 s
Initial gain constant for controller	=	0.0020
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.6274
Minimum rms phase error	=	0.00291
Maximum power delivered	=	232.6412
Proportional coef for real component	=	9.4114E+00
Integral coef for real component	=	3.7646E-04
Proportional coef for imag component	=	3.7646E+01
Integral coef for imag component	=	3.0117E-03



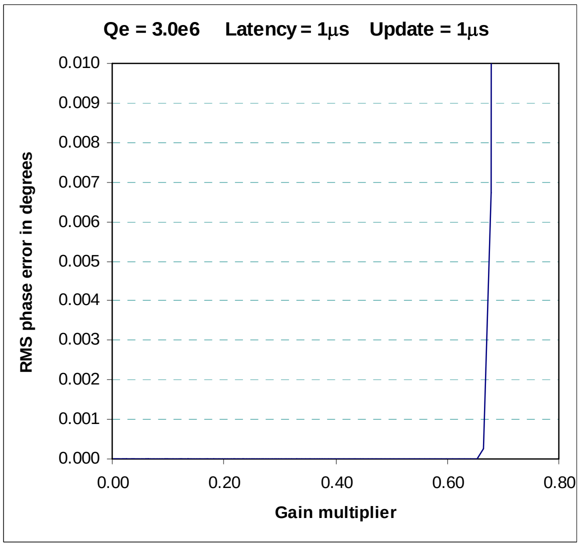
Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	3.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	1235.476 kV
Maximum beam offset	=	0.0 mm
Maximum bunch phase error	=	0.0 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	600.000 Hz
Cavity vibration frequency	=	230.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00000 deg
Fractional err in amplitude measurement	=	0.00000
Time delay (latency) for control system	=	1.0000E-06 s
Control update interval	=	1.0000E-06 s
Initial gain constant for controller	=	0.0020
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.6181
Minimum rms phase error	=	0.00292
Maximum power delivered	=	121.8580
Proportional coef for real component	=	9.2721E+00
Integral coef for real component	=	3.7089E-04
Proportional coef for imag component	=	3.7089E+01
Integral coef for imag component	=	2.9671E-03

22.4 No microphonics



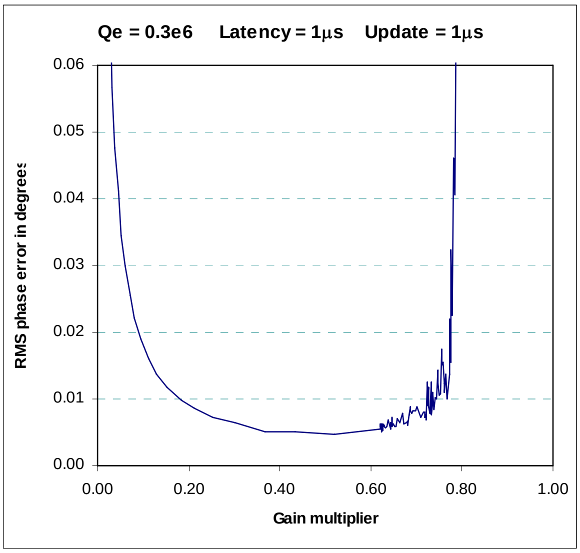
Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	3.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	1235.476 kV
Maximum beam offset	=	1.000 mm
Maximum bunch phase error	=	1.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	0.000 Hz
Cavity vibration frequency	=	230.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00000 deg
Fractional err in amplitude measurement	=	0.00000
Time delay (latency) for control system	=	1.0000E-06 s
Control update interval	=	1.0000E-06 s
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.4965
Minimum rms phase error	=	0.00045
Maximum power delivered	=	211.2583
Proportional coef for real component	=	7.4479E+00
Integral coef for real component	=	2.9792E-04
Proportional coef for imag component	=	2.9792E+01
Integral coef for imag component	=	2.3833E-03

22.5 No beam, no microphonics

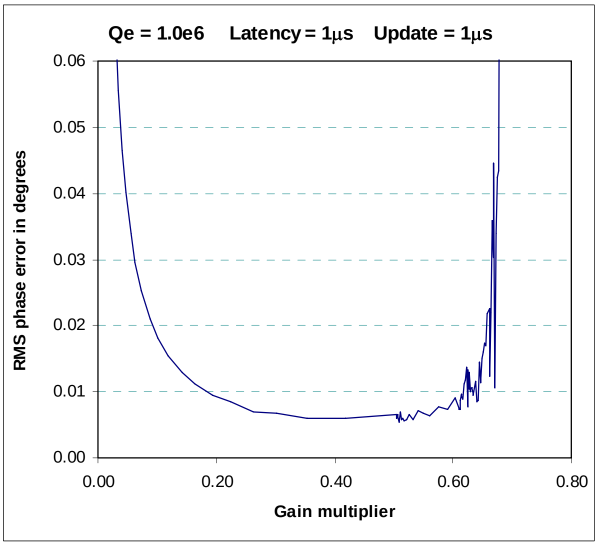


Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	3.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	1235.476 kV
Maximum beam offset	=	0.000 mm
Maximum bunch phase error	=	0.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	0.000 Hz
Cavity vibration frequency	=	230.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00000 deg
Fractional err in amplitude measurement	=	0.00000
Time delay (latency) for control system	=	1.0000E-06 s
Control update interval	=	1.0000E-06 s
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.3929
Minimum rms phase error	=	0.00000
Maximum power delivered	=	71.9769
Proportional coef for real component	=	5.8929E+00
Integral coef for real component	=	2.3571E-04
Proportional coef for imag component	=	2.3571E+01
Integral coef for imag component	=	1.8857E-03

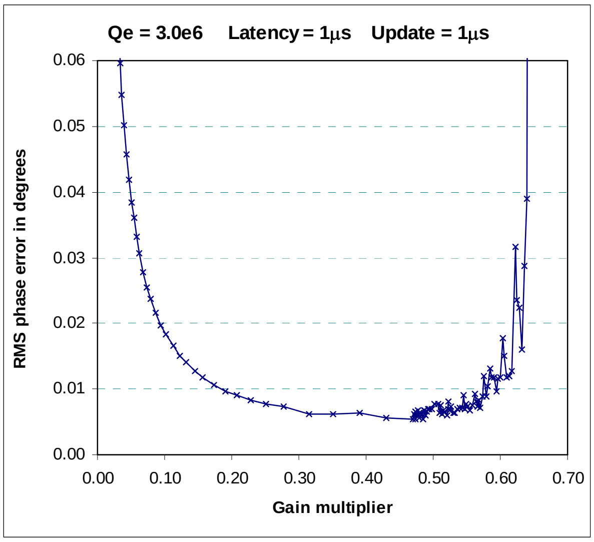
22.6 Measurement error = 0.005 degrees, Latency = 1 μs, Update = 1 μs, vary Q



Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	0.3000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	390.692 kV
Maximum beam offset	=	1.000 mm
Maximum bunch phase error	=	1.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	600.000 Hz
Cavity vibration frequency	=	230.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00500 deg
Fractional err in amplitude measurement	=	0.00200
Time delay (latency) for control system	=	2.0000E-06 s
Control update interval	=	1.0000E-06 s
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.5195
Minimum rms phase error	=	0.00547
Maximum power delivered	=	828.5186
Proportional coef for real component	=	7.7925E-01
Integral coef for real component	=	3.1170E-05
Proportional coef for imag component	=	3.1170E+00
Integral coef for imag component	=	2.4936E-04

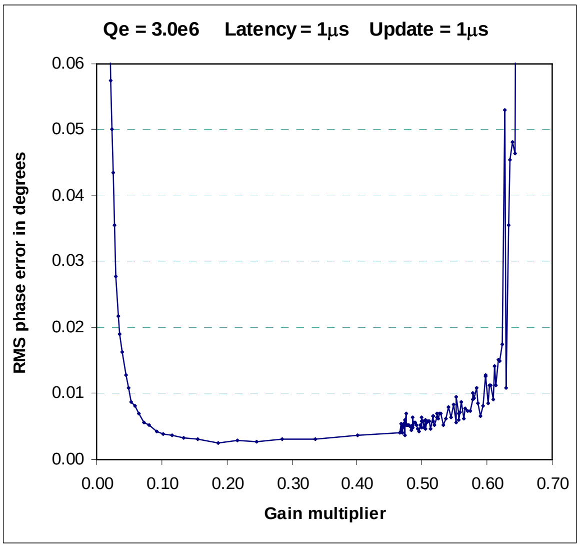


Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	1.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	713.302 kV
Maximum beam offset	=	1.000 mm
Maximum bunch phase error	=	1.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	600.000 Hz
Cavity vibration frequency	=	230.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00500 deg
Fractional err in amplitude measurement	=	0.00200
Time delay (latency) for control system	=	1.0000E-06 s
Control update interval	=	1.0000E-06 s
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.5089
Minimum rms phase error	=	0.00569
Maximum power delivered	=	343.8641
Proportional coef for real component	=	2.5447E+00
Integral coef for real component	=	1.0179E-04
Proportional coef for imag component	=	1.0179E+01
Integral coef for imag component	=	8.1429E-04



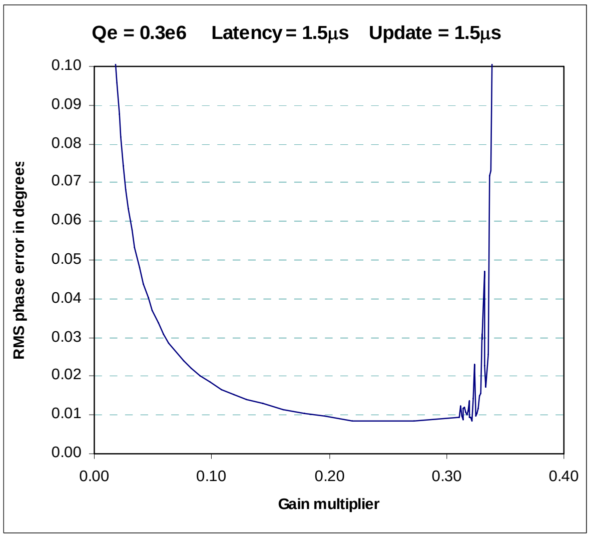
Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	3.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	713.302 kV
Maximum beam offset	=	1.000 mm
Maximum bunch phase error	=	1.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	600.000 Hz
Cavity vibration frequency	=	230.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00500 deg
Fractional err in amplitude measurement	=	0.00200
Time delay (latency) for control system	=	1.0000E-06 s
Control update interval	=	1.0000E-06 s
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.4707
Minimum rms phase error	=	0.00625
Maximum power delivered	=	238.9978
Proportional coef for real component	=	7.0600E+00
Integral coef for real component	=	2.8240E-04
Proportional coef for imag component	=	2.8240E+01
Integral coef for imag component	=	2.2592E-03

22.7 Reduce Microphonic frequency from 230 Hz to 23 Hz

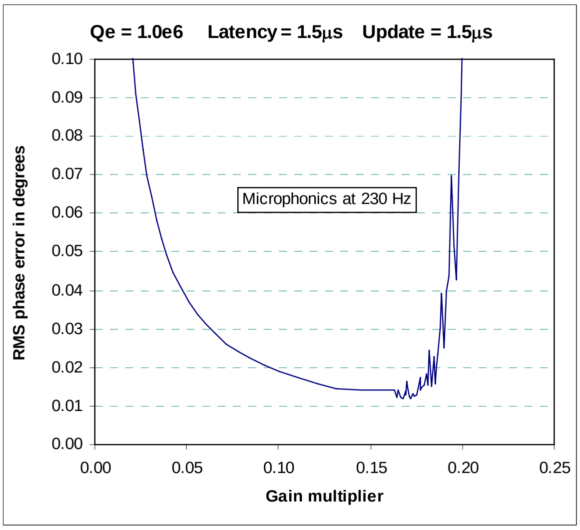


Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	3.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	1235.476 kV
Maximum beam offset	=	1.000 mm
Maximum bunch phase error	=	1.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	600.000 Hz
Cavity vibration frequency	=	23.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00500 deg
Fractional err in amplitude measurement	=	0.00200
Time delay (latency) for control system	=	1.0000E-06 s
Control update interval	=	1.0000E-06 s
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.1862
Minimum rms phase error	=	0.00280
Maximum power delivered	=	242.5511
Proportional coef for real component	=	2.7924E+00
Integral coef for real component	=	1.1170E-04
Proportional coef for imag component	=	1.1170E+01
Integral coef for imag component	=	8.9357E-04

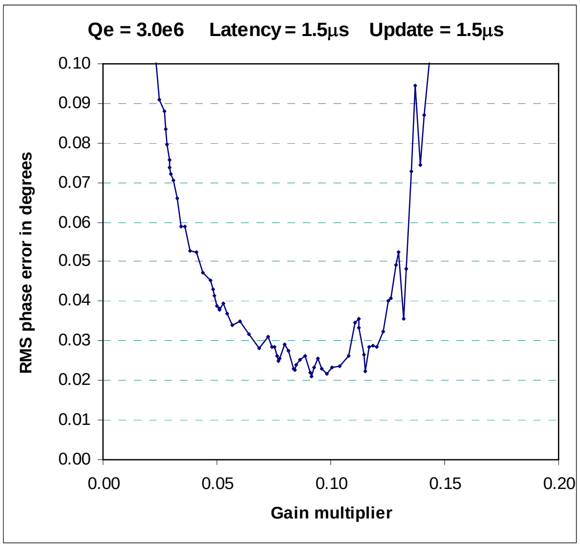
22.8 Measurement error = 0.005 degrees, Latency = 1.5 μs, Update = 1.5 μs, vary Q



Drive frequency in GHz = 3.900 GHz
Centre cavity frequency in GHz = 3.900 GHz
Cavity Q factor = 1.0000E+09
External Q factor = 0.3000E+06
Cavity R over Q (2xFNAL=53 per cell) = 53.000 ohms
Energy point ILC crab~0.0284J per cell) = 28.400 mJ
Amplitude set point = 301.675 kV
Maximum Amplifier Power per cell = 1200.000 W
Maximum voltage set point (no beam) = 390.692 kV
Maximum beam offset = 1.000 mm
Maximum bunch phase error = 1.000 deg
Beam offset frequency = 2000.000 Hz
Bunch charge (ILC=3.2 nC) = 3.200 nC
RF cycles between bunches = 1200
Bunch train length = 1.000 ms
Cavity frequency shift from microphonics= 600.000 Hz
Cavity vibration frequency = 23.000 Hz
Initial vibration phase (degrees) = 20.000 deg
Phase measurement error(degrees) = 0.00500 deg
Fractional err in amplitude measurement = 0.00200
Time delay (latency) for control system = 1.5000E-06 s
Control update interval = 1.5000E-06 s
Amplifier bandwidth = 1.0000E+07
Optimal gain constant for controller = 0.2523
Minimum rms phase error = 0.00788
Maximum power delivered = 827.3736
Proportional coef for real component = 3.7852E-01
Integral coef for real component = 1.5141E-05
Proportional coef for imag component = 1.5141E+00
Integral coef for imag component = 1.2112E-04

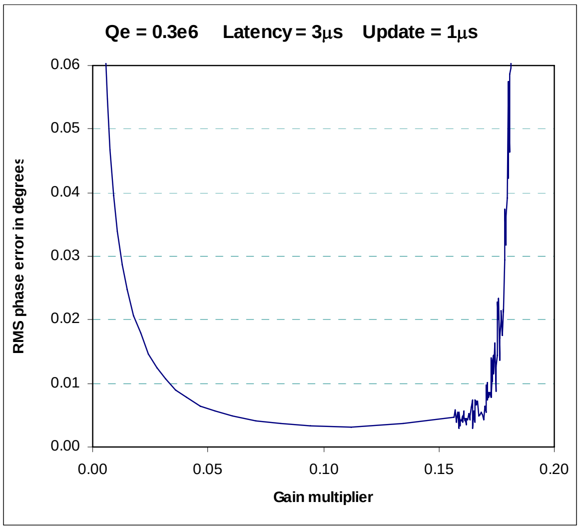


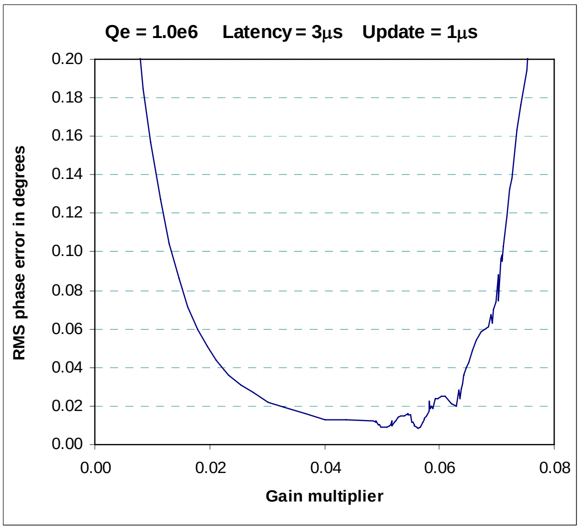
Drive frequency in GHz = 3.900 GHz
Centre cavity frequency in GHz = 3.900 GHz
Cavity Q factor = 1.0000E+09
External Q factor = 1.0000E+06
Cavity R over Q (2xFNAL=53 per cell) = 53.000 ohms
Energy point ILC crab~0.0284J per cell) = 28.400 mJ
Amplitude set point = 301.675 kV
Maximum Amplifier Power per cell = 1200.000 W
Maximum voltage set point (no beam) = 713.302 kV
Maximum beam offset = 1.000 mm
Maximum bunch phase error = 1.000 deg
Beam offset frequency = 2000.000 Hz
Bunch charge (ILC=3.2 nC) = 3.200 nC
RF cycles between bunches = 1200
Bunch train length = 1.000 ms
Cavity frequency shift from microphonics= 600.000 Hz
Cavity vibration frequency = 23.000 Hz
Initial vibration phase (degrees) = 20.000 deg
Phase measurement error(degrees) = 0.00500 deg
Fractional err in amplitude measurement = 0.00200
Time delay (latency) for control system = 1.5000E-06 s
Control update interval = 1.0500E-06 s
Amplifier bandwidth = 1.0000E+07
Optimal gain constant for controller = 0.1525
Minimum rms phase error = 0.00408
Maximum power delivered = 343.4789
Proportional coef for real component = 7.6246E-01
Integral coef for real component = 3.0499E-05
Proportional coef for imag component = 3.0499E+00
Integral coef for imag component = 2.4399E-04



Drive frequency in GHz	= 3.900 GHz
Centre cavity frequency in GHz	= 3.900 GHz
Cavity Q factor	= 1.0000E+09
External Q factor	= 3.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	= 53.000 ohms
Energy point ILC crab~0.0284J per cell)	= 28.400 mJ
Amplitude set point	= 301.675 kV
Maximum Amplifier Power per cell	= 1200.000 W
Maximum voltage set point (no beam)	= 1235.476 kV
Maximum beam offset	= 1.000 mm
Maximum bunch phase error	= 1.000 deg
Beam offset frequency	= 2000.000 Hz
Bunch charge (ILC=3.2 nC)	= 3.200 nC
RF cycles between bunches	= 1200
Bunch train length	= 1.000 ms
Cavity frequency shift from microphonics	= 600.000 Hz
Cavity vibration frequency	= 23.000 Hz
Initial vibration phase (degrees)	= 20.000 deg
Phase measurement error(degrees)	= 0.00500 deg
Fractional err in amplitude measurement	= 0.00200
Time delay (latency) for control system	= 1.5000E-06 s
Control update interval	= 1.5000E-06 s
Amplifier bandwidth	= 1.0000E+07
Optimal gain constant for controller	= 0.1242
Minimum rms phase error	= 0.02200
Maximum power delivered	= 241.4754
Proportional coef for real component	= 1.8627E+00
Integral coef for real component	= 7.4510E-05
Proportional coef for imag component	= 7.4510E+00
Integral coef for imag component	= 5.9608E-04

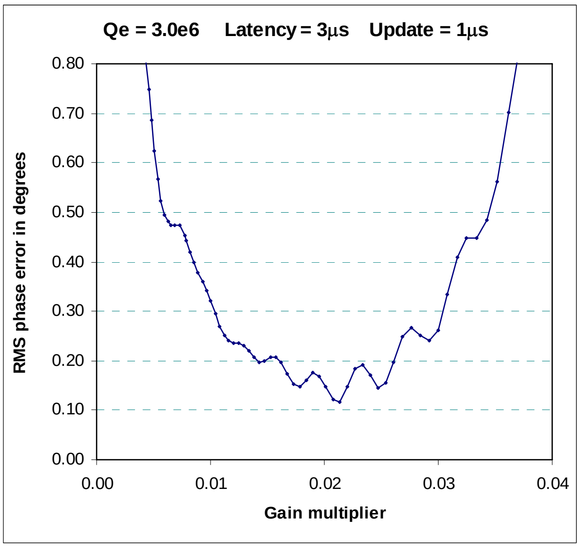
22.9 *Measurement error = 0.005 degrees, Latency = 3.0 μs, Update = 1.0 μs, vary Q*





Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	0.3000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	390.692 kV
Maximum beam offset	=	1.000 mm
Maximum bunch phase error	=	1.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	600.000 Hz
Cavity vibration frequency	=	23.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00500 deg
Fractional err in amplitude measurement	=	0.00200
Time delay (latency) for control system	=	3.0000E-06 s
Control update interval	=	1.0000E-06 s
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.1648
Minimum rms phase error	=	0.00446
Maximum power delivered	=	827.3696
Proportional coef for real component	=	2.4719E-01
Integral coef for real component	=	9.8876E-06
Proportional coef for imag component	=	9.8876E-01
Integral coef for imag component	=	7.9101E-05

Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	1.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	713.302 kV
Maximum beam offset	=	1.000 mm
Maximum bunch phase error	=	1.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	600.000 Hz
Cavity vibration frequency	=	23.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00500 deg
Fractional err in amplitude measurement	=	0.00200
Time delay (latency) for control system	=	3.0000E-06 s
Control update interval	=	1.0000E-06 s
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.0563
Minimum rms phase error	=	0.00703
Maximum power delivered	=	354.8504
Proportional coef for real component	=	2.8150E-01
Integral coef for real component	=	1.1260E-05
Proportional coef for imag component	=	1.1260E+00
Integral coef for imag component	=	9.0079E-05



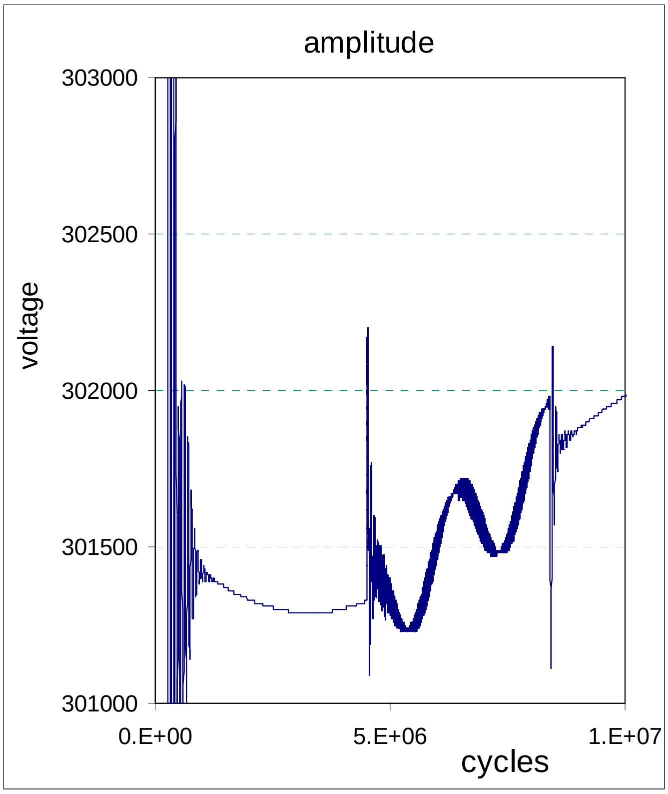
Drive frequency in GHz	=	3.900 GHz
Centre cavity frequency in GHz	=	3.900 GHz
Cavity Q factor	=	1.0000E+09
External Q factor	=	3.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	=	53.000 ohms
Energy point ILC crab~0.0284J per cell)	=	28.400 mJ
Amplitude set point	=	301.675 kV
Maximum Amplifier Power per cell	=	1200.000 W
Maximum voltage set point (no beam)	=	1235.476 kV
Maximum beam offset	=	1.000 mm
Maximum bunch phase error	=	1.000 deg
Beam offset frequency	=	2000.000 Hz
Bunch charge (ILC=3.2 nC)	=	3.200 nC
RF cycles between bunches	=	1200
Bunch train length	=	1.000 ms
Cavity frequency shift from microphonics	=	600.000 Hz
Cavity vibration frequency	=	23.000 Hz
Initial vibration phase (degrees)	=	20.000 deg
Phase measurement error(degrees)	=	0.00500 deg
Fractional err in amplitude measurement	=	0.00200
Time delay (latency) for control system	=	3.0000E-06 s
Control update interval	=	1.0000E-06 s
Amplifier bandwidth	=	1.0000E+07
Optimal gain constant for controller	=	0.0214
Minimum rms phase error	=	0.11827
Maximum power delivered	=	403.8752
Proportional coef for real component	=	3.2060E-01
Integral coef for real component	=	1.2824E-05
Proportional coef for imag component	=	1.2824E+00
Integral coef for imag component	=	1.0259E-04

23. Three Mode Simulation for ILC Cavities

Section 11 developed the theory for the case where several modes can be excited by the RF power amplifier or the beam. A proposal for the ILC crab cavity is to use a nine cell cavity. In this design the nearest two modes to the operating mode are the $8\pi/9$ mode at + 2 MHz and the $7\pi/9$ mode at + 9 MHz. Modes with frequencies further from the operating mode have less influence on control of the phase and amplitude of the operating mode. In this section calculations with the first 3 modes are made and can be compared with previous results where only one mode was included. Figures 41(a-c) of section 23.1 show the amplitude control, the phase control and the power requirement for near optimal gain. RMS amplitude control is less than 0.1% and this is far better than the ILC requirement. RMS phase control has significantly deteriorated with respect to the single mode case, it has deteriorated from 7 milli-degrees (see figure 42) to 26 milli-degrees.

Section 23.2 gives more realistic performance for measurements made with 3.9 GHz to 1.3 GHz dividers and 1.3 GHz digital phase detector. Measurements made in this way would introduce jitter of 8 milli-degrees in a bandwidth of 1 MHz. The simulation is not completely realistic as the noise introduced by the program has not been tailored to the actual noise spectrum of the digital phase detectors.

23.1 Microphonics and oscillatory beamloading but no measurement error



Drive frequency in GHz	= 3.900 GHz
Centre cavity frequency in GHz	= 3.900 GHz
Number of cavity modes	= 3
Cavity Q factor	= 1.0E+09
External Q factor	= 3.0E+06
Cavity R over Q (2xFNAL=53 per cell)	= 53.0 ohms
Energy point ILC crab~0.0284J per cell)	= 28.400 mJ
Amplitude set point	= 301.675 kV
Maximum Amplifier Power per cell	= 1200.000 W
Maximum voltage set point (no beam)	= 1235.476 kV
Maximum beam offset	= 0.6 mm
Maximum bunch phase jitter	= 1.0 deg
Beam offset frequency	= 2000.0 Hz
Bunch charge (ILC=3.2 nC)	= 3.2 nC
RF cycles between bunches	= 1200.0
Bunch train length	= 1.0 ms
Cavity frequency shift from microphonics	= 600 Hz
Cavity vibration frequency	= 230 Hz
Initial vibration phase (degrees)	= 20 deg
Phase measurement error(degrees)	= 0 deg
Fractional err in amplitude measurement	= 0
Time delay (latency) for control system	= 1.0E-06 s
Control update interval	= 1.0E-06 s
Gain constant for controller	= 0.7
Amplifier bandwidth	= 1.0E+07
maximum power delivered	= 167.34
In pulse rms phase err	= 0.02560 degrees
In pulse rms amplitude err	= 0.07966 %
Relative excitation of 2nd mode	= 0.03260 %
Relative excitation of 3rd mode	= 0.01756 %
Proportional coef for real component	= 4.20E+01
Integral coef for real component	= 1.26E-03
Proportional coef for imag component	= 4.20E+01
Integral coef for imag component	= 1.26E-03

Figure 41 Amplitude control performance of a multi-cell crab cavity when three modes are considered

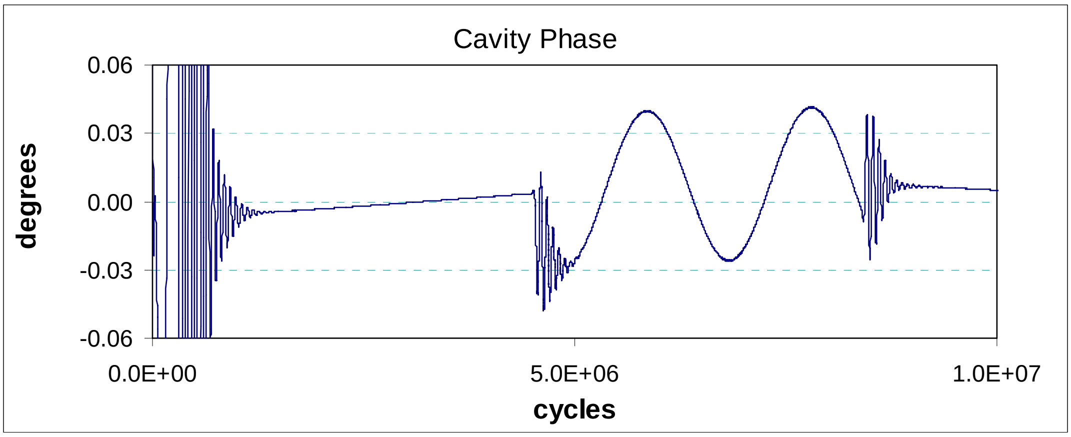


Figure 41b Phase control performance for computation of figure 41a

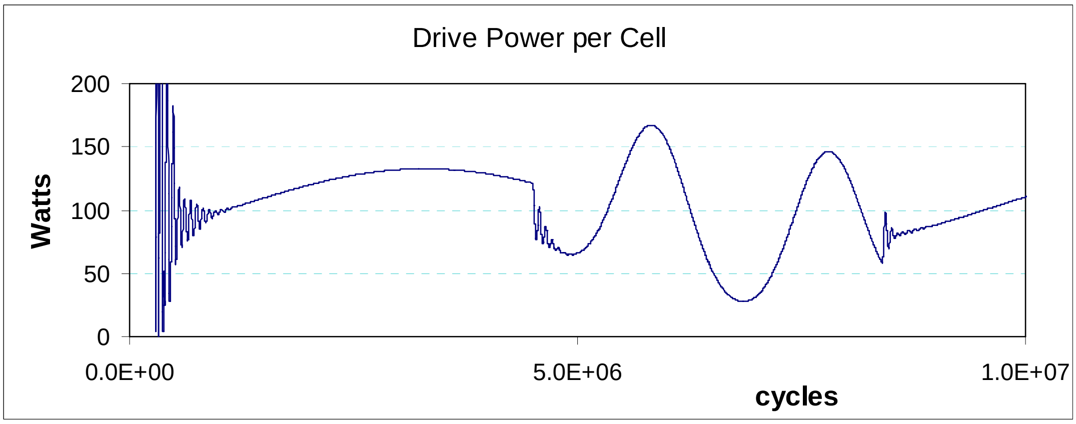


Figure 41c Drive power requirement for computation of figure 41a

The striking feature of figure 41b is that when 3 modes are present the coupling to the beam is much stronger. Comparison can be made with figure 34 paying attention to the scale. Figures 41c and 34 were made with slightly differing control parameters. Figure 42 is the phase control for an identical calculation used for figure 41 with the exception that only the operating mode is included.

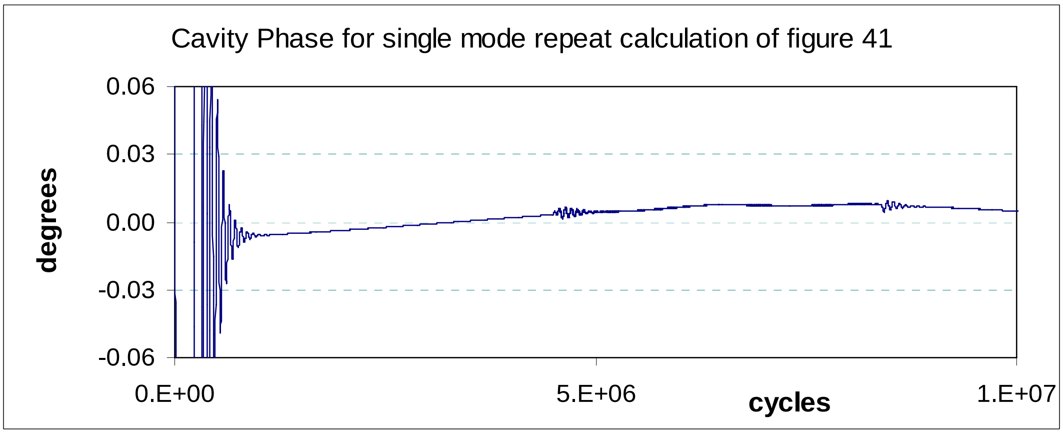


Figure 42 Phase control single mode case

We make the supposition that the increased error comes from the fact that amplitude and phase is determined from the total field in the cavity rather than that for the operating mode. This is easily examined by modifying the measurement model in the code. Results are shown in figure 43.

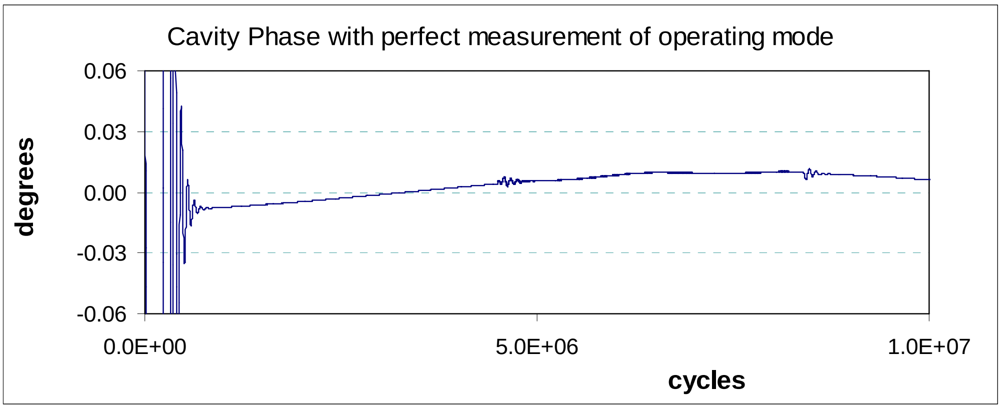
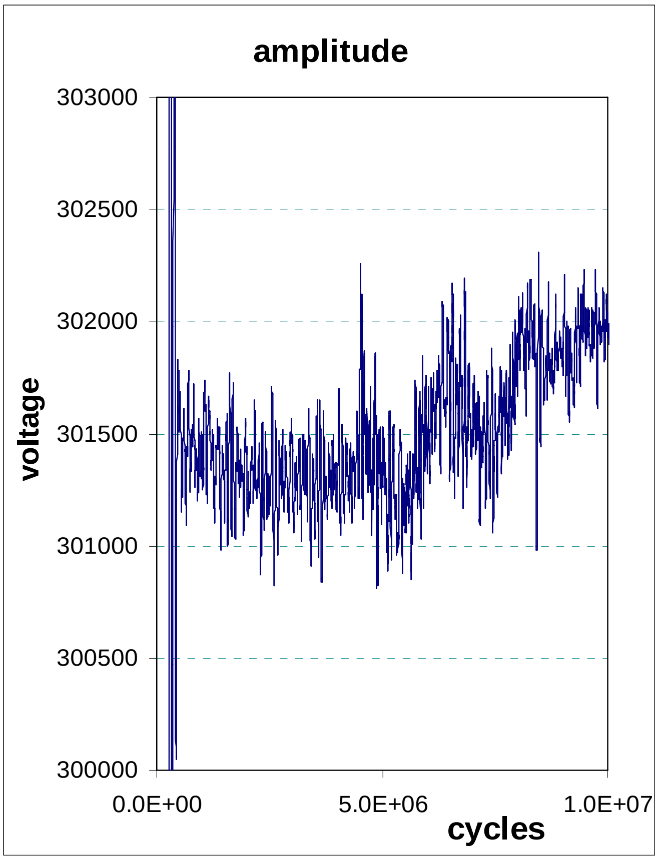


Figure 43 Phase control for three mode case when by some clever filter one determines the amplitude and phase of just the operating mode.

23.2 Measurement error, microphonics and oscillatory beamloading



Drive frequency in GHz	= 3.900 GHz
Centre cavity frequency in GHz	= 3.900 GHz
Number of cavity modes	= 3
Cavity Q factor	= 1.0000E+09
External Q factor	= 3.0000E+06
Cavity R over Q (2xFNAL=53 per cell)	= 53.000 ohms
Energy point ILC crab~0.0284J per cell)	= 28.400 mJ
Amplitude set point	= 301.675 kV
Maximum Amplifier Power per cell	= 1200.000 W
Maximum voltage set point (no beam)	= 1235.476 kV
Maximum beam offset	= 0.600 mm
Maximum bunch phase jitter	= 1.000 deg
Beam offset frequency	= 2000.000 Hz
Bunch charge (ILC=3.2 nC)	= 3.200 nC
RF cycles between bunches	= 1200.000
Bunch train length	= 1.000 ms
Cavity frequency shift from microphonics	= 600.000 Hz
Cavity vibration frequency	= 230.000 Hz
Initial vibration phase (degrees)	= 20.000 deg
Phase measurement error(degrees)	= 0.02000 deg
Fractional err in amplitude measurement	= 0.00100
Time delay (latency) for control system	= 1.0000E-06 s
Control update interval	= 1.0000E-06 s
Gain constant for controller	= 0.5500
Amplifier bandwidth	= 1.0000E+07
Measurement filter bandwidth	= 5.0000E+05
maximum power delivered	= 193.34
In pulse rms phase err	= 0.02967 deg
In pulse rms amplitude err	= 0.10155 %
Relative excitation of 2nd mode	= 0.03289 %
Relative excitation of 3rd mode	= 0.01768 %
Proportional coef for real component	= 3.3000E+01
Integral coef for real component	= 9.9000E-04
Proportional coef for imag component	= 3.3000E+01
Integral coef for imag component	= 9.9000E-04

Figure 43a Amplitude control performance of a multi-cell crab cavity when three modes with measurement errors are considered

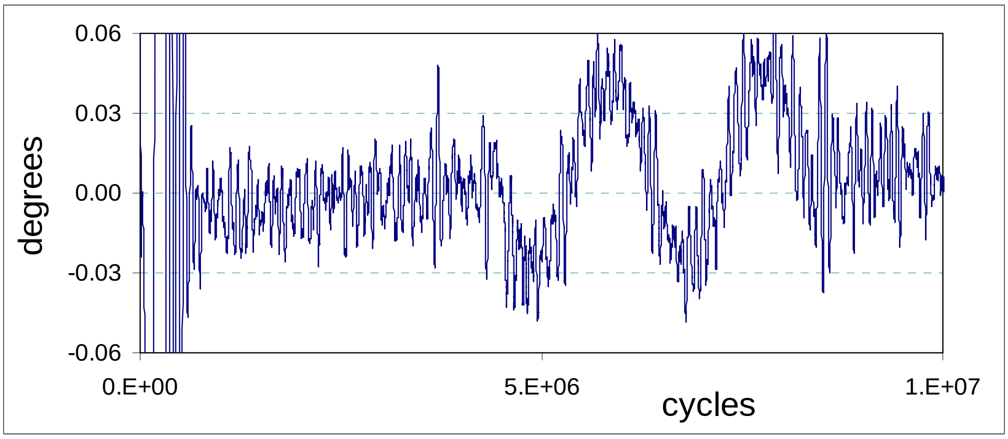


Figure 43b Phase control performance for computation of figure 43a

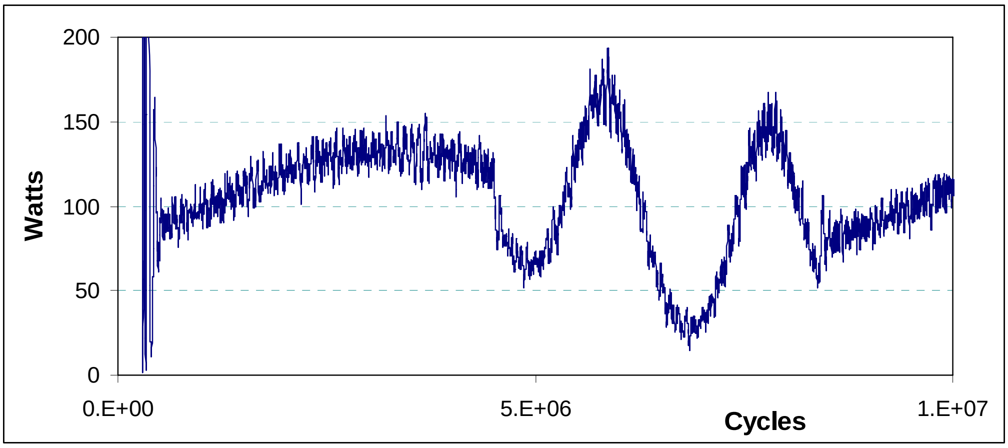


Figure 43c Power per cell for computation of figure 43a

24. Conclusions

Particle accelerators and light sources have a range of amplitude and stability requirements for the control of their RF cavities. The theory and code developed in this report has far more general application than to just the ILC crab cavities. None the theory developed in sections 3-16 is essentially new however we are not aware of another publication where it is conveniently grouped together. New results are numerical prediction of phase stabilisation performance of superconducting crab cavities. The simulations assume a digital control system where the updated control output has a fixed delay with respect the input. By nature of the numerical solution, realistic measurement errors and incremental beamloading are easily included.

A highly significant result from this work is the added difficulty of getting precise phase control for an operating mode when adjacent modes have even quite small levels of excitation. For a single mode crab cavity operated with ILC beam parameters and in the absence of measurement errors, the phase stability performance is determined by how well the control system compensates microphonics which are present; beam-loading has no significant effect on the phase. This ceases to be true for the multi-mode cavity when the measurement of amplitude and phase detects modes other than the operating mode.

A second result which was new to us (but not necessarily new to the community) is that the optimum phase control performance is almost independent of the cavity external Q for significant range of beamloading, microphonic, digital time delay and measurement error parameters around the preferred operating point.

A third result which may be new are some the formulae given in section 21.2 for the stability limit of the PI algorithm and in particular the empirical approximations.

A final concern requiring further work is the simplistic measurement error model we have used. Preliminary data from cavity control experiments indicate that the random errors we have in our model do not give representative fluctuations on a kHz timescale. Accurate measurements of the noice spectrum need to be made for incorporation into the model.

25. References

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26. Acknowledgements

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27. Appendix 1 Code for direct integration (see section 6)

```
c      Driven oscillator with beam load
c      ***********************************
c      Last modified 26th Oct 2006

c      This program solves the equation

c      C*dV/dt+V*(1/Zext+1/R)+(1/L)*Integral(V*dt)=2*Forw*cos(wd*t+psi)/Zext

c      Where the voltage V takes a step change at t = 0 and V0 and psi can be
c      changed to minimise recovery time. Forw is the voltage in the forward wave.

c      For solution the differential equation is written in the form

c      d2V/dt2+(wc/QL)*dV/dt+wc**2*V=-2*Forw*(wd*wc/QE)*sin(wd*t+psi)

c      in this program the time derivative Forw is neglected as Forw is usually
c      and when it changes it will change over about 100 cycles hence its values is
c      on hundredth of the other term.

c      The differential equation is solved by 4th or Runge Kutta
c      In the solution E~V, F~dV/dt, Vd0 = Forw for t<0, Vd1 = Forw for t>0

c      The solution starts at a time before t=0 so equilib is established

c2345678901234567890123456789012345678901234567890123456789012

      real*8 Vdr0, Vdr1, Vdi0, Vdi1, Vd0, Vd1, vjump
      real*8 vdr, Vdr_last, vdi, Vdi_last, vdr_dot, vdi_dot
      real*8 wc0, wc1, wc, wd0, wd1, wd
      real*8 QE, QC, QL0, QL1, QE0, QE1, QC0, QC1
      real*8 bL0, bL1, bE0, bE1, bE, bL
      real*8 pi, period, t0, dt, t1, t2, tstart, hdt, dt6, ts, time, tx
      real*8 psi0, psi1, psi0_degrees, psi1_degrees
      real*8 fc0, fc1, fd0, fd1, dfc, dfd
      real*8 E0, E1, E2, E3, E4, F0, F1, F2, F3, F4
      real*8 DF1, DF2, DF3, DF4
      real*8 amplitude, frequency, shift, phase, phase_deg
      real*8 xamplitude
      real*8 driver, drivei, drive
      real*8 tjump, beamloadi, beamloadr

      integer n, ns, nf, its, j, it, nprint, nwrite
      integer ivalue0, ivalue1, ivalue2

c      logical ljump

      intrinsic abs, atan, cos, sin

      open(file='data.txt',unit=42, status='modify')

      pi=4.0d00*atan(1.0d00)

      print*, 'Give forward wave amplitude for t<0'
      read(*,*) vd0
      write(42,949) vd0
949   format('Forward wave amplitude for t<0         =', 1pe11.4)

      print*, 'Give forward wave amplitude for t>0'
      read(*,*) vd1
      write(42,950) vd1
950   format('Forward wave amplitude for t>0         =', 1pe11.4)
```

```
      print*, 'Give beam induced amplitude jump at t=0'
      read(*,*) vjump
      write(42,9501) vjump
9501  format('Voltage jump at t=0                   =', 1pe11.4)

      fd0=3.90e09
      write(42,951) fd0
951   format('Inital drive frequency t<0            =', 1pe11.4,' Hz')
c     initial drive frequency

      shift = 0.0d00
c     print*, 'Give drive frequency shift at t=0 in MHz'
c     read(*,*) Shift
      dfd=1.0d06*shift

      fd1=fd0+dfd
      write(42,952) fd1
952   format('Drive frequency for t>0               =', 1pe11.4,' Hz')
c     new drive frequency

      psi0_degrees=0.0d00
      write(42,953) psi0_degrees
953   format('Initial drive phase                   =', 1pe11.4,' deg')
      psi0=psi0_degrees*pi/180.0d00

      shift=0.0d00
      print*, 'Give forward wave phase shift at t=0 in degrees'
      read(*,*) Shift
      psi1_degrees=psi0_degrees+shift
      psi1=psi1_degrees*pi/180.0d00
      write(42,954) psi1_degrees
954   format('Phase shift at t=>0 =', 1pe11.4,' deg')

      fc0 =3.9e09
      write(42,955) fd0
955   format('Inital cavity frequency               =', 1pe11.4,' Hz')
c     initial cavity frequency

      dfc=0.0d00
      fc1 = fc0 + dfc
      write(42,956) fd0
956   format('Cvity frequency for t>0               =', 1pe11.4,' Hz')
c     new cavity frequency

      wd0=2.0d00*pi*fd0
      wd1=2.0d00*pi*fd1
      wc0=2.0d00*pi*fc0
      wc1=2.0d00*pi*fc1

      write(*,*) 'Give Cavity Q factor'
      read(*,*) QC
      write(42,958) QC
958   format('Cavity Q factor                       =', 1pe11.4)
      QC0=QC
      QC1=QC

      write(*,*) 'Give External Q factor'
      read(*,*) QE
      write(42,959) QE
959   format('External Q factor                     =', 1pe11.4)

      QL0=1.0d00/(1.0d00/QE+1.0d00/QC0)
      QL1=1.0d00/(1.0d00/QE+1.0d00/QC1)
      QE0=QE
      QE1=QE
```

```
      Close(unit=42,status='keep')


      write(*,*) '     '

      bL0=wc0/QL0
      bL1=wc1/QL1
      bE0=wc0/QE0
      bE1=wc1/QE1

      ns=10000
      nf=20000
      its=180
      ns=100000
      nf=200000
      its=180
      it=0

      period=2.0d00*pi/wd0

      tjump=0.25d00*period
      tjump=0.0d00

      beamloadr=0.319324157d00*vjump*QE
      beamloadi=0.0d00

      tstart=-ns*period
      dt=period/its
      n=its*(ns+nf)

      open(file='results_os.txt',unit=40,status='modify')
      open(file='wave_os.txt',unit=41,status='modify')

c     ljump=.false.

      t0=tstart
      ts=t0
      amplitude =0.0d00
      xamplitude=amplitude
      vdr0 = vd0*cos(psi0)
      vdi0 = vd0*sin(psi0)
      vdr1 = vd1*cos(psi1)
      vdi1 = vd1*sin(psi1)

      phase=psi0+0.5d00*pi
c  ************************************************************************
c  *    The phase is to be reference to a cosine function.                *
c  *    Add 90 degrees to the initial phase as we measure at a going       *
c  *    positive zero rather than at the peak field.                      *
c  ************************************************************************

      write(40,930)
930   format(10x,'time',3x,'amplitude',6x,'frequency',7x,'phase')

      write(41,931)
931   format(8x,'time',5x,'    field')

      hdt=dt*0.5d00
      dt6=dt/6.0d00

      E0 = 0.0d00
      F0 = 0.0d00

      nprint=0
      nwrite=10
```

```
      if(t0.gt.0.0d00)then
        ivalue0=0
        ivalue1=1
      else
        ivalue0=1
        ivalue1=0
      end if

      vdr=ivalue0*vdr0+ivalue1*vdr1
      vdi=ivalue0*vdi0+ivalue1*vdi1

      vdr_last=vdr
      vdi_last=vdi
      vdr_dot=(vdr-vdr_last)/period
      vdi_dot=(vdi-vdi_last)/period

      do 1 j=1,n

        if(t0.gt.0.0d00)then
          ivalue0=0
          ivalue1=1
        else
          ivalue0=1
          ivalue1=0
        end if

        ivalue2=0
        if((t0.gt.tjump-0.25*period).and.(t0.lt.tjump+0.25*period))then
          ivalue2=1
        end if

        Vdr=ivalue0*Vdr0 + ivalue1*Vdr1 + ivalue2*beamloadr
        Vdi=ivalue0*Vdi0 + ivalue1*Vdi1 + ivalue2*beamloadi
        wd=ivalue0*wd0 + ivalue1*wd1
        wc=ivalue0*wc0 + ivalue1*wc1
        bL=ivalue0*bL0 + ivalue1*bL1
        bE=ivalue0*bE0 + ivalue1*bE1

        driver=(vdr_dot+wd*Vdi)*cos(wd*t0)
        drivei=(vdi_dot-wd*Vdr)*sin(wd*t0)
        drive=driver+drivei

        t1 =t0+hdt
        E1 =E0+hdt*F0
        DF1=-bL*F0+2.0d00*bE*drive-wc**2*E0
        F1 =F0+hdt*DF1

        driver=(vdr_dot+wd*Vdi)*cos(wd*t1)
        drivei=(vdi_dot-wd*Vdr)*sin(wd*t1)
        drive=driver+drivei

        E2 =E0+hdt*F1
        DF2=-bL*F1+2.0d00*bE*drive-wc**2*E1
        F2 =F0+hdt*DF2

        E3 =E0+dt*F2
        DF3=-bL*F2+2.0d00*bE*drive-wc**2*E2
        F3 =F0+dt*DF3

        t2 = t0+dt

        driver=(vdr_dot+wd*Vdi)*cos(wd*t2)
        drivei=(vdi_dot-wd*Vdr)*sin(wd*t2)
        drive=driver+drivei

        E4 = E0 + dt6*(F0 + 2.0d00*(F1+F2) + F3)
```

```
        DF4=-bL*F3+2.0d00*bE*drive-wc**2*E3
        F4 = F0 + dt6*(DF1 + 2.0d00*(DF2+DF3) + DF4)

        if(abs(E4).gt.amplitude) amplitude = abs(E4)


        if((E0.lt.0).and.(E4.gt.0.0d00))then
          tx = (t0*E4-t2*E0)/(E4-E0)
          period = tx-ts
          frequency=1.0d-09/period
          phase=phase + wd0*period-2.0d00*pi
          phase_deg=phase*180.0d00/pi

          time = t2*1.0d09

          if(it.gt.nprint)then
            write(*,901) time, amplitude, frequency, phase_deg
901         format('time=',f11.3,' amplitude=',f8.4,
     +              '  frequency=', f13.9,'  phase=', e11.4)
            nprint=nprint+2000
          end if

          if(it.gt.nwrite)then
            write(40,902) time, amplitude, frequency, phase_deg
902         format(3x,f11.3,3x,f9.5,3x,f13.9,3x,e13.6)
            nwrite=nwrite+100
          end if

          ts=tx
          amplitude=0.0d00
          it=it+1
        end if

        if((t2.gt.-3.0d00*period).and.(t2.lt.3.0d00*period))then
          time = t2*1.0d09
          write(41,903) time, E4
903       format(3x,f11.3,3x,f11.5)
        end if

        t0=t2
        E0=E4
        F0=F4

c       Old code for introducing a voltage jump without a driving current.
c       ****************************************************************
c       if(.not.ljump)then
c         if(t0.gt.(tjump-hdt))then
c            ljump=.true.
c            E0=E0+0.0d00*Vjump
c         endif
c       endif

1     continue

      close(unit=40,status='keep')
      close(unit=41,status='keep')

      stop

      end
```

28. Appendix 2 Code for direct integration with controller

```
c      Driven oscillator with beam load and controller
c      **********************************************
c      Last modified 15th Oct 2006

c      This program solves the envelope equations for

c      C*dV/dt+V/Zext+(1/L)*Integral(V*dt)=2*Forw*cos(wd*t+psi)/Zext

c      which are formed by seting V=(Ar+j*Ai)*exp(-j*wd*t) and neglecting second
c      derivatives of Ar and Ai

c      Where the voltage V takes a step change at t = 0 and V0

c      Forw is the amplitude of the forward wave and is determined by a PI controller

c      For solution the differential equation is written in the form

c      d2V/dt2+(wc/QL)*dV/dt+wc**2*V=d/dt(2*Forw*wc/QE*cos(wd*t+psi))

c      The differential equation is solved by 4th or Runge Kutta
c      In the solution E~V, F~dV/dt, Vd0 = Forw for t<0, Vd1 = Forw for t>0

c      The solution starts at a time before t=0 so equilib is established

c23456789012345678901234567890123456789012345678901234567890123456789012

      real*8 V_set_point, vjump, Vmax
      real*8 vdr, Vdr_last, vdi, Vdi_last, vdr_dot, vdi_dot
      real*8 wc0, wc1, wc, wd0, wd1, wd, QC, QC0, QC1, QL0, QL1
      real*8 QE0, QE1, QE
      real*8 pi, period, t0, dt, t1, t2, tstart, hdt, dt6, time
      real*8 time_step
      real*8 fc0, fc1, fd0, fd1, dfc, dfd, fc, fd
      real*8 g10, g11, g20, g21, g30, g31, g40, g41, g50, g51
      real*8 f1, f10, f11, f2, f20, f21, f3, f30, f31, f4, f40, f41
      real*8 DAR1, DAR2, DAR3, DAR4, DAI1, DAI2, DAI3, DAI4
      real*8 AR0, AR1, AR2, AR3, AR4, AI0, AI1, AI2, AI3, AI4
      real*8 amplitude, shift, phase, phase_deg
      real*8 driver, drivei
      real*8 cpr, cir, cpi, cii, sumr, sumi, Rerror, Ierror, summax
      real*8 its, it, nprint, nwrite
      real*8 tjump

      integer n, ns, nf, j
      integer ivalue0, ivalue1

      logical ljump

      intrinsic abs, atan, cos, sin, sqrt

      open(file='data.txt',unit=42, status='modify')

      pi=4.0*atan(1.0d00)

      print*, 'Give voltage set point'
      read(*,*) V_set_point
      write(42,949) V_set_point
949   format('Amplitude set point            =', 1pe11.4)

      print*, 'Give voltage jump at t=0'
      read(*,*) vjump
```

```
      write(42,9501) vjump
9501  format('Voltage jump at t=0            =', 1pe11.4)

      fd0=3.90e09
      write(42,951) fd0
951   format('Inital drive frequency         =', 1pe11.4,' Hz')
c     initial drive frequency

      shift = 0.0d00
c     print*, 'Give drive frequency shift at t=0 in MHz'
c     read(*,*) Shift
      dfd=1.0d06*shift

      fd1=fd0+dfd
      write(42,952) fd1
952   format('Drive frequency for t>0        =', 1pe11.4,' Hz')
c     new drive frequency

      fc0 =3.9e09
      write(42,955) fd0
955   format('Inital cavity frequency        =', 1pe11.4,' Hz')
c     initial cavity frequency

      dfc=0.0d00
      fc1 = fc0 + dfc
      write(42,956) fd0
956   format('Cavity frequency for t>0       =', 1pe11.4,' Hz')
c     new cavity frequency

      wd0=2.0d00*pi*fd0
      wd1=2.0d00*pi*fd1
      wc0=2.0d00*pi*fc0
      wc1=2.0d00*pi*fc1

      write(*,*) 'Give Cavity Q factor'
      read(*,*) QC
      write(42,958) QC
958   format('Cavity Q factor                =', 1pe11.4)
      QC0=QC
      QC1=QC

      write(*,*) 'Give External Q factor'
      read(*,*) QE
      write(42,959) QE
959   format('External Q factor              =', 1pe11.4)


      QL0=1.0d00/(1.0d00/QE+1.0d00/QC0)
      QL1=1.0d00/(1.0d00/QE+1.0d00/QC1)
      QE0=QE
      QE1=QE

      Close(unit=42,status='keep')


      write(*,*) '     '

      ns=100000
      nf=200000
      its=0.1
      it=0

      period=2.0d00*pi/wd0

      tjump=0.0d00
```

```
      tstart=-ns*period
      dt=period/its
      n=its*(ns+nf)

      open(file='results_os.txt',unit=40,status='modify')
      open(file='wave_os.txt',unit=41,status='modify')

      ljump=.false.

      t0=tstart
      amplitude =0.0d00
      vdr = 0.0d00
      vdi = 0.0d00
      time_step = 1.0d00

      write(40,930)
930   format(10x,'time',3x,'amplitude',6x,'phase',
     +       9x,'Rcontrol',9x,'Icontrol')

      write(41,931)
931   format(8x,'time',5x,'    field')


      hdt=dt*0.5d00
      dt6=dt/6.0d00

      AR0 = 0.0d00
      AI0 = 0.0d00

      nprint=0
      nwrite=1

      cpr=5.0d00
      cir=5.0d-04
      cpi=5.0d00
      cii=5.0d-04
      sumr=0.0d00
      sumi=0.0d00
      summax=0.6d00*V_set_point/cir
      Vmax=0.55d00*V_set_point

      g10=(2.0d00*wd0/wc0)**2+(1.0d00/QL0)**2
      g11=(2.0d00*wd1/wc1)**2+(1.0d00/QL1)**2

      g20=((wd0/wc0)**2+1.0d00)*(1.0d00/QL0)
      g21=((wd1/wc1)**2+1.0d00)*(1.0d00/QL1)

      g30=((1.0d00/QL0)**2-(2.0d00*wd0/wc0)*(wc0/wd0-wd0/wc0))*(wd0/wc0)
      g31=((1.0d00/QL1)**2-(2.0d00*wd1/wc1)*(wc1/wd1-wd1/wc1))*(wd1/wc1)

      g40=2.0d00/(QE0*QL0)
      g41=2.0d00/(QE1*QL1)

      g50=4.0d00*(wd0/wc0)*(1.0d00/QE0)
      g51=4.0d00*(wd1/wc1)*(1.0d00/QE1)

      f10=g20/g10
      f11=g21/g11

      f20=g30/g10
      f21=g31/g11

      f30=g40/g10
      f31=g41/g11

      f40=g50/g10
```

```
      f41=g51/g10


      do 1 j=1,n

        if(t0.gt.0.0d00)then
          ivalue0=0
          ivalue1=1
        else
          ivalue0=1
          ivalue1=0
        end if

        wd=ivalue0*wd0 + ivalue1*wd1
        wc=ivalue0*wc0 + ivalue1*wc1
        fd=ivalue0*fd0 + ivalue1*fd1
        fc=ivalue0*fc0 + ivalue1*fc1
        f1=ivalue0*f10 + ivalue1*f11
        f2=ivalue0*f20 + ivalue1*f21
        f3=ivalue0*f30 + ivalue1*f31
        f4=ivalue0*f40 + ivalue1*f41

c       PI Control
c       ********************
        vdr_last=vdr
        vdi_last=vdi
        Rerror=V_set_point-AR0
        Ierror=-AI0
        sumr=sumr+fd*Rerror*dt
        sumi=sumi+fd*Ierror*dt
        if(sumr.gt.summax)  sumr=summax
        if(sumr.lt.-summax) sumr=-summax
        if(sumi.gt.summax)  sumi=summax
        if(sumi.lt.-summax) sumi=-summax

        Vdr=cpr*Rerror+cir*sumr
        Vdi=cpi*Ierror+cii*sumi

        if(Vdr.gt.Vmax) Vdr=Vmax
        if(Vdr.lt.-Vmax) Vdr=-Vmax
        if(Vdi.gt.Vmax) Vdi=Vmax
        if(Vdi.lt.-Vmax) Vdi=-Vmax

c       development code fixd stimulus
c       ******************************
c       vdr=10.0d00
c       vdi=0.0d00

        vdr_dot=(vdr-vdr_last)/time_step
        vdi_dot=(vdi-vdi_last)/time_step

        driver=(f3*(vdr_dot+wd*Vdi)-f4*(vdi_dot-wd*Vdr))/wc
        drivei=(f3*(vdi_dot-wd*Vdr)+f4*(vdr_dot+wd*Vdi))/wc

        t1  = t0+hdt
        DAR1=-f1*AR0-f2*AI0+driver
        AR1 = AR0+wc*hdt*DAR1
        DAI1=-f1*AI0+f2*AR0+drivei
        AI1 = AI0+wc*hdt*DAI1

        DAR2=-f1*AR1-f2*AI1+driver
        AR2 = AR0+wc*hdt*DAR2
        DAI2=-f1*AI1+f2*AR1+drivei
        AI2 = AI0+wc*hdt*DAI2

        DAR3=-f1*AR2-f2*AI2+driver
```

```
        AR3 = AR0+wc*dt*DAR3
        DAI3=-f1*AI2+f2*AR2+drivei
        AI3 = AI0+dt*DAI3

        t2 = t0+dt
        DAR4=-f1*AR3-f2*AI3+driver
        AR4 = AR0+wc*dt6*(DAR1+2.0d00*(DAR2+DAR3)+DAR4)
        DAI4=-f1*AI3+f2*AR3+drivei
        AI4 = AI0+wc*dt6*(DAI1+2.0d00*(DAI2+DAI3)+DAI4)

        amplitude = sqrt(AR4*AR4+AI4*AI4)
        phase = atan(AI4/AR4)
        phase_deg=phase*180.0d00/pi

        time = t2*1.0d09

        if(it.gt.nprint)then
          write(*,901) time, amplitude, phase_deg, vdr, vdi
901       format('time=',f11.3,' amp=',f10.3,
     +           ' pha=', e11.4,' Rcntr=', f7.3,' Icntr=', f7.3)
          nprint=nprint+400*its
        end if

        if(it.gt.nwrite)then
          write(40,902) time, amplitude, phase_deg, vdr, vdi
902       format(3x,f11.3,3x,f9.3,3x,e13.6,3x,f9.5,3x,f9.5)
          nwrite=nwrite+10*its
        end if

        it=it+its

        if((n.gt.0).and.(n.lt.5000))then
          time = t2*1.0d09
          write(41,903) time, AR0
903       format(3x,f11.3,3x,f11.5)
        end if

        time_step=t2-t0
        t0=t2
        AR0=AR4
        AI0=AI4

        if(.not.ljump)then
          if(t0.gt.tjump)then
             ljump=.true.
             AR0=AR0+Vjump
          endif
        endif

1     continue

      close(unit=40,status='keep')
      close(unit=41,status='keep')

      stop

      end
```

29. Appendix 3 Code to integrate envelope equations

```
c      Driven multi-mode cavity with beamloading, microphonics & delayed PI control.
c      *****************************************************************************
c      Program last modified 29th October 2007

c      29/10/2007 adjustment to mode beam couplings.
c      The 8pi/9 mode is +2MHz, The ratio of its QE to that of the pi mode is 1.667
c      hence the voltage coupling is root this value = 1.29. The relative beam to
c      cavity voltage coupling for the 8pi/9 mode is 0.032

c      The 7pi/9 mode is +9MHz, The ratio of its QE to that of the pi mode is 4.5
c      hence the voltage coupling is root this value = 2.12. The relative beam to
c      cavity voltage coupling for the 7pi/9 mode is 0.024

c      The 6pi/9 mode is +20.6MHz, The ratio of its QE to that of the pi mode is 0.67
c      hence the voltage coupling is root this value = 0.816. The relative beam to
c      cavity voltage coupling for the 6pi/9 mode is 0.0259

c      1/9/07  add measurement filter bandwidth as an input (previously determined from
c              the update interval.)

c      Program still requires a more realistic amplifier model (current modelled as a
c      band pass filter with gain).

c      Program now includes measurement model, amplifier model, multi-mode cavity
c      and beamloading of all modes (the bunch arrival can be an arbitrary phase
c      with respect to the peak mode voltage).

c      This program solves the envelope equations for
c      C*dV/dt+V/Zext+(1/L)*Integral(V*dt)=2*Forw*cos(wd*t+psi)/Zext

c      which are formed by seting V=(Ar+j*Ai)*exp(-j*wd*t) and neglecting second
c      derivatives of Ar and Ai

c      Forw is the amplitude of the forward wave and is determined by a PI controller

c      Two differential equations per mode are solved by 4th order Runge Kutta

c2345678901234567890123456789012345678901234567890123456789012345678901234567890012

      integer*4 jic, jm
      parameter(jic=100000)
      parameter(jm=3)
c      jm sets maximum number of modes
c      jic sets the maximum delay for the control system in cycles

      real*8 V_set_point(jm), vjump, drive_amp, drive_max, V_max_point
      real*8 energy_set_point
      real*8 Vkick, Max_power, drive_pow, drive_phase, bunch_phase, c
      real*8 Offset, bunch_phase_err, cos_err, sin_err, bunch_charge
      real*8 bunch_phase_jitter
      real*8 vdr, Vdr_last, vdi, Vdi_last, vdr_dot, vdi_dot
      real*8 wd, wc(jm), wc0(jm), QC(jm), QE(jm)
      real*8 wa, QA, ROQ, amp_bandwidth, meas_bandwidth
      real*8 pi, period, t0, dt, t2, tstart, hdt, dt6, time
      real*8 time_step, time_last, time_step1, time_step2
      real*8 fc(jm), fd
      real*8 f0(jm), fe(jm), g1(jm), g2(jm), g3(jm)
      real*8 ga1, ga2, ga3, ga4, ga5
      real*8 fa1, fa2, fa3, fa4
      real*8 AR(jm), AI(jm), AR_sum, AI_sum
      real*8 amp_AR0, amp_AR
      real*8 amp_AI0, amp_AI
```

```
      real*8 amp_AR_dot, amp_AI_dot
      real*8 outr(jm), outi(jm)
      real*8 AR0_meas, AI0_meas, rand_phase, rand_mag, mag_factor
      real*8 meas_phase_jitter, meas_amp_jitter, meas_phase_jitter_deg
      real*8 amplitude(jm), phase(jm), phase_deg(jm)
      real*8 driver(jic), drivei(jic)
      real*8 dr_ramp, di_ramp, dr_flat, di_flat
      real*8 cntr_delay, update_interval
      real*8 c_prop_Ar, c_intl_Ar, c_prop_Ai, c_intl_Ai
      real*8 sumr, sumi, Rerror, Ierror, summax
      real*8 cycle
      real*8 xrandom
      real*8 random
      real*8 aa
      real*8 offset_freq, w_offset
      real*8 pulse_length
      real*8 drive_freq, cavity_freq, cavity_vib_freq, cavity_freq_shift
      real*8 initial_vib_phase
      real*8 wshift, wcav, vph
      real*8 sum_sq_ph_err(jm), sum_sq_amp_err(jm), amp_err(jm), value
      real*8 gain, max_power_used
      real*8 Qmeas, AR0_filter, AI0_filter
      real*8 testr, testi
      real*8 mode_coupler_coupling(jm), mode_beam_coupling(jm)
      real*8 cycles_between_bunches, cycle_of_next_bunch
      real*8 cycle_offset_first_bunch

      integer*4 n_iterations, ncycle, icycle, j, js, nprint, nwrite
      integer*4 its_per_cycle, idelay, ic_delay, ic
      integer*4 nbeam_on, nbeam_off, ndetail
      integer*4 cycles_per_train
      integer*4 count, control_update, cnt_param, nsettle
      integer*4 normalise(jm)
      integer*4 n_vec_mod
      integer*4 nerror, modes

      character*80 anything

      logical lrandom

      intrinsic abs, atan, cos, sin, sqrt, acos, asin
      external random, RK, RKM

      pi=4.0*atan(1.0d00)

      c=2.998d08



c     Read input data from file
c     *************************
      open(file='indata.txt',unit=45,status='old')
      open(file='outdata.txt',unit=46,status='modify')

900   format(a)

      read(45,900) anything
      nerror=952
      read(anything(41:),*,err=3000) drive_freq
      write(*,952) drive_freq
      write(46,952) drive_freq
      fd=drive_freq*1.0d09
952   format('Drive frequency in GHz                  =',f11.3,' GHz')

      read(45,900) anything
      nerror=953
```

```
      read(anything(41:),*,err=3000) cavity_freq
      write(*,953) cavity_freq
      write(46,953) cavity_freq
      fc(1)=cavity_freq*1.0d09
953   format('Centre cavity frequency in GHz          =',f11.3,' GHz')

      modes=3
      write(46,9539) modes
9539  format('Number of cavity modes                  =',i2)

      fc(2)=fc(1)+2.0d+06
      fc(3)=fc(1)+9.0d+06

      mode_coupler_coupling(1)=1.0d00
      mode_coupler_coupling(2)=1.29d00
      mode_coupler_coupling(3)=2.12d00

      mode_beam_coupling(1)=1.0d00
      mode_beam_coupling(2)=0.032d00
      mode_beam_coupling(3)=0.024d00

      wd=2.0d00*pi*fd
      do 61 js=1,modes
        wc0(js)=2.0d00*pi*fc(js)
61    continue

      aa=0.5d00*c*pi/wd
c     reference radius to get voltage setpoint (aa*w/c)=pi/2

      read(45,900) anything
      nerror=954
      read(anything(41:),*,err=3000) QC(1)
      write(*,954) QC(1)
      write(46,954) QC(1)
954   format('Cavity Q factor                         =',1pe11.4)
      do 62 js=1,modes
       QC(js)=QC(1)
62    continue

      read(45,900) anything
      nerror=955
      read(anything(41:),*,err=3000) QE(1)
      write(*,955) QE(1)
      write(46,955) QE(1)
955   format('External Q factor                       =',1pe11.4)
      do 63 js=1,modes
        QE(js)=QE(1)/mode_coupler_coupling(js)**2
63    continue

      read(45,900) anything
      nerror=956
      read(anything(41:),*,err=3000) ROQ
      write(*,956) ROQ
      write(46,956) ROQ
956   format('Cavity R over Q   (2xFNAL=53 per cell)  =',f11.3,' ohms')

      read(45,900) anything
      nerror=957
      read(anything(41:),*,err=3000) energy_set_point
      value=energy_set_point*1000.0d00
      write(*,957) value
      write(46,957) value
957   format('Energy point  ILC crab~0.0284J per cell)=',f11.3,' mJ')

      do 52 js=1,modes
        v_set_point(js)=0.0d00
```

```
52    continue

      v_set_point(1)=(aa*wd/c)*sqrt(energy_set_point*wd*ROQ)
      value=v_set_point(1)/1000.0d00

      write(*,958) value
      write(46,958) value
958   format('Amplitude set point                     =',f11.3,' kV')

      read(45,900) anything
      nerror=959
      read(anything(41:),*,err=3000) Max_power
      write(*,959) Max_power
      write(46,959) Max_power
959   format('Maximum Amplifier Power per cell        =',f11.3,' W')

      drive_max= sqrt(2.0d00*QE(1)*ROQ*Max_power)
c****** drive_max is the maximum amplitude of the forward wave ********

      V_max_point=2.0d00*drive_max
      value=V_max_point/1000.0d00
      write(*,960) value
      write(46,960) value
960   format('Maximum voltage set point (no beam)     =',f11.3,' kV')

      read(45,900) anything
      nerror=961
      read(anything(41:),*,err=3000) offset
      write(*,961) offset
      write(46,961) offset
961   format('Maximum beam offset                     =',f11.3,' mm')

      read(45,900) anything
      nerror=962
      read(anything(41:),*,err=3000) bunch_phase_jitter
      write(*,962) bunch_phase_jitter
      write(46,962) bunch_phase_jitter
962   format('Maximum bunch phase jitter              =',f11.3,' deg')

      read(45,900) anything
      nerror=963
      read(anything(41:),*,err=3000) xrandom

      lrandom=.false.
      if(xrandom.gt.0.5d00) lrandom=.true.
      if(lrandom)then
        read(45,900) anything
c       this line contains a offset period which is not used
        offset_freq=0.0d00
        write(*,963)
        write(46,963)
963     format('Random offset')
      else
        read(45,900) anything
        nerror=964
        read(anything(41:),*,err=3000) offset_freq
        write(*,964) offset_freq
        write(46,964) offset_freq
964     format('Beam offset frequency                   =',
     +  f11.3,' Hz')
      end if
      w_offset=2.0d00*pi*offset_freq

      read(45,900) anything
      nerror=965
      read(anything(41:),*,err=3000)  bunch_charge
```

```
      value=bunch_charge*1.0d09
      write(*,965)  value
      write(46,965)  value
965   format('Bunch charge (ILC=3.2 nC)               =',f11.3,' nC')
      cos_err=cos(bunch_phase_jitter*pi/180.0d00)
      sin_err=sin(bunch_phase_jitter*pi/180.0d00)
      vjump=0.5d00*(aa*wd/c)*(offset*1.0d-03*wd/c)*wd*ROQ*bunch_charge
      write(*,966) vjump
966   format('Voltage jump at t=0                     =', 1pe11.4,' V')

      read(45,900) anything
      nerror=967
      read(anything(41:),*,err=3000) cycles_between_bunches
      write(*,967) cycles_between_bunches
      write(46,967) cycles_between_bunches
967   format('RF cycles between bunches               =', f11.3)

      read(45,900) anything
      nerror=969
      read(anything(41:),*,err=3000) pulse_length
      value=pulse_length*1000.0d00
      write(*,969) value
      write(46,969) value
969   format('Bunch train length                      =',f11.3,' ms')

      read(45,900) anything
      nerror=970
      read(anything(41:),*,err=3000) cavity_freq_shift
      write(*,970) cavity_freq_shift
      write(46,970) cavity_freq_shift
970   format('Cavity frequency shift from microphonics=',f11.3,' Hz')
      wshift=2.0d00*pi*cavity_freq_shift

      read(45,900) anything
      nerror=971
      read(anything(41:),*,err=3000) cavity_vib_freq
      write(*,971) cavity_vib_freq
      write(46,971) cavity_vib_freq
971   format('Cavity vibration frequency              =',f11.3,' Hz')
      wcav=2.0d00*pi*cavity_vib_freq

      read(45,900) anything
      nerror=972
      read(anything(41:),*,err=3000) initial_vib_phase
      write(*,972) initial_vib_phase
      write(46,972) initial_vib_phase
972   format('Initial vibration phase (degrees)       =',f11.3,' deg')
      vph=initial_vib_phase*pi/180.0d00

      read(45,900) anything
      nerror=973
      read(anything(41:),*,err=3000) meas_phase_jitter_deg
      write(*,973) meas_phase_jitter_deg
      write(46,973) meas_phase_jitter_deg
973   format('Phase measurement error(degrees)        =',f11.5,' deg')
      meas_phase_jitter=meas_phase_jitter_deg*2.0d00*pi/360.00d00

      read(45,900) anything
      nerror=974
      read(anything(41:),*,err=3000) meas_amp_jitter
      write(*,974) meas_amp_jitter
      write(46,974) meas_amp_jitter
974   format('Fractional err in amplitude measurement =',f11.5)

      read(45,900) anything
      nerror=968
```

```
      read(anything(41:),*,err=3000) cntr_delay
      idelay=cntr_delay*fd
      if(idelay.gt.jic) idelay=jic
      if(idelay.lt.1) idelay=1
      write(*,968) cntr_delay
      write(46,968) cntr_delay
968   format('Time delay (latency) for control system =',1pe11.4,' s')

      read(45,900) anything
      nerror=975
      read(anything(41:),*,err=3000) update_interval
      write(*,975) update_interval
      write(46,975) update_interval
975   format('Control update interval                 =',1pe11.4,' s')

      control_update=fd*update_interval

      read(45,900) anything
      nerror=9766
      read(anything(41:),*,err=3000) gain
      write(*,9766) gain
      write(46,9766) gain
9766  format('Gain constant for controller            =',f9.4)

      read(45,900) anything
      nerror=9767
      read(anything(41:),*,err=3000) amp_bandwidth
      write(*,9767) amp_bandwidth
      write(46,9767) amp_bandwidth
9767  format('Amplifier bandwidth                     =',1pe11.4)

      meas_bandwidth=1.0d00/update_interval
      read(45,900,end=101) anything
      nerror=9768
      read(anything(41:),*,err=3000) meas_bandwidth
      write(*,9768) meas_bandwidth
      write(46,9768) meas_bandwidth
9768  format('Measurement filter bandwidth            =',1pe11.4)

101   cycle_offset_first_bunch=0.0d00
c     first bunch can arrive part way through an RF cycle

      n_vec_mod=20
c     number of RF cyles over which the vector modulator ramps to a new value

      QA=fd/amp_bandwidth
c     Amplifier Q factor


c     PI Controller coefficients
c     **************************
      cnt_param=3
      if(cnt_param.eq.1)then
c       OK from QE=6e6 to high QE=1.75e7 with 3900 delay
        c_prop_ar= 250.00d-8*QE(1)
        c_intl_ar=   0.01d-8*QE(1)*0.2
        c_prop_ai= 250.00d-8*QE(1)
        c_intl_ai=   0.01d-8*QE(1)*0.2
      else if(cnt_param.eq.2)then
        c_prop_Ar= 500.00d-8*QE(1)*gain
        c_intl_Ar=   0.02d-8*QE(1)*gain
        c_prop_Ai=2000.00d-8*QE(1)*gain
        c_intl_Ai=   0.16d-8*QE(1)*gain
      else
c       increased values for Ai
        c_prop_Ar=2000.00d-8*QE(1)*gain
```

```
        c_intl_Ar=   0.06d-8*QE(1)*gain
        c_prop_Ai=2000.00d-8*QE(1)*gain
        c_intl_Ai=   0.06d-8*QE(1)*gain
      endif

      write(46,976) c_prop_ar
976   format('Proportional coef for real component    =',1pe11.4)
      write(46,977) c_intl_ar
977   format('Integral coef for real component        =',1pe11.4)
      write(46,978) c_prop_ai
978   format('Proportional coef for imag component    =',1pe11.4)
      write(46,979) c_intl_ai
979   format('Integral coef for imag component        =',1pe11.4)

      close(unit=45,status='keep')


      write(*,*) '    '

      icycle=0

      cycles_per_train=1.0e-3*fd
c     number of cycle in a bunch train = 1ms * frequency

      if(QE(1).lt.3.0d06)then
        nbeam_on=2.0d00*QE(1)
      else
        nbeam_on=1.5d00*QE(1)
      end if
      if(nbeam_on.gt.10000000)then
        nbeam_on=10000000
c       limit size for plotting in Excel
      end if
      nbeam_on=4500000
      nbeam_off=nbeam_on+cycles_per_train
      nsettle=0.05d00*cycles_per_train
      ncycle=nbeam_off+0.5d00*cycles_per_train
      ndetail=0.5d00*(nbeam_on+nbeam_off)
c     cycle when next bunch is due

      its_per_cycle=1
c     time iterations per cycle

      period=2.0d00*pi/wd

      tstart=0.0d00
      dt=period/its_per_cycle
      n_iterations=its_per_cycle*ncycle

      open(file='results_os.txt',unit=40,status='modify')
      open(file='wave_os.txt',unit=41,status='modify')

      t0=tstart
      amplitude(1) =0.0d00
      vdr = 0.0d00
      vdi = 0.0d00
      time_step = 1.0d00

      write(40,930)
930   format(9x,'cycle',10x,'time',5x,'amplitude',6x,'phase',
     +       11x,'Rcontrol',6x,'Icontrol',6x,'DrvPower',6x,'DrvPhase')

      write(41,931)
931   format(8x,'time',5x,'    field')
```

```
      hdt=dt*0.5d00
      dt6=dt/6.0d00

      Qmeas=dt*meas_bandwidth
c     filter parameter for measurement model

      do 31 js=1,modes
       AR(js) = 0.0d00
       AI(js) = 0.0d00
31    continue
      amp_AR = 0.0d00
      amp_AI = 0.0d00
      AR0_filter=0.0d00
      AI0_filter=0.0d00

      nprint=0
      nwrite=1

      sumr=0.0d00
      sumi=0.0d00
      summax=drive_max/c_intl_Ar

      cycle=0.0d00
c     when cycle gets to one the number of cycles=icycle increments

      do 10 ic=1,idelay
        driver(ic)=0.0d00
        drivei(ic)=0.0d00
10    continue

      cycle_of_next_bunch=nbeam_on + cycle_offset_first_bunch
      max_power_used=0.0d00


      do 70 js=1,modes
        sum_sq_ph_err(js)=0.0d00
        sum_sq_amp_err(js)=0.0d00
        normalise(js)=0
70    continue



      ic=2
      ic_delay=1
      count=control_update

c     evaluate amplifier coefficients
c     ******************************
      wa=wd
      ga1=(2.0d00*wd/wa)**2+(1.0d00/QA)**2
      ga2=((wd/wa)**2+1.0d00)*(1.0d00/QA)
      ga3=((1.0d00/QA)**2-(2.0d00*wd/wa)*(wa/wd-wd/wa))*(wd/wa)
      ga4=1.0d00/(QA*QA)
      ga5=2.0d00*(wd/wa)*(1.0d00/QA)

c********** the coefficients of ga4 and ga5 need checking              ***********
c********** and also the QA of the amplifier with respect to QE and QL  ***********

      fa1=wa*ga2/ga1
      fa2=wa*ga3/ga1
      fa3=ga4/ga1
      fa4=ga5/ga1

      do 1 j=1,n_iterations

        t2=t0+dt
```

```
c       set instantaneous cavity frequency and evaluate cavity coefficients
c       ******************************************************************
        do 30 js=1,modes
          wc(js)=wc0(js)+wshift*sin(wcav*t0+vph)

          f0(js)=wc(js)/(4.0d00*QC(js))
          fe(js)=wc(js)/(4.0d00*QE(js))

          g1(js)=(1.0d00+(wc(js)/wd)**2)
          g2(js)=(wc(js)**2-wd**2)/(2.0d00*wd)
          g3(js)=wc(js)/(QE(js)*wd)
30     continue


c       Measurement Model
c       *****************
c       Note that the actual measurement model deteriotes at the flash ADC
c       combination points. Still need to put this in.

c       Low Pass Filter to average cavity fields over period between DSP
c       re-calculating the control action
c       ***********************************************************************
c       There will be some sort of low pass filter between the cavity probe and the
c       device that measures amplitude and phase. If the device that measures
c       amplitude and phase delivers its output by means of an ADC to a DSP then it
c       is sensible to chose the low pass cut-off equal to the sample rate. The next
c       two lines implement a first order low pass filter. We also suppose that the
c       filter can select the fundamental mode and discrimate the other modes.

        AR_sum=0.0d00
        AI_sum=0.0d00
        do 73 js=1,modes
          AR_sum=AR_sum+AR(js)*mode_coupler_coupling(js)
          AI_sum=AI_sum+AI(js)*mode_coupler_coupling(js)
73      continue

        AR0_filter=AR0_filter*(1.0d00-Qmeas)+AR_sum*Qmeas
        AI0_filter=AI0_filter*(1.0d00-Qmeas)+AI_sum*Qmeas


        if(count.ge.control_update)then
c         measurement errors depend on bandwidth. The measurement errors that should
c         be inserted are those appropriate to the control update frequency.

          count=1
          rand_phase=meas_phase_jitter*(2.0d00*random()-1.0d00)
          rand_mag=1.0d00 + meas_amp_jitter*(2.0d00*random()-1.0d00)
          mag_factor=rand_mag/sqrt(1.0d00+rand_phase**2)
          AR0_meas=mag_factor*(AR0_filter+rand_phase*AI0_filter)
          AI0_meas=mag_factor*(AI0_filter-rand_phase*AR0_filter)

c         PI Control
c         **********
          vdr_last=vdr
          vdi_last=vdi
          time_last=t0
          Rerror=V_set_point(1)-AR0_meas
          Ierror=-AI0_meas
          sumr=sumr+fd*Rerror*dt*control_update
          sumi=sumi+fd*Ierror*dt*control_update
          if(sumr.gt.summax)  sumr=summax
          if(sumr.lt.-summax) sumr=-summax
          if(sumi.gt.summax)  sumi=summax
          if(sumi.lt.-summax) sumi=-summax
```

```
          Vdr=c_prop_Ar*Rerror+c_intl_Ar*sumr
          Vdi=c_prop_Ai*Ierror+c_intl_Ai*sumi

c         limit drive output
c         ******************
          drive_amp=sqrt(Vdr**2+Vdi**2)
          if(drive_amp.gt.drive_max)then
            Vdr=vdr*drive_max/drive_amp
            Vdi=vdi*drive_max/drive_amp
          end if

c         evaluate driver terms in differential equations
c         ***********************************************
          time_step1=n_vec_mod*dt
          time_step2=t0-time_last
          if(time_step1.lt.time_step2)then
            time_step=time_step1
          else
            time_step=time_step2
          end if
          if(time_step.le.1.0e-12) time_step=1.0d00
          vdr_dot=(vdr-vdr_last)/time_step
          vdi_dot=(vdi-vdi_last)/time_step

c         evaluate driver while change is being made and after completion
c         ****************************************************************
c         everytime the DSP re-calculates the drive, the vector modulator
c         ramps the drive to the new value.
          dr_ramp=(fa3*(vdr_dot+wd*Vdi)-fa4*(vdi_dot-wd*Vdr))
          di_ramp=(fa3*(vdi_dot-wd*Vdr)+fa4*(vdr_dot+wd*Vdi))
          dr_flat=( fa3*wd*Vdi+fa4*wd*Vdr)
          di_flat=(-fa3*wd*Vdr+fa4*wd*Vdi)
        end if


c       evaluate drive - this stays the same if the controller has not recalculated
c       ***************************************************************************
        if(count.le.20)then
          driver(ic_delay)=dr_ramp
          drivei(ic_delay)=di_ramp
        else
          driver(ic_delay)=dr_flat
          drivei(ic_delay)=di_flat
        endif
c       *********************************************************
c       *  The drive action is delayed by the processing time  *
c       *********************************************************

        count=count+1

c       amplifier model
c       ***************
c       The amplifier is modelled as another cavity hence one needs to solve a
c       second set of envelope functions.


c       4th order Runge Kutta integration for amplifier
c       ***********************************************
        amp_AR0=amp_AR
        amp_AI0=amp_AI

        call RK(t0,hdt,dt6,amp_AR,amp_AI,fa1,fa2,driver(ic),drivei(ic))

        testr=amp_AR-vdr
        testi=amp_AI-vdi
```

```
        amp_AR_dot=(amp_AR-amp_AR0)/dt
        amp_AI_dot=(amp_AI-amp_AI0)/dt

        do 32 js=1,modes
          outr(js)=-g3(js)*(amp_AI_dot-wd*amp_AR)
          outi(js)= g3(js)*(amp_AR_dot+wd*amp_AI)
32      continue


c       4th order Runge Kutta integration for cavity
c       ********************************************
        call RKM(t0,hdt,dt6,AR,AI,f0,fe,g1,g2,outr,outi,modes)

        do 51 js=1,modes
          amplitude(js) = sqrt(AR(js)*AR(js)+AI(js)*AI(js))
          if(abs(AR(js)).gt.1.0d-9)then
            phase(js) = atan(AI(js)/AR(js))
          else
            phase(js)=0.5d00*pi
          end if
          phase_deg(js)=phase(js)*180.0d00/pi
          if((j.gt.nbeam_on+nsettle).and.(j.lt.nbeam_off))then
            sum_sq_ph_err(js)=sum_sq_ph_err(js)+phase_deg(js)**2
            amp_err(js)=amplitude(js)-v_set_point(js)
            sum_sq_amp_err(js)=sum_sq_amp_err(js)+amp_err(js)**2
            normalise(js)=normalise(js)+1
          endif
51      continue

        time = t2*1.0d09

        ic=ic+1
        ic_delay=ic_delay+1
        if(ic_delay.eq.idelay+1) ic_delay=1
        if(ic.eq.idelay+1) ic=1

c       output data to files and screen
c       *******************************
        if(j.gt.nprint)then
          write(*,901) time, amplitude(1), phase_deg(1), vdr, vdi
901       format('t=',f11.3,' amp=',f11.1,
     +           ' ph=', e11.4,' Rcntr=', f10.1,' Icntr=', f9.1)
          nprint=nprint+100000*its_per_cycle
        end if

        if(j.gt.nwrite)then
          drive_amp=sqrt(Vdr**2+Vdi**2)
          if(drive_amp.gt.0.0d00)then
             drive_phase=180.0d00*asin(Vdi/drive_amp)/pi
          else
            drive_phase=0.0d00
          end if
          drive_pow=0.5d00*drive_amp**2/(QE(1)*ROQ)
          write(40,902) icycle, time, amplitude(1), phase_deg(1),
     +    vdr, vdi,drive_pow,drive_phase
902       format(2x,i12,3x,1pe11.4,3x,1pe11.4,3x,e13.6,3x,1pe11.4,
     +          3x,1pe11.4,3x,1pe11.4,3x,1pe11.4)

          if(icycle.gt.nbeam_on)then
           if(drive_pow.gt.max_power_used)then
             max_power_used=drive_pow
           end if
          end if

          if((icycle.gt.ndetail).and.(icycle.lt.ndetail+20000))then
            nwrite=nwrite+20*its_per_cycle
```

```
          elseif(icycle.lt.nbeam_on)then
            nwrite=nwrite+2000*its_per_cycle
          elseif((icycle.gt.nbeam_on).and.
     +           (icycle.lt.(nbeam_on+400000)))then
            nwrite=nwrite+100*its_per_cycle
          else
            nwrite=nwrite+2000*its_per_cycle
          end if
        end if

        if((j.gt.0).and.(j.lt.5000))then
          time = t2*1.0d09
          write(41,903) time, AR(1), testr, testi
903       format(3x,f11.3,3x,f11.5,3x,1pe11.5,3x,1pe11.5)
        end if

c       Provision for more than one time step per cycle
c       ***********************************************
        cycle=cycle+1.0d00/its_per_cycle
        if(cycle.gt.0.9999d00)then
          icycle=icycle+cycle*1.00001d00
          cycle=0.0d00
        end if

c       Beam loading of modes
c       *********************
        if(icycle.gt.cycle_of_next_bunch)then
         if((icycle.gt.nbeam_on).and.(icycle.le.nbeam_off))then
           bunch_phase_err=bunch_phase_jitter*pi/180.0d00
           Vkick=Vjump*cos(w_offset*t2)
           if(lrandom)then
             bunch_phase_err=bunch_phase_err*random( )
             Vkick=Vjump*random( )
           end if

c          add in systematic phase advance (zero if bunches synchronised to drive)
c          ***********************************************************************
           value=cycle_of_next_bunch-icycle
           bunch_phase=bunch_phase_err+2.0d00*value*pi
           cos_err=cos(bunch_phase)
           sin_err=sin(bunch_phase)

           do 152 js=1, modes
            AR(js)=AR(js)-Vkick*mode_beam_coupling(js)*cos_err
            AI(js)=AI(js)-Vkick*mode_beam_coupling(js)*sin_err
152        continue

           cycle_of_next_bunch = cycle_of_next_bunch
     +                               + cycles_between_bunches
          endif
        endif

        t0=t2
1     continue


      write(*,9977) max_power_used
      write(46,9977) max_power_used
9977  format('maximum power delivered                 =', f9.2)

      value=sqrt(sum_sq_ph_err(1)/normalise(1))
      write(*,9978) value
      write(46,9978) value
9978  format('In pulse rms phase err                  =', f10.5,
     +       ' degrees')
```

```
      if(v_set_point(1).gt.0.0d00)then
      value=
     +  100.0d00*sqrt(sum_sq_amp_err(1)/normalise(1))/v_set_point(1)
      write(*,9979) value
      write(46,9979) value
9979  format('In pulse rms amplitude err              =', f10.5,' %')
      value=
     +  100.0d00*sqrt(sum_sq_amp_err(2)/normalise(2))/v_set_point(1)
      write(*,9980) value
      write(46,9980) value
9980  format('Relative excitation of 2nd mode         =', f10.5,' %')

      value=
     +  100.0d00*sqrt(sum_sq_amp_err(3)/normalise(3))/v_set_point(1)
      write(*,9981) value
      write(46,9981) value
9981  format('Relative excitation of 3rd mode         =', f10.5,' %')

      else

      value=sqrt(sum_sq_amp_err(1)/normalise(1))
      write(*,9939) value
      write(46,9939) value
9939  format('In pulse rms amplitude err              =', f10.5,' V')

      value=sqrt(sum_sq_amp_err(2)/normalise(2))
      write(*,9930) value
      write(46,9930) value
9930  format('Relative excitation of 2nd mode         =', f10.5,' V')

      value=sqrt(sum_sq_amp_err(3)/normalise(3))
      write(*,9931) value
      write(46,9931) value
9931  format('Relative excitation of 3rd mode         =', f10.5,' V')
      end if

      close(unit=40,status='keep')
      close(unit=41,status='keep')
      close(unit=46,status='keep')

1000  stop

3000  print*,'error ',nerror,' reading value'
      go to 1001

1001  close(unit=45,status='keep')
      close(unit=46,status='keep')
      go to 1000
      end

      subroutine RK(t0,hdt,dt6,AR0,AI0,f1,f2,outr,outi)
c     *************************************************
      real*8 AR0, AR1, AR2, AR3, AR4, AI0, AI1, AI2, AI3, AI4
      real*8 DAR1, DAR2, DAR3, DAR4, DAI1, DAI2, DAI3, DAI4
      real*8 f1, f2, t0, t1, t2, dt6, hdt, outr, outi

      t1  = t0+hdt
      DAR1=-f1*AR0-f2*AI0+outr
      AR1 = AR0+hdt*DAR1
      DAI1=-f1*AI0+f2*AR0+outi
      AI1 = AI0+hdt*DAI1

      DAR2=-f1*AR1-f2*AI1+outr
      AR2 = AR0+hdt*DAR2
      DAI2=-f1*AI1+f2*AR1+outi
```

```
      AI2 = AI0+hdt*DAI2

      DAR3=-f1*AR2-f2*AI2+outr
      AR3 = AR0+2.0d00*hdt*DAR3
      DAI3=-f1*AI2+f2*AR2+outi
      AI3 = AI0+2.0d00*hdt*DAI3

      t2 = t1+hdt
      DAR4=-f1*AR3-f2*AI3+outr
      AR4 = AR0+dt6*(DAR1+2.0d00*(DAR2+DAR3)+DAR4)
      DAI4=-f1*AI3+f2*AR3+outi
      AI4 = AI0+dt6*(DAI1+2.0d00*(DAI2+DAI3)+DAI4)

      AR0=AR4
      AI0=AI4

      return
      end


      subroutine RKM(t0,hdt,dt6,AR0,AI0,f0,fe,g1,g2,outr,outi,modes)
c     **************************************************************
      integer*4 jm
      parameter(jm=3)

      real*8 AR0(jm), AR1(jm), AR2(jm), AR3(jm), AR4(jm)
      reAL*8 AI0(jm), AI1(jm), AI2(jm), AI3(jm), AI4(jm)
      real*8 DAR1(jm), DAR2(jm), DAR3(jm), DAR4(jm)
      real*8 DAI1(jm), DAI2(jm), DAI3(jm), DAI4(jm)
      real*8 f0(jm), fe(jm), g1(jm), g2(jm)
      real*8 outr(jm), outi(jm)
      real*8 t0, t1, t2, dt6, hdt, sum

      integer*4 j1, j2, modes

      t1      = t0+hdt

      do 11 j1=1, modes
         sum=0.0d00
         do 12 j2=1, modes
            sum=sum+g1(j2)*AR0(j2)
12       continue
         sum=f0(j1)*g1(j1)*AR0(j1)+fe(j1)*sum
         DAR1(j1)= - sum + g2(j1)*AI0(j1) + outr(j1)
         AR1(j1) =   AR0(j1) + hdt*DAR1(j1)

         sum=0.0d00
         do 13 j2=1, modes
            sum=sum+g1(j2)*AI0(j2)
13       continue
         sum=f0(j1)*g1(j1)*AI0(j1)+fe(j1)*sum
         DAI1(j1)= - sum - g2(j1)*AR0(j1) + outi(j1)
         AI1(j1) =   AI0(j1)+hdt*DAI1(j1)
11    continue

      do 21 j1=1, modes
         sum=0.0d00
         do 22 j2=1, modes
            sum=sum+g1(j2)*AR1(j2)
22       continue
         sum=f0(j1)*g1(j1)*AR1(j1)+fe(j1)*sum
         DAR2(j1)= - sum + g2(j1)*AI1(j1) + outr(j1)
         AR2(j1) =   AR0(j1) + hdt*DAR2(j1)

         sum=0.0d00
         do 23 j2=1, modes
```

```
            sum=sum+g1(j2)*AI1(j2)
23        continue
          sum=f0(j1)*g1(j1)*AI1(j1)+fe(j1)*sum
          DAI2(j1)= - sum - g2(j1)*AR1(j1) + outi(j1)
          AI2(j1) =   AI0(j1)+hdt*DAI2(j1)
21      continue

        do 31 j1=1, modes
          sum=0.0d00
          do 32 j2=1, modes
            sum=sum+g1(j2)*AR2(j2)
32        continue
          sum=f0(j1)*g1(j1)*AR2(j1)+fe(j1)*sum
          DAR3(j1)= - sum + g2(j1)*AI2(j1) + outr(j1)
          AR3(j1) =   AR0(j1) + 2.0d00*hdt*DAR3(j1)

          sum=0.0d00
          do 33 j2=1, modes
            sum=sum+g1(j2)*AI2(j2)
33        continue
          sum=f0(j1)*g1(j1)*AI2(j1)+fe(j1)*sum
          DAI3(j1)= - sum - g2(j1)*AR2(j1) + outi(j1)
          AI3(j1) =   AI0(j1) + 2.0d00*hdt*DAI3(j1)
31      continue

        t2 = t1+hdt

        do 41 j1=1, modes
          sum=0.0d00
          do 42 j2=1, modes
            sum=sum+g1(j2)*AR3(j2)
42        continue
          sum=f0(j1)*g1(j1)*AR3(j1)+fe(j1)*sum
          DAR4(j1)= - sum + g2(j1)*AI3(j1) + outr(j1)
          AR4(j1) = AR0(j1) +
     +             dt6*(DAR1(j1)+2.0d00*(DAR2(j1)+DAR3(j1))+DAR4(j1))

          sum=0.0d00
          do 43 j2=1, modes
            sum=sum+g1(j2)*AI3(j2)
43        continue
          sum=f0(j1)*g1(j1)*AI3(j1)+fe(j1)*sum
          DAI4(j1)= - sum - g2(j1)*AR3(j1) + outi(j1)
          AI4(j1) = AI0(j1) +
     +             dt6*(DAI1(j1)+2.0d00*(DAI2(j1)+DAI3(j1))+DAI4(j1))
41      continue

        do 5 j1=1, modes
         AR0(j1)=AR4(j1)
         AI0(j1)=AI4(j1)
5       continue

        return
        end
```