Response to Guerino Mazzola

Alan Marsden

Lancaster Institute for the Contemporary Arts, Lancaster University, UK

A.Marsden@lancaster.ac.uk

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This response to Guerino Mazzola defends the use of imprecise concepts in some circumstances, particularly in the light of the impossibility of a precise definition of the domain of music. A possible contribution of mathematics to music through the demonstration of relationships between formulations of different theories is envisaged, specifically those of species counterpoint and functional harmony.

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Guerino Mazzola is rightly admired for the richness of his mathematical formulations of musical concepts, in particular for the way they allow the basis of one concept in another to be demonstrated, and for extensions of musical concepts to be envisaged. Here is one of the powers of the mathematical approach to music: because mathematics looks towards universals, it allows us to talk meaningfully about possible musics different from those we know. I would like to take a different tack, though, and ask two questions about such mathematical formulations: how do we know if a formulation is adequate, and are precise mathematical formulations necessarily better than the imprecisely formulated—and probably imprecise—concepts customarily used by musicologists and music analysts?

Precise and imprecise concepts

I am fond of emphasising to students the distinction between precision and accuracy. A watch with a second hand can give us a more precise measurement of time than a clock with no second hand, but it is less accurate if the watch is five minutes slow. A mathematical formulation can similarly be very precise (e.g., we can know for all chords which Riemannian function—tonic, dominant or subdominant—they belong to) but its accuracy depends on how well it describes the phenomenon in question. The problem here is that music, as pointed out by Geraint Wiggins also, is not a fixed phenomenon: we do not know where exactly to draw its boundaries, and more music is always being created.

We could try to avoid this by formulating descriptions of a circumscribed subset of music, such as Bach chorales. In such cases, however, the scores (or whatever sources are used to define the subset) are themselves a kind of theory because they too constitute a description of the music. A system to harmonise melodies in the style of Bach chorales could, given a melody, simply find that melody in a database of Bach's entire output, and reproduce one of the harmonisations found there. What we generally seek, instead, is a theory which is distinctly 'smaller' than the phenomenon it aims to describe. Obviously, when the phenomenon is infinite we can only seek to understand it through a theory which is smaller. So, for example, we understand the relations between the sides of right-angle triangles—an infinite set—through Pythagoras' theorem rather than a list of proportions such as 3:4:5, 5:12:13, etc. Even when the phenomenon is finite, we do not regard it as being explained by a list of its cases; we seek to generalise. What, though, is the use of a theory which is larger, which takes longer to state or to learn, than the set of instances it seeks to explain? This is why we generally seek smaller theories.

Ebcioğlu's CHORAL system for harmonising Bach chorale melodies [1] contained more than 300 rules, a number strikingly close to the 371 chorales in the collection by Albert Riemenschneider which is the most commonly used source. The rules are probably each 'smaller' than an entire chorale harmonisation, so Ebcioğlu's theory is smaller than the phenomenon it describes, but the difference in size is far less than that between, say, Newtonian mechanics and the universe of physical bodies in motion. The CHORAL system does something which a mere database of Bach's chorales cannot, of course: it is capable of harmonising melodies which Bach did not harmonise himself. How are we to judge whether the harmonisations it produces are in the style of Bach? There is no way to do this without reference either to human judgement, which is imprecise, or to something which is itself effectively a theory of Bach-style chorale harmonisation, which is circular.

To illustrate further the role of imprecise concepts in musicology, let me return once again to the topic of motivic analysis, and in particular to its manifestation in the kind of paradigmatic analysis exemplified by Nattiez's analysis of Varèse *Density 21.5* [2]. This is an avowedly systematic approach to analysis, but also one which is explicitly not mathematical. Nattiez sometimes gives a detailed discussion of the basis for aligning notes in 'paradigms' (roughly equivalent to motives), as for example in his discussion of whether his second paradigm depends on the rhythmic pattern long-short-long (in fact 'long-short-not shorter' would have been a better formulation) or is based on the pitch sequence C sharp to G. In general, though, the discussion is less detailed, and the reader is left either to simply 'read' the similarity from the paradigmatic diagrams or to infer the basis for the paradigms. Nattiez is clear that no single set of criteria (rhythmic pattern, intervallic pattern, etc.) defines a paradigm, and it also appears from the diagrams that the kind of criteria used in one paradigm would not succeed in defining another. The paradigms are *constructed* in the course of analysis on the basis of the analyst's *judgement*. This is a creative, and hence inherently unpredictable, enterprise. It might well be that Nattiez's imprecise concept of 'paradigm' is more accurate as a description of analysts' thinking, and even perhaps of listeners' listening, than a precise mathematical formulation. It allows the concept to adapt to the circumstances in analysing or hearing a piece rather than breaking down at some unpredictable point as, I suspect, a precise mathematical formulation is bound to do. Imprecise concepts, in my view, will always have an important role in our understanding of music.

Mathematics and explanation

The precision of mathematical formulations, on the other hand, has some distinct benefits. A mathematical formulation of species counterpoint, for example, can make the rules clear. (This is not the same as making them easier to understand. Musicians, unfortunately, are rarely equipped to understand a mathematical formulation. Sets of examples, which was essentially Fux's method, are generally much easier.) It also allows for computational implementation, and so for educational tools to correct student counterpoint exercises. (Note that this does not, however, lead necessarily to a system for *composing* species counterpoint. It is clear that there is more to composition than random selection from solutions to rules. Mazzola himself refers to a required 'semiotic culture'.) More importantly, it can demonstrate the derivation of the rules of species counterpoint from other principles. (Other scholars have attempted a derivation on psychological principles [3, 4].) We understand something better if we can see how it relates to other things we know.

In this regard, I look forward to a valuable contribution to music theory from mathematics through an explanation of how functional harmony evolved from species counterpoint (a possibility Mazzola himself envisages [5, pp. 636–37]). In the course of music history when this process took place (roughly in the seventeenth and eighteenth centuries), many human factors were involved, of course, but because mathematical music theory deals in possible musics, it is much better placed to explain the formal musical factors than other music theories. Schenker's claim that the free composition of composers up to Brahms derives from the principles of species counterpoint is shrouded in vaguely formed but rather polemical principles, and fails to convince. Brown [6] is more convincing, especially since he sketches how a modification of the rules of species counterpoint can lead to the rules of functional harmony and Schenkerian structure, following in part from the introduction of the concept of abstract triadic harmony distinct from consonance between concrete pitches, and from the introduction of virtual voices distinct from sequences of notes in a single part. Nevertheless, there remain many gaps in Brown's formulation.

A valuable contribution from mathematical music theory would be a demonstration that a mathematical formulation of the rules of species counterpoint becomes equivalent to the rules of functional harmony through specific modifications such as the addition of new principles and/or the modification of existing ones. That triads emerge from species counterpoint in three or more parts is clear: among all the possible three-note chords in the standard pitch universe, only major and minor triads contain no intervals classed as dissonances in species counterpoint. If a triad is defined as a collection of pitch classes, there are many fewer triads than three-note chords allowed by the rules of species counterpoint, and so to define possible contrapuntal/harmonic configurations in terms of triads rather than in terms of intervals allows a more parsimonious theory (making things simpler for composers and music teachers). Once triads have become the governing concept rather than intervals, we can envisage relaxations of the rules to allow skipping from one note of a triad to another without constraint, and so to the phenomenon of virtual voices. Furthermore, since triads can be formulated by a rule based on taking alternate notes from scales as well as by combinations of consonant intervals, we can envisage the emergence of freer treatment of diminished fifths and augmented fourths, since diminished triads containing these intervals follow from this scale-based principle.

The contribution of mathematics here would be to demonstrate that an adaptation of the rules of species counterpoint is equivalent to the rules of functional harmony in the sense that the sets of allowed pieces of music in each case are equal. (The adaptation of the rules of species counterpoint would ideally be distinct from the simple product of the rules of species counterpoint and those of functional harmony; otherwise the equivalence result might follow trivially.) This would allow the clear demonstration of the relation between species counterpoint and functional harmony in terms of musical *concepts* rather than musical practices.

References

[1] K. Ebcioğlu, *An expert system for harmonizing four-part chorales,* Computer Music Journal, 12 (1988), pp. 43–51.

[2] J.-J. Nattiez, *Varese's 'Density 21.5': a study in semiological analysis*, Music Analysis, 1 (1982), pp. 243–340.

[3] D. Huron, *Tone and voice: a derivation of the rules of voice-leading from perceptual principles*, Music Perception, 19 (2001), pp. 1–64.

[4] R. Parncutt, *Harmony: A psychoacoustical approach*, Springer Verlag, New York, 1989.

[5] G. Mazzola, *The Topos of Music: Geometric Logic of Concepts, Theory and Performance*, Birkhäuser Verlag, Basel, 2002.

[6] M. Brown, *Explaining Tonality: Schenkerian Theory and Beyond*, University of Rochester Press, 2005.