# Final response: Ontology, epistemology, and some research proposals

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Apparently contrasting ontologies of music as a psychological-cognitive entity or as having independent existence are shown not to be mutually exclusive. They are related to orthogonal dimensions of empirical and rational approaches in research. The best research employs both approaches. Mathematical and computational researchers are urged to explore research which moves away from the abstract, rational approach and makes greater use of empirical data. Suggestions are given for future research.

Keywords: musical ontology, empiricism, rationality, theory

Two themes have emerged from this exchange of views. In my final contribution to this issue, I aim to show that they are related, and that by using them to map the terrain of mathematical-computational-musical research we can see more clearly the sources of possible misunderstanding and the opportunities for productive research.

### Ontology of Music

A complete and universally agreed definition of music is ultimately unachievable, as Geraint Wiggins has powerfully argued in his response. However, this does not mean that we do not find useful information in at least the early stages of such a search. The article on 'Music' in the 1911 edition of the *Encyclopaedia Britannica* is sometimes quite broad-minded about the scope of music[1], and how it should be approached: 'the listener [...] will best learn to understand [music] by divesting his [or, we would now add, her] mind of prejudices and allowing the music to make itself intelligible by its own self-consistency. The understanding of music thus finally depends neither upon technical knowledge nor upon convention, but upon the listener's immediate and

[1] It is simultaneously chauvinistically ethnocentric: 'As a mature and independent art music is unknown except in the modern forms realized by Western civilization'.

familiar experience of it.' [1, vol. 19, p. 72] The ontological line from here to learning models of musical cognition, such as IDyOM (which Wiggins briefly describes in his response), is clear. However, earlier in the same paragraph, the author of the article ('D. F. T.', identified as the eminent British musicologist Donald Francis Tovey) stated 'while [music's] language has been wholly created by art, this language is yet so perfectly organised as to be in itself natural.' Even in the work of a single author in 1911 we see the two points of view which lead Wiggins to describe, in his response paper, music as 'primarily a psychological entity' and Mazzola, responding to the same point in Wiggins' position paper, to claim that 'musical works [...] are traces of a truth which far transcends individual human cognition.' The points of view are not necessarily mutually exclusive. If we remove the value-laden words and the directionality of 'traces', the quotations of my two fellow contributors could be meaningfully combined into a description of music as 'a psychological entity reflecting and reflected in an external truth.'

To take another tack (illustrated schematically in Figure 1), the 'psychological entity' of music is hidden in brains/minds, but we can infer its existence and at least some of its properties from the behaviour of those in whose brains/minds the entity resides. Others might want to see music as consisting of that behaviour (often communal rather than individual), and the 'psychological entity' as an image of music. Yet others instead regard music as the sound which is the focus of attention in such behaviours. (Indeed it is this last which we acquire when we purchase 'music': the modern form is a digital representation of a stereo sound field.) Not all sound is music, and we commonly find certain patterns and properties in musical sound not commonly found in other sounds, so some regard music to be determined by those properties (cf. Mazzola's 'truth'). This could come full circle, since the recognition of patterns and properties is a psychological entity, but, as I pointed out in my remarks on ‘gap-fill’ in my position paper, the properties recognised in music theory do not necessarily match those which direct musical behaviour.

[Figure 1 about here]

Furthermore, not all points on this apparent circle necessarily connect. For example, a piece of music which has been composed but not yet performed—and so has not connected with musical behaviour—nevertheless has a definite existence as a psychological entity in the mind of the composer. At the other end, novel types of musical pattern can be conceived by mathematical means, but the results are not necessarily accepted as music. (Mazzola’s objection to music without ‘semiotic depth’, section 2.2 of his position paper in this issue, is a case in point.) Perhaps we should consider both kinds of ‘disconnected’ cases as ‘potential music’.

Potential music is a good thing, because it is by means of it that new music is created. The contribution of research based on a quasi-Platonic ontology of music to potential music is clear. By exploring a universe of abstract music, in Tovey’s words ‘perfectly formed’ and having ‘its own self-consistency’, previously unrecognised possibilities can be discovered, and novel music can arise. In practice this seems to have been a rare occurrence, and I cannot think of any significant example where empirical factors have not also played a part. Serial composition is a good candidate, but even before Schoenberg codified the system he had been writing music in which continual use of all twelve pitch classes and recurrences of pitch patterns was important. (Furthermore, the musical success of the system is questioned by some, as we have seen.) It seems clear that mathematical pattern and constraint does not guarantee good music, as Mazzola pointed out in his position paper. The first musical computing I ever did was to find 12-note series which, when treated as circular, contained in the twelve sets of three consecutive pitch classes, all the twelve possible classes of trichord (up to the usual transpositional and inversional equivalence). There are only four such set classes:[2] 026A5389B741, 026AB9835147, 026A74B98351, and 026A145389B7. This is a tight constraint, and it is remarkable that such sets exist at all. However, though this seemed to promise a way of relating pitch class to harmonic content (in the sense that each trichord class has a distinctive harmonic character) different from the relations inherent in tonality, I never did find a convincing way to make use of this in composition.

Concern for potential music need not be absent from research based on a psychological-cognitive ontology, but it has been less prominent than other kinds of research. One could regard those computer-composition systems (e.g., that of David Cope [2]) which work by reorganising material derived from existing pieces as examples. As implied above, though, most actual composition operates on the basis of this ontology: most new musical ideas emerge from a composer’s experience of other music.

## Empiricism, rationalism, and the validity of concepts

Underlying these two ontological perspectives are the fundamental perspectives of empiricism and rationalism and their associated criteria for validity in concepts: on the one hand verifiable evidence, on the other axiomatic proof. (Note that I do not ascribe Wiggins and Mazzola simply to one camp or the other.) The difference in part relates to the issue of the freedom to define concepts which Mazzola raised as a difference between mathematics and humanities in his response to me. If the basis for concepts is axiomatic proof, then different axioms and a different direction of proof can quite

[2] ‘A’ and ‘B’ are used here for the tenth and eleventh pitch classes respectively.

legitimately lead to different and novel concepts, but the relation between those concepts and others is, essentially, predefined. If, on the other hand, the basis of concepts is empirical observation, which includes observation of the behaviour of others, there is an important constraint on the definition of concepts, but their relationships can be unclear and undetermined. Note, however, that music theory has displayed both kinds of concept formation: ‘sonata form’ emerged as a concept from analysts’ observations of common structures in pieces of music (and it was a controversial step for Rosen to use the plural ‘sonata forms’ in the title of his book [3]); the concept of ‘Z-related pitch-class sets’ [4], now an accepted part of some musicological discourse, on the other hand, was derived on a purely theoretical basis.

However, just as the two ontologies are not necessarily exclusive, so the two epistemologies of empiricism and rationalism need not necessarily be opposed. The best scientific research draws on both, and in the case of the Large Hadron Collider at CERN is spending a great deal of money to do so. Sometimes the empirical research follows the theory, as in the case of Eddington’s observation in 1919 of the apparent change in location of stars during an eclipse of the sun providing empirical evidence in support of Einstein’s theory of the effect of gravity on light. At other times, as in the case of the discovery of pulsars by Jocelyn Bell Burnell, the empirical observation precedes theoretical explanation.

We might then be better to think of empirical and rational approaches not as opposite poles but as orthogonal dimensions, as suggested in Figure 2. In a highly schematic fashion, I indicate the approximate locations in this space of some famous theories of tonality. To be more precise these are all theories which seek to explain the relation of the notes of a piece of music to the key of that piece to the extent that one can, to some degree, ‘compute’ the key from the notes. The work of Longuet-Higgins

[5] is an early example of computer software to determine key, and also an example of the many tonal theories which describe tonal relations in a spatial metaphor. (The space Longuet-Higgins used is isomorphic to the *Tonnetz*.) The theory is essentially axiomatic (the consonant intervals of fifths and thirds create a 'space' of pitches; keys consist of compact regions in this space) though Longuet-Higgins did present empirical tests using the themes of Bach fugues, and we do not know the degree to which the path he took to this theory was empirical. The 'tonal profiles' of Krumhansl and colleagues [6, 7] are empirical: these profiles were derived on the basis of tests of the 'fittingness' of pitches in relation to 'tonal contexts'. Leman's schema theory [8] effectively uses a similar concept to these 'tonal profiles', but the basis for this is not solely empirical. The theory also allows tonality to be explained on the basis of axioms about auditory perception and human learning. I do not want to make a specific claim that Leman's theory is better than either Krumhansl's or Longuet-Higgins'—more detailed investigation would be required for such a claim—but it seems incontestable that a theory which is grounded on both empirical and rational bases is preferable to one which is otherwise similar but grounded on only one.

Music theories are not necessarily grounded on either, unfortunately, and much as I admire other aspects of Schenker's ground-breaking theory, the 'chord in nature' [9, p. 10] which he advances as the basis for tonality seems to me to be an example of this. Empirically, it would imply that all musical cultures which made music with instruments based on resonating air columns or strings would use the system of western tonality, which is clearly not the case. Rationally, it fails to explain why there are minor keys or why musical intervals equivalent to integer frequency ratios involving 7 are absent from almost all music.

**Future research**

The same two dimensions, somewhat reinterpreted, can map not only ontologies of music and music theories, but also different views of the locus of music's power or value. According to some accounts, music derives its value and power from circumstances, i.e., in an empirical fashion. For example, 'optimal complexity' theory claims musical preference to be based on the complexity of a piece of music in relation to what listeners have heard before (a clear account of the theory is contained in [10, pp. 76–84]). In other accounts, the value and power is inherent in the relationships within music (within a piece and among pieces) without any necessary reference to circumstances (of composition, listening, etc.). A famous, though not mathematical, account of this kind is Hanslick's *Vom Musikalisch-Schönen* [11]. Readers will probably be familiar with less scholarly writing which aims to find the 'golden ratio' in pieces of music with the apparent aim of thereby explaining their beauty. (Such writing usually lacks any explanation of *why* the golden ratio should be beautiful, but an explanation in terms of fuzzy logic has been advanced [12].) Once again, these two points of view do not preclude each other, and the most beautiful music is probably that which has *both* rich relationships within itself *and* fits with the circumstantial criteria for beauty.

If mathematicians seek to explain musical beauty, then, they perhaps should not stick to the 'rational' axis, where they (perhaps naturally) have been most active. It is important to realise that there is mathematics in work with an empirical basis also (and often a failure of such research is not to apply sufficiently rigorous mathematical analysis to the statistical findings). Mathematical work of this nature does exist (e.g., [13]) but it is not common. In my view there is much useful work of this kind yet to be done, and I urge musically minded mathematicians to consider more empirical, statistical work.

This, in broad terms, is the answer I failed to give in my position paper to our editors' questions about 'unexplored fields'. To be more precise, here are some suggestions for future mathematical-computational-musical research:

1. Can 'cluster analysis' or some similar technique show a sound empirical basis for theories of catalogues of patterns underlying styles of music, such as 'topics' in Classical and Romantic music [14, 15], the importance of partimenti as models for Baroque and early Classical composers [16], or the idea of reusable contrapuntal 'modules' in Renaissance polyphony [17]. (Some work towards the last of these is presented in [18].) Related existing work on similarity among folk songs (e.g., [19]) might provide some models.
2. Information theory has recently received renewed interest in the field of music analysis [20] but I suspect it could benefit from greater mathematical sophistication. The essential theory of information in a sequence of symbols is well understood, but in music what constitutes a symbol is rarely simple. Music conveys information at multiple levels, but information-theoretic studies often work at only one. To consider only the sequence of notes in a melody is to ignore information carried in how those notes are performed. On the other hand, an information-theoretic approach which operated on recorded music at the level of the FFT frame would struggle to reveal anything about the structure of a melody. I suspect there is useful work to be done in information-theoretic approaches which operate on multiple levels and allow information to pass between levels also.

3. It is generally believed that dissonance has both acoustic and conventional components, as Mazzola has pointed out in his comment that the interval of a fourth (with the bass) is not a consonance in tonal music (response to Wiggins). If dissonance genuinely does have an acoustic component, then we should be able to see its influence in voice-leading in compositions, but analyses of actual pieces invariably proceeds only on the basis of the conventional principles. Mathematical and computational tools should now make it possible to infer, in each instance of an interval in a composition (especially if an audio recording is available), the ‘acoustic dissonance’ and thereby to test for its influence on voice-leading. For example, in the fourth bar of Mozart’s symphony in C major K. 551 (‘Jupiter’) (Figure 3, bar 2), convention would state that the G in the first violin is consonant (making a sixth with the bass) while the F is dissonant (making a diminished fifth). To my ears, however, the G seems nevertheless to resolve to the F. Is this because of the closeness of the second harmonic of the bass note B to the fundamental of the G?
4. Music theory suffers from too much taxonomy and too little explanatory power. Several theories do little more than categorise musical configurations and relationships (pitch-class set theory [4] is a prime example). If being able to categorise is the only objective of a theory, success is measured by the size of the set of phenomena which can be categorised. There is no constraint on the number of categories other than the limit of putting every phenomenon into its own category, analogous to the notional ‘theory’ of Bach-chorale harmonisation (mentioned in my response to Guerino Mazzola (this issue)) which consists simply of a record of all Bach’s actual chorale

harmonisations. As I pointed out with respect to the Bach example, we seek theories which *explain* musical phenomena. A simple characterisation of a theory's explanatory power might be the accuracy and precision by which it distinguishes those phenomena which are music from those which are not, perhaps using something like the f-measure from information-retrieval theory. In this sense, set theory has a perfect recall for its intended domain (atonal music), because every piece of music has a description in terms of set theory, but its precision is near zero because any configuration of notes at all in the twelve-note system has a description. Mathematicians and computer scientists could help musicians to derive useful means for judging the power of a music theory and so help guide theory formation.

The tenor of our discussion has been to consider how mathematics and computation can benefit musical research. Does the benefit have to be always one way? Music already helps mathematicians and computer scientists perhaps by suggesting interesting fields for study, and by offering entertainment while the brain grinds away. Could it have a more essential role in these disciplines? From an outsider's perspective, it seems to me that significant advances in both disciplines, while they might be verified through a definite procedure of reasoning and proof, are often first conceived through mental processes as obscure and apparently intuitive as those in composing a piece of music. Analogy and metaphor seem often to be aids to this intuition. Could music (or at least sound) provide a ground for such mathematical or computational analogy? There is existing work on the sonification of the evaluation of computer programs [21]. Could mathematical problems be converted to music? This is my final challenge. Marcus du Sautoy's book *The Music of the Primes* [22] sadly contains no actual music, but to make a quasi-musical sequence which lets one hear the (non-)pattern in prime numbers is not difficult. I am sure others can do better than my initial attempts (http://www.lancaster.ac.uk/staff/marsdena/software/primes) and maybe greater sophistication along these lines might spark the intuition which eventually leads to a proof of the hypothesis proposed by Bernhard Riemann and a better understanding of the distribution of prime numbers.

## References


[1] 'Music', *Encyclopaedia Britannica*, 11th edition, Cambridge U. Press, New York, 1911.

[2] D. Cope, *Experiments in Musical Intelligence*, A-R Editions, Maddison, WI, 1996.

[3] C. Rosen, *Sonata Forms*, Norton, New York, 1980.

[4] A. Forte, *The Structure of Atonal Music*, Yale U. Press, New Haven, 1973.

[5] H.C. Longuet-Higgins, *The perception of music*, Interdiscipilinary Science Reviews. 3 (1978), pp. 148–156.

[6] C.L. Krumhansl & R.N. Shepard, *Quantification of the Hierarchy of Tonal Functions Within a Diatonic Context*, Journal of Experiemental Psychology: Human Perception and Performance. 5 (1979), pp. 579–594.

[7] C.L. Krumhansl, *The Cognitive Foundations of Musical Pitch*, Oxford U. Press, New York, 1990.

[8] M. Leman, *Music and Schema Theory: Cognitive Foundations of Systematic Musicology*, Springer, Berlin, 1995

[9] H. Schenker, *Free Composition*, Longman, New York, 1979 (translated by E. Oster, original *Der Freie Satz*, 1935).

[10] A.C. North & D.J. Hargreaves, *The Social and Applied Psychology of Music*, Oxford U. Press, 2008.

[11] E. Hanslick, *The Beautiful in Music: A contribution to the revisal of musical aesthetics* (translated by G. Cohen), Da Capo Press, New York, 1974

[12] M. Koshelev, *Fuzzy logic explains the golden proportion*, Int.J. of Intelligent Systems. 12 (1997), pp. 415–417.

[13] D.K. Simonton, *Thematic fame, melodic originality, and musical zeitgeist: A bibliographical and transhistorical content analysis*, J. of Personality and Social Psychology 38 (1980), pp. 972–983.

[14] L.G. Ratner, *Classic Music: Expression, Form, and Style*, Schirmer, New York, 1980.

[15] V.K. Agawu, *Playing with Signs: a Semiotic Interpretation of Classic Music*, Princeton U. Press, 1991.

[16] R.O. Gjerdingen, *Partimento, Que Me Veux-Tu?*, J. of Music Theory. 52 (2009), pp. 85–135.

[17] P.N. Schubert, *Hidden forms in Palestrina's first book of four-voiced motets*, J. of the Am. Musicological Soc. 60 (2007), pp. 483–556.

[18] I. Knopke & F. Jürgensen, *A system for identifying common melodic phrases in the masses of Palestrina*, J. of New Music Research. 38 (2009), pp. 171–181.

[19] Z. Juhász, *Automatic segmentation and comparative study of motives in eleven folk song collections using self-organizing maps and multidimensional mapping*, J. of New Music Research. 38 (2009), pp. 71–85.

[20] K. Potter, G.A. Wiggins, & M.T. Pearce, *Towards greater objectivity in music theory: Information-dynamic analysis of minimalist music*, Musicae Scientiae 11 (2007), pp. 295–322.

[21] P. Vickers & J. L. Alty, *The well-tempered compiler: The aesthetics of program auralization*, in *Aesthetic Computing*, P. A. Fishwick, ed., MIT Press, Boston, MA, 2006, pp. 335–354.

[22] M. du Sautoy, *The Music of the Primes: Why an Unsolved Problem in Mathematics Matters*, Fourth Estate, London, 2003.

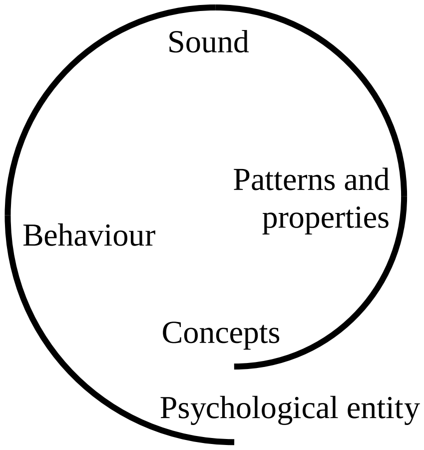


Figure 1. Schematic illustration of the loci of ‘music’.

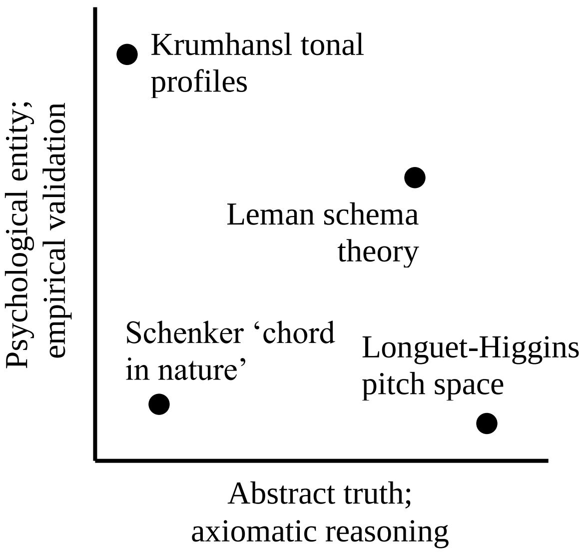


Figure 2. Dimensions of music ontology and epistemology.

Figure 3. Mozart symphony in C major, K.551, bars 3-4.