Real exchange rates and time-varying trade costs
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Abstract
This paper re-examines the empirical modeling of Purchasing Power Parity (PPP) deviations in the presence of commodity market frictions. First, we show that a specific type of smooth transition models can closely approximate the functional form of the theoretical adjustment mechanism derived by Dumas (1992) [Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World, Review of Financial Studies, 5: 2153–180] for the case of constant as well as changing trade costs. Second, we develop, for the first time, an empirical model of the real exchange rate which allows for changes in the degree of market integration. By employing a long span of data on the Dollar-Sterling real exchange rate and a micro-founded proxy for trade frictions, we provide novel evidence of a significant relationship between the persistence of the real exchange rate and the level of trade costs. This finding suggests that both the difficulty of detecting PPP and the extend of Rogoff’s puzzle vary over time with the degree of trade restrictiveness. Finally, we highlight policy repercussions of our results.

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1. Introduction

In the early 1990s, two decades after the break down of the Bretton-Woods system and the inception of floating exchange rates, the consensus view in international macroeconomics was that Purchasing Power Parity (PPP) did not hold to any meaningful degree. The hypothesis that the real exchange rate contained a unit root or that there was no long-run relationship between the nominal exchange rate and the aggregate domestic and foreign price levels could not be rejected. The lack of
evidence supporting PPP led researchers to focus on the identification of potential pitfalls concerning the empirical approaches employed till then as well as to provide theoretical justifications for the observed behavior of real exchange rates.

As noted by Frankel (1986), the tests typically employed during the 1980s to investigate whether real exchange rates are stationary may have had low power when applied to small spans of data during the floating rate period. Following Frankel a number of researchers provided support for the hypothesis by employing longer spans of data, panel unit root tests and Monte Carlo techniques (Abuaf and Jorion, 1990; Frankel and Rose, 1996; Lothian and Taylor, 1996). Even though these studies provided evidence that real exchange rates mean revert in the long run, the implied half life of deviations from PPP ranged from three to five years. The fact that real shocks cannot account for such a high degree of persistence gave rise to Rogoff’s (1996) PPP puzzle.

Perhaps the most important explanation of Rogoff’s (1996) puzzle is provided by theoretical models which demonstrate how transactions costs or the sunk costs of international arbitrage induce nonlinear but mean reverting adjustment of the real exchange rate (see, e.g., Dumas, 1992; Sercu et al., 1995; O’Connell and Wei, 2002; Berka, 2005). In his seminal paper Dumas (1992) summarized this position as follows:

“Linear equations are unlikely clearly to identify a process such as the one for \( \ln p \) …in which long-run behavior is very different from short-term behavior, since reversion manifests itself only when deviation from parity has become wide enough.”

Dumas (1992, p. 171)

This theoretical work on the properties of the real exchange rate process has motivated numerous empirical applications utilizing nonlinear econometric models. The most widely used family of models is the Smooth Transition Autoregressive (STAR) of Granger and Terasvirta (1993) and Teräsvirta (1994) (for a survey see Sarno and Taylor, 2002; Pavlidis et al., 2009). The appealing feature of these models is that they allow both the speed of mean reversion to increase with the size of the deviation from the equilibrium and the transition between high and low persistence regimes to occur smoothly and symmetrically. Moreover, the findings of the empirical literature support the presence of STAR nonlinearity for various real exchange rate series (Michael et al., 1997; Taylor et al., 2001; Kilian and Taylor, 2003). The impulse response functions derived from the estimated models suggest that large shocks mean revert much faster in the nonlinear models than the ones previously reported for linear models, for which the speed of mean reversion is independent of the size of the shock. These results appear to provide a framework for solving Rogoff’s (1996) puzzle.

The present paper aims to provide further insights regarding the behavior of PPP deviations by extending the nonlinear framework of analysis in two directions. First, we argue that no formal direct link has so far been established between the economic models with trade costs and the nonlinear econometric framework adopted in empirical applications. This deficiency can clearly result in sub-optimal empirical model specifications. We bridge the gap between theory and empirical modeling by employing the spatial economy model of Dumas (1992) to determine the theoretical functional form of the PPP adjustment mechanism and, in turn, investigate the ability of STAR models to approximate it. Interestingly, our findings illustrate that a relatively neglected STAR type model, the Quadratic Logistic STAR (QLSTAR), can approximate very accurately the theoretical Data Generating Process (DGP).

The second contribution of the paper is to relax the assumption of constant trade costs and to formally examine the effect of time-varying trade costs on the nonlinear analysis of real exchange rates. Our approach is motivated by a simple argument that dates at least to Friedman and Schwartz (1982, pp. 290–292) but has been widely ignored in the nonlinear PPP literature: trade frictions vary over time, which induces changes in the degree of market integration and the range of PPP.

1 Trade costs can exhibit significant economic magnitudes and can play an essential role in addressing several major puzzles in international economics (Obstfeld and Rogoff, 2000; Anderson and van Wincoop, 2004).

2 In \( p \) denotes the logarithm of the real exchange rate.
Intuitively, periods characterized by low levels of trade costs should be associated with a smaller range of PPP deviations than periods with high levels of trade costs. Hence, the persistence of real exchange rates depends largely on the magnitude of trade costs at each particular point in time. We theoretically illustrate this point by altering the degree of trade restrictiveness in the model of Dumas (1992). We also show that a simple modification of the QLSTAR model which incorporates time-varying trade costs can closely approximate the new adjustment mechanism.

From an empirical point of view, relaxing the assumption of constant trade costs is not straightforward due to problems related with the measurement of trade frictions. Anderson and van Wincoop (2004) highlight the lack of high-quality data on policy barriers, data on transportation costs are not publicly available, and other trade frictions, such as information barriers, are not directly observable. The limitations of individual trade friction data pose serious difficulties in drawing direct inferences regarding the total level of trade costs.

Here we follow the advances in the gravity literature and circumvent this problem by including a micro-founded index developed by Jacks et al. (2008) into the nonlinear real exchange rate model. The model is fitted to a long span of data (1830–2005) for the dollar-sterling real exchange rate. Our choice is based on data availability and primarily from the appealing hypothesis that the relationship between trade frictions and the persistence of the real exchange rate should become apparent over long time periods in which large fluctuations of trade costs are known to have occurred.

Our results provide strong evidence in support of an economically meaningful and statistically significant relationship between the persistence of the real exchange rate and the micro-founded measure of trade costs. On average, periods where the micro-founded measure takes small values are characterized by a narrow range of PPP deviations, low volatility, and relatively fast mean reversion. This finding contributes to the ongoing debate amongst international economists on the usefulness of the PPP doctrine by highlighting that the difficulty of detecting PPP empirically and the implied speed of adjustment to shocks in the Rogoff’s puzzle are not constant but vary over time with factors determining trade costs as well as the magnitude of shocks occurring. Moreover, it may provide valuable insights regarding the real exchange rate formation mechanism for policy makers and market participants.

The rest of the paper is structured as follows. Section 2 outlines the theoretical model of Dumas (1992), which facilitates the analysis of the impact of changes in trade costs on the real exchange rate process. This section deals also with the nonlinear econometric framework and the ability of STAR models to approximate the functional form of the theoretical adjustment mechanism with and without movements in trade costs. Section 3 discusses the micro-founded measure of trade restrictiveness proposed by Jacks et al. (2008). The following section describes the evolution of trade costs over time and presents the econometric results. Section 5 provides a discussion of the implications these results have for policy makers. Finally, the last section concludes.

2. Nonlinear real exchange rate models

PPP states that once expressed in a common currency prices of identical baskets of goods in different geographic locations should be the same. This simple theory is based on the fact that, under frictionless trade, deviations from PPP imply arbitrage opportunities. In reality, however, numerous impediments to trade can create a wedge between prices and induce nonlinearity in the deviations from PPP.

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3 We are grateful to the editor, James R. Lothian, for bringing this reference to our attention.
4 Numerous authors provide historical evidence supporting the existence of large changes in the magnitude of several constituents of trade costs, such as tariffs and freight rates (see, e.g., O’Rourke and Williamson, 1999; Hummels, 2007). This evidence is suggestive of changes in the total level of trade costs but not conclusive. The time evolution of major determinants of trade costs between the United States and the United Kingdom, which is the country pair examined in this study, is examined in Section 4.1.
5 The origins of this idea go back at least to Heckscher’s (1916) commodity points.
2.1. The theoretical model

Dumas (1992) provides theoretical support for a nonlinear adjustment mechanism through a dynamic general equilibrium model which incorporates trade frictions. The model consists of two equally-sized countries with identical preferences and technology separated by an ocean. The two spatially separated countries produce homogeneous goods which can be used in a stochastic constant-return-to-scale production process, transferred costly across countries, or consumed. In particular, the home, $H$, and foreign, $F$, capital stocks are governed by

$$
dKH_t = \left( aK^H_t - c^H_t \right) dt + \sigma K^H_t dz^H_t - dX^H_t + s dX^F_t,
$$

(1)

$$
dKF_t = \left( aK^F_t - c^F_t \right) dt + \sigma K^F_t dz^F_t + dX^H_t - dX^F_t,
$$

(2)

where $a$ is the expected rate of return and $\sigma$ the standard deviation characterizing the production processes, $z^H_t$ and $z^F_t$ are two independent Wiener processes that represent production shocks and, hence, cause capital imbalances, $c$ denotes consumption, $s \in (0, 1)$ is a shipping-loss factor so that $\tau = 1 - s$ is the unit iceberg cost (because a fraction of goods melts during transit) of transferring capital, and $X^H_t$ and $X^F_t$ denote the cumulative capital that has been shipped from $H$ and $F$, respectively.

By assuming no impediments to the exchange of assets and commodities other than $s$ and letting preferences in both countries be given by a constant relative risk aversion instantaneous utility function, the PPP adjustment mechanism can be obtained by solving the following central planning welfare problem

$$
V(\cdot) = \max_{c^H, c^F} \mathbb{E}_t \left[ \int_t^\infty e^{-\rho(u-t)} \left\{ \frac{1}{\tau} \left( c^H_u \right)^{\tau} + \frac{1}{\tau} \left( c^F_u \right)^{\tau} \right\} du \right], \quad \zeta < 1,
$$

(3)

subject to Constraints (1) and (2). $V$ denotes the maximum attainable present value of current and expected future rewards given the process is at state $(K^H, K^F)$, $\rho$ is the discount rate, and $1 - \zeta$ is the degree of risk aversion.

In this framework, risk averse agents have an incentive to mitigate any difference that arises between capital stocks for reasons of risk sharing. In fact, in the absence of market frictions, $s = 0$, any capital imbalance is corrected instantaneously and the real exchange rate (the price of $K^H$ relative to $K^F$), which equals the ratio of the first derivatives of the value function,

$$
Q = \left( \frac{\partial V}{\partial K^H} / \frac{\partial V}{\partial K^F} \right),
$$

(4)

is continuously given by the PPP rate.

The introduction of trade costs, however, creates a region within which no trade takes place since the costs of transferring capital from the country with the abundant resources to the other outweigh the benefits. Hence, capital imbalances can arise and persist as a result of a series of random production shocks. Shipping occurs only at the edges of the capital imbalance fluctuation band. Consequently, consumption levels between countries may differ and the real exchange rate can deviate from the PPP rate.

The solid line in Fig. 1 shows the conditional expected change (drift) of the log real exchange rate as a function of the deviation from the PPP rate, $q$, for the set of parameter values employed by Dumas (1992): $\rho = 0.15$, $\tau = 1 - 1/1.22$, $\zeta = 0.9$, $\sigma = 0.5$ and $a = 0.11$. A broad conclusion that emerges is that the incorporation of trade costs can generate rich dynamics in the economy. The log of the real

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6 Following Dumas, a number of authors have examined the impact of commodity market frictions on the real exchange rate (e.g., Sercu et al., 1995; Bec et al., 2004; Berka, 2005).

7 The utility levels of the two countries are equally weighted as a result of the symmetry between the two countries.
exchange rate, like the economic fundamentals, moves inside a band, similar to a target zone model (see, e.g., Svensson, 1990). In equilibrium the drift is zero and, hence, PPP deviations do not exhibit mean reversion. As the value of the absolute deviation from the equilibrium increases and the process approaches the edges of the band agents anticipate that shipping will occur. This causes the drift to become negative (positive) for large positive (negative) deviations and, hence, ensures global stability. Put it differently, the process is inherently nonlinear with low persistence at the edges of the band and high persistence at the equilibrium. It is important to note that there is a one-to-one mapping between the drift of the real exchange rate and its level. Finally, the transition between high and low persistence regimes is smooth and symmetric.8

The theoretical nonlinearity postulated above can clearly confound linear unit root and cointegration techniques as well as linear impulse response functions over short periods (Taylor et al., 2001). All the above motivates the use of nonlinear econometric techniques.

2.2. Smooth transition autoregressive models

A family of nonlinear econometric models which has been widely employed to test the implications of macroeconomic models with trade costs is the STAR of Teräsvirta (1994) and Granger and Teräsvirta (1993). The appealing feature of STAR models is that they allow the speed of mean reversion to increase with the absolute size of the deviation from the equilibrium and the transitions between regimes to occur smoothly and symmetrically. In other words, the properties of STAR processes resemble the theoretical properties of the real exchange rate.9 Nevertheless, the ability of different STAR models to adequately capture the properties of the real exchange rate derived from the theoretical analysis has not been examined so far. In this section we illustrate that a relative neglected STAR model, the Quadratic Logistic STAR (QLSTAR), can parsimoniously capture the type of nonlinearity postulated by the model of Dumas. Due to the fact that most empirical studies to date have employed the Exponential STAR (ESTAR), we also provide an analysis of the ESTAR model and the corresponding empirical results in Appendix I.

Fig. 1. The expected change of the log real exchange rate against the deviation from the PPP rate in the model of Dumas (1992) (solid line) and the QLSTAR approximation (circled line). The parameter values used are: $r = 0.15$, $s = 1-1/1.22$, $z = 0.9$, $\sigma = 0.5$ and $a = 0.11$.

8 Note that the fact that the process is not a martingale does neither contradict rational expectations nor market efficiency.

9 These properties are also demonstrated by Berka (2005). Teräsvirta (1994) argues, moreover, that if an aggregated process is observed, regime changes may be smooth rather than discrete as long as heterogeneous agents do not act simultaneously even if they individually make dichotomous decisions. All the above favor the use of STAR models over Threshold Autoregressive (TAR) models, in which changes of persistence occur abruptly. Moreover, as we will show the STAR model employed in this work nests the three regime TAR. Note, finally, that incorporating time-varying trade costs in TAR models is not straightforward. The difficulty lies in drawing statistical inferences on the significance of the time-varying threshold coefficients.
A QLSTAR model for the log real exchange rate process \( \{ q_t \} \) may be written as

\[
q_t - \mu = \sum_{i=1}^{p} \phi_i (q_{t-i} - \mu) G(q_{t-d}) + \epsilon_t, \tag{5}
\]

where \( \mu \) is a constant representing the long-run equilibrium, \( \epsilon_t \) is a white noise process with mean 0 and variance \( \sigma^2 \), and \( G(\cdot) \) is the transition function. In the presence of constant trade costs, the transition function is given by\(^{10}\)

\[
G(q_{t-d}) = 1 - \left( 1 + \exp \left( -\frac{\gamma^2}{2b^2} (q_{t-d} + b_1)(q_{t-d} + b_2) \right) \right)^{-1}, \tag{6}
\]

where \( \gamma \) is the smoothness parameter, \( d \) is the delay parameter, and \( b_1 = -\mu - b \) and \( b_2 = -\mu + b \) with \( b > 0 \) are the band coefficients. Given the autoregressive structure, \( \sum_{i=1}^{p} \phi_i \), the shape of \( G(\cdot) \) specifies completely the dynamics of the process. The attractiveness of Equation (5) lies in its flexibility to approximate narrow and wide bands of inaction and the fact that it can accommodate both smooth and abrupt transition between regimes. Therefore, the model nests numerous adjustment mechanisms such as the three regime TAR and the linear and can approximate closely the ESTAR.\(^{11}\)

The suitability of STAR models for testing the implications of the theoretical analysis remains an unexplored issue. We use Equation (5) to derive an econometric expression for the expected change of the real exchange rate process and, in turn, compare it with its theoretical counterpart. Motivated by the continuous time model of Dumas, where the equilibrium rate takes the value of zero and the drift is a function of a PPP deviation, we impose the following set of restrictions on the regression coefficients: \( \mu = 0, p = d = 1 \), and \( \phi_1 = 1 \). Under these restrictions, Equation (5) becomes

\[
q_t = q_{t-1} \left[ 1 - \left( 1 + \exp \left( -\frac{\gamma^2}{2b^2} (q_{t-1} - b)(q_{t-1} + b) \right) \right)^{-1} \right] + \epsilon_t. \tag{7}
\]

Subtracting \( q_{t-1} \) from both sides and taking expectations yields

\[
E_t(\Delta q_t) = -q_{t-1} \left( 1 + \exp \left( -\frac{\gamma^2}{2b^2} (q_{t-1} - b)(q_{t-1} + b) \right) \right)^{-1}. \tag{8}
\]

The above equation establishes the econometric relationship between the magnitude of PPP deviations and the expected change in the real exchange rate series.

We obtain a set of values for the theoretical conditional expected change of the real exchange rate by numerically solving the Dumas model and evaluating the drift function at 1000 equally spaced points on the real exchange rate line.\(^{12}\) Next, we employ an optimization method so as to fit the QLSTAR

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\(^{10}\) The QLSTAR parameterization that we consider is a modification of the one proposed by Jansen and Teräsvirta (1996). The difference between the two parameterizations is that the smoothness parameter \( \gamma^2 \) in Equation (5) is divided by \( b^2 \). This implies that the maximum values of \( G(\cdot) \) is independent of the value of \( b \) and also that changes in the persistence of the process become more abrupt as \( b \) decreases. As we show below, both of these properties are useful for incorporating changes in trade costs.

\(^{11}\) To illustrate this point, let the sum of the autoregressive coefficients equal unity, \( \sum_{i=1}^{p} \phi_i = 1 \). Consider, first, the scenario that regime changes occur abruptly rather than gradually (see Sercu et al., 1995), which favors the use of TAR models. If \( \gamma \rightarrow \infty \) and the real exchange rate is outside the band of inaction, \( q_{t-d} < b_1 \) or \( q_{t-d} > b_2 \), the value of the quadratic logistic function equals zero and \( q_t \) becomes white noise. In this case, deviations from the equilibrium are expected to be instantaneously eliminated. Whilst, inside the band of inaction, \( b_1 < q_{t-d} < b_2 \), \( G(\cdot) \) equals one and \( q_t \) behaves as a unit root process. Hence, the QLSTAR model nests the three regime TAR. At the other extreme, when \( \gamma = 0 \) the model becomes linear. For moderate values of \( \gamma \), the QLSTAR model can approximate the ESTAR model. The speed of mean reversion increases with the absolute deviation from the equilibrium. In the unlikely event that \( |q_{t-d} - \mu| \rightarrow \infty \) the process approaches the white noise regime (outer regime). Whilst, in the inner regime, \( q_{t-d} - \mu = 0 \), the degree of persistence is given by the maximum value of the transition function \( G(\cdot) \), which occurs at the equilibrium rate, and equals \( G(0) = 1 - (1 + \exp(\gamma^2))^{-1} \).

\(^{12}\) The dynamic general equilibrium model is numerically solved by a shooting method and the 4th order Runge-Kutta technique.
Model (8) to the theoretical values and obtain estimates for the coefficients $\gamma$ and $b$. These estimates are 2.553 and 0.247, respectively.

Fig. 1 displays the fitted values of the QLSTAR model (circled line). The high goodness-of-fit, as measured by the distance between the observed (theoretical) values and the expected (fitted) values, illustrates that the empirical adjustment mechanism can closely approximate its theoretical counterpart. This finding supports the application of the QLSTAR model in testing the implications of the theoretical analysis. We show in Appendix I that the ESTAR model cannot approximate as closely the theoretical adjustment mechanism. For relatively small PPP deviations, it falsely predicts a faster reversion toward the equilibrium, whilst for large deviations, it predicts a slower reversion rate.

2.3. Real exchange rate behavior in the presence of changing trade costs

So far we have assumed that trade costs, $\tau = 1 - s$, are constant and, consequently, the range of the band of inaction is time-invariant. However, there is a plethora of evidence suggesting that substantial changes in the degree of commodity market integration take place over time see, e.g., Findlay and O’Rourke (2003). To this end, we extend the analysis by investigating the impact of changes in trade costs.\(^{13}\)

Fig. 2 depicts the behavior of the expected change of the log real exchange rate in the spatial model for three levels of trade costs. The solid line represents low costs, $\tau = 1 - 1/1.22$, the dashed line moderate costs, $\tau = 1 - 1/1.26$, and the dotted line high costs, $\tau = 1 - 1/1.3$. Not surprisingly, as the level of trade costs increases the band of inaction widens and the real exchange rate can take larger absolute values. Hence, the unconditional volatility of the real exchange rate depends positively on trade costs. Fig. 2 also shows that the absolute expected change of the log real exchange rate is smaller for higher trade costs for all PPP deviations except $q = 0$. This suggests that the degree of persistence of the real exchange rate, like the unconditional volatility, increases with trade costs. Consequently, standard unit

![Graph](image-url)

Fig. 2. The theoretical expected change of the log real exchange rate against the deviation from the PPP rate for three levels of trade costs: $\tau = 1 - 1/1.22$ (solid line), $\tau = 1 - 1/1.26$ (dashed line), and $\tau = 1 - 1/1.3$ (dotted line). The values for the remaining parameter are: $\rho = 0.15$, $\xi = 0.9$, $\sigma = 0.5$ and $a = 0.11$.

\(^{13}\) A closely related study by Berka (2009) estimates TAR models for good-specific United States-Canada real exchange rates. The author builds on the results of Novy (2006) regarding movements in trade costs during the post Bretton-Woods era, and imposes a gradual decline in the absolute value of the thresholds. The imposed change in thresholds causes a reduction in the overall degree of persistence which leads to an improvement in the regression fit. This finding motivates further the formal investigation of the role of time-varying trade costs in the behavior of real exchange rates.
root and cointegration techniques are more likely to detect PPP for periods of relatively low rather than high trade costs.

As far as the nonlinear specification is concerned relaxing the constant trade costs assumption implies that there is no longer a one-to-one relationship between the expected change and the level of the real exchange rate. For the case of variable trade costs the speed of mean reversion depends not only on the level of the real exchange rate but also on the specific level of trade costs.\textsuperscript{14} Hence, the QLSTAR model of the previous section suffers from an omitted variable bias that is most likely to result in false inference regarding the speed of mean reversion. This is illustrated in Fig. 3 which displays the fitted values of the constant trade costs QLSTAR model (circled line) and the theoretical expected change of the real exchange rate (solid line) for the three levels of trade costs considered.\textsuperscript{15}

In order to resolve the missing variable problem, we propose a simple modification of the quadratic logistic transition function which allows the band coefficients to depend linearly on the level of trade costs. The dynamics in the Time-Varying Trade Costs QLSTAR (TVTC-QLSTAR) are governed by

\[
q_t - \mu = \sum_{i=1}^{p} \phi_i (q_{t-i} - \mu) \left[ 1 - \left( 1 + \exp \left( -\frac{\gamma^2}{(b + b_3 \tau_{t-d})} (q_{t-d} + b_3) (q_{t-d} + b_4) \right) \right) \right]^{-1} + \epsilon_t, \tag{9}
\]

where \(b_3 = -\mu - b - b_4 \tau_{t-d}\) and \(b_4 = -\mu + b + b_4 \tau_{t-d}\) with \(b_3 < b_4\).\textsuperscript{16} The econometric relationship between the expected change of the real exchange and the PPP deviation as well as the ability of the TVTC-QLSTAR model to approximate the theoretical adjustment mechanism can be obtained in a straightforward manner by employing the methodology of Section 2.2. Fig. 4 shows that the simple modification parsimoniously captures the effect of time-varying trade costs. On this basis, Equation (9) allows testing the effect of the evolution of commodity market integration on the behavior of the real exchange rate series. From an empirical point of view, the main difficulty to estimate the TVTC-QLSTAR model is the measurement of the aggregate level of trade costs, which is examined in the following section.

\textsuperscript{14} The only exception to this rule is the equilibrium of the process, where the drift equals zero irrespective of the level of trade costs.

\textsuperscript{15} The details of the methodology employed to fit the QLSTAR model is the same as in previous section.

\textsuperscript{16} The estimated trade costs index has been rescaled to have a minimum value of zero. Consequently, \(b\) reflects the lowest level of trade costs in time.
3. Measuring trade costs, the gravity literature

“Trade costs, broadly defined, include all costs incurred in getting a good to a final user other than the marginal cost of producing the good itself” (Anderson and van Wincoop, 2004, p. 691).

Trade costs break down into a vast number of components such as transportation costs (freight rates and time costs), policy barriers (tariffs and non-tariff barriers), informational costs and costs associated with the use of different currencies. The fact that several of these components are unobservable and data limitations pose serious problems in obtaining accurate estimates of the magnitude of total trade costs by direct atheoretical measures. The gravity literature circumvents this obstacle on the basis of theoretical models which enable measuring the degree of trade restrictiveness by extracting information from trade flows.

In this framework, Jacks et al. (2008) present a micro-founded measure of aggregate bilateral trade costs that captures trade frictions. The key idea in the derivation of their measure is that changes in trade barriers have an effect on both international and intranational trade. By establishing a relationship between countries’ average international trade barriers and intranational trade, trade costs can be obtained directly from observable trade data without imposing a particular trade cost function (Novy, 2008).

Consider a world consisting of N countries and a continuum of differentiated goods. Anderson and van Wincoop (2003) derive the following gravity equation of international trade

\[
x_{ij} = \frac{y_i y_j}{y_w} \left( \frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma_g},
\]

where \(x_{ij}\) are nominal exports from country i to j. Income levels of country i, country j and world income are denoted by \(y_i\), \(y_j\) and \(y_w\), respectively. The elasticity of substitution, \(\sigma_g\), is assumed to be constant and greater than unity. The cost of importing a good or, equivalently, the trade cost barrier (one plus the tariff equivalent) is \(t_{ij} \geq 1\). Finally, the price indexes (or outward and inward multilateral resistance variables) \(\Pi_i\) and \(P_j\) for countries i and j represent the average trade restrictiveness of the countries. Novy (2008) uses Equation (10) to obtain a bidirectional gravity equation, which includes inward and outward multilateral resistance variables for both countries,

![Figure 4. The theoretical expected change of the log real exchange rate against the deviation from the PPP rate for three levels of trade costs, \(\tau = 1-1/1.22\), \(\tau = 1-1/1.26\), and \(\tau = 1-1/1.3\), and \(\rho = 0.15\), \(\xi = 0.9\), \(\sigma = 0.5\) and \(a = 0.11\) (solid line) and the corresponding fitted values of the QLSTAR model with time-varying trade costs (circled line).](image-url)
\[ x_{ij}x_{j} = \left( \frac{y_i y_j}{y_w} \right)^2 \left( \frac{t_{ij} t_{j}}{\Pi_i \Pi_j} \right)^{1-\sigma} \tag{11} \]

In turn, the author makes use of the fact that intranational trade, like international trade, depends on the magnitude of trade barriers, \( x_{ii} = (y_i y_i)/(y_w) (t_{ii}/(\Pi_i \Pi_i))^{1-\sigma} \), so as to control for multilateral resistance. Substituting into the bidirectional gravity Equation (11) yields

\[ x_{ij}x_{j} = x_{ii}x_{j} \left( \frac{t_{ij} t_{j}}{t_{ii} t_{jj}} \right)^{1-\sigma} \tag{12} \]

The size variable in Equation (12) is intranational trade, \( x_{ij}x_{j} \), which controls for the countries’ economic size. The geometric average of the tariff equivalent can now be obtained by

\[ \tau \equiv \left( \frac{t_{ij} t_{j}}{t_{ii} t_{jj}} \right)^{1/2} - 1 = \left( \frac{x_{ij}x_{j}}{x_{ii}x_{j}} \right)^{1/(2(\sigma-1))} - 1 \tag{13} \]

The above equation states that a drop in trade flows between countries with respect to trade flows within countries is associated with higher trade costs. Note that the micro-founded measure evaluates bilateral trade costs against the domestic trade cost benchmark. Novy (2008) argues that \( \tau \) can be interpreted as international trade costs net of domestic distribution costs. The computation of \( \tau \) only requires data for bilateral exports and intranational trade, and the latter variable can be approximated by subtracting aggregate exports from a country’s Gross Domestic Product (Jacks et al., 2008). Consequently, this measure enables the estimation of the TVTC-QLSTAR over very long time series of historical data, during which substantial changes in the economy are more likely to occur.

4. Data and empirical results

Our data set consists of annual observations for the dollar-sterling real exchange rate and the corresponding trade costs index from 1830 to 2005. For the construction of the real exchange rate we use the International Financial Statistics database to update the nominal exchange rate and the price indexes analyzed in Lothian and Taylor (1996). International trade data are obtained by Mitchell (2008a,b) and GDP series for the United Kingdom and the United States are taken from Officer (2008) and Johnston and Williamson (2008), respectively.

4.1. Market integration in historical perspective

Fig. 5 shows the demeaned real exchange rate and the estimated trade costs series, \( \tau \). Overall, we observe that both series display substantial fluctuations. The general pattern of \( \tau \) is consistent with the findings of historical studies regarding commodity market integration (see, e.g., Findlay and O’Rourke, 2003, and the references therein).

During the second half of the 19th century, important innovations in the shipping and communications technology (most prominently the steam engine and the trans-Atlantic telegraph cable), the construction of railroads, and the development of mechanical refrigeration appear to have led to an enormous decline in transportation and communications costs (O’Rourke and Williamson, 1999). The decline in transportation costs, the Gold Standard and the free trade policy adopted by the United Kingdom since the repeal of the Corn Laws in 1846 accommodated a substantial increase in trade ratios and an unforeseen intercontinental commodity market integration in terms of price convergence (see, e.g., Estevadeordal et al., 2003; Goodwin et al., 2002; Findlay and O’Rourke, 2003).18

17 A regular trans-Atlantic steam service started in 1838.

18 Unlike Britain, tariffs in the United States had been high throughout the late 19th century, and especially during the years of the Civil War.
The inception of World War I marked the end of the first era of globalization. In Britain, the Gold Standard was abandoned in 1914 (until 1925) to fund military operations and the following year a turn to protectionism occurred with the introduction of the McKenna tariff. At the same time, a 25 percent rise in prices led to the imposition of price controls, mainly on food. The pound-sterling appreciated sharply at the beginning of the war, then it returned to its prewar parity and it remained pegged near that value for the rest of the war.

After World War I, a proliferation of tariffs, import quotas, and capital controls took place. The process of economic integration was set back especially during the years of the Great Depression. In 1930, the American Smoot-Hawley Act, the symbol of interwar protectionism, raised tariffs on a wide range of industrial products and agricultural goods. Similar policies were adopted in Britain. Madsen (2001) finds that 15 percent of the world trade implosion during the Great Depression can be attributed to tariff increases and 6 percent to the imposition of non-tariff barriers.

Similarly to World War I, World War II was characterized by price controls in both the United States and the United Kingdom. In the United States, a general price control was imposed in 1942 and suspended in 1946, whilst in the United Kingdom price controls lasted until 1953 (Friedman and Schwartz, 1982). The United Kingdom also imposed extensive foreign exchange controls.

The massive increase in the degree of trade restrictiveness during the first half of the 20th century is reflected in the behavior of the estimated trade costs index. The index reaches its two peaks during the Great Depression and the second World War. Interestingly, for these periods the range of PPP deviations (as measured by the distance of the real exchange rate from its mean) is, in general, larger than for the second half of the 19th century. This is in line with the findings of Friedman and Schwartz (1982).

Trade liberalization in the form of significant tariff reduction began in the second half of the 20th century with the General Agreement on Tariffs and Trade (GATT) and the Bretton Woods system. This period was also characterized by technological improvements in transportation and in communications. The findings of Hummels (2007) suggest that substantial reductions in air but not ocean freight rates took place, whilst, Levinson (2006) supports that containerization led to a sharp fall in transportation costs. The estimated trade costs index is in line with the evidence reported in the literature and displays a fall in the overall degree of trade restrictiveness between the United Kingdom and the United States in

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19 For a detailed description of commercial policies adopted in the United Kingdom see Friedman (1974).
20 According to the 1996 World Bank survey, price controls are one of the top fifteen “obstacles for doing business” and, therefore, a constituent of trade costs (Anderson and Marcouiller, 2002; Brunetti et al., 1997). Price controls do not only impact on trade costs but can also distort the measurement of price indexes and hence the real exchange rate since they imply that price increases can take indirect and concealed forms (see, e.g., Friedman and Schwartz, 1963).
21 For a discussion about the impact of GATT on world trade see Irwin (1995). It should be noted that despite the decline in industrial tariffs, agricultural protection remains high and non-tariff barriers play a more important role (e.g., antidumping actions).
the second half of the 20th century. However, the decline is not substantial enough for trade costs to reach their pre-World War I level.\footnote{Studies on financial markets also show a high degree of international integration in the period 1870–1914. Similarly to commodity markets, financial integration was interrupted in the war and interwar periods and the reconstruction of the world financial system did not start until the end of World War II. However, financial market integration appears to be greater in the recent float than in the period 1870–1914 (Lothian, 2000, 2002).} There are at least three possible explanations for this pattern.

First, global market integration may have been greater in the second half of the 19th century. Irwin (1996) argues that it is not clear that the postwar trade liberalization and changes in transportation and communications costs have resulted in as large convergence in commodities markets as did the transportation costs decline in the late 19th century. Notably, for the United Kingdom, trade as a percentage of output was not higher in the late 20th century than in the high Victorian period (see, e.g., Krugman, 1995). Friedman and Schwartz (1982) approach this matter from a different angle and find that the range of the dollar-sterling PPP deviations was substantially smaller for the period 1867–1930 than for the period 1931–1975. The authors conclude:

“The cliché is it has become one world. In the economic world, the reality is clearly the reverse”. Friedman and Schwartz (1982, p. 292)

Moreover, numerous empirical studies stress the difficulty of detecting PPP when only data for the post-Bretton Woods era are employed (Sarno and Taylor, 2002). The high persistence of the real exchange rate during the recent-floating rate period does not contradict the presence of high trade costs.

The second explanation is based on the fact that trade costs are inversely proportional to multilateral resistance variables (see Section 3). These variables are determined by the magnitude of the trade barriers of all the trading partners of a country (Anderson and van Wincoop, 2003). Hence, the slow rate at which the micro-founded measure has been falling may be due to the substantial decline of trade barriers of other countries, bilateral trade agreements, and the formation of trade blocs, such as NAFTA and the European Union. Finally, changes in the trade costs index may reflect shifts in supply and demand preferences rather than changes in the degree of commodity market integration. In this case it is a poor proxy for trade restrictiveness and, therefore, its incorporation in the nonlinear econometric model would most likely not contribute to the explanation of the behavior of the real exchange rate.

4.2. Econometric results

We are interested in examining whether the documented long swings in the trade costs proxy are associated with changes in the persistence of the real exchange rate. To this end, we employ the TVTC-QLSTAR and the standard QLSTAR models.\footnote{Before estimating the models, we run a battery of linearity tests on the real exchange rate series. Specifically, we employ the testing procedures proposed by Teräsvirta (1994), Harvey and Leybourne (2007), and Kapetanios et al. (2003). The first two are general procedures for testing linearity against smooth transition nonlinearity. The main difference between them lies in the fact that the null critical values for the test of Teräsvirta (1994) are based on the assumption of an I(0) process, whilst, the test of Harvey and Leybourne (2007) allows for both I(0) and I(1) processes. We find that the hypothesis of linearity can be rejected at the 5 and 10 percent significance levels, respectively. Finally, the test of Kapetanios et al. (2003) shows that the null hypothesis of a unit root in the real exchange rate against the alternative hypothesis of a globally stationary exponential smooth transition autoregressive process can be rejected at all conventional levels of significance. The results are available upon request. The two models are fitted to the demeaned real exchange rate. The lag length of the autoregressive part and the variables which enter the transition function are specified on the basis of residual diagnostics and, subsequently, the statistical significance of the coefficients of the models. In the estimation procedure we impose the restriction \( \phi_1 = 1 \). This choice is based on the fact that the autoregressive coefficient is not statistically different from unity in the TVTC-QLSTAR model. Further, for the standard QLSTAR model imposing this restriction allows convergence of the nonlinear least squares algorithm.}

Table 1 reports the estimated equations, regression standard errors, t-statistics, the bootstrap p-value for the coefficient on trade costs, the Ljung-Box Q-statistic for serial correlation in the residuals and the LM test statistic (ARCH) for conditional heteroskedasticity up to lags 1 and 5.

Both models appear to provide a parsimonious fit to the real exchange rate in terms of statistical significance and residual diagnostics. However, the incorporation of time-varying trade costs leads to a radically different adjustment process. The fact that the bootstrap p-value corresponding to the band

\[ \text{arch} \]
Table 1
Estimated nonlinear real exchange rate models.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Model</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>QLSTAR</td>
<td>$q_i = 0.014 \times (q_{i-1} + 0.014</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$s = 0.064; Q_1 = 0.141 [0.061]; Q_5 = -0.126 [0.219]; ARCH_1 = 0.535 [0.465]; ARCH_5 = 0.786 [0.561]$.</td>
</tr>
<tr>
<td>B</td>
<td>TVTC-QLSTAR</td>
<td>$q_i = 0.059 \times (q_{i-1} + 0.059</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$s = 0.063; Q_1 = 0.020 [0.787]; Q_5 = -0.154 [0.426]; ARCH_1 = 0.667 [0.411]; ARCH_5 = 0.344 [0.886]$.</td>
</tr>
</tbody>
</table>

Notes: Figures in parentheses and square brackets denote absolute t-statistics and p-values, respectively. The p-value for the coefficients on trade costs $\beta_i$ is obtained through a simulation exercise, where the bootstrap DGP is the fitted ESTAR model. $\hat{\mu} \pm \hat{\beta}_i$ is the standard error of the regression. $Q_1$ and $Q_5$ denote the Ljung-Box Q-statistic for serial correlation up to order 1 and 5, respectively. ARCH_1 and ARCH_5 denote the LM test statistic for conditional heteroskedasticity up to order 1 and 5, respectively.

In order to shed more light on the economic implications of the relationship between trade costs and the persistence of PPP deviations, we follow Lothian and Taylor (1996, 2000) and examine the reaction of the nonlinear process to shocks. The time profile of the impact of a shock on the future behavior of the series is obtained through the application of the Generalized Impulse Response Function (GIRF) proposed by Koop et al. (1996). The GIRF is defined as the average difference between two realizations of the stochastic process, $q_{t+h}$, which start with identical histories up to time $t-1$, but only the first realization is hit by a shock of magnitude $\delta_t$ at period $t$

$$GIRF(h, \delta_t, \omega_{t-1}) = E[q_{t+h} | q_t = \delta_t, \omega_{t-1}] - E[q_{t+h} | \omega_{t-1}],$$

where $h = 1, 2, \ldots$ denotes the horizon, $\delta_t = \omega_{t-1}$ is an arbitrary shock occurring at time $t$, and $\omega_{t-1}$ is the history set of $q_t$.

Fig. 6 depicts the GIRFs for the shock equal to the maximum PPP deviation (left) and half the maximum PPP deviation (right) for all levels of trade costs and a maximum horizon of 20 years. It is clear that the absorption time depends largely on both the level of trade costs and the size of the shock. Regarding the former, low levels of trade costs, and, thereby, narrow bands of inaction, are associated

24 We conduct a bootstrap experiment (the details are discussed in Appendix II) which indicates that the probability of selecting a misspecified TVTC-QLSTAR is smaller than selecting a misspecified TVTC-ESTAR, when the true DGP is given by the alternative model. This finding indicates that the TVTC-QLSTAR model is not only theoretically but also empirically superior to the TVTC-ESTAR.

25 In the computation of the variance of the real exchange rate, the unconditional mean is set equal to the TVTC equilibrium estimate.

26 Given that the GIRF($h, \delta, \omega_{t-1}$) is a function of $\delta_t$ and $\omega_{t-1}$, which are realizations of random variables, the GIRF($h, \delta, \omega_{t-1}$) itself is a realization of a random variable. It follows that various conditional versions of the GIRF can be defined. In this work we set the process initially at its equilibrium value, and we consider shocks of magnitude $\delta_t$ equal to the maximum absolute PPP deviation and half the maximum PPP deviation. Due to the fact that analytic expressions for the conditional expectations involved for the GIRF are not available for $h > 1$, we use bootstrap integration methods. In particular, 1000 repetitions are implemented to average out future shocks, where future shocks are drawn with replacement from the models residuals, and then the results are averaged.
with fast shock absorption and vice versa. While for the latter, in line with the implication of the theoretical analysis, large shocks push the process closer to its boundaries which makes agents anticipate shipment and increases the speed of mean reversion of the real exchange rate. To further illustrate these points as well as to make comparisons with the standard QLSTAR model, we compute half-lives for the maximum and half the maximum PPP deviation.\footnote{The half-life is defined as the minimum horizon beyond which the difference between the impulse responses at all longer horizons and the ultimate response is less than or equal to half of the difference between the initial impact and the ultimate response (van Dijk et al., 2007).}

In the case of the standard QLSTAR model, half-lives depend solely on the magnitude of the shock due to the assumption of time-invariant costs. In particular, as shown by the dashed lines in Fig. 7, the real exchange rate process absorbs 50 percent of the maximum PPP deviation in four years, and 50 percent of half the maximum PPP deviation in eight years. The former half-life estimate lies within the three to five years range usually reported in studies employing long spans of data and linear techniques (see, e.g., Abuaf and Jorion, 1990; Rogoff, 1996; Lothian and Taylor, 1996). Nonlinear real exchange rate studies than use higher frequency data find a faster expected rate of decay for PPP deviations. However, Paya and Peel (2006) illustrate that aggregation of nonlinear models affects persistence. This creates difficulties in comparing half-lives across different frequencies and, hence, comparing our results with monthly or quarterly studies.

Turning to the TVTC-QLSTAR model, year-specific half-lives for the two shocks considered are represented by the solid lines in Fig. 7. It is evident that relaxing the constancy assumption gives rise to substantial differences across historical periods, and the behavior of half-lives over time is similar to that of the estimated trade costs index.

The two world wars and the wave of protectionism characterizing the first half of the 20th century are associated with long half-lives. When trade costs reach their peak, i.e. during the Great Depression and around World War II, the estimated half-life is about 11 years for the maximum PPP deviation and about 19 years for half the maximum PPP deviation. In this economic environment, there is very low predictability of the real exchange rate and deviations from PPP, although volatile in the short run, can last for decades.\footnote{The TVTC-QLSTAR model predicts a finite band of inaction even for the highest level of trade costs. Hence, the real exchange rate process, although very persistent in some periods, is always globally mean-reverting. We show in Appendix I that this prediction is not shared by the TVTC-ESTAR model. For very high trade costs, such as the ones experienced during the Great Depression and World War II, the TVTC-ESTAR suggests near-unit root behavior of the real exchange rate (irrespective of the PPP deviation) which implies that the economy is in a state of autarky. This unappealing implication can be attributed to the inability of the ESTAR model to approximate the $q$ process when it behaves similar to a unit root for a wide range of PPP deviations around the equilibrium and outside this range it exhibits strong mean reversion, i.e. a process similar to a three-regime TAR with large thresholds.} Linear and nonlinear models which utilize data for the whole period (1830–2005) but do not take into account changes in the economic environment will falsely suggest relatively fast
mean reversion and a high degree of real exchange rate predictability. This can be seen by subtracting the half-life for the QLSTAR model from those for the TVTC-QLSTAR model. The difference is 7 (11) years for the large (half the large) shock and for the maximum level of trade costs. As we will discuss in the following section, this finding is important because knowledge of the degree of predictability of the real exchange rate is critical for economic decision making.

Focusing on the first era of globalization, the TVTC model predicts substantially narrower bands of inaction due to the lower degree of trade restrictiveness. In this period, reversion to PPP is more apparent and the real exchange rate less volatile than in the first half of the 20th century. For large shocks, the estimated half-lives always equal their minimum value of two years, which is below the range of half-lives suggested by linear and standard nonlinear models. Whilst, for the shock equal to half the maximum PPP deviation, half-lives range from two to seven years. In order to highlight the effect of trade costs on the real exchange rate process, we note that the difference between the minimum half-life, which occurs in the second half of the 19th century, and the maximum half-life, which occurs in the first half of the 20th century, is 17 years.

During the post-Bretton Woods era, the estimated half-lives for large PPP deviations are of the same magnitude as in the second half of the 19th century, two years. The effect of higher trade costs over the recent period on the real exchange rate becomes evident when we consider the shock equal to the maximum PPP deviation. In this case, PPP deviations decay at a much slower rate for the recent floating rate period than for the first era of globalization. This result indicates that the persistence of the real exchange rate, although not extreme, remains high in the late 20th century, which is in line with the findings of the empirical literature.

In summary, our results show that large swings in the level of trade costs over the last centuries have been associated with substantial changes in the range (band), and thereby the persistence, of PPP deviations. Hence, the difficulty of detecting PPP and the extent of Rogoff’s puzzle are both associated with time-varying factors that determine trade costs. Linear and standard nonlinear models cannot capture the documented behavior of the real exchange rate.

5. Policy implications

On the basis of the analysis reported in this paper and other recent contributions a policy maker can confidently assume that the real exchange rate is a stationary mean reverting process. Moreover, the magnitude of shocks to the process will determine in conjunction with trade costs how fast that mean reversion will take place. The policy implication is that a reduction of trade costs, ceteris paribus, will increase the speed with which the real exchange rate mean reverts and, thereby, its predictability and reduce its unconditional volatility. These properties are of potential importance in a number of areas because, as Jorion and Sweeney (1996) point out, from the perspective of multinational firms, policy makers, currency traders and others what is critical is the prediction of the future behavior of the real exchange rate.
In the context of monetary policy, when a central bank is concerned with the level of PPP deviations, the real exchange rate or its expected future deviation will appear on the right hand side of the Taylor rule equation (see, e.g., Svensson, 2000; Taylor, 1999, 2001). This implies that the behavior of the real exchange rate affects the frequency and magnitude of the response of the policy instrument and, hence, makes real exchange rate volatility or lower predictability undesirable.

PPP deviations appear to matter also for foreign exchange traders. Cheung and Chinn (2001) report that about 40 percent of the traders believe that PPP is relevant to exchange rate prediction for horizons exceeding six months. The length of the horizon will in part reflect the persistence of the real exchange rate.

At the micro level, in order to deal with exchange rate exposure (Adler and Dumas, 1984; Jorion, 1990) firms can devote resources to risk management either through financial derivatives or operation hedges or both (see, e.g., Allayannis et al., 2001). Of course the resources devoted to such practices depend on the volatility of the real exchange rate and, hence, the range of PPP deviations.

Some empirical work suggests that real exchange rate volatility has negative effects on the volume of exports (e.g., Chowdhury, 1993). Intuitively, because the exchange rate is agreed before the delivery date at which payment occurs, uncertainty about real exchange rate movements induces uncertainty about future profits. From this perspective higher volatility lowers the incentives for trade and makes trade policies aiming at export expansion less effective (Arize et al., 2000).

Regarding the labor market, Klein et al. (2003) illustrate how real exchange rate movements can impact on job reallocation and affect -through job destruction- aggregate net employment. In addition, Campa and Goldberg (2001) derive a theoretical model of labor demand where wages and employment depend on the degree of persistence of the real exchange rate. Focusing on the U.S. manufacturing industry, the authors provide empirical evidence that exchange rate movements have a small but statistically significant effect on different measures of labor market activity.

The examples above illustrate how greater predictability and lower unconditional volatility of the real exchange rate can help a variety of decision makers. It would appear that by incorporating the level of trade costs in empirical real exchange rate model we can enhance the decision making process of both practitioners and policy makers.

6. Conclusion

During the last decades the validity and usefulness of PPP as a long-run equilibrium condition has been a topic of debate amongst international economists. Perhaps the most important theoretical explanation for the well-documented persistence of PPP deviations is based on the presence of trade frictions. Trade frictions create a band of inaction around the equilibrium exchange rate in which PPP deviations are not instantaneously corrected because the benefits of trade do not outweigh the costs. In this setting, the real exchange rate is mean reverting in a nonlinear fashion. A typical assumption in the nonlinear PPP literature is that trade costs, and, thereby, the range of the band of inaction, are time-invariant.

This paper, for the first time in a time series analysis of the real exchange rate, relaxes this assumption and focuses on the often ignored role of changes in the degree of commodity market integration in the behavior of the real exchange rate. We first approach the issue theoretically. By utilizing the spatial economy model of Dumas (1992), we obtain the exact functional form of the PPP adjustment mechanism and its relation with the degree of trade restrictiveness. Not surprisingly, the theoretical model predicts that the range of the band of inaction is positively related with the degree of trade frictions.

Turning to the empirical modeling of PPP deviations, we show that a simple modification of a relatively neglected econometric model, namely the QLSTAR, can closely approximate the theoretical adjustment mechanism postulated by the analysis of Dumas (1992). By fitting the model to

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29 Theoretically, foreign exchange risk could be completely hedged by using forward contracts. In reality, however, there are limitations (e.g., traders do not have access to forward markets, the size of forward contracts is relatively large) and costs.
the Dollar-Sterling real exchange rate and a micro-founded index of trade restrictiveness for the period 1830–2005, we find a significant and economically meaningful empirical relationship between movements in trade costs and the size of the band of inaction. The empirical approach adopted is supported by a battery of statistical tests and simulation methods. Most importantly, supporting the theoretical insights, our empirical results reveal that the behavior of the real exchange rate has varied substantially with the degree of market integration. For instance, a given shock to the real exchange rate was absorbed much faster during the first era of globalization than during World War II, with the estimated difference in half-lives being around 17 years. This suggests that the difficulty of detecting long-run PPP empirically and the nature of the puzzle of Rogoff have varied substantially over time.

Although trade costs appear to have declined since the second World War, their magnitude is still significant. Consequently, our empirical results are also consistent with the documented high persistence of real exchange rates in the post-Bretton Woods era. These findings are useful for policy makers and practitioners who are interested in understanding the forces contributing to real exchange rate movements and also for predicting movements in future real exchange rates.

Acknowledgments

We are grateful to the editor, James R. Lothian, two anonymous referees, seminar audiences at Rutgers, East Carolina, and Sheffield, and participants at the 17th Annual Symposium of the Society for Nonlinear Dynamics and Econometrics, the 25th European Economic Association Congress, the Royal Economic Society 2010 Annual Conference, and the 13th Annual International Conference on Macroeconomic Analysis and International Finance for constructive comments. Ivan Paya acknowledges financial support from the Spanish Ministerio de Educacion y Ciencia Research Project ECO2008-05721/ECON.

Appendix I. The exponential smooth transition autoregressive model

The ESTAR model in the presence of constant trade costs

An Exponential STAR (ESTAR) model parameterization which has been used extensively in the PPP literature is given by

$$q_t - \mu = \sum_{i=1}^{p} \phi_i (q_{t-i} - \mu) F(\cdot) + \epsilon_t,$$

where $\mu$ is a constant representing the long-run equilibrium, $\epsilon_t$ is a white noise process with mean 0 and variance $\sigma^2$, and $F(\cdot)$ is the transition function. Like the QLSTAR model, for a given AR structure, $\sum_{i=1}^{p} \phi_i$, the transition function, $F(\cdot)$, specifies the degree of persistence of the real exchange rate at each point in time. The transition function is given by

$$F(q_{t-d}) = \exp\left(-\gamma^2 (q_{t-d} - \mu)^2\right),$$

where $q_{t-d}$ is the transition variable and $\gamma > 0$ is the smoothness (or transition) parameter. The exponential transition function shares many of the properties of the quadratic logistic. In particular, it implies symmetric adjustment for positive and negative deviations from the equilibrium with the speed of adjustment increasing with the smoothness parameter $\gamma$ and the absolute value of the past deviation from the equilibrium. For expositional reasons, let $\sum_{i=1}^{p} \phi_i = 1$. In this case, if in equilibrium $F(\cdot) = 1$ and the real exchange rate behaves as a unit root process, $q_t - \mu = \sum_{i=1}^{p} \phi_i (q_{t-i} - \mu) + \epsilon_t$. Whilst, for nonzero deviations $F(\cdot) \in (0, 1)$ and the process becomes mean reverting. Finally, in the extremely unlikely case that $|q_{t-d} - \mu| \rightarrow \infty$ the function value approaches zero and the process becomes a white noise, $q_t - \mu = \epsilon_t$. The speed of transition between regimes is specified by the smoothness parameter $\gamma$. If $\gamma$ is equal to zero the real exchange rate behaves as a linear unit root process irrespectively of the regime. Whilst, if $\gamma \rightarrow \infty$ the process is a white noise. Intermediate values of $\gamma$ imply smooth adjustment of the real exchange rate.
In order to examine the ability of the ESTAR model to approximate the theoretical adjustment mechanism of the model of Dumas (1992), we employ the procedure of Section 2.2. This yields the following econometric relationship between the magnitude of PPP deviations and the expected change in the real exchange rate series:

$$E_t(\Delta q_t) = C_0 q_t/C_1 e^{C_2 q_t/C_3}$$

The values for the theoretical expected change used in Section 2.2 (solid line) and the corresponding fitted values of the ESTAR model (circled line) are depicted in Fig. 8. As can be seen by comparing Figs. 1 and 8, the ESTAR model cannot approximate as closely as the QLSTAR model the nonlinearity of the real exchange rate postulated by the theoretical analysis. Specifically, the model predicts a faster (slower) reversion rate than the theoretical one for small (large) deviations.

The ESTAR model in the presence of time-varying trade costs

Time-varying trade costs can be incorporated into Equation (16) by allowing $\gamma$ to change over time depending on $\tau_{t-d}$. By assuming a linear relationship between the value of the smoothness parameter and trade costs, the transition function for the Time-Varying Trade Costs ESTAR (TVTC-ESTAR) is given by

$$G(q_{t-d}, \tau_{t-d}) = \exp(-C_4 q_{t-d})$$

where the coefficient, $\gamma$, on trade costs is greater than zero and $\gamma \geq C_4 \forall t$. The above equation allows both the degree of trade restrictiveness and the size of the deviation from the equilibrium to determine the speed of adjustment of the real exchange rate at a particular point in time.

We employ again the methodology of Section 2.2 to obtain the econometric relationship between the expected change of the real exchange and the PPP deviation as well as the ability of the TVTC-ESTAR model to approximate the theoretical adjustment mechanism. Fig. 9 displays the theoretical expected change (solid line) and the fitted values of the TVTC-ESTAR model (circled line). The difference between the two lines illustrates the inability of the TVTC-ESTAR model to approximate the time-varying band of inaction of the real exchange rate process. Like the ESTAR model, the TVTC-ESTAR predicts a higher (lower) degree of persistence than that predicted by the model of Dumas for large (small) PPP deviations.
Econometric results

The results for the ESTAR models with constant and time-varying trade costs are reported in Table 2. In the estimation procedure we impose the restriction $f_1 = 1$. This choice is based on the fact that the AR coefficient is not statistically different from unity in the estimated ESTAR and TVTC-ESTAR models. Most importantly, the statistical significance of the estimated coefficients $\hat{\gamma}$ and $\hat{\gamma}_s$ indicates that (i) the real exchange rate series is a nonlinear process and (ii) that movements in trade costs and the persistence of the real exchange rate are related.

We now turn to the investigation of the effect of changes in trade costs on the persistence of the real exchange rate. Fig. 10 depicts GIRFs for two shocks, one equal to the maximum PPP deviation and the other equal to half the maximum PPP deviation. We consider all levels of trade costs and set the maximum impulse response horizon to 20 years. Like for the TVTC-QLSTAR model, the broad conclusion is that the absorption time increases with the level of trade costs and decreases with the magnitude of the shock. It is worth noting, however, that when the level of trade costs is high, shocks fade out extremely slowly for the TVTC-ESTAR model.

This is also evident from the estimated year-specific half-lives for the TVTC-ESTAR model (solid lines) presented in Fig. 11. During the Great Depression and the mid 20th century, the TVTC-ESTAR model predicts unrealistic values for the half-lives irrespective of the size of the shock. The

Table 2

<table>
<thead>
<tr>
<th>Panel</th>
<th>Model</th>
<th>Equation</th>
<th>$s$</th>
<th>$Q_1$</th>
<th>$Q_5$</th>
<th>ARCH1</th>
<th>ARCH5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>ESTAR</td>
<td>$q_t + 0.016 = (q_{t-1} + 0.016) \exp(-0.505(q_{t-1} + 0.016)^2)$.</td>
<td>0.064</td>
<td>0.140 [0.642]</td>
<td>-0.127 [0.227]</td>
<td>0.557 [0.456]</td>
<td>0.802 [0.550]</td>
</tr>
<tr>
<td>B</td>
<td>TVTC-ESTAR</td>
<td>$q_t - 0.066 = (q_{t-1} - 0.066) \exp((-3.552 - 0.324 q_{t-2} - 0.016)^2)$.</td>
<td>0.043</td>
<td>0.035 [0.642]</td>
<td>-0.161 [0.314]</td>
<td>1.538 [0.217]</td>
<td>0.538 [0.747]</td>
</tr>
</tbody>
</table>

Notes: Figures in parentheses and square brackets denote absolute t-statistics and p-values, respectively. The $p$-value for the coefficients on trade costs $\hat{\gamma}_s$ is obtained through a simulation exercise, where the bootstrap DGP is the fitted ESTAR model. $s$ is the standard error of the regression. $Q_1$ and $Q_5$ denote the Ljung-Box Q-statistic for serial correlation up to order 1 and 5, respectively. ARCH1 and ARCH5 denote the LM test statistic for conditional heteroskedasticity up to order 1 and 5, respectively.
maximum value is about 70 years, which implies that shocks to the real exchange rate have essentially a permanent effect. This prediction seems unrealistic since it suggests that during these periods agents did not have an incentive to trade even for extremely large PPP deviations. From a market efficiency point of view, it implies that arbitrage opportunities could not occur irrespective of the magnitude of the PPP deviation because of the high level of trade costs.

We argue that this unappealing prediction is solely due to the econometric specification and, in particular, it can be attributed to the fact that the TVTC-ESTAR model cannot approximate the PPP

Fig. 10. GIRFs for the TVTC-ESTAR model. The left (right) graph corresponds to a shock equal to the maximum PPP deviation (half the maximum PPP deviation).

Fig. 11. Time series plots of half-lives for the ESTAR model with constant (dashed line) and time-varying (solid line) trade costs. The left (right) graph corresponds to shocks equal to the maximum PPP deviation (half the maximum PPP deviation).

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Fig. 12. Plots of the estimated exponential (left) and quadratic logistic (right) transition functions for $q_{t-2} - \mu \in \{-0.5, \ldots, 0.5\}$ and trade costs corresponding to the years 1900, 1950 and 2000.
adjustment mechanism when the real exchange rate exhibits near-unit root behavior for a wide range of PPP deviations around its equilibrium and outside this range it becomes mean reverting, i.e. similar (but not identical) to a three-regime TAR with large thresholds. This point is illustrated in Fig. 12, which displays the transition functions of the TVTC models for three time periods, namely 1900, 1950 and 2000. The period of interest is 1950 because of the very high trade costs level, while the remaining two periods exhibit moderate trade costs and are included for comparison purposes. As we can see from the figure, when trade costs are set equal to their mid-twentieth-century value, the exponential function takes a value close to unity irrespective of the magnitude of the PPP deviation. Hence, the real exchange rate behaves as a near-unit root process globally. On the contrary, the quadratic logistic model indicates that the range of the band of inaction although large is finite and mean reversion takes place for large PPP deviations.

Appendix II. Econometric model specification

We conduct two bootstrap experiments. For each experiment, we employ one of the estimated TVTC models (reported in Tables 1 and 2), the original trade costs series and the corresponding estimated residuals so as to generate 1000 artificial samples of size 176. We initialize the bootstrap DGP by using the first observations of the original real exchange rate series. In turn, we fit the rival model to each artificial sample and compute the \( t \)-statistic for the coefficient on trade costs (either \( \gamma_t \) or \( b_t \)), \( t_b \). This provides the empirical distribution for the \( t \)-statistic under the null hypothesis that the true DGP is given by the competing model. The probability of obtaining a \( t \)-statistic as extreme as the original is

\[
p_b = \frac{1}{B} \sum_{b=1}^{B} I(\hat{t}_b \leq t_b),
\]

where \( I(A) \) is the indicator function, which takes the value of 1 if event \( A \) occurs and 0 otherwise, and \( \hat{t}_b \) is the original \( t \)-statistic. When the DGP is the TVTC-ESTAR model, the probability of the \( t \)-statistic for \( b_t \) exceeding the actual one, 4.488, is only 13.8 percent. Whilst, when the DGP is given by the fitted TVTC-QLSTAR, there is a 52.1 percent probability that the value of the \( t \)-statistic for \( \gamma_t \) is greater than 3.145. Hence, it is very likely to obtain a \( t \)-statistic for the coefficient on trade costs in the TVTC-ESTAR model as extreme as the original when the DGP is given by the estimated TVTC-QLSTAR model. However, the opposite is not true. Put it differently, the probability of selecting a misspecified TVTC-ESTAR is greater than selecting a misspecified TVTC-QLSTAR, when the true DGP is given by the alternative model.

References


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