REPORT OF GROUP L2

e members of this group and authors of this report are:

Keith Austin, University of Sheffield

Christine Davies, Royal Holloway and Bedford New College

Ken Houston, University of Ulster

Peter Oates, Macmillan Education Ltd

Pip Surgey, BBC/Open University

Mike Thomlinson, Sheffield City Polytechnic

David Towers, University of Lancaster (Chairperson)

1. Introduction

The group agreed with the proposition of the brief that teaching and learning is dominated by exposition by the lecturer. However, it was less willing to accept that the exercises provided for consolidation were all closely imitative of the illustrative examples given by the lecturer. Indeed, it was felt that some exercises were not imitative enough and were therefore too hard and led to a loss of confidence in students.

Further, the group found that teaching and learning was by no means limitated to exposition and consolidation. Teaching and learning methods and activities which follow Cockcroft's recommendations were found already to be practised by members of the group. These are defined below.

In order to consider how the balance of these learning activities and the more traditional activities might be improved, the group first identified a number of aims which it felt all undergraduate courses in mathematics should meet. The group then tried to identify which learning activities could best be used to meet those aims.

2. Methods used in the teaching and learning of mathematics at undergraduate level

A **Exposition by the lecturer**, using a range of technologies, such as overhead projector, microcomputer, slides, audio and video tape, and combinations of these.

B **Worksheets** - routine exercises and non-routine problems.

C **Tutorial sessions** involving discussion between lecturers and students, and between students themselves.

D **Examples classes**, where students receive individual help with worksheets.

E **Reading course**

Type 1: self-paced learning course.

Type 2: Each student has a copy of the course text. The lecturer states what is to be read by the next contact hour and spends that hour discussing the material.

F Practical work/workshops - longish, student-centred practical sessions of an investigative nature (see appendix 1).

G Projects - individual and group projects. These are student-led activities involving investigation, modelling, problem-solving or precis writing, and require written reports (see appendix 2).

H Case studies - structured studies of the applications of mathematics to real scientific and industrial problems (see appendix 3).

I Critical analysis - criticism and comparative evaluation of school and undergraduate text books, articles (including research papers), historical texts, and other students' arguments in tutorials, homework, projects and case studies.

3. Aims and methods of achieving them

We list below a set of aims which all agreed were essential for every undergraduate mathematics course. We were guided by Shannon and Sleet (1), but rejected some of their aims as being desirable rather than essential, and modified others. Against each we have listed the learning activities which, taken together, we felt did most to further that aim in a reasonably economic way. We should emphasise that in each case there may be other activities which address the aim, but they were felt by the group to be inferior in some sense to those listed.

AIMS	LEARNING ACTIVITIES
KNOWLEDGE	
1. To learn certain basic facts in mathematics	exposition by lecturer, reading course, descriptive project.
2. To master certain basic techniques in mathematics	work sheets, examples classes.
3. To develop an understanding of the applications of mathematics in industry, to environmental problems, to other subjects, and to everyday experience	all, but particularly modelling and case studies
COGNITIVE SKILLS	
4. To develop an ability to learn independently	projects (perhaps supplemented by courses on study skills), reading course.
5. To develop logical thinking through the study of mathematics	all.
6. To learn to obtain relevant information from the literature and other sources	projects, discussion, reading course.
7. To develop the ability to apply mathematical principles to new situations	investigations, modelling, work sheets.
8. To develop an ability to visualise and work with situations in three dimensions	computer graphics, exposition, work sheets, projects.
9. To develop an ability to solve problems in mathematics using basic principles and concepts	investigations, work sheets, modelling, problem solving.
10. To develop the ability to criticise and evaluate a mathematical argument, and to learn to take criticism	critical analysis, case studies.
11. To learn to think in abstract terms (that is, to think about things that cannot be seen or touched) through the study of mathematics	all.

12. To develop an awareness of the processes involved in modelling	modelling, case studies.
13. To develop an ability to deduce relationships from data	investigations, workshops.
14. To develop an ability to communicate mathematics	project reports, small group discussions.
COMPUTING SKILLS	
15. To develop confidence and competence in using software packages and hardware	appropriate practical work.
16. To develop an ability to use a calculator	appropriate practical work.
COMBINED SKILLS	
17. To develop an ability to use mathematics in research investigations	projects.
18. To develop an ability to work in a team	group projects, group discussions.
ATTITUDES	
19. To develop a willingness to consider various careers	Need to experience a variety of teaching methods; maths club.
20. To enjoy mathematics	(as 19)
21. To develop a desire to know more about mathematics	(as 19)
22. To become aware of the importance of accuracy in reports	projects.
23. To become aware of the importance of accuracy in calculations	all, particularly consolidation and practice, work sheets, practical work. (Here, marking and feedback are important.)
24. To develop confidence in using mathematics	group work, well-structured work sheets, problem solving, investigations.

Summary

	A	B	C	D	E	F	G	H	I
1	✓				✓		✓		
2		✓		✓					
3	✓	✓	✓	✓		✓✓		✓✓	✓
4					✓		✓		
5	✓	✓	✓	✓	✓	✓	✓	✓	✓
6			✓		✓		✓		
7		✓				✓	✓	✓	
8	✓	✓				✓	✓		
9		✓				✓	✓	✓	
10			✓					✓	✓
11	✓	✓	✓	✓	✓	✓	✓	✓	✓
12	✓					✓	✓	✓	
13						✓	✓	✓	
14			✓				✓		
15						✓			
16						✓			
17							✓		
18			✓				✓		
19 } 20 } 21 }				VARIETY					
22							✓		
23	✓	✓✓	✓	✓	✓	✓✓	✓	✓	✓
24		✓	✓			✓	✓		

4. Recommendations

All of the learning activities outlined above should be used as described to achieve the aims. Hence we make the following recommendations.

(1) That present traditional exposition/work sheet courses be adapted by the removal of some lectures and their replacement by other activities like group mini projects and case studies. There will be a different mix of these activities for different courses. An example is given in appendix 4.

(2) The mode of assessment for a particular course must reflect the balance of activities in that course. For example, for an algebra course the weighting of coursework assessment to written examination could be 25:75, whereas for a course in applied mathematics and modelling the ratio could be reversed.

(3) All students must carry out an extended project.

(4) Assessment of projects poses several problems. There tends to be a narrower spread of marks, say, between 3 and 8, and a variation in marks awarded to projects of similar standards; a good project might be awarded 70% on one course, but 85% on another. There is also a problem in allocating the proportion of marks on a five scale; breaking down 100 marks into units of 5 marks is difficult. Therefore, the group recommends that projects be awarded one of the five degree classifications (First, Upper Second, Lower Second, Third and Pass).

The group further recommends that marking schemes should be devised which allocate marks on broader lines; for instance, 30% for the description, and 70% for the execution of a particular project. Within these, it should be clear to both lecturer and student what particular aspects are being assessed. For example, the 70% could include such things as originality, initiative, and research.

(5) Mathematics departments should evaluate the degree to which the aims listed above are achieved by the chosen teaching methods. The evaluation could be carried out by considering the results of assessments and by questionnaires and discussion.

(6) Consideration should be given to the introduction of student profi these would be helpful both to the student during the course, and to sh to an employer afterwards.

Reference

(1) Shannon, A.G. and Sleet, R.J., "Staff and student expectations of undergraduate mathematics courses", Int. J. Math. Sci. Technol. 9(1978), 239-247.

Appendix 1

Practical work/Workshops

Practical work can come in a variety of forms, either in the context of a "workshop", or perhaps of collecting data "in the field". Although workshops mean different things to different people, the consensus definition seems to be a longish session - probably a whole morning or afternoon - where the learning is student-centred and of an investigative nature. Many workshop sessions will involve the use of a micro, typically a standard piece of software which will enable students quickly to try out the effect of altering parameters in families of curves, or to try out different methods of numerical integration, etc. After a period of investigation - structured to a greater or lesser extent by accompanying worksheets - conclusions may be written up, or drawn together by discussion. In this type of exercise the lecturer acts very much as a consultant to be used as needed by the students. Alternatives to the use of a micro may be, for example, physical experimentation with models of surfaces, or experimentation with physical systems or analogue models.

Practical sessions include, as well as the workshop type of exercise, data collection and analysis. For instance, in a statistics course, real "Poisson type" data may be collected by observing cars passing a particular point, or data from a variety of distributions can be simulated using a micro. Experiments can also be done as a precursor for modelling sessions in which the experimental situation is described in terms of mathematics.

Appendix 2

Projects

A "project" is a learning activity which is more "student-led" than "teacher-led" in that the student has to take more responsibility for the development of the work. While the topic of the project may be outlined by the teacher, it may be suggested by the student, and certainly there should be discussion between student and teacher to detail and agree on the aims and objectives of the project. After that, however, the student should take charge of the activity and while he may use the teacher as a resource person, and while he may respond to criticism from the teacher, he must make his own decision.

The term "project" is used to describe several different, but closely related, activities. The most common activities are investigation, modelling, problem-solving or description, giving a precis of some topic. A project requires the student to communicate his findings in writing. There may or may not be a measure of open-endedness about the objectives of the project and it may be possible to terminate the exercise at different stages.

A "mini-project" usually describes an activity which can be completed in a relatively short period of time, say a fortnight, while an "extended project" might last for six months. A project could be a "group project" wherein a small group of people (say 3 $\pm$ 1) work together to the same end, perhaps sharing out the different tasks, or it could be an "individual project" wherein a student essentially works on their own, using their peers only as resource persons.

Appendix 3

Case studies

Case studies involve the study of a mathematical model, and the modelling process that has been used in the solution of a real problem from industry, or possibly the other sciences. The form taken by a study can vary considerably from the very open-ended, to the very tightly structured, largely depending on the time available for the exercise.

In an open-ended case study, the student will essentially be left with the original problem (in an appropriately watered-down version) that was treated in the industrial or scientific context. The lecturer may give a few hints on the solution method, but will act mainly as an information source, and the students (as individuals or in groups) will work out a solution to the problem from scratch. The final element of the case study will then often be a comparison of the solution method adopted by the students, and that taken when the problem was originally solved.

When operating a more closely structured case study, much more information and guidance will be given in the setting up of the model required for the solution of the problem. This help can be given in various ways, from leading students through by a series of questions to virtually presenting the whole model to them as a "finished product". Once the model is established, there is again a choice as to how open-ended to make its use in the solution of the original problem.

Appendix 4

MATHEMATICS UNIT M125

APPLIED MATHEMATICS AND MODELLING 1

Total Hours: 66		Lectures	35 hours
Total Weeks: 22		Tutorials	16 hours
		Practicals	15 hours

Teaching period: 1 and 2

The aims of this unit are:

(i) to develop skills of mathematical modelling and communication;

(ii) to introduce mathematical models of some phenomena studied in the physical, environmental and social sciences;

(iii) to present Newtonian mechanics as a model of the behaviour of simple systems.

SYLLABUS	Lecture Hours
Vectors and vector functions Vector Algebra in three dimensions including scalar triple product, vector triple product and quadruple products. Vector equations of straight line and plane. Differentiation of vector functions of one variable: displacement, velocity and acceleration of a particle, including radial and transverse components in a plane.	5
Static equilibrium of simple systems Force, moment of force and couple. Equilibrium of a particle and rigid body under the action of coplanar forces.	4
Dynamics of simple systems Axioms of Newtonian dynamics. Straight line motion of a particle under variable force which is (a) time dependent	

(b) displacement dependent (c) velocity dependent. Conservation of momentum; impulse; work; conservation of energy. Direct and oblique impact. Hooke's law. Plane motion of a mass point including use of polar coordinates: the projectile and motion in a vertical circle. Moment of inertia of a system of particles about an axis; calculation of moments of inertia of rigid bodies of simple shape. The balance equations of linear momentum, moment of momentum, and energy for particle systems. Applications to plane motion including the compound pendulum and rigid body motion without a fixed point. Introduction to normal mode vibrations with applications confined to simple systems with two degrees of freedom. 13

Mathematical Modelling

Philosophy and methodology. Analysis of simple systems and the construction, revision, interpretation and verification of mathematical models of these systems. Assimilation of problems amenable to mathematical modelling; identification of features, choice of variables, expression of relations. Solution of mathematical models using appropriate analytical or numerical techniques. Criticism and revision of models. Communication of results by a written report and an oral presentation by groups of students or by individual students as appropriate. 3

[Students will spend about 15 hours on the activity of modelling. Exercises will be like those in Burghes et al.].

Some mathematical models in the physical and social sciences

Population dynamics: single species, Malthus, Verhulst; two species, competition, predator-prey; effects of age structure. 2

Economics: demand, supply; cost, revenue, profit; elasticity, marginal quantities; optimization, equilibrium and stability. 2

Electric circuits: LCR circuits, components, quantities, equations. 2

Kinetic theory of gases: ideal gases, temperature, energy. 2

Heat: specific heat; thermal expansion of solids and liquids; heat conduction. 2

READING LIST

Dynamics, W E Williams (Van Nostrand, 1975)

Principles of Dynamics, M B Glauert (Routledge and Kegan Paul)

Mechanics, R C Smith and P Smith (Wiley and Sons, 1968)

Applying Mathematics, D H Burghes, I Huntley and J McDonald (Ellis Horwood, 1982)

New Uses of Mathematics, ed. J Lighthill, (Penguin, 1978)

Physics, Vols 1 and 2, H C Ohanian, (Norton and Co, 1985)

Concepts of Mathematical Modelling, W J Meyer (McGraw-Hill, 1984)