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Towards a unified conceptual model representation

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Abstract

A good documentation or representation is important for communicating simulation conceptual models between stakeholders effectively. A good conceptual model representation may be used as a legal document in a simulation project contract; it may be used as a model description, which is crucial in a distributed simulation environment for model reusability and interoperability and, finally, it can improve the credibility of the model. This paper proposes a unified representation of simulation conceptual models. A conceptual model comprises a number of components, which can be grouped into the problem domain and the model domain. A number of diagrams and textual representations are proposed to represent each of the conceptual model components. Our intention is to initiate the discussion and development of a unified conceptual model representation that will benefit stakeholders involved in a simulation project. A case study in health-care is used to show how the proposed unified conceptual modelling can be applied in practice.

Keywords: Conceptual model, conceptual model representation, unified conceptual model representation, simulation, health service.

Introduction

It is perhaps true that the phrase "a well thought out plan is half the battle" also applies to computer simulation projects. In simulation modelling, we select a certain portion of the real world system to be simulated for specific objectives. The process of capturing the essential elements of the system is referred to as conceptual modelling and the resulting model is referred to as a conceptual model (Pidd, 2004). In a typical simulation modelling process, the conceptual modelling stage is followed by computer implementation, validation, experimentation and implementation (Pidd, 2004).

The fact that conceptual modelling precedes most activities in a simulation modelling process shows the significance of a conceptual model. It shows that the conceptual model affects the subsequent stages in a simulation modelling process. People may argue that advances in computer simulation software, especially with the introduction of VIMS (Visual Interactive Modelling System), allow simulation modellers to build simulation models directly without the need to produce any conceptual model. Robinson (2006) disagrees and argues that VIMS enables more rapid model development, but it neither reduces nor removes the importance of model design. Despite its significant role, there is a paucity of research into conceptual modelling. The Journal of Simulation, however, recently released a special issue on conceptual modelling for simulation to raise awareness and interest in conceptual modelling. Robinson (2006) listed a number of possible areas for research in conceptual modelling. One of them is conceptual model representation.

This paper proposes a unified conceptual model representation, which specifies all conceptual model components both at the problem domain and the model domain. We hope this paper can initiate the discussion on a unified conceptual model representation and lead to the development of a standard conceptual model representation. Standardization is essential for communicating conceptual models effectively. The rest of this paper is organised as follows. Section 2 outlines the related works. Section 3 discusses the structure of a conceptual model and the proposed representation for each component in the conceptual model. We apply the proposed unified representation to a case study: District General Hospital Performance Simulation in Section 4. Finally, we present our conclusions in Section 5.

Related Works

Conceptual model representation is important for three reasons. Firstly, it is used as a tool for communicating conceptual models between stakeholders (simulation modellers, clients and domain experts). Secondly, it helps conceptual model validation by clients or domain experts. Hence, it may increase the validity and credibility of conceptual models. Lastly, it documents the conceptual model. This document can be very useful in a simulation project. It may, for example, be used as a legal document in the project contract, it can be used for change management should there be new requirements from clients and it can also be used for the project evaluation.

The main challenge in conceptual model representation is to devise a representation that can be understood by all stakeholders and yet which should be expressive enough to handle the varying levels of complexity in the conceptual model. This, in essence, is quite similar to some other areas of research, such as business process modelling in the context of IT within organizations. Nowadays, organizations are becoming dependent on IT. This creates possibilities for re-engineering existing business processes or opens the way for the creation of new business processes that have been made feasible owing to the advances in IT. To effectively (re)design complex business processes, a good process model (representation) is very useful. A number of models have been proposed, for example, Petri-Net (Aalst and Hee, 2002), UML (Eriksson and Penker, 2000), Business Process Modelling Notation or BPMN in short (http://www.bpmn.org/), etc.

Similar development may also be seen in simulation conceptual modelling research. Robinson (2007[b]) summarizes a number of possible representations for a conceptual model, namely: component list, process flow diagram, logic flow diagram and activity cycle diagram (ACD). The component list provides a textual representation of the model. The process flow diagram provides a visual representation of the process flow in the model. The logic flow diagram is a flow-chart showing the logic of the model. ACD is commonly used to represent a discrete-event simulation model. The model is decomposed into series of active and dead states. Pidd (2004) provides a number of good examples on how to represent a conceptual model using an activity cycle diagram. Pooley (1991) presents a good discussion on a number of diagrams that can be used to model process oriented discrete-event simulation.

Conceptual Model Structure

This paper follows the conceptual model definition proposed by Robinson (2007[a]). A conceptual model is a software independent description of the simulation model to be developed. A conceptual model comprises *problem-domain* elements and a *model-domain* element. The problem-domain elements are used as a means of communication mainly between clients/domain experts and a simulation modeller, or between clients and domain experts. These include objectives, inputs and outputs, contents (scope, level of detail, assumptions and simplifications). The model-domain element is mainly used as a means of communication between simulation modellers. At the problem-domain, a decision has to be made to decide whether simulation is appropriate for the objective at hand. At the model-domain, we need to further decide which simulation modelling approach should be used. The model-domain component represents the conceptual model using specific diagrams and terminologies commonly used in the chosen approach, for example, an activity cycle diagram (ACD) or event graph in discrete-event simulation, or stock and flow diagram in system dynamics. These diagrams may make the implementation of a conceptual model easier.

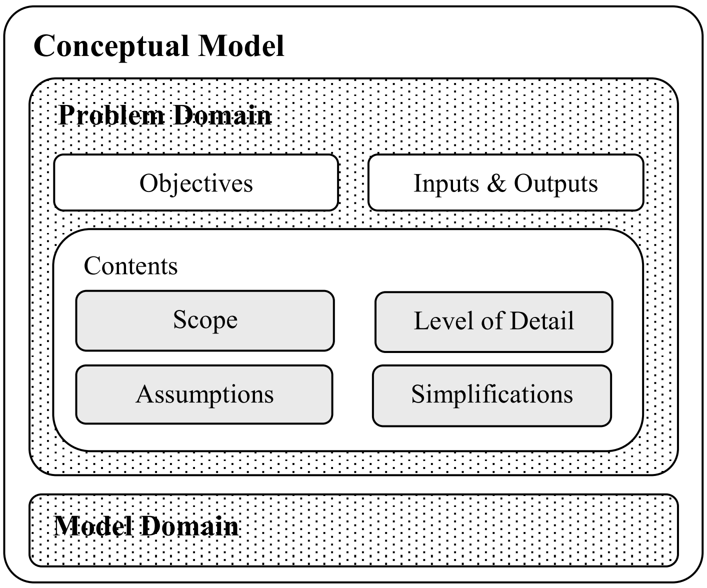

Figure 1: Conceptual Model Structure

Figure 1 shows the components of a conceptual model. Most of the components have been explained in detail by Robinson (2007[b]). The main difference is in the structure of the conceptual model. In this paper, the contents component comprises scope, level of detail, assumptions and simplifications, while Robinson puts assumptions and simplifications outside the contents. The rationale behind our structure is that the assumptions and simplifications are made in the context of scope and level of detail. They, together with the scope and level of detail, determine the structure of the model. The proposed representation of each component is explained in the following sections based on the explanation of the flow of the conceptual modelling framework in Robinson

(2007[b]). Hence, it will be easier for readers to see how these representations are used in the conceptual modelling process.

Objectives Component

The objective is the most important element in any modelling process, including simulation. Simulation is commonly used in the larger context of a problem solving exercise. In this context, objectives are used to judge the success of the exercise and to compare the quality of various decision alternatives. Therefore, the first component in a conceptual model is rightfully given to the objectives of the study.

The objectives component can be represented using the *objective diagram* (Keeley, 1992). An objective diagram is commonly used to structure objectives in decision science. It classifies objectives into *fundamental objectives* and *means objectives*. The fundamental objectives are the end result that we want to achieve and they are organized into hierarchies. In an objective diagram, each fundamental objective is represented as a node in a tree. The higher-level fundamental objectives represent the more general objectives and their measurement can be obtained from the lower-level fundamental objectives. Thus, the lowest-level fundamental objectives provide the basis on which various design alternatives are measured. Consequently, the highest-level fundamental objective provides the ultimate measurable consequence that will be used to evaluate and compare various design alternatives. Figure 2 shows an example of fundamental objectives in a hospital simulation project. The ultimate objective is to improve the overall hospital performance. Let us assume the performance refers to waiting time of patients at the hospital, which is the average of waiting time of patients in its departments: A&E (Accident and Emergency or Emergency Room in the US), outpatient and in-patient. This measurement will be used to compare alternatives. The second-level fundamental objectives can be further expanded if necessary.

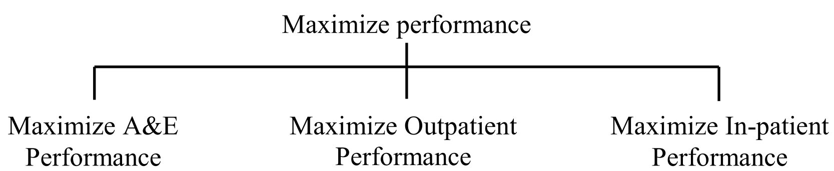

Figure 2: Hospital – Fundamental Objectives

Means objectives are important because they help us to achieve fundamental objectives and they are commonly used when the fundamental objectives are difficult to measure directly. In some cases, identifying means objectives can help us to characterize new alternatives. In the objective diagram, means objectives are organized into networks. Figure 3 shows an example of means objectives of an in-patient simulation project. The top-level objective is the fundamental objective, i.e., maximizing in-patient department performance. It can be further expanded into two fundamental objectives: minimizing the waiting time for emergency and elective admissions. The next two objectives are means objectives. Maximizing number of day cases and reducing the patient lengths of stay are important because they will increase the number of available beds. Hence, it may reduce the waiting time of both emergency and elective admissions.

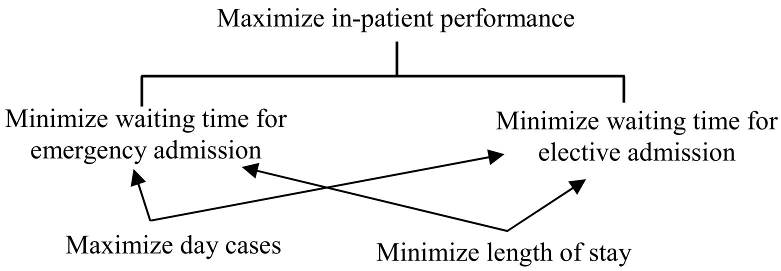

Figure 3: Hospital – Fundamental and Means Objectives

The fundamental objectives can be differentiated from means objectives by continuously asking the question why an objective is important. The objective is a means objective if it is important because it helps achieve another objective. The same question is repeated until we find an answer where an objective is important because it is important. Detailed explanation on the objective diagram can be found in Clemen and Terry (2005).

Inputs and Outputs Component

Once the objectives have been defined, we need to translate them into output variables that can be quantified and to identify the different alternatives (input variables) to achieve the objectives. The controllable input variables are sometimes referred to as the decision variables. The inputs, outputs and contents (the subject of the next section) are used to represent the top-level conceptual model where inputs are transformed into outputs, as shown in figure 4. The outputs can be directly inferred from the objectives. The inputs are sometimes specified explicitly in the objectives; otherwise, they can be obtained from the clients. Robinson (2007[a]) provides a more detailed figure of a conceptual model in the context of simulation project life cycle.

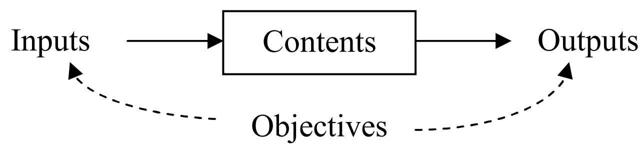

Figure 4: Top-level conceptual model

An influence diagram (Howard and Matheson, 1984) is one of the diagrams that can be used to represent inputs and outputs in conceptual modelling. An influence diagram is commonly used to structure decisions by representing the relationship among key variables. An influence diagram consists of certain elements as follows. *Decision variables* represent the decisions to be made (symbolized as rectangles in the diagram). *Uncontrollable variables* represent uncertainty or chance events (ovals). *Outputs* represent final consequences or payoffs (diamonds). *Intermediary variables* including calculation nodes and constants are used to compute the outputs (rounded rectangles). Relationships between nodes are represented using arcs. All arcs pointing to a rectangle (decision variable) show *sequences*. It means that the node at the beginning of the arc must be known before the decision can be made. All arcs pointing to ovals, diamonds or rounded rectangles (non-decision variables) show the *relevance* relations. The node at the beginning of the arc is relevant for the node at the end of the arc.

Figure 5 shows the inputs and outputs of an in-patient department simulation model represented using an influence diagram. The output of the model is the performance of the in-patient department, which is computed from the percentage of patients waiting up to a certain threshold for both types of admissions (emergency and elective). The inputs are as follows. The number of admissions from the A&E department (uncontrollable) directly affects the performance for emergency cases. When the number of beds is insufficient, some elective admissions can be cancelled; hence, indirectly, it also affects the performance for elective cases. The number of beds affects the performance of both admission types. The scheduling policy and the number of admissions from the outpatient department (uncontrollable) affect the performance of the elective cases. The objectives hierarchy is reflected in the sequence of output and intermediary variables in the influence diagram. From Figure 5 we know that the top-level objective is to maximize the performance of the in-patient department. This is followed by the second-level objective, i.e., to minimize the long waiting time for patients (in both emergency and elective admissions).

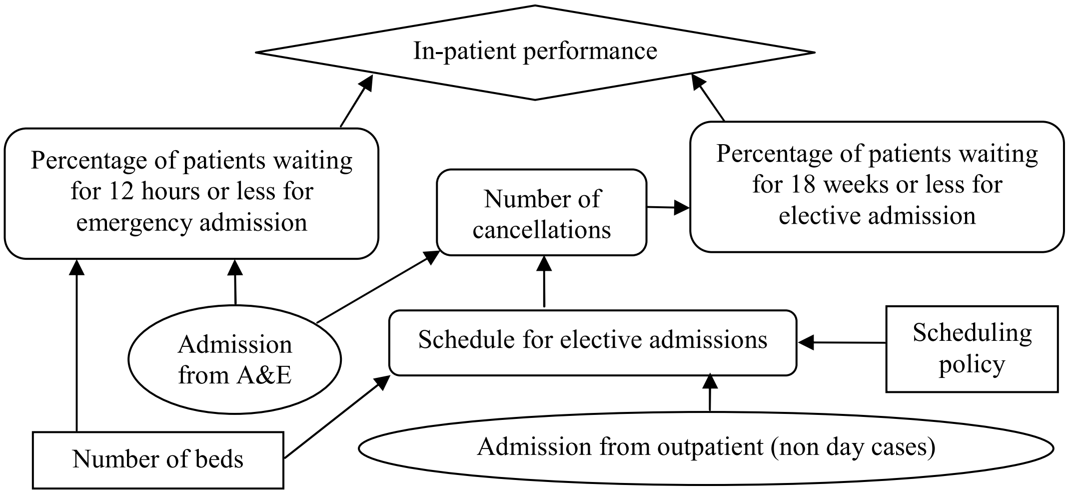

Figure 5: Hospital – Influence Diagram

A decision tree (Holloway, 1979) can be used as an alternative to represent inputs and outputs in a conceptual model. It shows more information than an influence diagram. In our opinion, this diagram is less suitable for conceptual model representation for two reasons. Firstly, as the level of detail is increased, the inputs and outputs become more complicated and the decision tree tends to become muddled much faster than the influence diagram. Secondly and most importantly, an influence diagram is easier to understand for people without mathematical training. Having said that, influence diagrams and decision trees are isomorphic, hence they can be converted to one another.

Contents Component

Once the inputs and outputs have been specified, the next step is to specify the transformation processes or the contents. The contents component of a conceptual model contains the scope of the model, the level of detail, assumptions and simplifications. The *scope of the model* specifies all relevant processes and their interactions within the boundary of the model. The *level of detail* specifies the required degree of detail for each process in the model and the required input data. Both scope and level of detail are determined based on the modelling objectives. *Assumptions* are necessary to address the uncertainty or unknown factors that may be important to the processes in the model. *Simplifications* are needed to handle the complexity of processes and other important elements (such as resources) in the model.

The contents component of a conceptual model is probably best described using BPMN's *business process diagram (BPD)*. Despite its simplicity, BPD has the ability to model complex business processes. It is easy to understand and widely used by non-simulation specialists. Moreover, it is designed with web services in mind, which make it very useful for component-based simulation, web-based simulation and any distributed simulation environment.

Figure 6 shows BPD's main elements: activities, events, gateways and connectors. BPD's activities are used to model processes in the real world. These processes can further be decomposed into sub-processes that are important in representing level of detail. The lowest-level process is called a task. BPD's events are used to model events that happen in the real world. An event can start the processes, be intermediary, or end the processes. BPD's connector is used to represent a sequence of processes. BPD's gateways are used to represent decisions in the process flow, i.e., joins, forks, and mergers. BPMN (http://www.bpmn.org/) provides a more detailed explanation on each element and other elements that are not mentioned here, such as pool and lane.

Figure 6: BPD's Main Elements

The scope of a conceptual model can easily be represented by specifying the relevant processes, events that start these processes and the process flows (including decisions or branching of flows). Figure 7 shows the scope of a hospital simulation project in which only three processes (A&E, Outpatient, and In-patient) are modelled. Patients arrive to the system through A&E and Outpatient. Depending on the condition, a patient can be admitted to hospital (in-patient) or discharged. The figure shows that General Practitioner (GP) is outside the scope of this project.

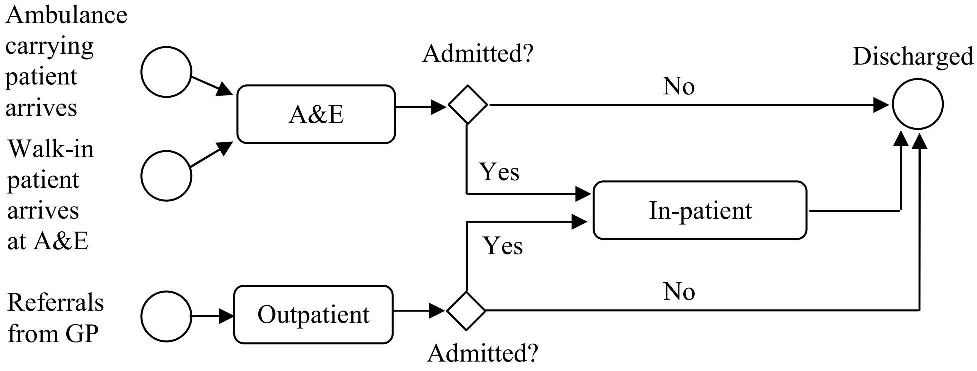

Figure 7: Hospital – Business Process Diagram

Figure 7 also shows the level of detail of the processes. It considers A&E, outpatient and in-patient as three black boxes. As a process in the diagram can contain sub-processes (which can be shown by another BPD), it is possible to show a more detailed model for the outpatient process in figure 7, as shown in figure 8. The outpatient department receives referrals from the GP. Based on their condition, a patient can be discharged, admitted to hospital (become an in-patient), or scheduled for re-attendance at the outpatient department. The level of detail of the input data is best described using a textual representation. For example, in figure 8, we can have a note to say that the duration of the first appointment will be sampled from a distribution function.

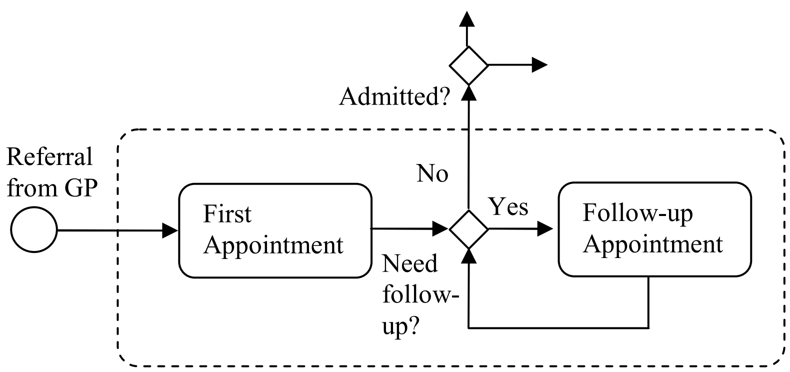

Figure 8: Outpatient – Business Process Diagram

Assumptions and simplifications are probably best represented using textual representation. For example, for Figure 8, we can have a list of assumptions such as, to take one example, the arrival data does not include referrals from hospitals. Similarly, we can list the simplifications, such as the number of no-show patients is sampled using a simple top-hat sampling.

Model Dependent Component

From the problem domain, modellers need to decide whether simulation is appropriate. At the model domain, modellers need to choose which simulation approach should be used in the modelling. Each simulation approach is suitable for certain conditions. In this paper, we discuss two approaches only, namely: *discrete-event simulation* and *system dynamics*.

Discrete-event simulation is suitable when it is necessary to track entities from their arrival to the system until they leave the system (or until the simulation is completed). The results from individual entities are aggregated in the simulation outputs. There are two widely used worldviews in discrete-event simulation. An *event-oriented worldview* specifies a simulation model using a set of events and states. When an event occurs, it may schedule a set of other events and it may also change the state of the system. An event graph is commonly used to represent this world-view. A *process-oriented worldview*, such as three-phase and process-interaction, specifies a simulation model using a set of processes. When an entity arrives at the system, it must go through a set of processes that may change the state of the entity (for example, in service or waiting) until the entity leaves the system. ACD is commonly used to represent this worldview. Pidd (2004) provides a good explanation on the similarities and differences between different worldviews. Figures 9 and 10 show the event graph and ACD of a hospital simulation, respectively.

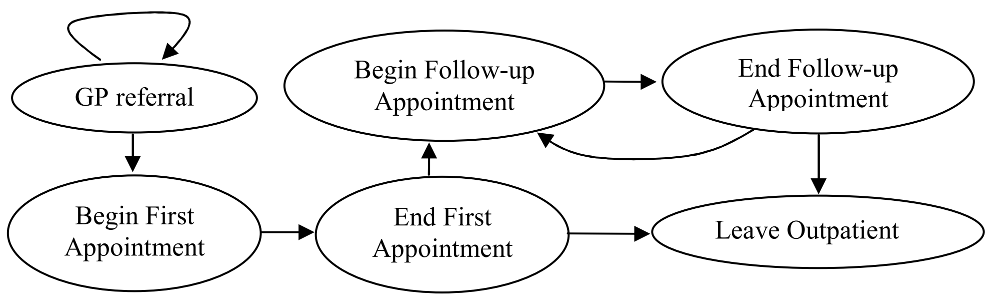

Figure 9: Outpatient – Event State Diagram

In event graphs, an oval represents an event and an arrow shows that an event at the start of the arrow generates an event at the end of the arrow. In the complete version, each arrow carries two more pieces of information: a Boolean condition for an event to be scheduled and the time delay until the generated event is scheduled to occur. Due to space limitation, figure 9 does not show the extra information. The figure shows that a "GP referral" event triggers the simulation process. This event serves as a bootstrap event that will generate the subsequent arrivals to the outpatient department (with a specified time delay, for example, using an exponential distribution with a mean of β). A "GP referral" event generates a "begin first appointment" event (with a specified condition, say when there is at least one senior consultant is available). The figure also shows that an "end first appointment" event may generate a "begin follow-up appointment" event or a "leave outpatient" event.

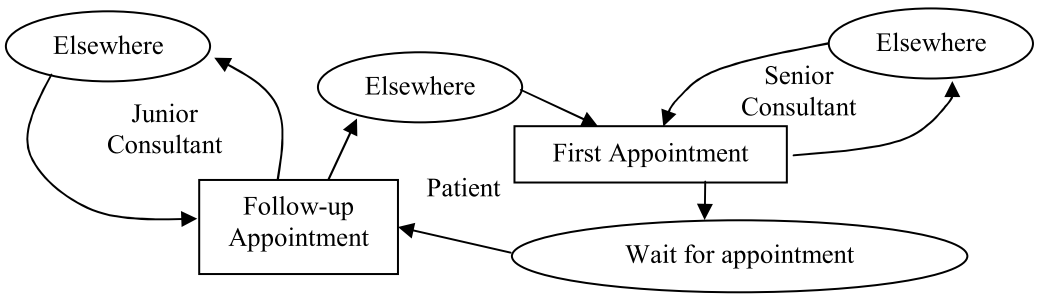

Figure 10: Outpatient – Activity Cycle Diagram

In ACD, an oval corresponds to a dead state that represents conditional waiting (until the required resources are available). A rectangle corresponds to an active state that represents a process with a specific duration (it may be sampled using a predefined distribution function). In most common cases, resources show a similar pattern (see junior and senior consultants in figure 10). Therefore, it is common not to show their cycle. Instead, we can write down the required resources next to the corresponding active states (see figure 15).

System dynamics is suitable when the population of entities and the rates of entities moving from one place to another are more important than the individual entities. Stock and Flow Diagram and Causal Loop Diagram are widely accepted as standards in representing system dynamics models (Sterman, 2004). Figure 11 shows the Stock and Flow diagram of the outpatient simulation drawn using Vensim™

(http://www.vensim.com), which has slightly different notations from the original version (Forrester, 1961). The rectangles represent the states of the system (i.e., population size), the valves represent the flow rate and the double line arrows show the direction of flows. The single line arrow shows that the node at the start of the arrow affects the node at the end of the arrow.

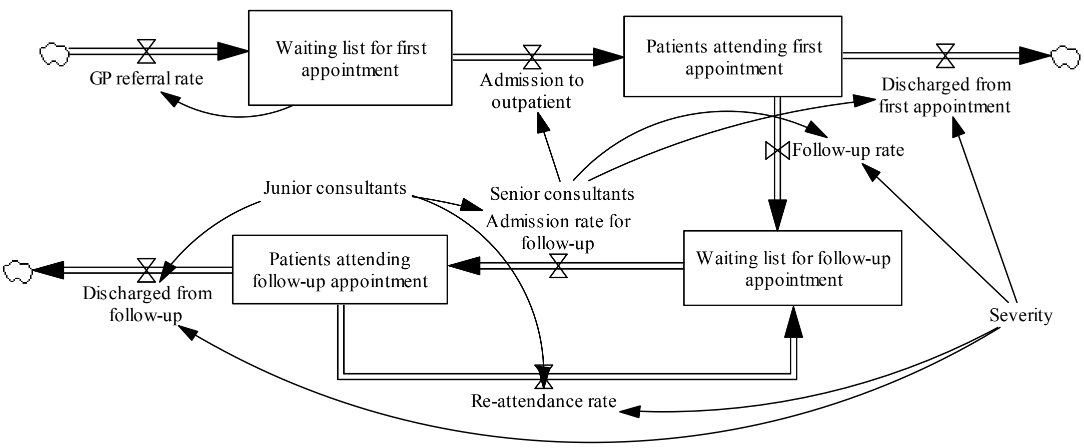

Figure 11: Outpatient – Stock and Flow Diagram

Case Study: District General Hospital Performance Simulation

This case study is based on the District General Hospital Performance Simulation (DGHPSim) which is a collaborative research project involving three British universities that aims to develop generic simulation models of entire acute hospitals so as to understand how hospital performance can be improved (Gunal and Pidd, 2006). For the case study in this paper, we focus only on one specific aspect of the A&E operation, i.e., to study the effect of the number of doctors, nurses and clerks on A&E performance. The A&E performance targets are based on patient total time (98% of patients must not spend more than four hours in A&E) and staff utilization.

Objectives Component

The objective of this simulation project is to maximize A&E performance, which is obtained from two components: patient total time and staff utilization as shown in the objectives diagram (figure 12).

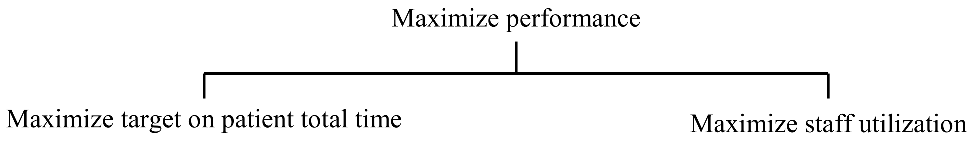

Figure 12: A&E – Objective Diagram

Inputs and Outputs Component

Based on the objectives, figure 13 shows the specification of inputs and outputs components represented using an influence diagram. The output (shown as a diamond) of the simulation should be the A&E performance measure. The A&E performance measure is calculated from two intermediary variables (shown as rounded rectangles), namely, the total number of patients who spend four hours or less in A&E and staff utilization. The decision variables (shown as rectangles) are the number of doctors, nurses and clerks. The uncontrollable variables (shown as ovals) are the arrival rate and severity of the condition of patients.

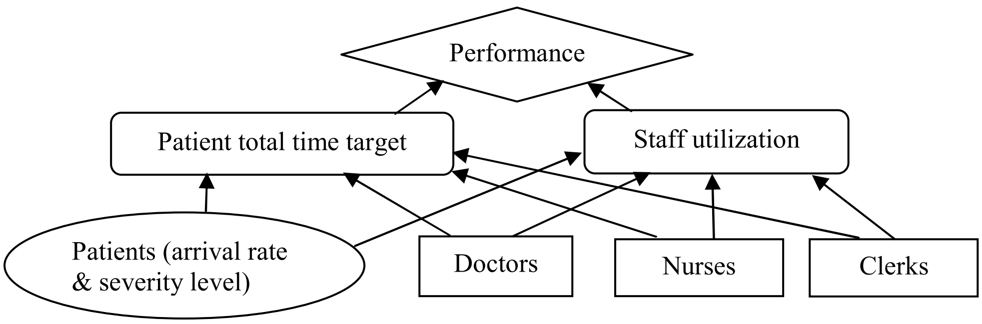

Figure 13: A&E – Influence Diagram

Contents Component

The contents component is shown in Figure 14. The figure shows that the process in A&E starts with patient arrivals. There are two types of patient arrival: voluntary and by ambulance. A patient who arrives voluntarily at the A&E will need to register before being evaluated by a nurse (triage) to determine the severity of the patient's condition. One who arrives by ambulance, however, may bypass the registration process (the triage is done on the way to the A&E). Next, the patient will be seen and treated by a doctor and nurse (either in the resuscitation room or cubicle). After the treatment, patients will either be discharged or admitted to the hospital. Some patients may need some tests and X-rays and these patients then need a second session with a doctor and nurse before discharge or admission.

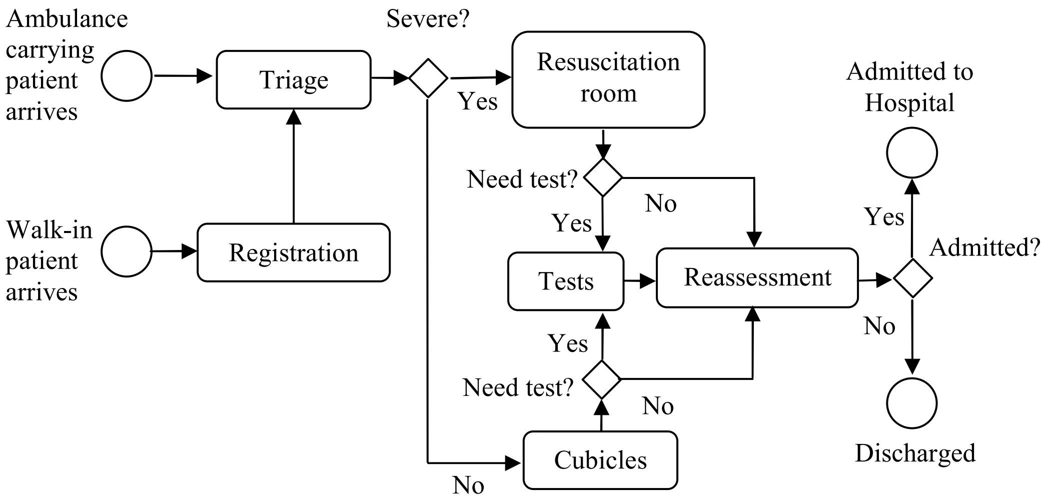

Figure 14: A&E – Business Process Diagram

The model includes six processes from registration to reassessment (scope) and each process is treated as a black box (level of detail of the processes). The level of detail of the input data is as follows.

- Arrivals of walk-in patients are modelled using an exponential distribution
- Arrivals by ambulance are modelled using an exponential distribution
- Service time for registration is modelled using a triangular distribution
- Service time for triage is modelled using a lognormal distribution
- Service time for treatment in the resuscitation room is modelled using a lognormal distribution
- Service time for treatment in the cubicles is modelled using a lognormal distribution
- Service time for tests is modelled using an empirical distribution
- Service time for reassessment is modelled using a uniform distribution. The service time for patients who do not need any test is zero.

The model's assumptions are:

- The severity of condition of patients is modelled as a simple top hat sampling
- The requirement for tests is modelled as a simple top hat sampling
- The admission to hospital is modelled as a simple top hat sampling

- Triage for patients arriving by ambulance is done on the way to hospital; hence, the service time is assumed to be zero.

The simplifications made are:

- The direct referrals from GPs are not included in the model
- The service time for tests does not differentiate between the type of test (X-Ray, blood test, etc)
- All queues have unlimited capacity
- Patients in the resuscitation rooms or cubicles do not release their bed when they go for tests. They only release their bed after the re-assessment process
- Fatalities are not included.
- Doctors and nurses attend one patient at a time (in the real world they switch quickly between patients).

Model Dependent Component

Literature shows that two approaches dominate the A&E modelling, namely, discrete-event simulation and system dynamics. The choice between the two depends on the objective of the study. The A&E system dynamics models reported in literature, such as Brailsford (2004), mostly include the social factors in the community served by the A&E department. The reported objectives are aimed mainly at studying the effect of different policies or external factors on the capacity of an A&E department. In this case study, the objective is to study the A&E department performance given a limited number of resources. Specifically, the performance is measured based on the percentage of patients leaving the department within four hours or less. In this case, discrete-event simulation is more suitable because we need to track each patient and resource in the system. The activity cycle diagram of the discrete-event simulation model is shown in Figure 15. Each process in the business process diagram (Figure 14) is mapped onto an active state in the activity cycle diagram. The diagram shows the main cycle, i.e., patient's cycle. The cycle for each resource is not shown in order to avoid cluttering the figure.

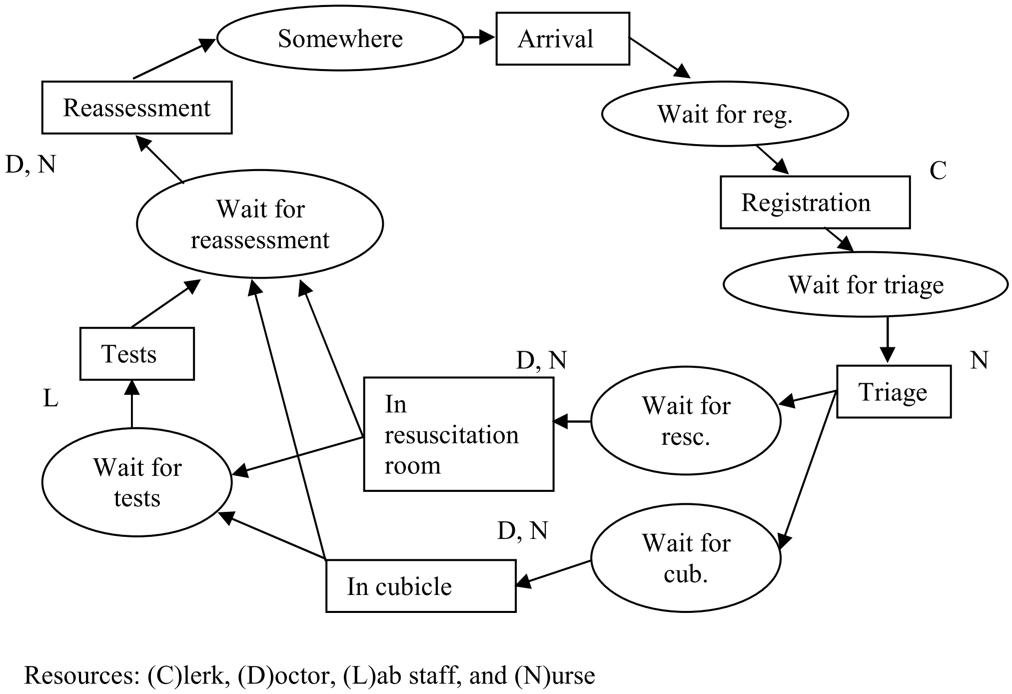

Figure 15: A&E – Activity Cycle Diagram

Conclusions

The main contribution of this paper is the proposal of a unified conceptual model representation. We have shown how conceptual model components can be represented using various visual and textual representations. The proposed representations are chosen carefully based on their simplicity and expressiveness. We hope this paper can initiate discussion that will eventually lead to a standard unified conceptual model representation. A few points need further analysis. Firstly, we need to evaluate critically a number of possible representations for each conceptual model component. Secondly, in the model dependent component, we need to address other approaches, such as agent-based simulation, micro-analytical simulation, multi-scales simulation model, etc. Finally, we need to explore the use of mark-up languages such as XML to exploit the full potential of the conceptual model documentation. This can be very beneficial for component-based simulation, web-based simulation and other distributed simulation environments.

Acknowledgement

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