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The impact of open source software on the strategic choices of firms developing proprietary software

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Abstract

Open source software (OSS) is now posing significant competition to proprietary or closed source software (CSS) in several software markets. In this paper, we characterize the response of a firm developing CSS to the presence of an OSS in its market. In particular, we look at the firm's choice of resource investments to improve quality and the firm's pricing decisions. We are primarly motivated by the following questions: Would a firm producing CSS produce higher-quality software when it faces competition from an OSS than when there is no OSS in its market? Would there be a change in the firm's response if the CSS faced competition from another CSS in addition to competition from the OSS? We show that the firm produces lower-quality CSS when it faces competition from an OSS than when it does not. Also, the quality of the CSS decreases as the quality of the OSS increases. This result holds true even if we consider network effects. When we consider competition from another CSS, in addition to competition from the OSS, then the quality of the CSS could increase or decrease as the quality of the OSS increases. The change in quality depends on how closely substitutable the two CSS are. We also extend our base model to consider: i) competition for resources, ii) uncertainty in resources available to the OSS, and iii) uncertainty about the software development process.

Keywords: Open source software, quality, resources, competition, network externality.

1 Introduction

Open source software (OSS) is now posing significant competition to proprietary or closed source software (CSS) in several software markets. The most well-known example of OSS, Linux, has a 23% market share in the enterprise server market, which is a threat to the market leader, Microsoft Windows operating system.[1] Apache, another OSS, has nearly 70% market share in the web server software market, significantly higher than Microsoft's IIS software.[2] OSS are emerging as significant alternatives to CSS in other software markets as well, such as office productivity tools (Open Office provides software tools similar to Microsoft Office), accounting software (GNU Cash, an OSS, is used to manage personal or business accounts similar to Intuit's Quicken), and database systems(the MySQL open source database products compete against CSS systems like Oracle).

The primary difference between OSS and CSS is that the source code of an OSS is accessible to everyone, while the source code is proprietary in the case of a CSS. One consequence of keeping the source code open is that the OSS can benefit from modifications and improvements made by programmers from all around the world. The CSS, on the other hand, can only be improved or modified by programmers hired by the firm developing the CSS (henceforth referred to as the firm when there is no risk of confusion). It is well known that OSS such as Linux and Apache have been developed and enhanced by contributions from thousands of volunteer programmers. Another consequence of keeping the source code open is that the firm cannot charge a price for purely selling the OSS - the open nature of the source code will drive the price down to zero. Most of the OSS such as Linux, Apache, SendMail, etc., can be obtained free of charge. In the case of Linux, there are several companies such as Red Hat, SuSe, MandrakeSoft etc., that distribute Linux for a price. However, they primarily choose a stable version of Linux and make money from selling support services and easy-installation utilties for their distributions. Thus, these firms could be thought of as

[1] Netcraft user survey, 2003.
[2] http://www.serverwatch.com/news/article.php/3524676.

firms selling complementary services to the Linux OSS. In most cases, a free version of their distribution (without the easy installation utilities) can be downloaded from their website.

The CSS thus faces competition from a product (OSS), that free-rides on voluntary contributions from programmers, and is also available for free.[3] The CSS on the other hand has to pay its programmers and also charge its customers a price for the CSS. Facing this two-pronged challenge, firms producing CSS (even if they are dominant in their markets) have begun to consider how to respond to the presence of OSS. In an internal email to Microsoft employees, Steve Ballmer, the Microsoft CEO, had this to say about Linux:

> *Noncommercial software products in general, and Linux in particular, present a competitive challenge for us and for our entire industry, and they require our concentrated focus and attention.*[4]

In this paper, we characterize the response of the firm to the presence of an OSS in its market. In particular, we look at the firm's choice of resource investments to improve quality and also look at the firm's pricing decisions. We are primarly motivated by the following questions: Would a firm producing a CSS produce higher-quality software when it faces competition from an OSS than when there is no OSS in its market? Also, how would the firm respond to an improvement in the quality of the OSS alternative? Would there be any change in the firm's response if it faced competition from another CSS in addition to competition from the OSS? In order to answer these questions, we consider a software market where there is a CSS and an open source alternative to the CSS. The firm improves the quality of the CSS by investing resources (paying programmers). The OSS is improved by voluntary contributions from programmers and also by contributions from paid programmers who are hired by firms that sell complementary products or services to the OSS. For example, companies such as IBM and Oracle that sell complementary products to

[3] We make a distinction here between OSS and freeware. Freeware is software that is available at zero price, but is closed source. For our purpose, this difference is significant because OSS can benefit from voluntary contributions from programmers (because of the open source code), while freeware cannot.

[4] http://www.itmweb.com/f060903.htm.

Linux are known to hire programmers to work on Linux.[5] The resources invested to improve the quality of the software and the price charged are strategic choices for the firm competing against the OSS. We also extend the base model to consider the impact of network effects (NE), and the impact of competition from another CSS. Finally, we also extend our base model to consider: i) competition for resources between the OSS and the CSS, ii) uncertainty in resources available to OSS, and iii) uncertainty about the software development process.

We show in this paper that the firm produces lower-quality CSS when it faces competition from an OSS than when it does not. Also, the quality of the CSS decreases as the quality of the OSS increases. The intuitive reason behind this result is that competition from the OSS lowers the market share of the CSS, which lowers the incentive of the firm to develop a better CSS. This result is robust even if we consider NE. Additionally, the resource investment by the firm increases with the increasing strength of the NE. We also find that, with the OSS in the market, the resource investment by the firm and the final quality of the CSS increase in the initial quality until a later stage in the software lifecycle compared to the case where there is no OSS in the market. Interestingly, if we consider competition from another CSS, in addition to competition from the OSS, then the quality of the CSS could be increasing or decreasing with incresing quality of the OSS. The change in quality depends on how closely substitutable the two CSS are. When the two competing CSS are not close substitutes, the results are similar to the case when there is one CSS and one OSS. The primary competition comes from the OSS. Hence, the higher the initial quality of the OSS, the lower the market share of the CSS, and hence the lower the incentive for each firm to invest resources to improve their respective CSS. However, when the two CSS are close substitutes, a higher intitial quality leaves a smaller market for the two CSS. In order to protect their respective market shares, it now becomes imperative for the two firms to invest more resources to improve the quality of their respective CSS. This contrasting result to the case when there is one CSS and one OSS, highlights the difference in the nature of competition with a passive

[5] http://www.consultingtimes.com/articles/ibm/frye/fryeinterview.html.

competitor (OSS) versus a more active competitor (another CSS). The results from the model are robust to several changes in the model specifications such as competition for resources between the OSS and the CSS, uncertainty regarding resources available to the OSS, and uncertainty regarding the software development process.

The rest of this paper is organized as follows: related literature is reviewed in Section 2. We state our assumptions and set up the base model in Section 3. We consider a benchmark case - no OSS in the market - in Section 4. The analysis of the base model and comparison with the benchmark case is presented in Section 5. The impact of NE and competition are considered in Sections 6 and 7, respectively. In Section 8, we consider three extensions to the base model: competition for resources between the CSS and the OSS in Section 8.1, uncertainty about the resources available to the OSS in Section 8.2, and uncertainty about the software development process in Section 8.3. Finally, managerial implications, limitations and directions for future research are discussed in Section 9.

2 Related Literature

For an introduction to the research issues on OSS see Lerner and Tirole (2001, 2002) and Schiff (2002). The open source literature has primarily focused on explaining the motivation of programmers to contribute to open source projects. Different explanations include: private provision of a public good (Johnson 2002) and a signaling incentive (Leppämäki and Mustonen 2003; Lerner and Tirole 2002). In a survey of Apache OSS programmers, Hann et al. (2004) found that the dominant motivations for participating in OSS projects are increasing contributor's use value, followed by the recreational value of task, and potential career impacts. Gutsche (2005) investigated why open source communities exist using an evolutionary model.

Another major stream of research has studied the development process of open source projects. Mockus et al. (2000) studied organizational issues for open source projects by reflecting on the development of the Apache Web server. Von Hippel and von Krogh (2003) investigated organizational issues in open source projects. Crowston et al. (2005) studied how to coordinate open source projects. Sagers (2004) analyzed the role of governance in OSS development.

Unlike the above two lines of research, our interest is in the strategic response of the firm producing the CSS to the presence of an OSS in the same market and in the impact on the quality of the CSS. Previous scholars who studied the strategic response of customers and firms to OSS focused on factors affecting adoption (Khalak 2000; Li et al. 2005), and ways in which CSS vendors can profit from the open source development methods (Nilendu and Madanmohan 2001; Hawkins 2004; Mustonen 2003). The question of how the existence of OSS affects the incentives of a CSS vendor to improve the quality of its software, which is the focus of the current paper, has not yet been addressed to the best of our knowledge.

3 Model Description

At time 0, there is a firm that produces a CSS of initial quality, q_c. There also exists at this time an imperfect open source substitute for this software which is of quality, q_o. Both the CSS and the OSS can be improved if resources are invested in them. Resources are typically programmers who work on adding new functionality to the software, or work on removing known problems from the software. All software programs, whether CSS or OSS, go through this incremental improvement over their lifecycle. Henceforth, we use the terms resources and programmers interchangably. The firm can hire programmers to work on the CSS. The OSS benefits from voluntary, as well as paid, contributions from programmers. Let the firm invest resources, r_c, in improving the CSS. The OSS, on the other hand, benefits from contributions from resources, r_o. The cost to the firm of investing resources, r_c, is $C(r_c)$. We make the following assumption about the cost function:

Assumption 1 *(i)* $C'(.) > 0$ *and (ii)* $C''(.) \geq 0$.

This cost could be thought of as the salary paid to the programmers. Hiring more programmers is thus more costly. Also, since the pool of programmers from which the firm can hire is limited, the marginal cost of hiring an additional programmer is increasing.

The final quality of the CSS after investing resources, r_c, is $Q_c(q_c, r_c)$. Similarly, the final quality of the OSS after resource, r_o, works on the software is $Q_o(q_o, r_o)$. We make the following assumption about the final qualities, Q_c and Q_o.[6]

Assumption 2 *(i)* $\frac{dQ_i}{dr_i} \geq 0$ *and (ii)* $\frac{dQ_i}{dq_i} \geq 0$*, where* $i = c, o$*.*

In general, an investment of resources increases the quality of the software. Programmers add code to implement new functions or to improve the working/performance of existing functions in the software. Improvements to a software are usually built on top of the existing software. For example, new releases of software such as the Windows operating system, have the same core components, with newly added modules runnning on top of the core. The same is true for new releases of OSS, such as Linux. Thus, the final quality of the software depends on its intial quality. For a given resource investment, we assume that the higher the initial quality of the software, the higher the final quality. Initially, we assume that there is no uncertainty regarding the software development process, i.e., the functions that determine the final qualities of the CSS and the OSS are deterministic. In an extension (Section 8.3) we look at the impact of uncertainty regarding these functions.

Let the firm set a price, p, for the CSS. The demand for the CSS is given by the demand function, $D(p, Q_c, Q_o)$. We make the following assumptions regarding the demand function:

Assumption 3

(i) $\frac{dD}{dp} \leq 0$*;*

(ii) $\frac{dD}{dQ_c} \geq 0$ *and* $\frac{dD}{dQ_o} \leq 0$*;*

(iii) All second-order derivatives are zero.

[6]The parameters are suppressed when there is no risk of confusion.

The demand function can be estimated using user groups and other commonly used approaches to estimate demand functions. A demand function that decreases in price is a standard assumption. The higher the quality of the CSS, the higher should be the demand for the CSS. Also, since the OSS competes with the CSS for consumers, a higher-quality of OSS leads to a lower demand for the CSS. Thus, both assumptions 3(i) and 3(ii) are natural assumptions to make. Assuming that there are no second-order effects is a common assumption to keep the analysis simple.

The timing is as follows:

Stage 0: Initial qualities of CSS and OSS are q_c and q_o, respectively.

Stage 1: The firm chooses to invest resources, r_c, and the OSS benefits from resources, r_o, which results in final qualities, Q_c and Q_o, for the CSS and the OSS, respectively.

Stage 2: The firm chooses the price, p, of the CSS.

Stage 3: Customers buy the CSS according to the demand function, $D(p, Q_c, Q_o)$.

Stage 0 is the time at which the firm developing the CSS decides how to respond strategically to the presence of the OSS. At this point in time, the initial qualities of the two software programs are given. We are not concerned with how the CSS and OSS got to this stage; we are only interested in what will happen when the firm makes strategic choices in response to competition from the OSS. The firm believes that the OSS will benefit from resources, r_o, and decides to invest resources, r_c, in the CSS. The firm's estimate of r_o could be based on past programmer contributions to the OSS. Many OSS communities publicly disclose information about contributions, and as such it is not hard for the firm to form an estimate of the resources that will be available to the OSS in the future. In the base model, we assume that the firm has perfect information about r_o. We relax this assumption in Section 8.2, where we show that our results do not change if the firm does not have perfect information on r_o, but has only an estimate of r_o. We initially assume that the CSS and the OSS do not compete for resources. This could happen when the programmers that develop the CSS and the OSS require different skill sets. We relax this assumption in Section 8.1. We make a distinction between stage 1 (quality choice) and stage 2 (price choice), because the price can be changed easily, while a change in the quality requires investment in both resources and time. The firm makes a decision on price only after it knows its own quality and that of its competitor OSS.[7] Hence, it is natural to assume that the pricing stage follows the quality choice stage. Our solution procedure is backward induction as is standard practice in such games.

4 Benchmark case: No OSS in the market

We first consider a benchmark case when there is no OSS in the market ($Q_o = 0$). We first plug the demand function into the profit function of the firm to calculate the optimal price for the CSS. Then, we plug this optimal price into the profit function of the firm to calculate the optimal investment in resources. The profit function of the firm is $\pi_b = pD(p, Q_c) - C(r_c)$. The optimal price, p_b^*, solves:

$$D(p_b^*, Q_c) + p_b^* \frac{dD}{dp} = 0. \tag{1}$$

The optimal price, p_b^*, has the following properties:

Lemma 1 *a.* $\frac{dp_b^*}{dQ_c} \geq 0$*; b.* $\frac{dp_b^*}{dQ_o} \leq 0$.

All proofs are presented in the appendix. As expected, the optimal price is increasing in Q_c and decreasing in Q_o. By substituting this optimal price, p_b^*, into the profit function and using (1), we determine the maximization problem for the firm to be:

$$\max_{r_c} -\frac{dD}{dp}(p_b^*)^2 - C(r_c).$$

[7]Mustonen (2003) makes a similar distinction between the two stages by describing a development stage and a pricing stage. These staged decisions are characterisic of the software industry, where firms first develop the software and then decide on its price.

Let r_c^{b*} be the optimal investment in resource by the firm. Then, r_c^{b*} solves:

$$-2p_b^*\frac{dp_b^*}{dQ_c}\frac{dQ_c}{dr_c}\frac{dD}{dp} - C'(r_c^{b*}) = 0. \tag{2}$$

For the profit funcion to be concave in r_c,

$$\pi_b^{''}(r_c) = -2(\frac{dp_b^*}{dQ_c})^2(\frac{dQ_c}{dr_c})^2\frac{dD}{dp} - 2p_b^*\frac{dp_b^*}{dQ_c}\frac{dD}{dp}(\frac{d^2Q_c}{dr_c{}^2}) - C''(r_c) < 0. \tag{3}$$

The first term is positive and the last term is negative from assumptions 1 and 3(i). Using lemma 1a, the profit function is concave if $\frac{d^2Q_c}{dr_c{}^2}$ is below a critical value which is postive.[8] Under the condition that the profit function is concave, r_c^{b*} is an interior maximum. We denote the final quality of the CSS at this optimal resource investment as $Q_c^{b*} = Q_c(q_c, r_c^{b*})$. The following proposition shows some comparitive statics results.

Proposition 1 *Denoting* $\frac{d^2Q_c}{dq_c dr_c}$ *by* μ,

$$\frac{dr_c^{b*}}{dq_c} = \begin{cases} \geq 0 & if \quad \mu \geq \hat{\mu}_b \\ < 0 & if \quad \mu < \hat{\mu}_b \end{cases} \quad and \quad \frac{dQ_c^{b*}}{dq_c} = \begin{cases} \geq 0 & if \quad \mu \geq \tilde{\mu}_b \\ < 0 & if \quad \mu < \tilde{\mu}_b \end{cases},$$

where $\hat{\mu}_b$ *solves* $\frac{dr_c^{b*}(\mu)}{dq_c} = 0$, $\tilde{\mu}_b$ *solves* $\frac{dQ_c}{dq_c}/\frac{dQ_c}{dr_c} + \frac{dr_c^{b*}(\mu)}{dq_c} = 0$, *and* $\tilde{\mu}_b < \hat{\mu}_b < 0$.

The term $\frac{dQ_c}{dr_c}$ is the resource effectiveness (RE). It captures the marginal improvement in quality for a unit of resource investment. Thus, μ is the responsivness of RE to a change in the initial software quality. A software that is in the early stages of its lifecycle would have a higher μ, which is positive most of the time. This is because, the software has a lot of room for improvement. A mature software, on the other hand, could have a negative μ, since it is more difficult to improve it. This result suggests that, when a software is in an early stage of its lifecycle (when μ is positive or greater than a critical value, $\hat{\mu}_b$, if it is negative), then a higher initial quality will induce the firm to invest more resources, which results in a software of higher final quality. Also, a firm that has a mature software will invest less resources in

[8]The upper limit of $\frac{d^2Q_c}{dr_c{}^2}$ can be calculated from (3).

response to a higher intial quality (since μ is negative), which results in a software of lower final quality.

The marginal effect of the intial quality on the final quality lags behind the marginal effect of the initial quality on the optimal resource investment. This lag can be seen in Figure 1. In the early stage of the software lifecycle, both marginal effects are positive. At a later stage in the lifecycle, the marginal effect of the initial quality on the optimal resource investment becomes negative, while the marginal effect of the initial quality on the final quality is still positive. At an even later stage in the lifecycle, both marginal effects are negative. This lag in the marginal change in the final quality with the intial quality of CSS results because of two effects that are evident from the following equation:

$$\frac{dQ_c^{b*}(\mu)}{dq_c} = \frac{dQ_c}{dq_c} + \frac{dQ_c}{dr_c}\frac{dr_c^{b*}(\mu)}{dq_c}.$$

First, there is a direct effect because of the change in the intial quality, which is always positive. Second, there is an indirect effect because of the change in the optimal resource investment with the initial quality. This indirect effect is positive when the software is early in its lifecycle (when μ is positive or greater than a critical value, $\hat{\mu}_b$, if it is negative), and negative when the software is mature (μ is less than the critial value, $\hat{\mu}_b$). The first, positive direct effect, counteracts the second indirect effect (when it is negative). Hence, there is a lag in the marginal effect of intial quality on final quality compared to the marginal effect of initial quality on optimal resource investment.

INSERT FIGURE 1 HERE

5 Base Model: OSS in the market

Next, we consider our base model when an OSS exists in the market. The profit function of the firm is $\pi = pD(p, Q_c, Q_o) - C(r_c)$. The optimal price, p^*, solves:

$$D(p^*, Q_c, Q_o) + p^* \frac{dD}{dp} = 0. \tag{4}$$

We now compare the optimal price, p^*, with the price in the benchmark case, p_b^*.

Proposition 2 $p^* \leq p_b^*$.

Due to competition from the OSS, the firm charges a lower price for the CSS. The following lemma shows some useful properties of p^*.

Lemma 2 *a.* $\frac{dp^*}{dQ_c} \geq 0$; *b.* $\frac{dp^*}{dQ_o} \leq 0$.

As expected, the optimal price is increasing in Q_c and decreasing in Q_o. By plugging this optimal price, p^*, into the profit function and using (4), we have the maximization problem for the firm:

$$\max_{r_c} -\frac{dD}{dp}(p^*)^2 - C(r_c).$$

The optimal resource investment by the firm, r_c^*, solves:

$$-2p^* \frac{dp^*}{dQ_c} \frac{dQ_c}{dr_c} \frac{dD}{dp} - C'(r_c^*) = 0. \tag{5}$$

For the profit funcion to be concave in r_c,

$$\pi''(r_c) = -2(\frac{dp^*}{dQ_c})^2(\frac{dQ_c}{dr_c})^2 \frac{dD}{dp} - 2p^* \frac{dp^*}{dQ_c} \frac{dD}{dp}(\frac{d^2Q_c}{dr_c{}^2}) - C''(r_c) < 0. \tag{6}$$

The first term is positive and the last term is negative from assumptions 1 and 3(i). From lemma 2a, the profit function is concave if $\frac{d^2Q_c}{dr_c{}^2}$ is below a critical value, which is postive.[9]

[9] The upper limit of $\frac{d^2Q_c}{dr_c{}^2}$ can be calculated from (6).

Under the condition that the profit function is concave, r_c^* is an interior maximum. We denote the final quality at this optimal resource investment as $Q_c^* = Q_c(q_c, r_c^*)$. We now compare the optimal resource investment by the firm with the resource investment in the benchmark case when there is no OSS in the market.

Proposition 3 $r_c^* \leq r_c^{b*}$.

Competition from the OSS lowers the market share of the firm. As a result, the firm has a lower incentive to develop a better CSS.

We next present some comparitive statics results:

Proposition 4 *Denoting* $\frac{d^2 Q_c}{dq_c dr_c}$ *by* μ,

a. $\frac{dr_c^*}{dq_o} \leq 0$ *and* $\frac{dQ_c^*}{dq_o} \leq 0$,

b. $\frac{dr_c^*}{dq_c} = \begin{cases} \geq 0 & \text{if} \quad \mu \geq \hat{\mu} \\ < 0 & \text{if} \quad \mu < \hat{\mu} \end{cases}$ *and* $\frac{dQ_c^*}{dq_c} = \begin{cases} \geq 0 & \text{if} \quad \mu \geq \tilde{\mu} \\ < 0 & \text{if} \quad \mu < \tilde{\mu} \end{cases}$,

c. $\frac{dr_c^*}{dr_o} \leq 0$ *and* $\frac{dQ_c^*}{dr_o} \leq 0$,

where $\hat{\mu}$ *solves* $\frac{dr_c^*(\mu)}{dq_c} = 0$, $\tilde{\mu}$ *solves* $\frac{dQ_c}{dq_c} / \frac{dQ_c}{dr_c} + \frac{dr_c^*(\mu)}{dq_c} = 0$ *and* $\tilde{\mu} < \hat{\mu} < 0$.

Competition from the OSS lowers the market share of the firm. An OSS of higher initial quality is more competitive and will further decrease the firm's incentive to develop a better CSS. Hence, the final quality of the CSS, and the resource investment, are decreasing in the initial quality of the OSS. Part b of proposition 4 is similar in nature to proposition 1. We illustrate the difference between the two results (which arises because of the presence of the OSS) in the next proposition. Firms like IBM pay their employees to work on OSS such as Linux. This increase in the resources available to the OSS will result in a higher-quality OSS. The firm producing the CSS competing against this OSS will thus have a lower incentive to invest resources, resulting in a lower-quality CSS.

Proposition 5 *a.* $\hat{\mu} < \hat{\mu}_b$ *and b.* $\tilde{\mu} < \tilde{\mu}_b$.

In the benchmark case, $\hat{\mu}_b$ ($\tilde{\mu}_b$) is a critical stage in the software lifecycle. Beyond $\hat{\mu}_b$ ($\tilde{\mu}_b$), the resource investment (final quality) is decreasing in the intial quality. Similarly, in the case when the OSS exists in the market, $\hat{\mu}$ ($\tilde{\mu}$) is a critical stage in the software lifecycle. Beyond $\hat{\mu}$ ($\tilde{\mu}$), the resource investment (final quality) is decreasing in the intial quality. With the OSS in the market, the resource investment by the firm and the final quality are increasing in the initial quality of the CSS until a later stage in the software lifecycle compared to the case when there is no OSS in the market. The result is intuitive. When there is no OSS in the market, the software users, if they do not buy the CSS, will have no software to use. With the presence of the OSS, users have more choice. This gives more bargaining power to the users and the firm needs to think twice before reducing the resource investment to improve quality given the initial software quality. Hence, it is natural that the presence of an OSS will defer the firm's decision to reduce resource investment, given the initial software quality, to a later stage in the software lifecycle compared to the case where there is no OSS in the market.

6 Network effects (NE)

Software is a product that exhibits NE. The larger the user base of a software, the greater is the utility to a consumer who uses that software. This increased utility could be because of more user groups for that software, more third-party applications/hardware that can interact with the software, etc. We extend our base model to incorporate NE. The demand for the CSS is now given by the demand function, $D(p, Q_c, Q_o, D^e)$, where p, Q_c and Q_o are the same as before, while D^e is the demand for the CSS in equilibrium, as *anticipated* by the consumers.

Assumption 4 *(i)* $\frac{dD}{dD^e} > 0$ *and (ii)* $\frac{d^2 D}{dD^e dx} = 0$*, where* $x = p, Q_c, Q_o, D^e$.

The higher the anticipated demand for the CSS, the higher the utility of a consumer for the CSS, and hence the higher the actual demand for the CSS. This assumption follows the long line of research in network economics (Katz and Shapiro 1985; Shy 2001). Assuming a linear demand function is common and is made here for simplicity. For ease of notation, we sometimes use γ to denote $\frac{dD}{dD^e}$. Thus, γ measures the strength of the NE. We assume that $\gamma < 2$.[10] We do no explicitly model the process through which consumers' expectations are formed, but we do, however, impose the restriction that, in equilibrium, consumers' expectations are fulfilled. This restriction is:

$$D^e = D(p_n^*, Q_c, Q_o, D^e), \tag{7}$$

where p_n^*, is the optimal price when the CSS benefits from NE. It is easy to show that $p_n^* > p^*$. The following lemma proves some properties of the optimal price, p_n^*:

Lemma 3 *a.* $\frac{dp_n^*}{dD^e} \geq 0$; *b.* $\frac{dp_n^*}{dQ_c} \geq 0$; *and c.* $\frac{dp_n^*}{dQ_o} \leq 0$.

As expected, the optimal price is increasing in the anticipated demand for the CSS. We denote the optimal resource investment as r_c^{n*} and the final quality under this optimal resource investment as $Q_c^{n*} = Q_c(q_c, r_c^{n*})$. We can show that $r_c^{n*} > r_c^*$, i.e., the firm will invest more resources when the CSS benefits from NE.[11] We next present some comparitive statics results:

Proposition 6 *Denoting* $\frac{d^2Q_c}{dq_c dr_c}$ *by* μ,

a. $\frac{dr_c^{n*}}{d\gamma} \geq 0$ *and* $\frac{dQ_c^{n*}}{d\gamma} \geq 0$

b. $\frac{dr_c^*}{dq_o} \leq 0$ *and* $\frac{dQ_c^{n*}}{dq_o} \leq 0$,

c. $\frac{dr_c^{n*}}{dq_c} = \begin{cases} \geq 0 & if \quad \mu \geq \bar{\mu} \\ < 0 & if \quad \mu < \bar{\mu} \end{cases}$ *and* $\frac{dQ_c^{n*}}{dq_c} = \begin{cases} \geq 0 & if \quad \mu \geq \ddot{\mu} \\ < 0 & if \quad \mu < \ddot{\mu} \end{cases}$,

d. $\frac{dr_c^{n*}}{dr_o} \leq 0$ *and* $\frac{dQ_c^{n*}}{dr_o} \leq 0$,

[10] Dranove and Gandal (2003) have estimated γ to lie between 0.18 and 0.25 for DVD's. In the home video game market, γ has been estimated to lie between 1.71 and 1.93 (Shankar and Bayus 2003).

[11] The proof is available from the authors upon request.

where $\bar{\mu}$ *solves* $\frac{dr_c^{n*}(\mu)}{dq_c} = 0$, $\ddot{\mu}$ *solves* $\frac{dQ_c}{dq_c} / \frac{dQ_c}{dr_c} + \frac{dr_c^{n*}(\mu)}{dq_c} = 0$ *and* $\ddot{\mu} < \bar{\mu} < 0$.

The greater the strength of the NE, the greater the optimal resources invested by the firm and the higher the final quality of the CSS. The other comparitive statics results are similar to the results in the base model (with no NE). Hence, the results of the base model are quite robust.

7 Competition

We now extend our base model to consider competition from another CSS. Thus, there are now three competing software programs in the market - one OSS and two CSS. This setting differs from our base model in the sense that the two firms developing the CSS can respond strategically to each others' choices, while the OSS in the base model is passive. The two firms developing the CSS are denoted as i and j. We will also use i and j to denote the CSS developed by firms, i and j, respectively. Firm, $i(j)$, has a software of initial quality, $q_c^i(q_c^j)$, and invests resources, $r_c^i(r_c^j)$, to get a software of final quality, $Q_c^i(Q_c^j)$. In doing so, firm, $i(j)$, incurs a cost, $C^i(r_c^i)(C^j(r_c^j))$. Firm $i(j)$ chooses a price, $p^i(p^j)$, for its software. Demand for the i CSS is given by the demand function $D^i(p^i, Q_c^i, p^j, Q_c^j, Q_o)$, while demand for j CSS is given by the demand function, $D^j(p^j, Q_c^j, p^i, Q_c^i, Q_o)$. We make the following assumptions about the demand function of the firms:

Assumption 5

(i) $\frac{dD^k}{dp^k} < 0$, $\frac{dD^k}{dp^l} > 0$, $\frac{dD^k}{dQ_c^k} > 0$, $\frac{dD^k}{dQ_c^l} < 0$, $\frac{dD^k}{dQ_o} < 0$;

(ii) $\mid \frac{dD^k}{dp^k} \mid > \mid \frac{dD^k}{dp^l} \mid$, $\mid \frac{dD^k}{dQ_c^k} \mid > \mid \frac{dD^k}{dQ_c^l} \mid$; *and*

(iii) $\mid \frac{dD^k}{dp^k} \mid > \mid \frac{dD^l}{dp^k} \mid$, $\mid \frac{dD^k}{dQ_c^k} \mid > \mid \frac{dD^l}{dQ_c^k} \mid$;

where $k, l = \{i, j\}$, *and* $k \neq l$

Assumption 5(i) states that demand is decreasing (increasing) in the firm's own price (own quality) and increasing (decreasing) in the cross-price (cross-quality). Also, demands for both firms are decreasing with increasing quality of OSS. Assumption 5(ii) states that the firm's own price (own quality) effect on demand is greater than the cross-price (cross-quality) effect. Assumption 5(iii) states that the marginal effect of own price (own quality) on the firm's own demand is greater than the marginal effect of firm's own price (own-quality) on the competitor's demand. All the assumptions are fairly standard assumptions to make when considering competition.

The timing is as follows: in stage 0, the initial qualities of i and j are q_c^i and q_c^j, respectively, while the initial quality of the OSS is q_o. In stage 1, firms i and j invest resources, r_c^i and r_c^j, respectively. The OSS benefits from contributions from resources, r_o. This results in final qualities of Q_c^i, Q_c^j and Q_o. In stage 2, firms i and j choose their prices and, finally, in stage 3, the consumers choose to buy either one of the CSS or use the OSS. Let p^{i*} and p^{j*} be the optimal prices charged by the firms, i and j, respectively. The following lemma shows some properties of the optimal prices charged by the two firms:

Lemma 4

a. $\frac{dp^{k*}}{dQ_c^k} \geq 0$,

b. $\frac{dp^{k*}}{dQ_c^l} = \begin{cases} < 0 & \text{if} \quad \psi_l^l < 2\psi_l^k \\ \geq 0 & \text{if} \quad \psi_l^l \geq 2\psi_l^k \end{cases}$, *and*

c. $\frac{dp^{k*}}{dQ_o} \leq 0$,

where $\psi_l^l = \mid \frac{dD^l}{dQ^l} \mid / \mid \frac{dD^l}{dp^l} \mid$ *and* $\psi_l^k = \mid \frac{dD^k}{dQ^l} \mid / \mid \frac{dD^k}{dp^l} \mid$; $k, l = \{i, j\}$, *and* $k \neq l$.

The optimal price charged by each firm is increasing in the quality of its software. This result is thus the same as in the base model (with no competing CSS). The optimal price could be increasing or decreasing in the quality of the competing CSS. ψ_i^i is i's demand sensitivity to quality, per unit own price sensitivity, while ψ_i^j is j's demand sensitivity to cross quality, per unit cross price sensitivity. So, when ψ_i^j is sufficiently greater than ψ_i^i (by a factor of 1/2), then software i and j are highly substitutable in the quality dimension. In this case, the optimal price charged by one firm is decreasing in the quality of the competing CSS. Just as in the base model, the optimal price charged by each firm is decreasing in the quality of the OSS. We next show some comparitive statics results:

Proposition 7 *Denoting* $\frac{dp^{k*}}{dQ_c^l}$ *by* φ^k*,*

a. $\frac{dr_c^{k*}}{dq_o} = \begin{cases} < 0 & \text{if} \quad \varphi^k > \hat{\varphi}^k \\ \geq 0 & \text{if} \quad \varphi^k \leq \hat{\varphi}^k \end{cases}$ *and* $\frac{dQ_c^{k*}}{dq_o} = \begin{cases} < 0 & \text{if} \quad \varphi^k > \tilde{\varphi}^k \\ \geq 0 & \text{if} \quad \varphi^k \leq \tilde{\varphi}^k \end{cases}$*, where* $\hat{\varphi}^k$ *solves* $\frac{dr_c^{k*}(\varphi^k)}{dq_o} = 0$*,* $\tilde{\varphi}^k$ *solves* $\frac{dQ_c^k}{dq_o} / \frac{dQ_c^k}{dr_c^k} + \frac{dr_c^{k*}(\mu)}{dq_o} = 0$*,* $\tilde{\varphi}^k > \hat{\varphi}^k$*, and* $\hat{\varphi}^k < 0$*;*

b. $\frac{dr_c^{k*}}{dr_o} = \begin{cases} < 0 & \text{if} \quad \varphi^k > \bar{\varphi}^k \\ \geq 0 & \text{if} \quad \varphi^k \leq \bar{\varphi}^k \end{cases}$ *and* $\frac{dQ_c^k}{dq_o} = \begin{cases} < 0 & \text{if} \quad \varphi^k > \bar{\varphi}^k \\ \geq 0 & \text{if} \quad \varphi^k \leq \bar{\varphi}^k \end{cases}$*, where* $\bar{\varphi}^k$ *solves* $\frac{dr_c^{k*}(\varphi^k)}{dr_o} = 0$*, and* $\bar{\varphi}^k < 0$*;*

c. $\frac{dr_c^{k*}}{dq_c^l} = \begin{cases} < 0 & \text{if} \quad \varphi^k < 0 \\ \geq 0 & \text{if} \quad \varphi^k \geq 0 \end{cases}$ *and* $\frac{dQ_c^{k*}}{dq_c^l} = \begin{cases} < 0 & \text{if} \quad \varphi^k < 0 \\ \geq 0 & \text{if} \quad \varphi^k \geq 0 \end{cases}$*;*

where $k, l = \{i, j\}$ *and* $k \neq l$*.*

The intuition behind Proposition 7a is the following: when φ^k is large (greater than $\hat{\varphi}^k$), the two software programs are not very substitutable and the primary competition to each CSS comes from the OSS. The greater the inital quality of the OSS, the greater the competion with the OSS, and thus the lower the market share of each CSS. As a result, each firm has less incentive to improve its own quality and thus the optimal resource investment decreases with the initial quality of the OSS. When φ^k is small (smaller than $\hat{\varphi}^k$), then the two firms compete head-to-head and with the OSS. An increase in the initial quality of the OSS leaves a smaller market for the two firms to share. Thus, it now becomes imperative for each firm to protect its individual share of the market. Thus each firm will invest more intensively. Result Proposition 7b has an intuition similar to result Proposition 7a.

8 Other Extensions

In this section, we consider a few extensions of our base model.

8.1 Competition for resources

Until now we have assumed that the CSS and the OSS do not compete for resources and that they only compete on the demand side. Anecdotal evidence suggests that most of the programmers who work on the OSS are hobbyists or enthusiasts who may have day jobs. Thus, the CSS and OSS get programmers from two different pools, the CSS from the 'wage earner' pool, and the OSS from a 'hobbyist' pool. It is therefore quite realistic to assume that the CSS and the OSS do not compete for resources. However, there could be specific situations, such as when programmers with specific skill sets are required by both the CSS and the OSS, and when the programming pool with those specific requirements is limited, when the CSS and the OSS could compete for resources. In this section, we consider the impact of this supply-side competition between the CSS and the OSS. The cost function of the firm developing the CSS is $C(r_c|r_o)$. Note that now the cost of investing resources, r_c, depends on the resources that are available to the OSS, r_o, as opposed to the base model, where there was no such dependence. We will further explain this dependence shortly. The cost function continues to be increasing and convex in r_c, as in assumption 1. We make the following additional assumption:

Assumption 6 *(i)* $\frac{dC}{dr_o} \geq 0$ *and (ii)* $\frac{dr_o(r_c)}{dr_c} \leq 0$.

The rationale behind assumption 6(i) is the following: when the resources that the OSS gets, r_o, increase, then the size of the programming pool from which the CSS can hire shrinks. Since programmers are in short supply, their wages are higher, which increases the cost to the firm for hiring them. Similarly, if the firm hires a lot of programmers, then fewer programmers are available to work on the OSS, hence assumption 6(ii).

The optimal price is the same as in the base model. The comparative statics of the optimal resource investment, r_c^*, with rescpect to q_o and q_c, are given in the following proposition:

Proposition 8 *Denoting* $\frac{d^2 Q_c}{dq_c dr_c}$ *by* μ*,*

a. $\frac{dr_c^*}{dq_o} \leq 0$ *and* $\frac{dQ_c}{dq_o} \leq 0$*,*

b. $\frac{dr_c^*}{dq_c} = \begin{cases} \geq 0 & if \quad \mu \geq \check{\mu} \\ < 0 & if \quad \mu < \check{\mu} \end{cases}$ *and* $\frac{dQ_c^*}{dq_c} = \begin{cases} \geq 0 & if \quad \mu \geq \acute{\mu} \\ < 0 & if \quad \mu < \acute{\mu} \end{cases}$,

where $\check{\mu}$ *solves* $\frac{dr_c^*(\mu)}{dq_c} = 0$*,* $\acute{\mu}$ *solves* $\frac{dQ_c}{dq_c} / \frac{dQ_c}{dr_c} + \frac{dr_c^*(\mu)}{dq_c} = 0$ *and* $\acute{\mu} < \check{\mu} < 0$*.*

We get results similar to the base model. Hence, we find that our results are quite robust.

8.2 Uncertainty about r_o

In the base model, we assumed that the firm developing the CSS has full knowledge about the resources available to the OSS, r_o. In actual practice, the firm may only have an estimate of r_o. In this section, we show that this uncertainty has no impact on the resource investment made by the firm.

The firm has the following information: it knows that the resource available to the OSS will be $r_o^L = r_o - \delta$, with probability 1/2, and $r_o^H = r_o + \delta$, with probability 1/2. Thus, δ is a measure of the uncertainty regarding r_o. At the time when the firm makes the decision regarding the resource investment, it is uncertain about the actual resources available to the OSS, and hence the final quality of the OSS. However, at the time when the price decision is made, the firm can observe the actual quality of the OSS, and so can the consumers. We can show that $\frac{dr_c^*}{d\delta} = 0$ (see Appendix), i.e., the uncertainty the firm has regarding r_o does not affect the optimal investment choice made by the firm. The intuitive reasoning behind this result is the following: the expected decrease in the resource investment if the firm overestimates the resources available to the OSS cancels out the expected increase in investments if the firm underestimates the resources available to the OSS. Hence, the optimal resource investment is not affected by this uncertainty. Thus, all that the firm needs is the estimate of r_o.

8.3 Uncertainty about the software development process

In this section, we consider the case when the firm is uncertain about the final qualities, given an initial quality and resource investment, i.e., it is uncertain about the software development process. For an intitial quality, q_c, and resource investment, r_c, let the final quality of the CSS be $Q_c(q_c, r_c) - \varepsilon$, with probabilty 1/2, and $Q_c(q_c, r_c) + \varepsilon$, with probability 1/2. Similarly, for an intitial quality, q_o, and resource investment, r_o, let the final quality of the OSS be $Q_o(q_o, r_o) - \epsilon$, with probabilty 1/2, and $Q_o(q_o, r_o) + \epsilon$, with probability 1/2. Thus, $\varepsilon(\epsilon)$ is a measure of the uncertainty regarding the final quality, $Q_c(Q_o)$. At the time when the firm makes the resource investment decision, it is uncertain about the resulting final qualities of the CSS and the OSS. However, at the time of making the price decision, the firm can observe the actual qualities of both the CSS and the OSS, and so can the consumers. We can show that $\frac{dr_c^*}{d\varepsilon} = 0$ and $\frac{dr_c^*}{d\epsilon} = 0$ (The proof is similar to the proof for Section 8.2), i.e., the uncertainty the firm has regarding the actual final qualities does not affect the optimal choices made by the firm. The intuitive reasoning is also similar to result in Section 8.2: the impact of overestimation and underestimation cancel each other out. Thus, all that the firm needs is an estimate of $Q_c(q_c, r_c)$ and $Q_o(q_o, r_o)$.

9 Discussion

We show in this paper that a firm produces lower-quality CSS when the only competition it faces is from an OSS. Also, the quality of the CSS decreases as the quality of the OSS increases. This result is robust even if we consider NE. Additionally, the resource investment by the firm is increasing with the strength of the NE. We also find that, with the OSS in the market, the resource investment by the firm and the final quality of the CSS increase in the initial quality until a later stage in the software lifecycle compared to the case when there is no OSS in the market. Interestingly, if we consider competition from another CSS, in addition to competition from the OSS, then the quality of the CSS could be increasing or decreasing with the increasing quality of the OSS. The answer depends on how closely substitutable the two CSS are. The results from the model are robust to several changes in the model specifications, such as competition for resources between the OSS and the CSS, uncertainty regarding resources available to the OSS, and uncertainty regarding the software development process.

The novel contribution of this paper is in the analysis of resource investment decision (which affects the quality of the software produced) of a CSS vendor when faced with competition from an OSS. We capture the special type of threat to the CSS from an OSS product. When CSS firms compete against one another, each firm chooses its strategies actively. The OSS, on the other hand, is produced by a volunteer-based community and will not behave strategically. The OSS is thus a passive type of threat to CSS firms. We find that when CSS faces competition only from an OSS, or faces competition from an OSS and another CSS that is not a close substitute, then the incentive of the firms to develop higher-quality products decreases when the quality of the OSS increases. However, when there is competition between two closely substitutable CSS and an OSS, then the incentive of the firms to develop higher-quality CSS increases as the quality of the OSS increases.

Although we cannot empirically test our results given that it is difficult to collect data on resource investments made by software firms, we can present our results in the context of several real-world settings. In the web server software market, competition is essentially between Microsoft's IIS software and the OSS Apache. Our results suggest that Microsoft would have a lower incentive to improve the quality of IIS as the quality of Apache increases. Thus, competition with an OSS negatively affects the quality of the CSS. In the personal finance and accounting software industry, CSS like Intuit's Quicken and Microsoft's Money compete with GNU Cash, an OSS. Since Intuit's Quicken and Microsoft's Money are close substitutes, our results suggest that both Intuit and Microsoft will invest more resources in improving their respective programs, as the quality of GNU Cash improves. Implications for managers at firms developing CSS are that when the primary competior of the CSS is an OSS, then it is optimal to reduce investment in improving the quality of the CSS. However, when the CSS faces competition from another CSS, in addition to the OSS, the resource investment to improve the quality of the CSS must be increased.

We have assumed that the firm makes an estimate about the software development process, i.e., the firm can estimate the function that determines the final quality of the CSS, given the initial quality and resources invested. The firm can form this estimate using historical data and/or software process engineering. With software process engineering, each task that needs to be accomplished can be quantified in terms of the programming hours required. Thus, the firm has a good idea of the final quality that can be achieved, given the current quality of the CSS and the resources invested. It is more difficult for the firm to estimate the function that determines the final quality of the OSS, since the firm may not be familiar with the processes in the virtual firm that develops the OSS. Familiarity with the OSS will help the firm better estimate the processes. We have shown that, given that the firm can form an estimate about the software development process, uncertainty does not affect the choice of the optimal resource investment.

Currently, there is a debate in the open source community on whether or not social planners should actively promote OSS.[12] Our initial thought on the social welfare analysis in the current setting is that the results would be dependent on the functional forms chosen. Moreover, our focus in this paper has been on the strategic choices made by the CSS firm, hence social welfare analysis is not central to the main analysis here. The social planner's policy choice of whether or not to promote OSS merits further study. The model in the paper captures the strategic response of a CSS firm to an OSS in a static framework. Future research may extend this model using evolutionary game theory to see how OSS impacts

[12] http://foss4us.org/node/126.

a CSS firms' strategic decisions over time. From a modeling standpoint, we restrict ourselves here to a linear demand function. Future research may also study non-linear demand functions.

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Appendix

Proof of Lemma 1: a. By differentiating (1) wrt Q_c, we get:

$$2\frac{dp_b^*}{dQ_c}\frac{dD}{dp}+\frac{dD}{dQ_c}=0. \tag{A-1}$$

Using (A-1), and from assumption 3, $\frac{dp_b^*}{dQ_c}\geq 0$.

b. The proof is similar to part a and is omitted. □

Proof of Proposition 1: Differentiating (5) wrt q_c and simplifying, we get:

$$\frac{dr_c^*}{dq_c}=\frac{-2\frac{dD}{dp}\frac{dp_b^*}{dQ_c}[\frac{dp_b^*}{dQ_c}\frac{dQ_c}{dq_c}\frac{dQ_c}{dr_c}+p_b^*\frac{d^2Q_c}{dq_c dr_c}]}{2(\frac{dp_b^*}{dQ_c})^2(\frac{dQ_c}{dr_c})^2\frac{dD}{dp}+2p^*\frac{dp_b^*}{dQ_c}\frac{dD}{dp}(\frac{d^2Q_c}{dr_c^2})+C''(r_c^{b*})}.$$

From assumption 3(i), lemma 1a and from the condition of the concavity of the profit function (6), the sign of $\frac{dr_c^{b*}}{dq_c}$ depends on the sign in the square brackets. Let $\hat{\mu}_b=-\frac{1}{p_b^*}\frac{dp_b^*}{dQ_c}\frac{dQ_c}{dq_c}\frac{dQ_c}{dr_c}$. From assumption 2 and lemma 1, $\hat{\mu}_b<0$. Thus,

$$\frac{dr_c^{b*}}{dq_c}=\begin{cases}\geq 0 & \text{if } \frac{d^2Q_c}{dq_c dr_c}\geq\hat{\mu}_b\\ <0 & \text{if } \frac{d^2Q_c}{dq_c dr_c}<\hat{\mu}_b.\end{cases}$$

Now, $\frac{dQ_c^{b*}}{dq_c}=\frac{dQ_c}{dq_c}+\frac{dQ_c}{dr_c}\frac{dr_c^{b*}}{dq_c}$. When $\frac{dr_c^*}{dq_c}\geq 0$, $\frac{dQ_c}{dq_c}\geq 0$.□

Proof of Proposition 2: By evaluating $\frac{d\pi_b}{dp}$ at p^*, we get:

$$\frac{d\pi_b(p^*)}{dp}=D(p^*,Q_c)+p^*\frac{dD}{dp}.$$

From (4),

$$\frac{d\pi_b(p^*)}{dp}=D(p^*,Q_c)-D(p^*,Q_c,Q_o)\geq 0. \tag{A-2}$$

From (1) and (A-2), $p^*\leq p_b^*$. □

Proof of Lemma 2: The proof is similar to lemma 1 and is omitted. □

Proof of Proposition 3: By evaluating $\frac{d\pi_b(p_b^*)}{dr}$ at r_c^*, we get:

$$\frac{d\pi_b(p_b^*(Q_c(r_c^*)))}{dr} = -2p_b^*(Q_c(r_c^*))\frac{dp_b^*}{dQ_c}\frac{dQ_c(r_c^*)}{dr_c}\frac{dD}{dp} - C'(r_c^*). \tag{A-3}$$

By differentiating (1) wrt Q_c, we get:

$$2\frac{dp_b^*}{dQ_c}\frac{dD}{dp} + \frac{dD}{dQ_c} = 0. \tag{A-4}$$

Similarly, by differentiating (4) wrt Q_c, we get:

$$2\frac{dp^*}{dQ_c}\frac{dD}{dp} + \frac{dD}{dQ_c} = 0. \tag{A-5}$$

By using (A-4), (A-5), and assumption 3(i), we get:

$$\frac{dp_b^*}{dQ_c} = \frac{dp^*}{dQ_c} > 0. \tag{A-6}$$

From (5), (A-3), and (A-6), we get:

$$\frac{d\pi_b(p_b^*(Q_c(r_c^*)))}{dr} = -2\frac{dp_b^*}{dQ_c}\frac{dQ_c(r_c^*)}{dr_c}\frac{dD}{dp}[p_b^*(Q_c(r_c^*)) - p^*(Q_c(r_c^*)]. \tag{A-7}$$

From assumptions 2(i) and 3(ii) and proposition 2, the right hand side of (A-7) is ≥ 0. Hence, $r_c^* \leq r_c^{b*}$. $\square$

Proof of Proposition 4:

a. By differentiating (5) wrt q_o, we get:

$$2(\frac{dp^*}{dQ_o}\frac{dQ_o}{dq_o} + \frac{dp^*}{dQ_c}\frac{dQ_c}{dr_c}\frac{dr_c^*}{dq_o})\frac{dp^*}{dQ_c}\frac{dQ_c}{dr_c}\frac{dD}{dp} + 2p^*\frac{dp^*}{dQ_c}\frac{dD}{dp}\frac{dr_c^*}{dq_o}\frac{d^2Q_c}{dr_c{}^2} + C''(r_c)\frac{dr_c^*}{dq_o} = 0. \tag{A-8}$$

From (A-8), we get:

$$\frac{dr_c^*}{dq_o} = \frac{-2\frac{dp^*}{dQ_o}\frac{dQ_o}{dq_o}\frac{dp^*}{dQ_c}\frac{dQ_c}{dr_c}\frac{dD}{dp}}{2(\frac{dp^*}{dQ_c})^2(\frac{dQ_c}{dr_c})^2\frac{dD}{dp} + 2p^*\frac{dp^*}{dQ_c}\frac{dD}{dp}(\frac{d^2Q_c}{dr_c^2}) + C''(r_c^*)}.$$

From assumptions 2 and 3(i) and lemma 2a, the numerator is negative. From the condition of the concavity of the profit function (6), the denominator is positive. Hence, $\frac{dr_c^*}{dq_o} \leq 0$.

Now, $\frac{dQ_c^*}{dq_o} = \frac{dQ_c}{dr_c}\frac{dr_c^*}{dq_o} \leq 0$.

b. The proof is similar to Proposition 1 and is omitted.

c. Along similar lines as part a, we can obtain that:

$$\frac{dr_c^*}{dr_o} = \frac{-2\frac{dD}{dp}\frac{dp^*}{dQ_c}\frac{dp^*}{dQ_o}\frac{dQ_o}{dr_o}\frac{dQ_c}{dr_c}}{2(\frac{dp^*}{dQ_c})^2(\frac{dQ_c}{dr_c})^2\frac{dD}{\partial p} + 2p^*\frac{dp^*}{dQ_c}\frac{dD}{dp}(\frac{d^2Q_c}{dr_c^2}) + C''(r_c^*)} \leq 0,$$

and $\frac{dQ_c^*}{dr_o} = \frac{dQ_c}{dr_c}\frac{dr_c^*}{dr_o} \leq 0$. □

Proof of Proposition 5: a. We know that $\hat{\mu}_b$ satisfies:

$$\frac{dp_b^*}{dQ_c}\frac{dQ_c}{dq_c}\frac{dQ_c}{dr_c} + p_b^*\hat{\mu}_b = 0. \tag{A-9}$$

Also, we know that $\hat{\mu}$ satisfies:

$$\frac{dp^*}{dQ_c}\frac{dQ_c}{dq_c}\frac{dQ_c}{dr_c} + p^*\hat{\mu} = 0. \tag{A-10}$$

Using (A-6) and proposition 2, (A-9) and (A-10) gives us $\hat{\mu} < \hat{\mu}_b < 0$.

b. Using the result from part a, it is straightforward to show that $\tilde{\mu} < \tilde{\mu}_b < 0$. □

Proof of Lemma 3: The first order condition (FOC) for determining the optimal price, p_n^*, is:

$$D(p_n^*, Q_c, Q_o, D^e) + p_n^*\frac{dD}{dp} = 0. \tag{A-11}$$

a. By differentiating (A-11) wrt D^e, we get:

$$2\frac{dp_n^*}{dD^e}\frac{dD}{dp}+\gamma=0. \tag{A-12}$$

From assumptions 3a and 4, $\frac{dp_n^*}{dD^e}\geq 0$.

b. By differentiating (A-11) wrt Q_c, we get:

$$2\frac{dD}{dp}\frac{dp_n^*}{dQ_c}+\gamma\frac{dD^e}{dQ_c}+\frac{dD}{dQ_c}=0. \tag{A-13}$$

By differentiating (7) wrt Q_c, we get:

$$\frac{dD}{dp}\frac{dp_n^*}{dQ_c}+(\gamma-1)\frac{dD^e}{dQ_c}+\frac{dD}{dQ_c}=0 \tag{A-14}$$

Solving (A-13) and (A-14) yields

$$\frac{dp_n^*}{dQ_c}=\frac{\frac{dD}{dQ}}{-\frac{dD}{dp}(2-\gamma)}. \tag{A-15}$$

From assumption 3 and using the fact that $\gamma<2$, $\frac{dp_n^*}{dQ_c}\geq 0$.

c. The proof is similar to part b and is omitted. □

Proof of Proposition 6: a. By substituting the optimal price, p_n^*, into the profit function, and using (A-11), we get:

$$\pi_n=-(p_n^*)^2\frac{dD}{dp}-C(r_c).$$

The optimal resource investment, r_c^{n*}, satisfies the FOC:

$$-2p_n^*\frac{dD}{dp}\frac{dp_n^*}{dQ_c}\frac{dQ_c}{dr_c}-C'(r_c)=0. \tag{A-16}$$

By differentiating (A-16) wrt γ, we obtain:

$$\frac{dr_c^{n*}}{d\gamma} = \frac{-2p_n^* \frac{d^2 p_n^*}{d\gamma dQ_c} \frac{dQ_c}{dr_c} \frac{dD}{dp}}{2(\frac{dp_n^*}{dQ_c})^2 (\frac{dQ_c}{dr_c})^2 \frac{dD}{dp} + 2p_n^* \frac{dp_n^*}{dQ_c} \frac{dD}{dp} (\frac{d^2 Q_c}{dr_c^2}) + C''(r_c^{n*})}.$$

From the second-order condition (SOC), the denominator is positive. From (A-15), we can get $\frac{d^2 p_n^*}{d\gamma dQ_c} \geq 0$. Using this, and from assumptions 2 and 3, $\frac{dr_c^{n*}}{d\gamma} \geq 0$. Now, $\frac{dQ_c^{n*}}{d\gamma} = \frac{dQ_c}{dr_c} \frac{dr_c^*}{d\gamma} \geq 0$.

b. Using lemma 3a, the proof is similar to proposition 4a and is omitted.

c. The proof is similar to proposition 4b and is omitted.

d. The proof is similar to proposition 4c and is omitted. □

Proof of Lemma 4: p^{i*} and p^{j*} are optimal prices charged by firm i and firm j, respectively. The FOC for calculating the optimal price charged by firm i is:

$$\frac{d\pi^i}{dp^i} = D^i + p^{i*} \frac{dD^i}{dp^i} = 0. \tag{A-17}$$

We will first check the SOC. The Hessian matrix is:

$$\begin{pmatrix} 2\frac{dD^i}{dp^i} & \frac{dD^i}{dp^j} \\ \frac{dD^j}{dp^i} & 2\frac{dD^j}{dp^j} \end{pmatrix}.$$

From assumptions 5(i) and 5(ii), the Hessian matrix is negative semi-definite. Hence, the SOC are satisified. By differentiating (A-17) wrt Q_c^i, and Q_c^j, we get:

$$\frac{dD^i}{dQ_c^i} + 2\frac{dD^i}{dp^i} \frac{dp^{i*}}{dQ_c^i} + \frac{dD^i}{dp^j} \frac{dp^{j*}}{dQ_c^i} = 0, \tag{A-18}$$

and

$$\frac{dD^i}{dQ_c^j} + 2\frac{dD^i}{dp^i} \frac{dp^{i*}}{dQ_c^j} + \frac{dD^i}{dp^j} \frac{dp^{j*}}{dQ_c^j} = 0. \tag{A-19}$$

The FOC for calculating the optimal price charged by firm j is:

$$\frac{d\pi^j}{dp^j} = D^i + p^{j*}\frac{dD^j}{dp^j} = 0. \tag{A-20}$$

By differentiating (A-20) wrt Q_c^j and Q_c^i, we get:

$$\frac{dD^j}{dQ_c^j} + 2\frac{dD^j}{dp^j}\frac{dp^{j*}}{dQ_c^j} + \frac{dD^j}{dp^i}\frac{dp^{i*}}{dQ_c^j} = 0 \tag{A-21}$$

and

$$\frac{dD^j}{dQ_c^i} + 2\frac{dD^j}{dp^j}\frac{dp^{j*}}{dQ_c^i} + \frac{dD^j}{dp^i}\frac{dp^{i*}}{dQ_c^i} = 0. \tag{A-22}$$

a. Solving (A-18),(A-19),(A-21) and (A-22) yields

$$\frac{dp^{i*}}{dQ_c^i} = \frac{-2\frac{dD^j}{dp^j}\frac{dD^i}{dQ_c^i} + \frac{dD^i}{dp^j}\frac{dD^j}{dQ_c^i}}{4\frac{dD^j}{dp^j}\frac{dD^i}{dp^i} - \frac{dD^j}{dp^i}\frac{dD^i}{dp^j}}.$$

From assumption 5(ii), the denominator is positive. From assumption 5(iii), the numerator is also positive. Hence, $\frac{dp^{i*}}{dQ_c^i} \geq 0$. By symmetry, $\frac{dp^{j*}}{dQ_c^j} \geq 0$.

b. Solving (A-18), (A-19), (A-21) and (A-22):

$$\frac{dp^{i*}}{dQ_c^j} = \frac{-2\frac{dD^j}{dp^j}\frac{dD^i}{dQ_c^j} + \frac{dD^i}{dp^j}\frac{dD^j}{dQ_c^j}}{4\frac{dD^j}{dp^j}\frac{dD^i}{dp^i} - \frac{dD^j}{dp^i}\frac{dD^i}{dp^j}}.$$

From assumption 5(ii), the denominator is positive. If $\psi_j^j < 2\psi_j^i$, then numerator is negative, and hence $\frac{dp^{i*}}{dQ_c^j} < 0$. By symmetry we can obtain sign of $\frac{dp^{j*}}{dQ_c^i}$.

c. By differentiating (A-17) and (A-20) wrt Q_o, we get:

$$\frac{dD^i}{dQ_o} + 2\frac{dD^i}{dp^i}\frac{dp^{i*}}{dQ_o} + \frac{dD^i}{dp^j}\frac{dp^{j*}}{dQ_o} = 0, \tag{A-23}$$

and

$$\frac{dD^j}{dQ_o} + 2\frac{dD^j}{dp^j}\frac{dp^{j*}}{dQ_o} + \frac{dD^j}{dp^i}\frac{dp^{i*}}{dQ_o} = 0. \tag{A-24}$$

Solving (A-23) and (A-24):

$$\frac{dp^{i*}}{dQ_o} = \frac{-2\frac{dD^j}{dp^j}\frac{dD^i}{dQ_o} + \frac{dD^i}{dp^j}\frac{dD^j}{dQ_o}}{4\frac{dD^j}{dp^j}\frac{dD^i}{dp^i} - \frac{dD^j}{dp^i}\frac{dD^i}{dp^j}}.$$

From assumption 5(ii), the denominator is positive. From assumptions 5(i), the numerator is negative. Hence, $\frac{dp^{i*}}{dQ_o} \leq 0$. By symmetry, $\frac{dp^{j*}}{dQ_o} \leq 0$.

□ **Proof of Proposition 7**: r_c^{i*} and r_c^{j*} are the optimal resources invested by firms i and j, respectively. The FOC of firm i is:

$$-2p^{i*}\frac{dp^{i*}}{dQ_c^i}\frac{dQ_c^i}{dr_c^i}\frac{dD^i}{dp^i} - C'(r_c^{i*}) = 0. \tag{A-25}$$

a. By differenting (A-25) wrt q_o, we get:

$$\overbrace{[2(\frac{dp^{i*}}{dQ_c^i})^2(\frac{dQ_c^i}{dr_c^i})^2\frac{dD^i}{dp^i} + 2p^{i*}\frac{dp^{i*}}{dQ_c^i}\frac{d^2Q_c^i}{dr_c^{i\,2}}\frac{dD^i}{dp^i} + C''(r_c^{i*})]}^{A}\frac{dr_c^{i*}}{dq_o} + \overbrace{[2\frac{dp^{i*}}{dQ_c^i}\frac{dp^{i*}}{dQ_c^j}\frac{dQ_c^i}{dr_c^i}\frac{dQ_c^j}{dr_c^j}\frac{dD^i}{dp^i}]}^{B}\frac{dr_c^{j*}}{dq_o} + \underbrace{2\frac{dp^{i*}}{dQ_o}\frac{dp^{i*}}{dQ_c^i}\frac{dQ_o}{dq_o}\frac{dQ_c^i}{dr_c^i}\frac{dD^i}{dp^i}}_{C} = 0 \tag{A-26}$$

Similarly, by differentiating the FOC of firm j wrt q_o, we get:

$$\overbrace{[2(\frac{dp^{j*}}{dQ_c^j})^2(\frac{dQ_c^j}{dr_c^j})^2\frac{dD^j}{dp^j} + 2p^{j*}\frac{dp^{j*}}{dQ_c^j}\frac{d^2Q_c^j}{dr_c^{j\,2}}\frac{dD^j}{dp^j} + C''(r_c^{j*})]}^{A'}\frac{dr_c^{j*}}{dq_o} + \overbrace{[2\frac{dp^{j*}}{dQ_c^j}\frac{dp^{j*}}{dQ_c^i}\frac{dQ_c^j}{dr_c^j}\frac{dQ_c^i}{dr_c^i}\frac{dD^j}{dp^j}]}^{B'}\frac{dr_c^{i*}}{dq_o} + \underbrace{2\frac{dp^{j*}}{dQ_o}\frac{dp^{j*}}{dQ_c^j}\frac{dQ_o}{dq_o}\frac{dQ_c^j}{dr_c^j}\frac{dD^j}{dp^j}}_{C'} = 0 \tag{A-27}$$

By solving (A-26) and (A-27), we get: $\frac{dr_c^{i*}}{dq_o} = \frac{BC'-A'C}{AA'-BB'}$ and $\frac{dr_c^{j*}}{dq_o} = \frac{B'C-AC'}{AA'-BB'}$. The Hessian matrix is:

$$\begin{pmatrix} -A & -B \\ -B' & -A' \end{pmatrix}.$$

For the Hessian matrix to be negative semi-definite $A > 0$, $A' > 0$ and $AA' - BB' > 0$. Also, from assumption 2, assumption 5(i) and lemma 4, $C > 0$ and $C' > 0$. Note that if $\frac{dp^{i*}}{dQ_c^j} > 0$, then $B < 0$, and so $\frac{dr_c^{i*}}{dq_o} < 0$. Denote $\frac{dp^{i*}}{dQ_c^j}$ by φ^i. Then, if $\varphi^i \geq \hat{\varphi}^i$, $\frac{dr_c^{i*}}{dq_o} \leq 0$, where $\hat{\varphi}^i$ solves $\frac{dr_c^{i*}(\varphi^i)}{dq_o} = 0$.

Now, $\frac{dQ_c^i}{dq_o} = \frac{\partial Q_c^i}{\partial q_o} + \frac{dQ_c^i}{dr_c^i}\frac{dr^{i*}}{dq_o}$. If $\varphi^i \geq \tilde{\varphi}^i$, $\frac{dQ_c^{i*}}{dq_o} \leq 0$, where $\tilde{\varphi}^i$ solves $\frac{dQ_c^i}{dq_o} / \frac{dQ_c^i}{dr_c^i} + \frac{dr_c^{i*}(\varphi^i)}{dq_o} = 0$.

b. Along similar lines as in part a, we get $\frac{dr_c^{i*}}{dr_o} = \frac{BD' - A'D}{AA' - BB'}$ and $\frac{dr_c^{j*}}{dr_o} = \frac{B'D - AD'}{AA' - BB'}$, where $D = 2\frac{dp^{i*}}{dQ_o}\frac{dp^{i*}}{dQ_c^i}\frac{dQ_o}{dr_o}\frac{dQ_c^i}{dr_c^i}\frac{dD^i}{dp^i}$, $D' = 2\frac{dp^{j*}}{dQ_o}\frac{dp^{j*}}{dQ_c^j}\frac{dQ_o}{dq_o}\frac{dQ_c^j}{dr_c^j}\frac{dD^j}{dp^j}$ and A,A',B and B' are same as in part a. From assumption 2, assumption 5(i) and lemma 4, $D > 0$ and $D' > 0$. For $\varphi^i \geq \bar{\varphi}^i$, $\frac{dr^{i*}}{dr_o} \leq 0$, where $\bar{\varphi}^i$ solves $\frac{dr^{i*}(\varphi^i)}{dr_o} = 0$.

Now $\frac{dQ_c^i}{dr_o} = \frac{dQ_c^i}{dr_c^i}\frac{dr_c^{i*}}{dr_o}$. Thus, from assumption 2, for $\varphi^i \geq \bar{\varphi}^i$, $\frac{dQ^{i*}}{dr_o} \leq 0$.

c. By differentiating the FOC of firms i and j wrt to q_c^j and solving, we get: $\frac{dr_c^{i*}}{dq_c^j} = \frac{BE' - A'F}{AA' - BB'}$, where $E' = 2(\frac{dp^{j*}}{dQ_c^j})^2\frac{\partial Q_c^j}{\partial q_c^j}\frac{dQ_c^j}{dr_c^j}\frac{dD^j}{dp^j}$ and $F = 2\frac{dp^{i*}}{dQ_c^j}\frac{dp^{i*}}{dQ_c^i}\frac{dQ_c^j}{dq_c^j}\frac{dQ_c^i}{dr_c^i}\frac{dD^i}{dp^i}$. From assumption 2 and assumption 5, $E' < 0$. The denominator of $\frac{dr_c^{i*}}{dq_c^j}$ is positive. The numerator can be written as:

$$BE' - A'F = 2\frac{dp^{i*}}{dQ_c^j}\frac{dp^{i*}}{dQ_c^i}\frac{dQ_c^i}{dr_c^i}\frac{dD^i}{dp^i}[E'\frac{dQ_c^j}{dr_c^j} - A'\frac{\partial Q_c^j}{\partial q_c^j}].$$

From assumption 2 and the fact that $E' < 0$ and $A' > 0$, the term in the square brackets is negative. From lemma 4 and assumptions 2 and 5, the sign of $\frac{dr_c^{i*}}{dq_c^j}$ is the same as the sign of $\frac{dp^{i*}}{dQ_c^j}$. Also, $\frac{dQ_c^i}{dr_c^j} = \frac{dQ_c^i}{dr_c^i}\frac{dr_c^{i*}}{dq_c^j}$. Hence, the sign of of $\frac{dQ_c^{i*}}{dq_c^j}$ is the same as the sign of $\frac{dp^{i*}}{dQ_c^j}$.

By symmetry, we get the result that the signs of $\frac{dr_c^{j*}}{dq_c^i}$ and $\frac{dQ_c^{j*}}{dq_c^i}$ are the same as the sign of $\frac{dp^{j*}}{dQ_c^i}$. $\square$

Proof of Proposition 8: a. The FOC for the optimal resource investment is:

$$-2p^*\frac{dD}{dp}[\frac{dp^*}{dQ_c}\frac{dQ_c}{dr_c}+\frac{dp^*}{dQ_o}\frac{dQ_o}{dr_o}\frac{dr_o}{dr_c}]-\frac{dC(r_c^*|r_o)}{dr_c}=0. \quad \text{(A-28)}$$

Differentiating (A-28) wrt q_o, and simplifying, we get:

$$\frac{dr_c^*}{dq_o}=\frac{-2\frac{dD}{dp}\frac{dp^*}{dQ_o}\frac{dQ_o}{dr_o}[\frac{dp^*}{dQ_c}\frac{dQ_c}{dr_c}+\frac{dp^*}{dQ_o}\frac{dQ_o}{dr_o}\frac{dr_o}{dr_c}]}{-\pi''(r_c^*)}.$$

From the SOC, the denominator is positive. By using assumptions 2, 3 and 6, and lemma 2, we get $\frac{dr_c^*}{dq_o}\le 0$. Also, $\frac{dQ_c}{dq_o}=\frac{dQ_c}{dr_c}\frac{dr_c^*}{dq_o}\le 0$.

b. By differentiating (A-28) wrt q_c and simplifying, we get:

$$\frac{dr_c^*}{dq_c}=\frac{-2\frac{dD}{dp}\frac{dp^*}{dQ_c}[\frac{dp^*}{dQ_c}\frac{dQ_c}{dq_c}\frac{dQ_c}{dr_c}+\frac{dp^*}{dQ_o}\frac{dQ_o}{dr_o}\frac{dr_o}{dr_c}\frac{dQ_c}{dr_c}+p^*\frac{d^2Q_c}{dq_c dr_c}]}{-\pi''(r_c^*)}.$$

The rest of the proof is similar to that of proposition 4b and is omitted. □

Proof for Section 8.2: At the pricing stage, the OSS could have two qualities (states): High (if the resources available to the OSS are, r_o^H) and Low (if the resources available to the OSS are, r_o^L). We will represent these two states as $m=H,L$. The optimal price charged by the firm in state m is p^{m*}. Then, the profit function for the firm is:

$$-\frac{1}{2}\sum_{m=H,L}(p^{m*})^2\frac{dD}{dp}-C(r_c).$$

The FOC is:

$$-\sum_{m=H,L}p^{m*}\frac{dp^{m*}}{dQ_c}\frac{dQ_c}{dr_c}\frac{dD}{dp}-C'(r_c^*)=0. \quad \text{(A-29)}$$

By differentiating (A-29) wrt δ and simplifying we get:

$$\frac{dr_c^*}{d\delta}=\frac{-\sum_{m=H,L}\frac{dp^{m*}}{dQ_o^m}\frac{dQ_o^m}{dr_o^m}\frac{dr_o^m}{d\delta}}{\sum_{m=H,L}(\frac{dp^{m*}}{dQ_c})^2(\frac{dQ_c}{dr_c})^2\frac{dD}{dp}+p^{m*}\frac{dp^{m*}}{dQ_c}\frac{d^2Q_c}{dr_c{}^2}\frac{dD}{dp}+C''(r_c^*)}.$$

The denominator is positive from the second-order condition. Now, $\frac{dp^{H*}}{dQ_o^H} = \frac{dp^{L*}}{dQ_o^L}$ and $\frac{dQ_o^H}{dr_o^H} = \frac{dQ_o^L}{dr_o^L}$. Also, $\frac{dr_o^L}{d\delta} = -1$, while $\frac{dr_o^H}{d\delta} = +1$. Thus, the numerator is zero, and so $\frac{dr_c^*}{d\delta} = 0$. □

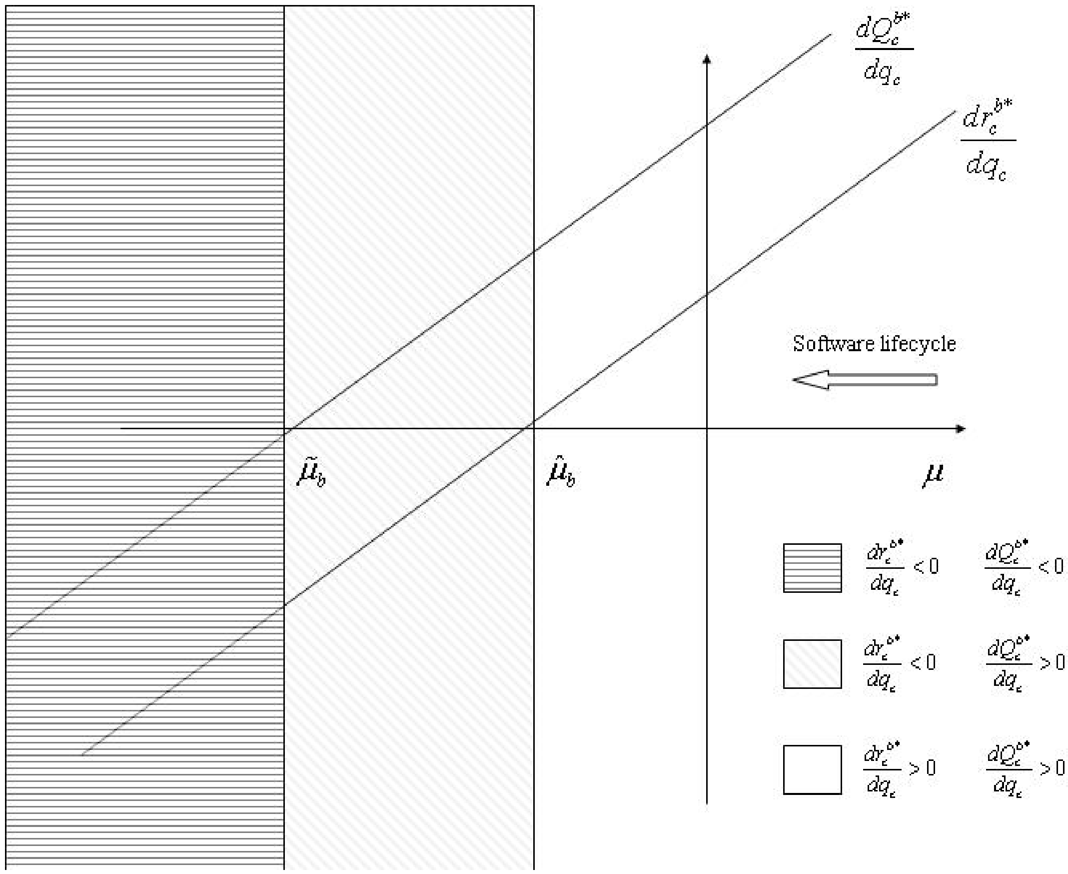

Figure 1: Marginal change in the final quality and the optimal resource investment with the initial quality.