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Integrated Equilibrium in a Heckscher-Ohlin-Ricardo model*

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Abstract

This paper shows that, unlike in the Heckscher-Ohlin model, the integrated equilibrium in the Davis (1995) Heckscher-Ohlin-Ricardo model depends crucially on demand patterns. The area defining the integrated equilibrium is smaller, the greater is the weight placed by consumers on the good that has different technologies across countries.

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1 Introduction

In an important paper, Davis (1995) showed that intra-industry trade, that is, trade in similar goods, can be the outcome of a model based on constant returns to scale and comparative advantage. The basic idea of the Davis Heckscher-Ohlin-Ricardo model is that, when there are three goods, two of which are intra-industry in the sense of sharing identical production techniques, and one of the trading partners has an absolute (technological) advantage in producing one of these intra-industry goods, then if this country has sufficient factor endowments, it will produce the entire world's output of that good. However, the remaining resources may result in the other country producing most of the world's output of the other intra-industry good. There will then be two-way trade in this intra-industry good.

Davis (1995) showed that the integrated equilibrium can be replicated when the country that has a technological advantage in producing one of the intra-industry goods, is able to produce the entire integrated equilibrium supply of that good. However, what Davis (1995) did not show, is that the integrated equilibrium depends on consumer demand. We show in a simple version of the Heckscher-Ohlin-Ricardo model, how demand patterns influence the size of the area in which the integrated equilibrium can be replicated.

We first develop the basic model. Then we allow for consumers to place different weights on each good which they consume, and observe how this affects the integrated equilibrium. Finally, we broaden the discussion to compare this model, with the standard Heckscher-Ohlin and Ricardian models.

2 The model

There are two countries, $c = H, F$ (Home and Foreign), four goods in two pairs, $i \in \{X_1, X_2, Y_1, Y_2\}$. Each pair of products is produced using a pair-specific type of factor input, capital (K) and labour (L). Capital is used exclusively in producing type X goods, and labour in type Y goods, but each factor is perfectly mobile between each pair of goods. The production functions take the form:

$$\begin{array}{ccc} Q_{X_1H} = 2K_{X_1H} & Q_{X_1F} = K_{X_1F} & Q_{X_2c} = K_{X_2c} \\ Q_{Y_1H} = L_{Y_1H} & Q_{Y_1F} = 2L_{Y_1F} & Q_{Y_2c} = L_{Y_2c} \end{array} \qquad (1)$$

where Q_{ic} indicates production of good i in country c, and K_{ic} is the capital used to produce good i in country c, and so on. That is, Home is twice as productive in good X_1 as Foreign, while Foreign is twice as productive in good Y_1 as Home. Both countries are equally productive in the other two goods X_2 and Y_2. All markets are perfectly competitive.

The model combines elements of Davis (1995) and Krugman (1981). From Davis (1995) we combine factor endowment and technological differences across countries (although we introduce symmetric technological differences across both types of goods), while from Krugman (1981) we adopt the simple setup of industry-specific factors of production. The production technology also bears some resemblance to that used in Ruffin (1988), since each good can be produced using only a single factor of production, but each factor can be used in the production of more than one good.

One alternative interpretation of the model is that it is a modified, general equilibrium version of Falvey (1981). In that paper, Falvey showed that, within a single industry with different qualities, countries will specialise in different qualities based on the capital-intensity of different qualities, and the capital-abundance of different countries. This results in two-way trade in different qualities in the same industry. In our model, we may interpret each pair of goods as representing different qualities of the good, but in this case, countries will specialise in different varieties of each pair of goods based on technological comparative advantage.

Returning to the model, consumer utility takes a Cobb-Douglas form:

$$U = \sum \alpha_i \log c_i \qquad \sum \alpha_i = 1 \qquad (2)$$

so that the consumer spends a share α_i of his income on each of the four goods. Finally, endowments of each country are perfectly symmetric with one another, following Krugman (1981) and Soo (2005):

$$\left.\begin{array}{lcc} \text{Home:} & K_H = 2-\gamma & L_H = \gamma \\ \text{Foreign:} & K_F = \gamma & L_F = 2-\gamma \end{array}\right\} \quad 0 \leq \gamma \leq 1 \qquad (3)$$

where K_c is the capital stock in country c. Therefore, Home is relatively abundant in capital, and Foreign in labour. Total world endowment of each factor of production is equal to 2. The parameter γ represents the relative endowments of the factors of production; larger values of γ imply increasing similarity in relative endowments across countries.

3 Integrated equilibrium

Consider what happens when we allow for free trade in goods between the two countries. Following Davis (1995), we ask: what values of γ are consistent with replicating the integrated equilibrium, that is, the resource allocation that would occur if both goods and factors of production are freely traded. In the integrated equilibrium factor price equalisation (FPE) holds across countries. To replicate the integrated equilibrium, it must be the case that each country produces the world output of the good(s) in which it has a technological advantage. Ricardian technology implies that, if Home produces both goods X_1 and X_2, the price ratio is equal to the opportunity cost of production: $\left(\frac{p_{X_1}}{p_{X_2}}\right)_H = \frac{1}{2}$; and, if Foreign produces both goods Y_1 and Y_2, then $\left(\frac{p_{Y_1}}{p_{Y_2}}\right)_F = \frac{1}{2}$.

Suppose that the consumer income share on each good is the following: $\alpha_{X_1} = \frac{4}{10}$, $\alpha_{X_2} = \frac{3}{10}$, $\alpha_{Y_1} = \frac{2}{10}$, $\alpha_{Y_2} = \frac{1}{10}$. That is, consumers place the greatest share of their income on consumption of good X_1, followed by X_2, Y_1 and Y_2. For expenditure on X_1 to be $\frac{4}{3}$ that of expenditure on X_2 when the price of good X_1 is half that of X_2, it must be that output of X_1 is $\frac{8}{3}$ that of X_2, so that, at the world level, the capital used in producing X_1 is $\frac{4}{3}$ that of the capital used in producing X_2. Home has the capital endowment needed to produce the world output of X_1 if $\gamma \leq \frac{6}{7}$. Using a similar argument for goods Y_1 and Y_2, we can show that Foreign has the labour endowment needed to produce the world output of Y_1 if $\gamma \leq \frac{2}{3}$.

We can represent this graphically using the Dixit-Norman-Helpman-Krugman (DNHK) rectangle[1] in Figure 1. Here, the world endowment of capital and labour are given by K and L, and the two origins are for Home and Foreign, respectively. Point D is the mid-point of the DNHK rectangle. The line KD represents changing values of the relative endowment parameter γ between 0 and 1. Point D corresponds to the endowment such that $\gamma = 1$, while point K corresponds to the endowment such that $\gamma = 0$.

In the case discussed here, the integrated equilibrium is represented by the area $KABC$. This area is not symmetric around the line KD; if endowment is along this line, then the binding constraint on the integrated equilibrium is the relative endowment of labour between the two countries. This is so, because the weight which consumers place on good Y_1 relative to Y_2, is greater than the weight they place on X_1 relative to X_2; it can be shown that setting these relative weights equal to one another, would result in a symmetric integrated equilibrium. That is, it is the weight which consumers place on the technologically differentiated goods relative to the identical-technology goods within the same product pair, which determines the size of the integrated equilibrium.

Figure 1 shows that a greater weight placed on the technologically differentiated good, reduces the size of the integrated equilibrium. This is because, to replicate the integrated equilibrium, countries with a technological advantage in producing a good, must produce the world output of that good. The greater the weight that consumers place on these goods, the greater the resources required to produce the world output of these goods, hence the more restricted is the possible allocation of resources that can replicate the integrated equilibrium.[2]

The relative weights which consumers place on the X-goods relative to the Y-

[1] First popularised by Dixit and Norman (1980), then used in a variety of contexts by Helpman and Krugman (1985).

[2] Note that the model in Davis (1995) assumes identical technologies in the Y-sector across countries. This would be equivalent in our model to setting the weight on the technologically differentiated good Y_1 equal to zero, thus extending the integrated equilibrium to include the entire labour endowment.

goods, does not matter for the size of the integrated equilibrium. What matters, are the relative weights placed on goods which are substitutable in their factor inputs. Also, since there are no trade barriers across countries, it is world relative demands that matter, not individual country demands; a home bias in consumption does not change the size of the integrated equilibrium.

4 Discussion and conclusions

It is useful to compare the results on replicating the integrated equilibrium in the Heckscher-Ohlin-Ricardo (HOR) model above, with the standard Heckscher-Ohlin (HO) model. Consider the above model, when consumers place the following weights on the demand for each of the four goods: $\alpha_{X_1} = \alpha_{Y_1} = 0$, $\alpha_{X_2} = \alpha_{Y_2} = \frac{1}{2}$; that is, consumers consume only goods X_2 and Y_2, where technologies are identical across countries. The integrated equilibrium will then be the entire DNHK rectangle $O_H K O_F L$. Different weights on goods X_2 and Y_2 will change the relative goods and factor prices, but the integrated equilibrium can always be replicated because there are no constraints on the location of production since technologies are identical across countries.

On the other hand, if we reverse the weights on demand, to $\alpha_{X_1} = \alpha_{Y_1} = \frac{1}{2}$, $\alpha_{X_2} = \alpha_{Y_2} = 0$, then consumers only demand the goods that have technological differences across countries, so that we revert to a type of Ricardian model. In this case, factor price equalisation cannot occur. Drawing on the example in the previous section, as the weight on the goods with identical technologies across countries decreases relative to the weight on the technologically differentiated goods, the integrated equilibrium shrinks and eventually vanishes.

These comparisons make it clear why the integrated equilibrium is more restricted in the HOR model than in the HO model. The HOR model imposes more constraints than in the HO model, because it specifies that one country has a technological advantage in producing certain goods, and therefore to replicate the integrated equilibrium this country must be the one that produces the world output of the good. In the HO model, in contrast, because of identical technologies, it doesn't matter where goods are produced. The restrictions that the HOR model places on the integrated equilibrium, are tighter the greater is the weight placed by consumers on the goods which have different technologies across countries.

References

[1] Davis, D. R., 1995, Intra-industry Trade: A Heckscher-Ohlin-Ricardo Approach, Journal of International Economics, 39(3-4), 201-226.

[2] Dixit, A. K. and V. Norman, 1980, Theory of International Trade (Cambridge University Press, Cambridge).

[3] Falvey, R. E., 1981, Commercial Policy and Intra-Industry Trade, Journal of International Economics, 11(4), 495-511.

[4] Helpman, E. and P. R. Krugman, 1985, Market Structure and Foreign Trade (MIT Press, Cambridge, MA).

[5] Krugman, P. R., 1981, Intraindustry Specialization and the Gains from Trade, Journal of Political Economy 89(5), 959-974.

[6] Ruffin, R. J., 1988, The Missing Link: The Ricardian Approach to the Factor Endowments Theory of Trade, American Economic Review 78(4), 759-772.

[7] Soo, K. T., 2005, Explaining International Trade Among China, India and the US, mimeo, Lancaster University.

Figure 1: Dixit-Norman-Helpman-Krugman rectangle showing the integrated equilibrium.