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Scheduling fixtures for basketball New Zealand

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SCHEDULING FIXTURES FOR BASKETBALL NEW ZEALAND

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Statement of Scope and Purpose

This paper describes a real complex sports scheduling application with few constraints but many diverse objectives. Issues are highlighted concerning the problem formulation, the solution procedure and implementation details. Experimental runs are described concerning the partial use of a specific solution structure during metaheuristic search. Results are presented showing that this approach may considerably reduce the cost of the final solution.

Abstract

This paper describes the problem faced every year by Basketball New Zealand in scheduling the National Basketball League fixtures. This is a combinatorial optimization problem with few constraints but many objectives, which are described in detail. Two features of the problem cause particular difficulty – the requirement that every team plays two matches in at least two rounds during the season and the fact that stadium availability is far from certain at the start of the process and must be negotiated once a draft schedule has been produced, necessitating an iterative process with possibly many drafts before the final schedule is confirmed.

A variant of Simulated Annealing is used to solve this problem, producing one or more schedules of high quality. The system will be used in practice for the 2004 season.

The paper also reports the results of experiments regarding the use of a potentially useful, but very restrictive, solution structure for the initial solution and possibly beyond. The results of the experiments show that, for this problem at least, it appears best to stick with this restrictive structure for part of the metaheuristic search procedure, but then to remove this restriction for the remainder of the process.

Key Words – Scheduling, timetabling, sport, basketball, metaheuristics, simulated annealing, multiple objectives, subcost guided search, initial solution

Scheduling of sports fixtures

The academic literature contains several papers concerned with the scheduling of sports fixtures. The sports include American Football [1], baseball [2], chess [3], cricket [4], [5], [6], dressage [7], ice hockey [8], [9], tennis [10] and even the Olympic Games [11]. Some of these applications have used highly problem-specific approaches, but others have used more general techniques including goal programming [1], simulated annealing [5], subcost-guided search [6], integer programming [8], tabu search [9], genetic algorithms [9] and local search [10].

Basketball scheduling has also received some consideration. Nemhauser and Trick [12] described an integer programming approach to scheduling the fixtures of the Atlantic Coast Conference; later Henz [13] tackled the same problem using constraint programming.

This paper concerns the scheduling of fixtures for the most important basketball league in New Zealand. This is a problem with few genuine constraints but several objectives of various kinds, some of which pose particular challenges to any potential solver.

The first part of the paper describes the problem in detail and the general solution approach adopted. The second part then describes experiments carried out concerning the precise implementation of this approach.

The National Basketball League of New Zealand

Basketball New Zealand (BNZ), based in Wellington, must schedule the fixtures of various basketball leagues. The most important and the most complex of these is the National Basketball League (NBL), which in 2003 featured ten teams based throughout the country.

The league follows a standard structure in that every team plays twice against every other team, once at home and once away. In 2003 there were fifteen available weekends for these matches, for which matches could be scheduled on any of Friday, Saturday and Sunday. There was also the possibility of matches on the Wednesday immediately prior to the first weekend.

Each team therefore was required to play eighteen matches in sixteen rounds, which inevitably meant a certain amount of "doubling up". This feature will be discussed in greater detail later, since it represents one of the most important and difficult facets of any acceptable solution.

The teams involved in 2003 were:

- Harbour (based in Auckland)
- Auckland
- Waikato (Hamilton)
- Taranaki (New Plymouth)
- Hawkes Bay (Napier)
- Manawatu (Palmerston North)
- Wellington

- Nelson
- Canterbury (Christchurch)
- Otago (Dunedin)

Since travel distances are of vital importance when considering a team's "double" rounds, it was necessary to create an approximate travel matrix (see Table 1) for journeys between these teams' headquarters. The matrix entries are not strictly proportional to distance; they also incorporate some measure of "difficulty".

********** insert Table 1 about here ************

Note that the teams divide neatly into pairs such that the distance between paired teams is no more than one unit. These pairs are:

- Harbour / Auckland
- Waikato / Taranaki
- Hawkes Bay / Manawatu
- Wellington / Nelson
- Canterbury / Otago

This is to some extent artificial – after all, Canterbury and Otago have home venues a considerable distance apart and there is a substantial body of water between Wellington and Nelson. However, these pairings are important.

As noted earlier, there will be rounds where a team must play twice. Generally this means playing two away matches – teams are not keen on having a home match and an away match in the same round, and are even less keen on having two home matches. It is important to the teams that their two away venues should not be too far from one another. This is why the teams have been "paired". For example, Canterbury and Otago are paired, meaning that when a team travels for a match in Canterbury, then it is desirable that they also have an away game against Otago in the same round. While this may not happen for every single team, an acceptable timetable will feature several such pairings of matches. Other pairings may be acceptable (e.g. Manawatu / Wellington), but the pairings listed above represent the "ideal".

This issue will be discussed further when variations of the program are described.

Stadium availability

One complicating factor is that, at the time the scheduling procedure begins, there is a great deal of uncertainty about the availability of stadia. Every team has its preferred venue for home matches, though for some teams there may be acceptable alternatives for occasional use. However, the stadia are all used for other purposes as well as the NBL. There may well be other sports, such as netball, volleyball etc. which use the same stadia; also non-sporting events such as concerts may be held in these stadia. Sometimes priority users may be clear, but often there is a process of negotiation to go through.

Thus a first draft of the schedule is produced which takes account of a best guess as to availability. This draft then forms the basis for local negotiation. As a result of this negotiation, some of the availability details are changed and an amendment to the draft is produced. This continues until all availability issues have been settled. This can take a considerable amount of time (generally several months).

Thus the user of the program inputs availability details for each home team and each applicable date during the season. A number is entered – this at first should range from zero (almost certainly available) to 20 (almost certainly not available). If unavailability is absolutely certain, which may not become clear until the second or succeeding drafts, this figure can be raised to 50, which should be enough to ensure that the program will not pick a home match for that team on that date as part of the final solution

These numbers can also reflect concerns other than stadium availability. For example, if the New Zealand All Black rugby union team has a match in a particular city on a particular day, it would not make any sense to schedule an NBL match in that city on that day. Thus a high number can be entered so as to "pretend" that the stadium is not available.

Alternatively, these numbers can be amended so as to reflect teams' preferences for different days of the week on which to stage home matches. Probably the ideal day for every team to stage a home match is Saturday; in addition, some teams may have locally-based reasons for preferring Friday to Sunday or vice versa. In every case, the user at BNZ has control over the importance granted to all such preferences, by means of these non-availability numbers.

Constraints and objectives

The formulation of this problem contains no hard constraints apart from the most basic: every match must be scheduled somewhere and no team can play two matches on the same day. However, there are invariably several preferences, which can be regarded as fuzzy constraints or objectives. Many of these relate to requests input by the program user.

Each preference relates to a subcost within the overall cost function – this subcost is non-zero when its associated preference is not complied with. The formulations and relative weightings of these subcosts are inevitably highly subjective; they have been chosen after a certain amount of trial and error, after several consultations with the users.

The costs are expressed in "units". It is expected that any single subcost of around 30 units or higher is unlikely to be included in the eventual final schedule, though this will inevitably depend on the number and interaction of the requests made by the user. However, such high costs should not be considered as representing infeasibilities; in particular it can be very useful to enable the search process to visit solutions involving such costs *en route* to a final solution. This is why the careful determination of such subcosts can be very important in ensuring the efficacy of the program.

Illegality – the user can specify that a particular match must, or must not, be scheduled in, before or after a particular round. The penalty for non-compliance is 50 units.

Specific request denied – the user may request, for any team and any round, any of :

- Home match
- Away match
- Bye
- No home match (i.e. away match or bye)
- No away match (i.e. home match or bye)
- No bye
- Two away matches
- Not two away matches

The user also inputs the penalty cost (usually between 1 and 20) for non-compliance with each request.

Availability – every match scheduled may give rise to an availability cost – this is the number entered by the user in the availability file (see above).

Not enough / too many byes for a team – the user may specify a preferred minimum and/or maximum number of byes for any team. The penalty is $4\boldsymbol{D}^2$ or $\boldsymbol{E}^2$, where $\boldsymbol{D}$ is the deficiency and $\boldsymbol{E}$ the excess.

Not enough / too many games in a round – the user may specify a preferred minimum and/or maximum number of games to be played in any round. The penalty is $5\boldsymbol{D}^2$ or $55\boldsymbol{E}^2$.

Byes in a round where not wanted – the user can specify that certain rounds (usually the first and last) should involve all teams. If a team has a bye in such a round, the penalty cost is 75.

Run of home matches too long – the user may specify a preferred maximum number of consecutive weekends for which a team should have a home match – typically this may be 3. The penalty is $\boldsymbol{E}^2$, plus an extra penalty of 15 if this run occurs at the start of the season.

Run of non-home rounds too long – the user may specify a preferred maximum number of consecutive weekends for which a team should have no home match – typically this may be 2. The penalty is $\boldsymbol{E}^2$, plus an extra penalty of 20 if this run occurs at the start of the season. There is also a penalty of 26 if a team is not at home in either of the last two rounds.

Two home matches in a round – the cost for this happening for any team is 90.

Three matches in a round – the cost for this happening for any team is 1800 (i.e. it could never possibly be even thinkable).

Home match followed by away match in same round – cost is 45.

Away match followed by home match in same round – cost is 70 (higher because the benefit of home advantage is effectively reduced).

One of the above four events happening more than once during the season to the same team – extra cost is $270(\boldsymbol{N}-1)^2$, where $\boldsymbol{N}$ is the number of times it happens.

Return matches too close – it is not a good idea to schedule the match ***A*v*B*** very close in time to match ***B*v*A***. If they are in consecutive rounds, the cost is 50; if there is one intervening round the cost is 10; if there are two intervening rounds the cost is 2.

Double rounds too close – teams like their double rounds to be reasonably well spread out. If a team has two matches in each of two consecutive rounds, the cost is 30; if there is just one intervening round, in which the team does not have a bye, the cost is 6; if the team has a bye in the intervening round there is no cost.

TV – the main sports TV network likes to be able to show an NBL match on specific dates, usually Sundays in the second half of the season. There are only certain venues which they deem suitable. Thus the user can specify that at least one of a given set of teams must have a home match on a specific date. The penalty cost is 40.

Unevenness of schedule – at any stage during the season it is desirable that every team should have played approximately the same number of matches. After any round, let X be the difference between the highest and lowest number of matches played by a team to date. Let Y be the number of teams who have played either the maximum or the minimum number of matches at this stage. Then if $X = 2$, the cost is $2Y$; but if $X > 2$, the cost is $10(X-1)^2 + 2Y$. This cost is calculated after every round and the total subcost for unbalanced distribution is the sum of these costs.

First or second home team – when a team plays two away matches in one round, it is thought that the second home team will have a strong advantage, as the away team may be tired after their first game. Thus is it desirable that this advantage be spread about reasonably evenly, though this can only be a minor consideration. Let $\boldsymbol{F}$ be the number of occasions that a team is the first of two home teams, and $\boldsymbol{S}$ the number of times it is the second of two home teams. Then, if $|\boldsymbol{F}\text{-}\boldsymbol{S}| > 1$, the cost is $0.1(|\boldsymbol{F}\text{-}\boldsymbol{S}|\text{-}1)^2$.

Travel saved – in a good timetable, this appears as a benefit or negative cost. Let d_{ij} be the distance from i to j as recorded in the distance matrix. Then, if team $\boldsymbol{x}$ plays two away matches in one round, the first at $\boldsymbol{y}$ and the second at $\boldsymbol{z}$, the distance saved is $(\boldsymbol{d}_{yx} + \boldsymbol{d}_{xz} - \boldsymbol{d}_{yz})$. However, the cost is not simply proportional to the distance saved; an extra incentive is given to a good pairing of $\boldsymbol{y}$ and $\boldsymbol{z}$, and an extra disincentive to a poor pairing, so the overall cost (which will be negative for a good pairing) is $(2\boldsymbol{d}_{yz}^2 - \boldsymbol{r}(\boldsymbol{d}_{yx} + \boldsymbol{d}_{xz}))$, where $\boldsymbol{r} = 1$ if $\boldsymbol{d}_{yz} > 1$, $\boldsymbol{r} = 2$ otherwise. This formulation explains why every team has a pair which is a distance of either 0 or 1 units away; thus every team is in a good pairing which the cost formulation will encourage.

Friday/Sunday – where a team has two matches in a round, there should not be a day between matches, to save on hotel costs and so that teams do not have to be away from home for too long. The cost of a Friday/Sunday pairing is five times the longest distance between any pair of teams.

Solution method

The solution approach used is subcost-guided simulated annealing (SGSA). This is the same as standard simulated annealing (SA), except that the cost increase is adjusted before being used in the acceptance criterion. The acceptance criterion is that a worsening perturbation is accepted if $R < e^{-(C'/T)}$, where R is a random number between 0 and 1, T is the temperature and C' is an amended cost increase, defined as $C' = Ce^{-\theta B/C}$, where C is the overall cost increase, B is the best decrease for any individual subcost and θ is a parameter (here set to 2.5). See Wright [14], [15] for a fuller explanation of SGSA.

Four slight variations of the precise system have been developed. The description here refers to just the first of these; the others are described later, when the experimental runs are discussed.

The initial solution for the first version of the system uses a totally random initial solution, satisfying the hard constraints but not concerned with any of the fuzzy constraints or objectives. This is easily achieved since there are 46 allowable dates (fifteen weekend rounds with three dates per round plus the initial Wednesday) and only eighteen matches involving each team.

Three different types of neighbourhood perturbation are used for the first version of the system, as follows.

Perturbation Type 1 – move one match from its current round and day (R_1,D_1) to (R_2,D_2). R_2 may or may not be equal to R_1. If $R_2 = R_1$ then D_1 and D_2 must not be equal; otherwise they may or may not be equal.

Perturbation Type 2 – move match M_1 from (R_1,D_1) to (R_2,D_2) **and** match M_2 (involving at least one of the same teams as M_1) from (R_2,D_3) to(R_3,D_4). If $R_1 = R_2$ then D_1 must not equal

D_2; if $R_2 = R_3$ then D_2, D_3 and D_4 must all be different; otherwise D_1, D_2 and D_3 may or may not be equal.

Perturbation Type 3 – swap all matches between (R_1,D_i) and (R_2,D_i), for all i = 1,2,3, $R_1 \neq R_2$.

At each iteration a random number is used to determine the type of perturbation, such that the probabilities of each type of perturbation are 0.2, 0.55 and 0.25 respectively. These probabilities have been chosen so as to reflect the fact that there are more perturbations available of some types than of others, but also to allow a reasonably even mix of perturbation types. Another random number is then used to determine the precise perturbation to be considered.

The cooling scheme is geometric, with the temperature being multiplied by a constant at every iteration. The starting temperature is 5, the ending temperature 1 and the number of iterations 2,500,000. The multiplicative parameter can thus be calculated automatically to be about 0.99999936. The starting and ending temperatures were chosen after a fair amount of experimentation; likewise the number of iterations – it needs to be high because neighbourhoods are large.

Since the best solution achieved by any Simulated Annealing method, whether SA or SGSA, may not be a local optimum, a simple descent is carried out from the best SA solution until a local optimum is reached. This is then the solution output.

The time taken to run the system on BNZ's computer is about ten minutes. However, since it is important to them that they obtain as good a solution as possible, they will probably be running the system overnight, thus obtaining about 100 different solutions. The system lists the top ten solution files in ascending order of total cost, thereby giving guidance as to which potential schedules to consider first.

When BNZ have chosen a first draft schedule, this will be presented to the teams. It will definitely include fixtures on days where stadium availability is not certain, so there will then be some negotiation. As a result of this negotiation, issues will have become clearer – some of the required dates will now definitely be OK, but others will definitely not be OK, while others may remain uncertain for the time being.

BNZ will then run the system again with a slightly amended availability file; possibly the requests file will have changed by this time also as a result of feedback from the teams. At this stage it should be possible to reduce some availability costs to zero and to increase others to 20 or beyond – an absolute definite non-availability should be reflected by a cost of 50 units.

The first draft schedule will be input as the initial schedule and the user will be asked to specify a minimum and maximum number of changes to be made from the original schedule. Going outside this range will incur a penalty cost to be added to the cost function; this penalty cost is 25 units times the square of the distance from the range of allowable number of changes. In addition there is a small cost equal to the number of changes, so as to encourage the program to aim for the lower end of the range *ceteris paribus*.

Variations on the original method

Although the solutions produced by the program during the test runs on the 2003 data were found satisfactory by BNZ, it was observed that the program achieved a reasonable "double round" structure only by rather a hit and miss approach, and, moreover, the final structure achieved was often perhaps not as good as it could be.

The ideal structure, if there were no other considerations, would be for every team to have nine home weekends, one away weekend, four double away weekends and two bye rounds (including the midweek round). The double away weekends would involve away matches at a "paired" set of venues (see above).

This could be achieved by constructing the fifteen rounds shown in Table 2, for example, where each team is abbreviated by the first three letters of its name. Note that the chronological order of rounds is not being considered at this stage.

************ Insert Table 2 about here ***********

In each of the first ten rounds, six teams have a home match, three teams have two away matches against "paired" opponents and one team has a bye. In rounds 11 to 15, six teams have a home match, two teams have two away matches against paired opponents and two teams have a single away match against their pair. Matches between paired opponents are shown in **bold**.

It is possible to construct this idealised structure because of the numbers of teams and rounds involved. Although sixteen rounds were available for matches in 2003, BNZ would have been quite happy if the first round, which was a midweek round, had been empty. If the numbers are different in future years it will not be possible to use this particular structure, though it may be possible to construct an alternative idealised structure.

Of course there are plenty of other considerations to take into account when constructing a schedule, as listed earlier. But it was conjectured that it might be worthwhile starting with this idealised structure and then possibly continuing with it for some or all of the search procedure.

It has been found that the careful construction of an initial solution can sometimes helpfully influence the quality of the final solution; for example, this was found by Perttunen [16] for Travelling Salesman problems and by Emden-Weinert and Proksch [17] in their study of the use of Simulated Annealing for airline crew scheduling problems. While other studies have been inconclusive, it was thought that the special structure of this problem might make the construction of an initial solution an important part of the solution process.

Three variations of the original method were thus created and tested, as follows.

Variation 1 – use this structured solution as the initial solution and maintain the structure throughout apart from the final descent to a local optimum.

Variation 2 – use this structured solution as the initial solution but then proceed as in the original version of the program.

Variation 3 – use this structured solution as the initial solution and maintain the structure for part of the annealing process, then switch to the original version for the remainder of annealing and the final descent.

Maintenance of the structure requires a change to the definition of an allowable perturbation. Some perturbations would not maintain the structure, so they need to be removed. Other perturbations are then introduced so that the neighbourhood does not become too small.

The perturbation types when the structure is being maintained are of five types.

Perturbation Type 1 – as before, except that either the match being moved must be between paired teams or $R_1 = R_2$.

Perturbation Type 2 – as before except that either both matches involved must be between paired teams or $R_1 = R_2 = R_3$.

Perturbation Type 3 – as before.

Perturbation Type 4 – swap three entire rounds – all matches in (R_1,D_i) move to (R_2,D_i), all matches in (R_2,D_i) move to (R_3,D_i), all matches in (R_3,D_i) move to (R_1,D_i), for all i = 1,2,3, $R_1 \neq R_2 \neq R_3$.

Perturbation Type 5 – swap the entire schedule for two teams and, unless these two teams are paired with each other, for the pairs of these teams.

This now gives a wide variety of types of perturbation, each of which maintains the solution structure shown in Table 2. The probabilities of using these types of perturbation are now 0.2, 0.2, 0.4, 0.15 and 0.05 respectively.

Since the new types of perturbation are likely to have a larger impact on solution costs than the previous ones, it seemed likely that the temperature scheme might need to be revised upwards (indeed it was observed in practice that very little progress was made for variation 1 between about a quarter of the way through the annealing procedure and the final descent, using the original temperature scheme). Therefore subvariations on variation 1 were devised as follows:

Variation 1a: temperature descends from 5 to 1.

Variation 1b: temperature descends from 10 to 2.

Variation 1c: temperature descends from 15 to 3.

Variation 1d: temperature descends from 20 to 4.

Four subvariations were also considered of Variation 3:

Variation 3a: switch made after 10% of annealing; temperature descends from 5 to 1.

Variation 3b: switch made after 20%; temperature descends from 5 to 1.

Variation 3c: switch made after 20%; temperature descends from 10 to 2.

Variation 3d: switch made after 20%; temperature descends from 10 until the switch, then is halved at the switch, so that the final temperature is 1.

Between 36 and 65 runs of each (sub)variation were made on 2003 data, using availability and request files that approximated to those that would have been used if the system had been in practical use when scheduling this season's matches.

The results are shown in Table 3. Variation 0 is the original method; Nsol is the number of solutions; Av_{ann} and SD_{ann} are the average and standard deviation of the best cost achieved during annealing; Av_{des} and SD_{des} are the average and standard deviation of the cost after descent; and Best is the cost of the best solution found.

************ Table 3 about here ***************

Summaries of the availability details and requests made, together with the best schedule ever found (with a total cost of 557), are given in Appendices 1, 2 and 3 respectively. It can be seen that this schedule adheres quite closely, but not completely, to the idealised structure.

It can be seen from Table 3 that, for all subvariations of Variation 1, the final descent made significant improvements, even when the final temperature was relatively low. This was because there were almost always gains to be made from relaxing the structure constraint.

The best results were achieved by Variation 3d. It was far better than all subvariations of Variation 1 at a 95% level of significance (assuming normality) in terms of the cost after annealing, and better than all except 1c in terms of the cost after descent; however, even in this case, 3d was better than 1c at a 90% level of significance.

It seems reasonable to suppose, therefore, that the approach adopted in Variation 3d (relaxing the structural constraint part way through annealing and making a careful adjustment of the temperature at the same time) might be the best approach to take.

If each method were to be run 100 times and the best solution of all taken, how often would 3d be better than 1c? Assuming normality, this was tested by taking 100,000 random samples of 100 variates each from the two distributions; the average {best of 100} for 3d was found to be 535, with a standard deviation of 24, whereas the average {best of 100} for 1c was found to be 559, with a standard deviation of 23. This implies that 3d will give a better solution overall than 1c with a probability of about 76%.

Of course, there are plenty of other variations that could have been tried, including different temperature schemes, different changeover points at which the structural constraints are relaxed, etc. In any case, this result should be treated with caution, since the assumption of normality is most likely to be invalid when looking at the tails of the distributions (which is what we are doing when considering the best of 100 variates), but the results are still indicative that Variation 3d is probably a good choice to make for this particular problem with this particular set of input data.

Clearly, if the cost parameters were changed such that structure became of overriding importance, or alternatively of very little importance, the results would probably indicate a different conclusion, so caution should be exercised when extrapolating these findings to other data or other problems. However, the results are sufficiently interesting to suggest that there may well be other circumstances when it is worth considering the option of starting with a structurally constrained solution and relaxing these constraints during the metaheuristic search procedure.

APPENDIX 1 - STADIUM AVAILABILITY COSTS

			Har	Auc	Wai	Tar	Haw	Man	Wel	Nel	Can	Ota
1	Wed	23 Apr	2	20	2	2	20	0	0	0	7	1
2	Fri	25 Apr	20	1	0	20	0	3	0	0	0	0
	Sat	26 Apr	6	20	0	0	0	0	0	20	0	0
	Sun	27 Apr	3	7	0	20	10	4	0	20	20	0
3	Fri	2 May	20	20	0	20	0	10	1	0	2	1
	Sat	3 May	0	1	0	0	0	20	20	0	2	20
	Sun	4 May	10	20	5	5	20	5	1	0	2	1
4	Fri	9 May	8	20	20	20	0	10	3	0	2	0
	Sat	10 May	1	1	0	20	6	10	7	0	2	0
	Sun	11 May	20	20	0	20	0	20	3	0	2	0
5	Fri	16 May	10	0	20	20	5	0	3	0	2	0
	Sat	17 May	0	0	0	0	20	0	10	0	20	0
	Sun	18 May	20	10	0	5	20	0	0	0	20	0
6	Fri	23 May	20	20	20	20	20	20	2	0	2	0
	Sat	24 May	0	0	0	0	0	20	2	0	20	0
	Sun	25 May	2	20	5	20	20	20	0	0	20	0
7	Fri	30 May	5	0	20	20	0	0	2	20	20	0
	Sat	31 May	20	0	0	20	0	20	0	0	3	0
	Sun	1 Jun	5	20	10	5	5	20	0	0	6	0
8	Fri	6 Jun	20	20	5	20	0	0	2	20	2	0
	Sat	7 Jun	0	0	0	20	4	0	2	20	2	0
	Sun	8 Jun	10	0	0	3	10	0	2	20	20	0
20	Fri	13 Jun	5	0	20	20	0	0	3	20	20	0
	Sat	14 Jun	0	0	0	20	0	0	20	20	20	0
	Sun	15 Jun	2	5	5	10	20	0	1	20	20	0
10	Fri	20 Jun	5	0	20	20	0	5	2	0	20	0
	Sat	21 Jun	0	0	20	0	0	7	2	0	10	0
	Sun	22 Jun	5	10	0	5	5	20	2	0	10	0
11	Fri	27 Jun	20	0	20	20	0	0	0	0	3	0
	Sat	28 Jun	0	0	0	0	0	0	0	0	20	0
	Sun	29 Jun	0	20	0	0	20	0	0	0	20	0
12	Fri	4 Jul	20	0	5	20	0	0	0	0	20	0
	Sat	5 Jul	0	0	0	20	0	10	0	0	20	0
	Sun	6 Jul	5	6	0	20	10	0	0	0	20	0
13	Fri	11 Jul	20	0	20	20	0	0	2	20	2	0
	Sat	12 Jul	20	0	0	0	0	0	2	5	2	0
	Sun	13 Jul	20	20	0	0	20	0	2	20	5	0
14	Fri	18 Jul	10	0	0	20	0	0	0	0	2	0
	Sat	19 Jul	20	0	0	0	0	0	0	0	20	0
	Sun	20 Jul	10	20	0	0	20	0	0	0	20	0
15	Fri	25 Jul	20	0	0	20	0	0	2	3	2	0
	Sat	26 Jul	0	0	0	0	0	0	2	10	2	0
	Sun	27 Jul	20	10	10	0	20	0	2	20	20	0
16	Fri	1 Aug	20	0	20	20	0	0	2	20	2	0
	Sat	2 Aug	20	0	0	0	0	0	2	20	2	0
	Sun	3 Aug	20	20	0	0	20	0	2	20	20	0

These are the costs before amendment by details in the requests file.

APPENDIX 2 - SPECIAL REQUESTS

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Preferred minimum number of byes for Ota/Can/Auc/Har/Tar/Haw - 2
Preferred minimum number of byes for other teams - 1
Preferred maximum number of byes for all teams - 2
Preferred maximum run of home rounds - 3
Preferred maximum run of non-home rounds - 2
Prefer no team has a bye in round 2 or 16
Prefer at least one of Har/Auc/Wel/Can to be at home on each Sunday
      from Round 9 onwards.
Minimum number of games in Round 1 – 0
Minimum number of games in any other round – 3
Maximum number of games in any round - 7
Add 2 to availability costs for Fridays for all teams
Add 3 to availability costs for Sundays for all teams
Add 5 to availability costs for Round 1 for all teams
Add another 3 to availability costs for Fridays for Taranaki and Manawatu
Add another 3 to availability costs for Sundays for Hawkes Bay
Add another 8 to availability costs for Sundays for Waikato
Canterbury doesn't want to be at home in Round 1 – penalty cost 10
Nelson doesn't want to be at home in Round 1 – penalty cost 10
Wellington wants a double-away in Round 2 – penalty cost 5
Nelson v Waikato to be before round 5
No match in Round 1 between opponents more than 2 distance units apart
```

APPENDIX 3 – Best solution ever found

The number after each fixture is the availability cost, in some cases after amendment by details in the requests file.

Matches between paired opponents are in **bold**.

National Basketball League 2003

Wed 23 Apr	Harbour	v	Waikato	7
	Wellington	v	Manawatu	5
	Auckland	- BYE		
	Taranaki	- BYE		
	Hawkes Bay	- BYE		
	Nelson	- BYE		
	Canterbury	- BYE		
	Otago	- BYE		
Fri 25 Apr	**Auckland**	**v**	**Harbour**	3
	Waikato	v	Canterbury	2
	Hawkes Bay	v	Otago	2
	Nelson	**v**	**Wellington**	2
Sat 26 Apr	Taranaki	v	Canterbury	0
	Manawatu	v	Otago	0
Fri 2 May	Waikato	v	Hawkes Bay	2
	Wellington	v	Auckland	3
Sat 3 May	Taranaki	v	Hawkes Bay	0
	Nelson	v	Auckland	0
	Canterbury	v	Harbour	2
Sun 4 May	Otago	v	Harbour	3
	Manawatu	- BYE		
Fri 9 May	Harbour	v	Taranaki	10
	Wellington	v	Waikato	5
	Otago	v	Manawatu	2
Sat 10 May	Auckland	v	Taranaki	1
	Nelson	v	Waikato	0
	Canterbury	v	Manawatu	2
	Hawkes Bay	- BYE		
Fri 16 May	Auckland	v	Wellington	2
	Canterbury	v	Nelson	4
Sat 17 May	Harbour	v	Wellington	0
	Taranaki	**v**	**Waikato**	0
	Manawatu	**v**	**Hawkes Bay**	0
	Otago	v	Nelson	0
Sat 24 May	Auckland	v	Hawkes Bay	0
	Taranaki	v	Otago	0
	Nelson	v	Manawatu	0
Sun 25 May	Harbour	v	Hawkes Bay	4
	Waikato	v	Otago	13
	Wellington	- BYE		
	Canterbury	- BYE		

Sat 31 May	Auckland	v	Waikato	0
	Hawkes Bay	**v**	**Manawatu**	0
	Wellington	v	Taranaki	0
	Otago	**v**	**Canterbury**	0
Sun 1 Jun	Nelson	v	Taranaki	2
Harbour	- BYE			
Fri 6 Jun	Hawkes Bay	v	Auckland	2
	Wellington	v	Canterbury	4
Sat 7 Jun	Waikato	v	Harbour	0
	Manawatu	v	Auckland	0
	Nelson	v	Canterbury	20
Sun 8 Jun	Taranaki	v	Harbour	5
Otago	- BYE			
Fri 13 Jun	Auckland	v	Nelson	2
Sat 14 Jun	Harbour	v	Nelson	0
	Hawkes Bay	v	Wellington	0
	Otago	v	Taranaki	0
Sun 15 Jun	Manawatu	v	Wellington	2
	Canterbury	v	Taranaki	22
Waikato	- BYE			
Fri 20 Jun	Auckland	v	Canterbury	2
	Manawatu	v	Waikato	10
Sat 21 Jun	Harbour	v	Canterbury	0
	Hawkes Bay	v	Waikato	0
	Nelson	v	Otago	0
Sun 22 Jun	Wellington	v	Otago	4
Taranaki	- BYE			
Fri 27 Jun	Canterbury	v	Hawkes Bay	5
Sat 28 Jun	Auckland	v	Manawatu	0
	Waikato	v	Wellington	0
	Taranaki	v	Nelson	0
	Otago	v	Hawkes Bay	0
Sun 29 Jun	Harbour	v	Manawatu	2
Sat 5 Jul	Hawkes Bay	v	Taranaki	0
	Nelson	v	Harbour	0
	Otago	v	Waikato	0
Sun 6 Jul	Manawatu	v	Taranaki	2
	Wellington	v	Harbour	2
	Canterbury	v	Waikato	22
Auckland	- BYE			
Sat 12 Jul	Waikato	v	Auckland	0
	Hawkes Bay	v	Nelson	0
	Otago	v	Wellington	0
Sun 13 Jul	Taranaki	v	Auckland	2
	Manawatu	v	Nelson	2
	Canterbury	v	Wellington	7
Harbour	- BYE			
Fri 18 Jul	Hawkes Bay	v	Canterbury	2
Sat 19 Jul	Auckland	v	Otago	0
	Waikato	v	Nelson	0
	Taranaki	v	Wellington	0
	Manawatu	v	Canterbury	0
Sun 20 Jul	Harbour	v	Otago	12

Fri 25 Jul	Waikato	v	Manawatu	2
Sat 26 Jul	**Harbour**	**v**	**Auckland**	0
	Taranaki	v	Manawatu	0
	Nelson	v	Hawkes Bay	10
	Canterbury	**v**	**Otago**	2
Sun 27 Jul	Wellington	v	Hawkes Bay	4
Sat 2 Aug	**Waikato**	**v**	**Taranaki**	0
	Hawkes Bay	v	Harbour	0
	Canterbury	v	Auckland	2
Sun 3 Aug	Manawatu	v	Harbour	2
	Wellington	**v**	**Nelson**	4
	Otago	v	Auckland	2

******** Table 4 about here **************

The top line in Table 4 gives the total cost in each category; then follow the costs in each category for each team. Costs relating to a matches rather than teams (e.g. return matches being too close) are shown against the first team involved. Some costs (e.g. unevenness of schedule) are not specific to any team, so are shown only in the top line.

The cost categories are:

ILL – match in "illegal" round
WAN – "want" not satisfied
AVA – (un)availability
MBY – a team having too many byes
FBY – a team having too few byes
MGA – a round having too many games
FGA – a round having too few games
DIS – distance
HRU – home run too long
NHR – non-home run too long
UNE – uneven schedule
RET – return games too close
DHO – two homes in same round
HAW – home and away in same round
AWH – away and home in same round
DBC – double rounds too close
FRS – imbalance between being first and second opponents for teams having a double away round
TV – TV requirement not met
CHA – "change" cost (not applicable here since this schedule was created from scratch)
WBY – bye where not wanted
TOT – total

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TABLES

	Har	Auc	Wai	Tar	Haw	Man	Wel	Nel	Can	Ota
Harbour	*	0	2	5	4	5	5	7	10	12
Auckland	0	*	2	5	4	5	5	7	10	12
Waikato	2	2	*	1	3	4	4	6	10	12
Taranaki	5	5	1	*	6	4	5	7	10	12
Hawkes Bay	4	4	3	6	*	1	3	7	10	12
Manawatu	5	5	4	4	1	*	2	4	9	12
Wellington	5	5	4	5	3	2	*	1	6	9
Nelson	7	7	6	7	7	4	1	*	3	6
Canterbury	10	10	10	10	10	9	6	3	*	1
Otago	12	12	12	12	12	12	9	6	1	*

Table 1 – "distances" between teams

Round							**Bye**
1	Har v Wai	Auc v Wai	Haw v Ota	Man v Ota	Wel v Can	Nel v Can	Tar
2	Har v Tar	Auc v Tar	Haw v Nel	Man v Nel	Can v Wai	Ota v Wai	Wel
3	Har v Haw	Auc v Haw	Wai v Wel	Tar v Wel	Can v Man	Ota v Man	Nel
4	Har v Man	Auc v Man	Wel v Haw	Nel v Haw	Can v Tar	Ota v Tar	Wai
5	Har v Wel	Auc v Wel	Wai v Nel	Tar v Nel	Haw v Can	Man v Can	Ota
6	Har v Can	Auc v Can	Wai v Man	Tar v Man	Wel v Ota	Nel v Ota	Haw
7	Wai v Har	Tar v Har	Haw v Wel	Man v Wel	Can v Nel	Ota v Nel	Auc
8	Wai v Haw	Tar v Haw	Wel v Auc	Nel v Auc	Can v Har	Ota v Har	Man
9	Wai v Ota	Tar v Ota	Haw v Auc	Man v Auc	Wel v Har	Nel v Har	Can
10	Haw v Wai	Man v Wai	Wel v Tar	Nel v Tar	Can v Auc	Ota v Auc	Har
11	Haw v Tar	Man v Tar	Wel v Wai	Nel v Wai	**Har v Auc**	**Can v Ota**	-----
12	Har v Nel	Auc v Nel	Can v Wel	Ota v Wel	**Tar v Wai**	**Man v Haw**	-----
13	Har v Ota	Auc v Ota	Wai v Can	Tar v Can	**Haw v Man**	**Nel v Wel**	-----
14	Wel v Man	Nel v Man	Can v Haw	Ota v Haw	**Auc v Har**	**Wai v Tar**	-----
15	Wai v Auc	Tar v Auc	Haw v Har	Man v Har	**Wel v Nel**	**Ota v Can**	-----

Table 2 – An example of the "ideal" schedule structure

Variation	Nsol	Av_{ann}	SD_{ann}	Av_{des}	SD_{des}	Best
0	**65**	**850**	**80**	**845**	**80**	**720**
1a	**64**	**780**	**32**	**730**	**45**	**611**
1b	**50**	**740**	**24**	**701**	**32**	**588**
1c	**50**	**741**	**22**	**690**	**52**	**563**
1d	**36**	**745**	**22**	**704**	**38**	**558**
2	**64**	**780**	**80**	**770**	**77**	**614**
3a	**65**	**725**	**75**	**713**	**69**	**587**
3b	**37**	**699**	**52**	**694**	**52**	**591**
3c	**50**	**739**	**39**	**696**	**43**	**586**
3d	**50**	**683**	**59**	**671**	**54**	**557**

Table 3 – Experimental results

0	5	235	0	0	0	0	138	3	31	46	74	0	0	0	18	7.2	0	0	0	557	TOT
0	0	35	0	0	0	0	8	0	0	0	2	0	0	0	0	0.4	0	0	0	45	Har
0	0	10	0	0	0	0	8	1	26	0	32	0	0	0	0	1.6	0	0	0	79	Auc
0	0	19	0	0	0	0	16	1	0	0	0	0	0	0	6	0.9	0	0	0	43	Wai
0	0	7	0	0	0	0	8	0	0	0	4	0	0	0	6	0.9	0	0	0	26	Tar
0	0	6	0	0	0	0	8	1	4	0	22	0	0	0	0	0.9	0	0	0	42	Haw
0	0	18	0	0	0	0	18	0	0	0	10	0	0	0	0	2.5	0	0	0	48	Man
0	5	31	0	0	0	0	24	0	0	0	2	0	0	0	0	0.0	0	0	0	62	Wel
0	0	34	0	0	0	0	32	0	0	0	2	0	0	0	0	0.0	0	0	0	68	Nel
0	0	68	0	0	0	0	8	0	1	0	0	0	0	0	6	0.0	0	0	0	83	Can
0	0	7	0	0	0	0	8	0	0	0	0	0	0	0	0	0.0	0	0	0	15	Ota
ILL	WAN	AVA	MBY	FBY	MGA	FGA	DIS	HRU	NHR	UNE	RET	DHO	HAW	AWH	DBC	FRS	TV	CHA	WBY	TOT	

Table 4 - Cost summary for this schedule