When can pessimism add value? How real option disinvestment flexibility augments project NPV.

Abstract

In this article by drawing together existing option pricing models we demonstrate how different degrees of managerial flexibility can affect the value of a project. In particular we also include new option pricing results that allow us to properly take into account a firm's exit option. Our paper illustrates that these disinvestment options can be valuable suggesting that managerial pessimism (i.e. consideration of outcomes if the project does not perform) can add value.

Key Words: Real options, NPV, project valuation, option to disinvest.

Aneel Keswani

Mark Shackleton
(corresponding author)
Department of Accounting & Finance
Lancaster University
LA1 4YX, UK
Tel.:+44(1524)59.41.31
Fax:+44(1524)84.73.21
E-mail: m.shackleton@lancaster.ac.uk

1 Introduction

When a new project is initiated, considering what happens if that project fails to perform in the future may seem an entirely pessimistic thing to do. However a large proportion of the value of a project may be attributable to the option to close the project at some time in the future. It is possible to show that consideration of this option may lead to the adoption of projects that would otherwise have been overlooked as too risky or offering too low a return.

When considering whether to pursue an investment project, it is typical to use a decision rule to determine whether the project should be undertaken or not. One particular approach, the Net Present Value (NPV) rule, states that if the current risk adjusted value of expected cash inflows exceed the value of cash outflows, then a project should be undertaken. It has emerged as the dominant decision rule owing to the shortcomings of other rules.[1]

Whilst the standard NPV approach allows future costs and revenues to depend on future states of the world, it assumes that managers will remain passive even if the circumstances change. Thus even if market conditions worsen dramatically, the NPV rule assumes that managers will not alter their level of production in response and will never, for example, close.[2] In other words, the conventional NPV method treats the investment decision as a static, one off affair. In practice managers can and do "undo" past decisions.

How can this future managerial freedom or optionality be valued? Modern finance theory values this optionality by using the ideas based on the pricing model of Black and Scholes (1973) and applying them to the valuation of real world projects. Thinking about how future optionality affects the value of projects has therefore come to be known as the area of real options.[3]

This paper will show how the value of projects can increase dramatically with increasing degrees of future flexibility. A project that has a classical NPV of \$1,000 can be shown to have a net present value many times that amount if a sufficient amount of flexibility is allowed for in future managerial decision making.

We will also show that the option to disinvest is as important as the option to invest in enhancing project value. Thus we will also consider the case where we have the option to disinvest over both a finite and infinite time horizon. We will show that the important variable for determining project value in this case is the recovery rate if the project is

terminated.

1.1 Classical NPV

The normal approach that is used to judge whether to undertake a project is to calculate the net present value of the project and proceed if it is greater than zero. As an example of how to calculate NPV's, let us suppose that we wish to value an investment project. It starts out paying cash at an annual rate of v_0 at time zero. For the moment it is assumed that this pay out rate is growing continuously at a certain annual growth rate of g

$$\frac{dv_t}{v_t} = gdt \iff v_t = v_0 e^{gt}.$$

In this certain case the value of the project today, V_0, is the risk free discounted sum of cashflows paid out

$$V_0 = \int_0^\infty e^{-rt} v_t dt = \int_0^\infty v_0 e^{(g-r)t} dt = \frac{v_0}{r-g}$$

where r is the risk free discount rate. If the project has known investment costs today of $\overline{X}$ then the NPV of the project, taking into account the costs of the project is

$$NPV_1 = \frac{v_0}{r-g} - \overline{X}. \tag{1}$$

So far we have assumed that the investor can either choose to invest today in a risk free project or never do so again. However we have not allowed the investor to choose his time of investment optimally. In the remaining sections of the paper we will show how within an uncertain environment the option to decide on this in the future can dramatically alter the value of a project.

2 Forward start NPV

2.1 Riskless case

Let us assume that the investor has the ability to pre–commit to a given forward start time, denoted T, in the future. What are the effects of delaying the start date of a project on its NPV?

If an investor defers investing in a project then the present value of costs will be lower; but the present value of revenues will be lower as well. Thus there may be benefits to an investor of delaying the start time of a project if the rate at which the present value of costs falls is faster than the rate at which the present value of revenues fall.

If the costs are known for certain today, then the rate at which costs fall is proportional to the risk free rate. If we commit to start at T then the payout rate of certain revenues at date T is $v_o e^{gT}$. The present value of the revenues received after this date can be calculated as

$$\int_T^\infty e^{-rt} v_t dt = V_0 e^{(g-r)T}.$$

The NPV from immediately committing to start at date T is therefore $V_0 e^{(g-r)T} - \overline{X} e^{-rT}$ where again V_0 is the value of the project if started today.

As the project is riskless the required rate of return on the project is the risk free rate. The required return on the project can be split up into this cashflow yield and a capital gain component g. Thus we can write the total required return on the project r as

$$r = \frac{v_0}{V_0} + g = \delta + g \tag{2}$$

where the dividend yield, $\delta = \frac{v_0}{V_0}$, is the cash flow yield on the project and g is the capital gain component. Thus using this definition of the dividend yield the NPV of the delayed project can be written

$$NPV_2 = V_0 e^{-\delta T} - \overline{X} e^{-rT}. \tag{3}$$

This expression makes sense. When the project is delayed, the present value of costs fall in proportion to the risk free rate because they are known. On the revenues side, by delaying the project, what is not received is the cash yield over the period until T. Thus it is understandable that the present value of revenues fall in proportion to the cash yield δ when the project is delayed.

Will it then pay to delay starting the project for ever? No, even if $r > \delta$ there will be a finite start time beyond which it does not pay to delay the project further. The reason why this is the case is that although initially the PV of costs fall faster than the PV of revenues they both fall over time at a decreasing rate so that eventually the rate at which both fall becomes equal.

2.2 Risky case

If future revenues or costs are unknown at the time the investment decision is made then the discount rate used to calculate the NPV of the project is then greater than that which would be applied to value a corresponding risk free cashflow by an amount known as the risk premium.[4]

Suppose we want to value the project when the revenues are risky. As before, assume cashflows are growing continuously at rate g but that now they have a volatility of σ in their growth rate so that they are random and depend on a Brownian variable $d\widetilde{W}_t$

$$\frac{d\widetilde{v}_t}{\widetilde{v}_t} = gdt + \sigma d\tilde{W}_t. \tag{4}$$

As in the riskless case, the value of the project is the discounted value of its future cash flows. If μ is the appropriate discount rate to use to value the project taking into account its riskiness (where $\mu \geqslant r > g$), then the value of the project if started today is[5]

$$V_0 = E\left[\int_0^\infty e^{-\mu t}\widetilde{v}_t dt\right] = \int_0^\infty v_0 e^{(g-\mu)t} dt = \frac{v_0}{\mu - g}$$

where the real world, time zero, expectations operator is denoted $E[\cdot]$.

If we allow the project to start at the forward time T instead of time 0, the present value of revenues today is

$$E\left[\int_T^\infty e^{-\mu t}\tilde{v}_t dt\right] = \int_T^\infty v_0 e^{(g-\mu)t} dt = v_0\left[\frac{e^{(g-\mu)t}}{(g-\mu)}\right]_T^\infty = V_0 e^{(g-\mu)T} = V_0 e^{-\delta T}.$$

This is a fraction of V_0 and this fraction depends on the amount of time that the project is delayed and the cash yield. Thus the net present value of the project is

$$NPV_2 = V_0 e^{(g-\mu)T} - \overline{X}e^{-rT}. \tag{5}$$

Given its risk, the required rate of return on this project is μ. As before we can define δ as the cash flow yield on the project and g as the capital gain component. Splitting up the required rate of return into cashflow yield and capital gain, now in the risk priced (not risk neutral) world the relationship between total return, yield and gain is

$$\mu = \delta + g. \tag{6}$$

Thus even with uncertainty, the forward NPV_2 can still be rewritten as $V_0 e^{-\delta T} - \overline{X} e^{-rT}$ i.e. Equation (3). If $r > \delta > 0$ then there will be an optimal forward start time. We can solve for this by maximising the NPV with respect to the time of start T. We can show that the optimal start time $T_{\max}$ which maximises the current NPV_2 is given by the expression

$$T_{\max} = \frac{-\ln\left(\delta V_0 / r\overline{X}\right)}{r - \delta}. \tag{7}$$

It is interesting to note that what matters for determining the optimal deferral time is not the riskless discount rate nor the payout rate of cashflows but the relative size of the two.

2.3 Numerical example

As an example consider a project that costs $\overline{X} = \$15,000$ to initiate. Suppose the riskless rate is $r = 6\%$, the risk adjusted discount rate is $\mu = 10\%$, the expected growth rate is $g = 5\%$ and that cashflows start initially at $v_0 = \$800$. In that case, $\delta = 5\%$ and NPV_1 if we start today and NPV_2 if we start in ten years time are

$$\begin{aligned} NPV_1 = & \quad \tfrac{800}{0.1-0.05} - 15,000 & = \$1,000 \\ NPV_2 = & \quad 16,000e^{-0.05*10} - 15,000e^{-0.06*10} & = \$1,472. \end{aligned}$$

The forward start is more valuable in this case.[6] Moreover the value is sensitive to the forward start time.

3 European option NPV

Suppose that in the forward start case we found at the intended date of project launch that the expected value of the project was less than the value of the costs of the project. In that case we would regret having committed to that particular forward start time and it is likely that we would not want to pursue the project.

In this section we consider how project value might be affected if the manager had the option to not start the project at T if conditions were unfavourable i.e. if $\tilde{V}_T < \overline{X}$ and captured the net benefit only if $\tilde{V}_T > \overline{X}$.

To do so we must evaluate the expected discounted value of the project in those states of the world where the net benefit is positive. We denote the expected value of realisations of $\tilde{V}_T$ that exceed $\overline{X}$, as $E\left[\left(\tilde{V}_T - \overline{X}\right)^+\right]$.

Boness (1964) correctly evaluated the *undiscounted* expected value[7] of the project at time T, $E\left[\left(\tilde{V}_T - \overline{X}\right)^+\right]$, but it was not until approximately ten years later that financial economists were able to correctly discount this expected value to the present.

It was shown by Cox and Ross (1976) that for options and non–linear claims it is easier to take expectations in a world where the underlying risks are assumed to be hedged away and then discount the cashflows on these instruments at the risk free rate rather than develop a complicated discount rate expression for discounting the value of the project back to date zero. We can call this world where risks are hedged away the **risk neutral** world. If expectations are taken in this world we can then discount payoffs at the risk free rate (we denote the **risk neutral** expectations operator $\hat{E}$). This technique was first used by Black and Scholes (1973) to value European call options (a right but not obligation to acquire an asset at time T alone, which has a positive payoff at date T if the value of the underlying at this time exceeds the exercise price).

Under risk neutral expectations it is possible to value our project (using a volatility of $\sigma = 20\%$). The pay–off on our project is identical to the pay-off on a European call option with maturity T and strike $\overline{X}$. Therefore we can use the formula for this option to value our project. For a ten year option (close to the optimal maturity in the forward start case of 11.8 years), the value of our project is now[8]

$$NPV_3 = \hat{E}\left[e^{-rT}\left(V_T - \overline{X}\right)^+\right] = V_0 e^{-\delta T} N\left(d_1\right) - \overline{X} e^{-rT} N(d_2) = \$3,034. \tag{8}$$

It is evident that the value of our project is now considerably greater than its value if we commit today to start at date T. The option to wait and compare the relative size of the present value of revenues and costs more than doubles the expected discounted value of the project to $\$3,034$. This is because in those circumstances when the costs of the project exceed the realised future value, we are no longer committed to proceed.[9]

If we are also allowed to choose the maturity of the option at will then we can solve for the value of T that maximises the value of the option to start the project at T and hence the current project value.[10]

4 American option NPV

4.1 Finite case

Instead of having to wait until date T and then deciding whether to pursue the project we now consider the situation where managers are allowed even greater flexibility. In this case, managers are now able to make their decision about whether to invest at any time up to and including date T (when they choose to invest, we still assume that the costs they incur to start the project are $\overline{X}$).

The value of the project in this case can be valued in the same way as an American call option (these options allow the holder to buy the underlying at any time up to and including the maturity date T for the strike price). Managers face a trade–off in their decision to invest just like the exercise decision in the case of American options. If they invest today they get the current difference between the present value of the revenues of the project and the cost of the project. However by delaying the project its NPV may rise further yielding a greater difference between the present value of revenues and the present value of costs. This difference will be received at a later date and therefore this amount will have to be discounted over a longer time period.

It can be shown that in this case the optimal strategy for managers to pursue is to invest at date t as soon as the value of the firm hits a time dependent threshold $\overline{V}_t$. This value threshold gets lower over time making investment more likely as maturity approaches reflecting the fact that the option value of waiting falls over time as there is less time remaining until the option to invest expires.

Unfortunately, it is not possible to express the values of this option in a succinct mathematical form. However Geske and Johnson (1984) developed a series solution for the coefficients w_1 and w_2 on the value and the strike in the expression (since there are two new NPVs introduced in this section, we label them 4.1 and 4.2)

$$NPV_{4.1} \approx w_1 V_0 - w_2 \overline{X}. \tag{9}$$

Like the European option NPV, the project value expressions has two components. The first is a linear function of V_0 and the second is a linear function of $\overline{X}$. In this particular case the

values w_1 and w_2 are dynamic fractions but the functional form of the NPV is the same as in the previous cases.[11]

The American option value can be shown to be about $NPV_{4.1} = \$3,304$, clearly adding more future flexibility increases value from the initial level of \$1,000 to about three times that value.

4.2 Perpetual case

We now consider the case where there is no time before which the option to invest expires and the manager is free to invest at anytime at a cost of $\overline{X}$ realising a pay-off at the time of investing of $\widetilde{V}_t - \overline{X}$. It is possible to show that the optimal exercise strategy involves investing as soon as the value of the project hits a time independent barrier, $\overline{V}$ (this option when exercised generates no further options, i.e. no investment funds can be recouped in the future). The value of $\overline{V}$ is time independent because the time available for future exercise is not diminishing, thus the payoff on exercise of the perpetual option is always $\overline{V} - \overline{X}$ where $\overline{V}$ may be a choice variable. The random time at which $\widetilde{V}_t$ first hits $\overline{V}$ is labelled $\widetilde{t}$.

In this environment, we can write the value of the expected NPV of the project as a function of $\overline{V}$ by taking expectations in the risk neutral world of the payoff at the random time $\widetilde{t}$

$$NPV_{4.2} = \left(\overline{V} - \overline{X}\right) \hat{E}\left[e^{-r\tilde{t}}\right] = \left(\overline{V} - \overline{X}\right) \left(\frac{V_0}{\overline{V}}\right)^a \tag{10}$$

In Equation (10), the term $\left(\overline{V} - X\right)$ is outside the expectations operator reflecting the fact that the amount received when investment occurs $\left(\overline{V} - X\right)$ is known in advance. However what is not known in advance is the random investment time, $\widetilde{t}$ and its associated stochastic discount factor $e^{-r\tilde{t}}$. However, for $V_0 < \overline{V}$, the risk neutral expectation of this stochastic discount factor has the simple expression

$$\widehat{E}\left[e^{-r\tilde{t}}\right] = \left(\frac{V_0}{\overline{V}}\right)^a$$

where a represents an elasticity constant calculated from a fundamental quadratic[12].

The stopping time or threshold $\overline{V}$ can be chosen at will. In this case raising $\overline{V}$ raises the eventual payoff on exercise but makes the likely time that exercise will occur later.

Conversely lowering $\overline{V}$ makes exercise more likely but also reduces the payoff when exercise occurs. The presence of this trade–off allows us to solve for the value of $\overline{V}$ that maximises the value of this perpetual option to invest. It can be shown that the optimal value of $\overline{V}$[13] has the following form[14]

$$\overline{V} = \frac{a}{a-1}\overline{X} = \$30,000.$$

Using this value of $\overline{V}$ the optimal NPV is then

$$NPV_{4.2} = V_0\left(\frac{V_0}{\overline{V}}\right)^{a-1} - \overline{X}\left(\frac{V_0}{\overline{V}}\right)^{a} = \$4,267. \tag{11}$$

Likewise this NPV expression has the same familiar structure as the cases above with a linear term in V_0 and a linear term in $\overline{X}$ and of course since even more flexibility is embedded, the value continues to increase.

5 Costly reversible NPV

5.1 Perpetual case

So far we have examined cases where we have allowed managers to time their investment decisions. Once their decision is made, there is no going back on it and once started, projects yield a perpetual stream of cashflows. As a result we would expect managers to be cautious in exercising their option to invest.

We now consider cases where managers can reverse their initial decision and disinvest should circumstances deteriorate. The value of the project if disinvestment occurs will have a number of elements.

First, if managers disinvest they will no longer incur the costs of running the project. In addition the project may have a liquidation value. This liquidation value may derive from selling the assets of the project on the market or utilising them for another project.

Let us suppose that after exercising the option to invest in the case above and incurring costs of $\overline{X}$ the manager is then told that he can disinvest the project once at any time in the future. He is told that if he does so, a known amount $\underline{X}$ $\left(< \overline{X}\right)$ will be realised.

After being told this, what is his optimal strategy? Without this reversibility, the manager compared the value of the project net of investment cost with the value of the option to open. Now because of this new shutting option that he has been given, the manager will compare the value of the project plus the value of the option to close with the disinvestment proceeds.

It turns out that in this case his optimal strategy involves disinvesting when the value of the project hits another time independent threshold[15] $\underline{V}$. By selecting the value of $\underline{V}$ to maximise the value of the subsequent option to close, we can show that $\underline{V}$ has the following form (b is defined similarly to a in Section 4.2)

$$\underline{V} = \frac{b}{b-1}\underline{X} = 0.6\underline{X}.$$

This allows the one time closure option to be evaluated but what would happen if the manager knew in advance that he would have the option to disinvest at $\underline{X}$ as well as the option to invest for $\overline{X}$ at the very beginning?

In that case the optimal thresholds are determined *simultaneously* rather than sequentially as in the case above. These thresholds will depend on both the costs of investment and disinvestment. Denoting the costs of investment $\overline{X}$ and the disinvestment proceeds as $\underline{X}$, we can write these two thresholds as $\overline{V}\left(\overline{X},\underline{X}\right)$ and $\underline{V}\left(\overline{X},\underline{X}\right)$. (We can rewrite $\overline{V}$ from the irreversible case as $\overline{V}\left(\overline{X},0\right)$ as the proceeds from disinvestment in this case were zero. Likewise $\underline{V}$ from above where reinvestment was not possible after disinvesting can be rewritten as $\underline{V}\left(\infty,\underline{X}\right)$ because the costs of subsequent reinvestment in this case are infinite.) The simultaneously determined thresholds differ from the previous thresholds i.e.$\overline{V}\left(\overline{X},\underline{X}\right) < \overline{V}\left(\overline{X},0\right)$ and $\underline{V}\left(\overline{X},\underline{X}\right) > \underline{V}\left(\infty,\underline{X}\right)$. The reason why the interval between upper and lower exercise thresholds is smaller now is because the decision to invest and disinvest is not one off and is therefore more readily made.[16]

In the case where the manager is always able to reverse his decision then the decision to invest or disinvest must be taken in anticipation of the subsequent option to reverse his decision. Thus if reversibility is permitted at all times, when deciding what to do, the manager compares the value of the project closed plus the option to open with the value of the project open with the option to close.[17]

The key pricing variable in this case is the degree of reversibility $\alpha = \underline{X}/\overline{X}$ and the ratio of the optimal investment thresholds $\gamma = \underline{V}\left(\overline{X}, \underline{X}\right)/\overline{V}\left(\overline{X}, \underline{X}\right)$ depends critically on α.[18]

Let us suppose that now we set $\underline{X} = \$14,000$ given our previous choice of $\overline{X} = \$15,000$. In this case the NPV can be shown to be \$4,734. In addition the exercise thresholds are $\overline{V}\left(\overline{X}, \underline{X}\right) = \$24,347$ and $\underline{V}\left(\overline{X}, \underline{X}\right) = \$12,347$.

The value threshold at which managers invest is now lower than in the perpetual case where disinvestment is not allowed (in that case the value threshold at which managers invest at was \$30,000). This reflects the fact that the option to reverse their decision makes managers less cautious about their initial investments and they therefore invest earlier. The value associated with this strategy is $NPV_5 = \$4,734$.

Finite reversibility can also be handled although analytic tractability is not available and numerical solutions are necessary.

6 Costless reversible NPV

6.1 Perpetual case

An increase in the disinvestment proceeds from zero to \$14,000 led the project option to grow from \$4,267 ($NPV_{4.2}$) to \$4,734 (NPV_5) but the NPV of the project could be further increased if the disinvestment proceeds were increased further. The limiting case is where the amount raised when disinvestment occurs is equal to the cost of setting up the project i.e. $\overline{X} = \underline{X} = \underline{\overline{X}} = \$15,000$, i.e. $\alpha = 1$.

In this case, as there are zero round trip costs of investment and disinvestment, the thresholds at which investment and disinvestment will occur are now no longer separated i.e. $\underline{\overline{V}} = \overline{V}\left(\overline{X}, \underline{X}\right) = \underline{V}\left(\overline{X}, \underline{X}\right)$.

At any point in time the manager chooses between being invested and being disinvested. The cashflows from being invested are the cash yield i.e. $\delta\widetilde{V}_t$. The cashflows from being disinvested consist of the risk free return on $\underline{\overline{X}}$ (i.e. $\underline{\overline{X}}$ is invested in a risk free account whilst we are disinvested. Thinking about the NPV in this way, it is evident that the NPV

NPV case and section	$\underline{X}$	$\overline{X}$	$\underline{V}$	$\overline{V}$	NPV
Classic NPV_1		$15,000$		$15,000$	$1,000$
Forward NPV_2		$15,000$		$18,000$	$1,472$
European call option NPV_3	0	$15,000$	0	$19,829$	$3,034$
American call option $NPV_{4.1}$	0	$15,000$	0	$\approx 29,582$	$\approx 3,816$
Perpetual American call $NPV_{4.2}$	0	$15,000$	0	$30,000$	$4,267$
Perpetual American put	$14,000$	∞	$8,400$	∞	
Perpetual costly reversible NPV_5	$14,000$	$15,000$	$12,347$	$24,347$	$4,734$
Perpetual costless reversible $NPV_{6.1}$	$15,000$	$15,000$	$18,000$	$18,000$	$5,079$
Finite costless reversible $NPV_{6.2}$	$15,000$	$15,000$	$18,000$	$18,000$	$3,860$

Table 1: Summary of different NPV formulations and values

is a continuum of infinitesimal Black-Scholes options which enables us to evaluate the NPV[19]

$$NPV_{6.1} = \hat{E}\left[\int_0^{\infty} e^{-rt}\left(\delta\widetilde{V}_t - r\overline{\underline{X}}\right)^+ dt\right] = \$5,079. \tag{12}$$

This amount is probably the largest NPV that could be feasibly be extracted from the parameter set. Note that for the numbers chosen this is a factor of five larger than the Classic NPV.

6.2 Finite case

The analysis presented is also tractable for finite maturity, that is where the project can be perfectly reversed for a limited period of time alone. In this case we would think of a finite integral of infinitesimal Black Scholes flow options followed by one final Black Scholes value option on terminal values at time $T = 10$ years.

$$NPV_{6.2} = \hat{E}\left[\int_0^{T} e^{-rt}\left(\delta\widetilde{V}_t - r\overline{\underline{X}}\right)^+ dt + e^{-rT}\left(\widetilde{V}_T - \overline{\underline{X}}\right)^+\right] = \$3,860 \tag{13}$$

7 Conclusions

In Table (1) we summarise the findings of Sections 1 to 6. For the parameters shown in Table (2) these indicate that increasing the flexibility that managers have in the future can

dramatically increase the expected NPV of a project from \$1,000 to \$5,079. Thus future flexibility should definitely be considered in project valuation.

We examined two types of flexibility. In Sections 2 to 4 we considered the value of managerial flexibility in the decision to invest. We showed that NPV's can be raised by delaying a project if the risk free rate exceeds the cashflow yield. In this case when making an investment decision, managers should compare the value of a project with its own delayed launch. Table 1 illustrates that in our example by committing to start in ten years, it was possible to raise the project NPV from \$1,000 to \$1,472.

More realistically, if managers have the option to decide on whether to start a project in the future, this will usually be even more valuable than a situation where the manager is tied to a forward start date. This situation is examined in Sections 3 and 4. With an option to invest at a fixed future start date, we showed that the NPV rose to \$3,034 whilst if managers were allowed to start the project at any time the NPV can rise up to \$3,816 for a ten year option and $\$4,267$ in the perpetual case.

In Sections 5 and 6 we considered how flexibility in the disinvestment decision may also add value and enhance project NPV's. We considered cases where managers faced the option to invest knowing in the future they would have the option to disinvest at a cost. This resulted in an NPV of \$4,734. We showed that in the limit where there were no costs of disinvestment and investment and switching was possible at all times that project value would rise to an NPV of \$5,079. Finally this special switching case together with the final European option can be evaluated for the ten year time horizon yielding a value of \$3,860.

Figure 1 shows these results graphically as a function of time to the flexibility horizon. The Classic NPV is shown at zero maturity. The forward start value varies with the time horizon, rising initially due to the effect of the risk free rate reducing the PV of investment in the near term but eventually falling due to the effect of the dividend opportunity cost in the long term. Likewise the European option PV rises in the short term but falls in the long term for the same reasons but it is slightly displaced (to the left) because of the option probabilities effect the discount factors on the option components. The American value only increases with time horizon because early exercise is always possible, waiting can be avoided and a longer final maturity can never disadvantage the option holder. The American value

Variable	**Notation**	**Parameter value**
Fixed horizon time	T	10 year
Risk free interest rate	r	6% pa
Required project rate	μ	10% pa
Project dividend yield	δ	5% pa
Project capital gain	g	5% pa
Project uncertainty	σ	20% pa
Random (initial) project cashflow at time t	$\widetilde{v}_t$	$v_0 = \$800$ pa
Random (inital) project value at time t	$\widetilde{V}_t$	$V_0 = \frac{v_0}{\mu - g} = \$16k$
Project investment cost	$\overline{X}$	$\$15k$
Project divestment amount	$\underline{X}$	$\$0k$ then $\$14k$
Fully reversible investment cost	$\overline{\underline{X}}$	$\$15k$
Elasticity parameters	a, b	$2.0, -1.5$

Table 2: Notation and parameter values

approaches the perpetual asymptote for long horizons. The costly reversible ($\underline{X} = \$14,000$) cases are not shown but the most flexible situation, the costless reversible ($\underline{X} = \$15,000$) is shown for the finite and infinite cases. Like the American case, the fully reversible only ever increases with final time horizon as extra future flexibility cannot destroy value, the reversible case also acts as an upper bound for the American since its set of exercise benefits contain those of the American.

For valuation purposes when modelling the time path of future cashflows of a company, what is normally examined in detail are the cashflows over the near future, typically 5 or 10 years and then a terminal value is inserted that reflects the present value of cashflows after that point. This number normally assumes that the firm will remain open thereafter in perpetuity. We have shown here that another possibility is to value this perpetuity recognising that firms have the option to close, thus replacing the terminal value calculation with the value of the termination option.

Thus managers may wish to over ride the NPV measure from any particular model in order to reflect the strategic flexibility that a project may generate.

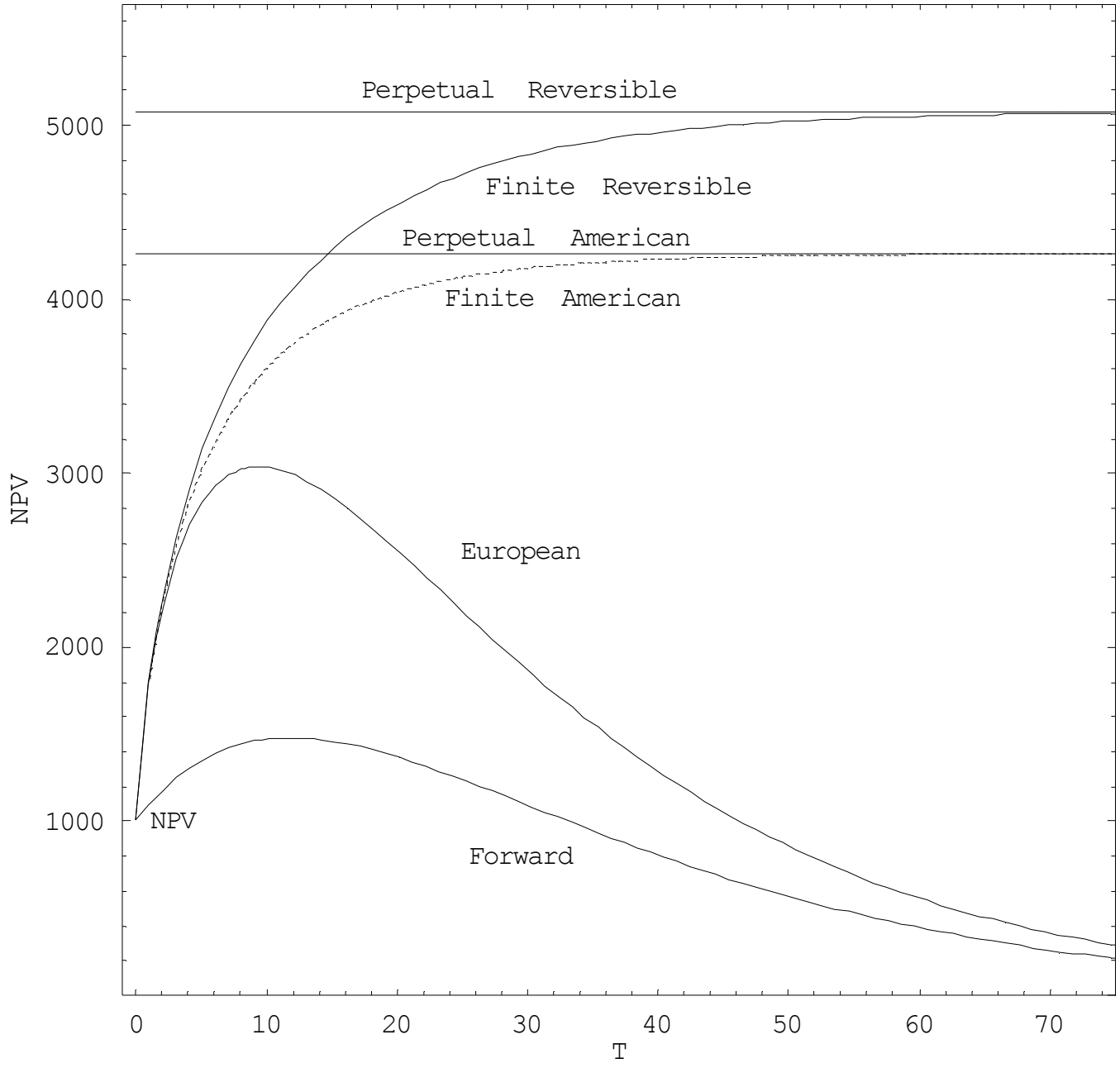

Figure 1: NPVs by time horizon for different levels of flexibility.

Notes

[1]Another decision rule that might be used for selecting projects is to choose the project that pays back its costs fastest; this is known as the payback rule. Another similar rule is the Internal Rate of Return (IRR). Both these rules fail to capture the size of the project.

[2]These problems had been considered by Hertz (1964) and Magee (1964) who modelled the impact of uncertainty on the future decisions that managers might make using simulation and decision trees.

[3]The idea of real options was first discussed and named by Myers (1977).

[4]Modern finance has developed a variety of models for determining how large the risk premium should be when valuing risky projects. One commonly used model for determining the size of project risk premia as a function of the market risk premium is the capital asset pricing model (CAPM), see Sharpe (1964).

[5]To evaluate this expectation we use the fact that $\widetilde{v}_t = v_0 e^{\left(g-\frac{1}{2}\sigma^2\right)t+\sigma\tilde{W}_t}$ which requires use of a growth correction of $\frac{1}{2}\sigma^2 t$ in the exponential representation of flows for the expected growth rate.

[6]In fact for the parameters in this paper, the conditions for the break even (be) and optimal (i.e. max NPV_2) forward start times are given by

$$\begin{array}{cc} NPV_{2\text{be}} = 0 & T_{\text{be}} = \frac{-\ln(V_0/X)}{r-\delta} < 0 \\ NPV_{2\max} = \$1,480 & T_{\max} = \frac{-\ln(\delta V_0/rX)}{r-\delta} = 11.8 \text{ years} \end{array}$$

[7]This can be shown to equal to

$$\begin{aligned} E\left[\left(V_T - \overline{X}\right)^+\right] &= V_0 e^{(\mu-\delta)T} N\left(d_3\right) - \overline{X} N\left(d_4\right) \\ d_{3,4} &= \frac{\ln V_0 - \ln \overline{X} + \left(\mu - \delta \pm \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}} \end{aligned}$$

For the parameter values already chosen the expected positive part of the forward payoff is $E\left[\left(V_T - \overline{X}\right)^+\right] = \$12,622$. Samuelson (1965) and others tried in vain to correctly value this discounted payoff by postulating an arbitrary (non–equilibrium) discount rate.

[8]Note that the expressions within the cumulative normal functions

$$d_{1,2} = \frac{\ln V_0 - \ln \overline{X} + \left(r - \delta \pm \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}}$$

do not depend on μ as in the Boness (1964) formula.

[9]We can also consider a higher level of flexibility where the holder has the right to choose today between exercising today and taking the European option. The NPV in this case is $NPV_{3.1} = \max\left(NPV_3, V_0 - \overline{X}\right)$. The reason why it might pay in certain circumstances to exercise today rather than hold the option is if the dividend yield is sufficiently high. It is possible to solve for the initial NPV that equals the option value. Above this trigger current NPV level, the holder would want to invest today. For the parameters chosen, the critical value of V_0 is \$19,829.

[10]There is an optimal time that maximises the value of the option because increasing the time to maturity increases the value of the option due to the greater volatility of $\widetilde{V}_T$ but at the same time increasing the time to maturity increases the cost of waiting. The optimal European option maturity is $T_{\max} = 9.41$ years in which case the NPV_3 is \$3,036.

[11]This result however is not tractable or indeed the most practical and therefore other methods are preferred for the pricing of American options. Here we use the Barone–Adesi, Whalley (1987) and MacMillan (1986) approximations (although not the most accurate these methods are amongst the simplest to implement).

[12]$a > 1$ (and another elasticity parameter $b < 0$) actually solves the fundamental quadratic

$$\frac{1}{2}\sigma^2\beta\left(\beta - 1\right) + \left(r - \delta\right)\beta - r = 0$$

[13]Shackleton and Wojakowski (2002) solve this problem without taking adjusted expectations $\widehat{E}\left[\cdot\right]$ showing that this perpetual case may be one of the few options which have a constant expected rate of return, allowing real world discounting under expectations $E\left[\cdot\right]$.

[14]This optimality condition is equivalent to smooth pasting. What is meant by this is that at the optimum, the payoff function of the option $\bar{V}\left(\bar{X}, 0\right) - \bar{X}$ meets the value function of the option tangentially.

[15]This consistent notation is required because the closure once exercised is not reversible i.e. an infinite amount would have to be spent to reopen. Thus the one way opening threshold used in this paper is equivalent to a "reversible" with an unattainable zero reverse threshold $\overline{V}\left(\overline{X},0\right)$ and the one way closing $\underline{V}\left(\infty,\underline{X}\right)$ has an unattainable reopening option at ∞. The next paragraph utilises the truly reversible thresholds $\overline{V}\left(\overline{X},\underline{X}\right),\underline{V}\left(\overline{X},\underline{X}\right)$.

[16]This is a situation of investment **hysteresis** (Dixit (1989), Brennan and Schwartz (1985)) where investment and dis–investment can occur at upper and lower thresholds.

[17]More formally we get a pair of value matching equations at exercise (and two associated smooth pasting conditions)

$$\begin{aligned}\overline{X}+\text{open option at }\overline{V}\left(\overline{X},\underline{X}\right) &\longrightarrow \overline{V}+\text{closing option at }\overline{V}\left(\overline{X},\underline{X}\right)\\ \underline{X}+\text{open option at }\underline{V}\left(\overline{X},\underline{X}\right) &\longleftarrow \underline{V}+\text{closing option at }\underline{V}\left(\overline{X},\underline{X}\right)\end{aligned}$$

[18]Using a four by four matrix to represent the value matching and smooth pasting conditions, Shackleton and Wojakowski (2001) show that the hysteresis system is solved by the one non–linear equation, a difficult analytic but trivial numerical task

$$\alpha\left(\gamma\right)=\frac{\left(ab-a\right)\gamma^{b+1}+\left(b-ab\right)\gamma^{a+1}+\left(a-b\right)\gamma^{a+b}}{\left(ab-b\right)\gamma^{b}+\left(a-ab\right)\gamma^{a}-\left(a-b\right)\gamma}$$

[19]In fact the perpetual costless reversible can be shown to have value that depends on the exercise amount $\overline{X}$ added to the opening option or the project value V_0 plus the shutting option

$$NPV_{6.1}=\begin{cases}\overline{X}+\overline{V}\frac{b-1}{a(b-a)}\left(\frac{V_0}{\overline{V}}\right)^{a} & V_0<\overline{V}\\ V_0+\underline{V}\frac{a-1}{b(b-a)}\left(\frac{V_0}{\underline{V}}\right)^{b} & V_0>\underline{V}\end{cases}$$

References

Barone-Adesi, G. and Whaley, R. E. 1987. Efficient analytic approximation of American option values. Journal of Finance 42(2), 301–320.

Black, F. and Scholes, M. 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–659.

Boness, A. J. 1964. Elements of a theory of stock option value. Journal of Political Economy 72(2), 163–175.

Brennan, M. J. and Schwartz, E. S. 1985. Evaluating natural resource investments. Journal of Business 58(2), 135–157.

Cox, J. C. and Ross, S. A. 1976. The valuation of options for alternative stochastic processes. Journal of Financial Economics 3, 145–166.

Dixit, A. K. 1989. Entry and exit decisions under uncertainty. Journal of Political Economy 97, 620–638.

Geske, R. and Johnson, H. E. 1984. The American put option valued analytically. Journal of Finance 39(5), 1511–1524.

Hertz, D. B. 1964. Risk analysis in capital investment. Harvard Business Review 42(Jan), 95–106.

MacMillan, L. 1986. An analytic approximation for the American put price. Advances in Futures and Options Research 1(1), 119–139.

Magee, J. F. 1964. How to use decision trees in capital investment. Harvard Business Review 42(Sept), 79–96.

Myers, S. C. 1977. Determinants of corporate borrowing. Journal of Financial Economics 5, 147–176.

Samuelson, P. A. 1965. Rational theory of warrant pricing. Industrial Management Review 6(Spring), 13–31.

Shackleton, M. B. and Wojakowski, R. 2001. Reversible, flow options. Lancaster Working Paper 2001(007).

Shackleton, M. B. and Wojakowski, R. 2002. The expected return and time to exercise of Merton style Real Options. Journal of Business Finance and Accounting 29(3–4), 541–555.

Sharpe, W. F. 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance 19(3), 425–442.