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Abstract

In this paper we develop a general equity valuation model where abnormal earnings, book value, interest rates and the discount factor evolve stochastically. We demonstrate that book value, abnormal earnings, "other information" and two risk adjustment terms are sufficient for valuation, when pricing kernel risk depends on short term interest rates and one market risk factor. The valuation weights and the risk adjustment terms reflect the linear information dynamics parameters. The risk adjustment terms also depend on accounting betas, reflecting the covariances between abnormal earnings and (i) the short interest rate, and (ii) the market risk factor.

Key Words: Risk-adjusted valuation, clean surplus accounting, abnormal earnings, accounting betas.

1 Introduction

Edwards and Bell (1961), Peasnell (1982), Ohlson (1995), Feltham and Ohlson (1995, 1996) and others have shown that no-arbitrage and the clean surplus accounting relation imply that a firm's market value can be expressed as the sum of book value and the present value of expected future abnormal earnings (or residual income). However, the issue of how to incorporate risk into accounting-based equity valuation has not been fully explored. One approach to risk adjustment is to discount projected abnormal earnings at a risk-adjusted rate (e.g. Peasnell, 1982). Recent empirical research that has applied the residual income valuation model and the Ohlson (1995) and Feltham and Ohlson (1995, 1996) models adopts this approach to risk adjustment (e.g. Dechow, Hutton and Sloan, 1998; Frankel and Lee, 1997; Myers, 1999). However, consistent with asset pricing theory, Feltham and Ohlson (1999) (FO) show that the correct method of dealing with risk is to adjust expected abnormal earnings for risk and to use the term structure of riskless interest rates in discounting. FO also explain that risk adjustments to expected abnormal earnings should depend on the correlation between abnormal earnings and economy-wide risk factors. But they do not explicitly model the risk-adjusted valuation process under stochastic interest rates. This paper addresses the form such risk-adjusted valuation should take.

We develop a general valuation model where abnormal earnings, book values, market risk factors and interest rates evolve stochastically. We extend FO's analysis in two respects. First, we model the interest rate risk of abnormal earnings and its impact on valuation. Second, we specify the properties of the pricing kernel in an arbitrage-free market and explicitly model the systematic risks of abnormal earnings in relation to the pricing kernel. Assuming a single systematic risk factor, our model shows that the value of the firm is equal to the sum of (i) a multiple of book value; (ii) a multiple of expected abnormal earnings discounted at appropriate risk-free spot interest rates; and (iii) two risk adjustment terms. The first risk adjustment term depends on an accounting beta reflecting the covariance between abnormal earnings and interest rates. The second risk adjustment term depends on an accounting beta reflecting the covariance between abnormal earnings and the systematic market risk factor.

Our analysis suggests a general framework for accounting-based valuation in which financial statement information fulfills three main informational roles. First, as in Ohlson (1995) book value serves as an anchor for firm value. The valuation weight on book value depends on the degree of accounting conservatism, as in Feltham and Ohlson (1995). Second, book value and abnormal earnings are informative in projecting future expected abnormal earnings, as in Ohlson (1995) and Feltham and Ohlson (1995). Third, the covariances between (abnormal) earnings and systematic risk factors are fundamental to the risk adjustment process, as suggested by Feltham and Ohlson (1999) and by earlier empirical work on accounting-determined risk measures (e.g., Beaver, Kettler and Scholes, 1970; Beaver and Manegold, 1975; Hill and Stone, 1980).

The remainder of the paper is organized as follows: in section 2 we discuss the notion of no-arbitrage and accounting-based valuation when market risk and interest rates are stochastic; in section 3 we present our results on risk-adjusted accounting valuation; in section 4 we discuss our results in relation to prior research; finally, in section 5, we conclude.

2 No Arbitrage and Accounting-Based Valuation

2.1 Dividend-based valuation

Suppose there are N tradable securities (firms) defined by the N adapted dividend processes $d = (d_1, ..., d_N)$. The respective ex-dividend value processes are $V = (V_1, ..., V_N)$. Assume there are $T - t + 1$ dates: t, $t + 1, ..., T$. Denote $\theta_\tau = (\theta_{1\tau,...,}\theta_{N\tau})$ the portfolio held after trading at time τ.

Market equilibrium requires the absence of arbitrage. An arbitrage opportunity arises if there is a trading strategy θ such that $d_\tau^\theta \equiv \theta_{\tau-1}(V_\tau + d_\tau) - \theta_\tau V_\tau > 0$. The assumption of no-arbitrage is equivalent to the existence of a stochastic discount factor (or pricing kernel), m, such that the value of any firm f $(f = 1, ..., N)$ at time t, $V_f(t)$, satisfies:[1]

$$V_f(t) = E_t\{\frac{m_{t+1}}{m_t}[V_f(t+1) + d_f(t+1)]\} \tag{1}$$

where $E_t[.]$ is the expectation operator conditional on time t information, $V_f(T) = 0$ and m reflects investors' beliefs and risk aversion (see Duffie, 1994).[2] By the property of iterated expectations, (1) reduces to the dividend valuation relation:[3]

$$V_f(t) = \frac{1}{m_t}E_t[\sum_{j=t+1}^{T} m_j d_f(j)] \tag{2}$$

For the representative firm f, we ignore the subscript in the following analysis.

[1] This representation is equivalent to the FO assumption of a strictly positive state-price deflator.

[2] The stochastic discount factor can be related to a consumption based asset pricing model by defining $\frac{m_{t+1}}{m_t} = \pi\frac{u'(c_{t+1})}{u'(c_t)}$, where π is the subjective discount factor captures investor impatience and $u'(.)$ is the marginal utility of consumption and c is consumption.

[3] In a discrete-time setting with certainty and a flat term structure of interest rates, this is equivalent to using the discount factor $\frac{1}{1+r}$ and (2) would be equivalent to $V(t) = \sum_{j=t+1}^{T} \frac{d(j)}{(1+r)^{j-t}}$.

The role of the "riskless" interest rate is fundamental to the model. We assume that the term structure of interest rates allows identification of the one period continuously compounded forward rate for the interval $(s, s+1)$, $(s \geq t)$, denoted $e^{r(s)}$. The no-arbitrage condition (1) implies:[4]

$$E_s[\frac{m_{s+1}}{m_s}] = e^{-r(s)} \tag{3}$$

By induction, (2) and (3) imply:

$$V(t) = \sum_{j=t+1}^{T} E_t\{L_{tj}[R_{tj}^{-1}d(j)]\} \tag{4}$$

where $R_{tj} \equiv \exp[\sum_{i=t}^{j-1} r(i)]$ is the continuously compounded risk-free rate from t to $t+j$ and $L_{tj} \equiv \frac{m_j}{m_t} R_{tj}$ is the state price density relevant to period $t+j$. Since m is strictly positive, $L > 0$ and $E_t[L] = 1$. Expression (4) states that the value of the firm is equal to the sum of the risk-adjusted present value of future dividends, discounted at the riskless spot rates.

If the subjective probability measure is denoted P, then $Q \equiv PL$ is the equivalent pricing kernel risk-neutral probability measure to P. Under the risk-neutral measure, we have:[5]

$$V(t) = E_t^Q[\sum_{j=t+1}^{T} R_{tj}^{-1}d(j)] \tag{5}$$

Expression (5) states that the value of the firm is equal to the value of all future dividends discounted under the risk neutral probability measure. Note that the valuation effects of market risk associated with the pricing kernel m are subsumed under the expectations operator E_t^Q. However, the term inside the expectations operator depends on spot rates, R_{tj}. If interest rates are stochastic we cannot move R_{tj} outside the expectation operator. Valuation

[4] See Duffie (1994, p.23). If the riskless rate does not exist over the interval $(s, s+1)$, one can *define* $m_s/E_s[m_{s+1}]$ as a single-period continuously compounded riskless rate (also called the "zero-beta" rate).

[5] See, also, Duffie (1994, pp.27-28).

will depend on the stochastic relation between $d(j)$ and R_{tj}. One cannot simply discount risk-adjusted expected dividends at the respective riskless spot rates. This is a key difference between our analysis and FO's.

2.2 Accounting-based valuation

Following Ohlson (1995), Feltham and Ohlson (1995, 1996) we assume that the clean surplus relation [CSR] applies for all time periods j, allowing us to write dividends in terms of accounting variables, as follows:

$$d(j) = x(j) + b(j-1) - b(j) \tag{6}$$

FO demonstrate that the capital charge in calculating abnormal earnings should be based on the risk-free short interest rate. Denote abnormal earnings for period j, $y(j) \equiv x(j) - [\exp(r(j-1)) - 1]b(j-1)$, where $x(j)$ is earnings in period j and $b(j-1)$ is beginning-of-period book value. Then from (2) and CSR we obtain a familiar expression for the value of a firm as a function of current period book value and all future expected abnormal earnings. This is stated in the following Lemma:

Lemma 1 *Assume that no-arbitrage and CSR hold and that $b(T) = 0$. The value of firm's equity is the sum of equity book value and the expected pricing kernel-discounted value of all future abnormal earnings. Equivalently, the value of firm's equity is the sum of equity book value and expected discounted value of all future abnormal earnings using relevant spot interest rates under a risk neutral probability measure, i.e.*

$$V(t) = b(t) + \frac{1}{m_t} E_t[\sum_{j=t+1}^{T} m_j y(j)] \tag{7}$$

$$= \quad b(t) + E_t^Q[\sum_{j=t+1}^{T} R_{tj}^{-1} y(j)]$$

Proof: See the Appendix.

3 Risk-Adjusted Valuation

3.1 General valuation expressions

Since $E_t[L_{tj}] = 1$, equation (4) implies that:[6]

$$V(t) = \sum_{j=t+1}^{T} \{E_t[R_{tj}^{-1}]E_t[d(j)] + cov_t[R_{tj}^{-1}, d(j)] + cov_t[L_{tj}, R_{tj}^{-1} d(j)]\} \tag{8}$$

The same risk-adjusted probability measures apply to valuation based on discounted dividends and on discounted abnormal earnings. Hence, (7) can be written as:

$$V(t) = b(t) + \sum_{j=t+1}^{T} \{E_t[R_{tj}^{-1}]E_t[y(j)] + cov_t[R_{tj}^{-1}, y(j)] + cov_t[L_{tj}, R_{tj}^{-1} y(j)]\} \tag{9}$$

Note that there are no changes in the risk adjustment state prices density L_{tj}, since the same set of state prices support (4) and (7). Therefore, in (9) the market value of the firm is determined by its book value, the expected discount factor, expected abnormal earnings, the covariance between interest rates and abnormal earnings and the covariance between the pricing kernel and discounted abnormal earnings. The two covariance terms adjust for

[6]Noting that $E_t[L_{tj}] = 1$, equation (4) can be expanded as follows:

$$\begin{aligned} V(t) &= \sum_{j=t+1}^{T} \{E_t\{L_{tj}[R_{tj}^{-1} d(j)]\} \\ &= \sum_{j=t+1}^{T} \{E_t[R_{tj}^{-1} d(j)] + cov_t(L_{tj}, R_{tj}^{-1} d(j))\} \end{aligned}$$

Equation (8) follows immediately.

systematic risks in expected (discounted) abnormal earnings, and underpin the accounting betas discussed in the following sections.

3.2 Accounting betas and valuation

We now obtain a closed form expression for firm value, assuming a specific structure for interest rate and pricing kernel dynamics. Our analysis yields a valuation expression in which accounting "betas" are fundamental in risk-adjusted valuation based on (9).

We make three assumptions concerning the model dynamics:

A1 *Term structure model*: The one-period continuously compounded interest rate (or the short rate), r, follows a mean reverting Vasicek (1977) process:

$$r(s+1) = \overline{r} + k(r(s) - \overline{r}) + \sigma_r \xi(s+1) \tag{10}$$

where $0 < k < 1$ and $\{\xi(j)\}_{j=t}^{T}$ are serially independent standard normals. The long run mean of the short rate is given by $\overline{r}$, the rate of mean reversion is given by k, and its volatility is given by σ_r.

A2 *Pricing kernel:* Suppose there are K market risk factors, such that expected returns are a function of K risk parameters, $\beta_1, ..., \beta_K$. We define the first factor as the interest rate risk factor. The following stochastic process for the pricing kernel is assumed, for $s = t, t+1, ...T-1$:

$$m_{s+1} = m_s \exp[-r(s) - \frac{1}{2}\sum_{i=1}^{K-1} \sigma_i^2 - \sum_{i=1}^{K-1} \sigma_i v_i(s+1)] \tag{11}$$

where $m_t = 1, \sigma_i$ is the volatility of the i'th risk factor and $\{v_i(j)\}_{j=t+1}^{T}$ $(i = 1, ..., K-1)$ are serially independent standard normals representing innovations in the respective risk factors. Equation (11) satisfies $E_s[\frac{m_{s+1}}{m_s}] = exp[-r(s)]$. Our analysis allows for dependence between ξ and v that will affect the relative pricing of equities and bonds.[7]

A3 *Linear information dynamics:* Abnormal earnings are mean-reverting and related to book value and "other information" via an information structure similar to Feltham and Ohlson (1995) and FO as follows:[8]

$$\begin{aligned} y(s+1) &= \overline{y} + \omega_{11}[y(s) - \overline{y}] + \omega_{12}b(s) + \vartheta(s) + \sigma_y \varepsilon_1(s+1) \qquad (12) \\ b(s+1) &= \omega_{22}b(s) + \sigma_b \varepsilon_2(s+1) \\ \vartheta(s+1) &= \gamma\vartheta(s) + \varepsilon_3(s+1) \end{aligned}$$

Here abnormal earnings have a long-run mean $\overline{y}$ and the rate of mean reversion is controlled by the parameter ω_{11}, $0 \leq \omega_{11} \leq 1$. Abnormal earnings at $s+1$ depend, generally, on book value at s and the scalar $\vartheta(s)$ represents "other information" relevant in the forecasting of future abnormal earnings. The parameter $\omega_{12} \geq 0$ captures the effects of accounting conservatism, and γ, $0 \leq \gamma < 1$, controls the relation between other information and future abnormal earnings. We also require a regularity condition on ω_{22} to rule out explosive growth in the accounting system. One sufficient condition is $1 \leq \omega_{22} < \min\{e^{r(s)} : t \leq s \leq T\}$. Innovations in the accounting dynamics are given

[7] Berk, Green and Naik (1999) discuss a similar risk factor representation in terms of cash flows. However, their analysis is based on an economy where the risk of a firm's opportunity set changes dynamically, and accounting items can play no role in valuation without first knowing the risk characteristics of individual investment projects. In contrast, in our model earnings play a direct role in risk assessment and valuation.

[8] For simplicity, we assume that the firm possesses only operating assets. Thus, there is no need to decompose book value and income into operating and financial activities for valuation, as in Feltham and Ohlson (1995). However, our model is easily adapted to accomodate such a decomposition.

by the serially independent standard normals $\{\varepsilon_1(j)\}_{j=t}^T$, $\{\varepsilon_2(j)\}_{j=t}^T$ and $\{\varepsilon_3(j)\}_{j=t}^T$. For simplicity, we also assume $cov_t(v, \varepsilon_i) = 0$ and $cov_t(\xi, \varepsilon_i) = 0$ $(i = 2, 3)$.[9] Without loss of generality, we assume $K = 2$, $v_1 \equiv v$ and $\sigma_1 \equiv \sigma_m$ in the following analysis.

From (12), by induction, for any $s \geq 1$:

$$\begin{aligned} y(t+s) &= \omega_{11}^s y(t) + (1-\omega_{11}^s)\overline{y} + \omega_{12}\phi(\omega_{11}, \omega_{22}, s)b(t) + \phi(\omega_{11}, \gamma, s)\vartheta(t) \\ &\quad + \eta_y(t,s) + \eta_b(t,s) + \eta_\vartheta(t,s) \end{aligned} \tag{13}$$

where η_y, η_b and η_ϑ are exponentially weighted sums of the innovations in, respectively, abnormal earnings, book value and other information, between times t and $t+s$; and $\phi(p, q, s) \equiv \sum_{i=1}^s p^{i-1}q^{s-i}$. Full details of the derivation and notation in (13) are provided in the Appendix.

Let the time t price of a pure discount bond maturing at time $t+s$ be $B[s, r(t)]$. If the short interest rate process is given by (10) then $B[s, r(t)] = E_t[\frac{m_{t+s}}{m_t}]$ has a closed form solution.[10] By applying Lemma 1, we show in the Appendix that firm value can be represented in the following Proposition:

Proposition 1. *The market value of a firm is the sum of (i) a multiple of book value; (ii) the present value of expected future abnormal earnings, discounted at the relevant spot interest rates; (iii) a discount reflecting the interest rate risk of the firm's abnormal earnings; and (iv) a discount reflecting the market risk of the firm's abnormal earnings. If interest rates, the*

[9] Other risk factors will arise when we allow $cov_t(v, \varepsilon_2) \neq 0$ or $cov_t(\xi, \varepsilon_2) \neq 0$. However, the spirit of the discussion on risk-adjusted valuation will still apply.

[10] Berk, Green and Naik (1999) show that $B[s, r(t)] = \exp[-\alpha_1(s)r(t) - \phi_1(s) + \frac{1}{2}\sigma_1^2(s)]$, where $\alpha_1(s), \phi_1(s)$ and $\sigma_1(s)$ are defined in the Appendix.

pricing kernel and abnormal earnings follow stochastic, mean-reverting processes, as defined in assumption A1-A3, the value of the firm value is given by:

$$V(t) = C_0(t)b(t) + C_1(t)\overline{y} + C_2(t)y(t) + C_3(t)\vartheta(t) - \sigma_y\sigma_r\Psi_r(t,\beta_r) - \sigma_y\sigma_m\Psi_m(t,\beta_m) \quad (14)$$

where:

$C_0(t) \equiv 1 + \omega_{12}\sum_{j=1}^{T-t}\phi(\omega_{11},\omega_{22},j)B[j,r(t)]$,

$C_1(t) \equiv \sum_{j=1}^{T-t}(1-\omega_{11})\phi(\omega_{11},1,j)B[j,r(t)]$,

$C_2(t) \equiv \sum_{j=1}^{T-t}\omega_{11}^{j}B[j,r(t)]$,

$C_3(t) \equiv \sum_{j=1}^{T-t}\phi(\omega_{11},\gamma,j)B[j,r(t)]$,

$\Psi_r(t,\beta_r) \equiv \sum_{j=2}^{T-t}\sum_{l=1}^{j-1}[\frac{1-k^{j-l}}{1-k}\omega_{11}^{j-l}]B[j,r(t)]\beta_r(t+l)$

$\Psi_m(t,\beta_m) \equiv \sum_{j=1}^{T-t}\sum_{l=1}^{j}\omega_{11}^{j-l}B[j,r(t)]\beta_m(t+l)$, and

β_r and β_m are risk parameters defined below.

In (14), the value of the firm is expressed as the sum of six components: a multiple, C_0, of book value, a multiple, C_1, of the long-run average level of abnormal earnings, a multiple, C_2, of current abnormal earnings, a multiple, C_3, of the "other information" variable and two risk adjustment terms, $\sigma_y\sigma_r\Psi_r(t,\beta_r)$ and $\sigma_y\sigma_m\Psi_m(t,\beta_m)$. The multiple on book value is increasing in the persistence of abnormal earnings, ω_{11}, and the conservatism parameter, ω_{12}. Also, it is an increasing function of bond prices (a decreasing function of spot interest rates). The multiples on long-run average abnormal earnings and current abnormal earnings are increasing in the persistence of abnormal earnings. The multiple on "other information" is increasing in abnormal earnings persistence and in the persistence of "other information", γ. Together, the sum of the second, third and fourth terms in (14) represents the present value of expected future abnormal earnings, given the linear information dynamics, discounted at the relevant risk-free spot rates prevailing at time t.

The two risk adjustment terms in (14) capture the valuation effects of the two relevant sources of risk associated with future abnormal earnings. The two risk parameters, or "accounting betas", $\beta_r(t+l) = cov_t[\varepsilon_1(t+l), \xi(t+l')]$, if $l' = l$, otherwise 0 and $\beta_m(t+l) \equiv cov_t[\varepsilon_1(t+l), v(t+l')]$ if $l' = l$, otherwise 0, capture the risk exposure of abnormal earnings with respect to the two pricing kernel risk factors in the model, interest rate risk and systematic (or market) risk.[11] The signs of the risk adjustments depends on the signs of the accounting betas and their magnitudes increase with the accounting betas, given $0 \leq k < 1$, $0 \leq \omega_{11} \leq 1$. When the betas are positive, the respective risk adjustments reduce firm value, i.e. they are risk discounts.

As well as depending on the accounting betas, the risk adjustment terms also depend on other model parameters. The magnitudes of both risk adjustments increase with abnormal earnings volatility, σ_y, and persistence, ω_{11}. They also increase with the price of pure discount bonds applicable to future periods (or equivalently they decrease as spot interest rates increase). Further, the magnitude of the interest rate-related risk adjustment increases with the volatility of the short interest rate, σ_r, and the persistence of the short interest rate process, k. Similarly, the magnitude of the market factor risk adjustment increases with the volatility of the market factor, σ_m.

[11] Note that the betas can also be thought of as capturing the covariances between earnings and the risk factors. In particular, it is straightforward to show that $cov_t(x(t+l), v(t+l)) = cov_t(y(t+l), v(t+l))$ and $cov_t(x(t+l), \xi(t+l)) = cov_t(y(t+l), \xi(t+l))$. Details are available from the authors on request.

3.3 Special cases

In this sub-section we examine two special cases of the model: (a) when abnormal earnings are unpredictable; and (b) when abnormal earnings follow a random walk. In both cases we make the simplifying assumption that the accounting betas are constant over time.

3.3.1 Unpredictable abnormal earnings

When abnormal earnings are unpredictable, $\omega_{11} = \omega_{12} = \overline{y} = \vartheta = 0$. Under these conditions from (14) we obtain:

$$V(t) = b(t) - \sigma_y \sigma_m \sum_{j=1}^{T-t} B[j, r(t)] \beta_m$$

If interest rates are constant and equal to r, then the value of the firm reduces to:

$$V(t) = b(t) - \sigma_y \sigma_m \frac{1 - e^{-r(T-t)}}{e^r - 1} \beta_m$$

When abnormal earnings are unpredictable, the value of the firm is equal to risk-adjusted book value. The magnitude of the risk adjustment to book value depends only on the pricing kernel risk of abnormal earnings, β_m. Thus risk in abnormal earnings is relevant to valuation, even when abnormal earnings have no persistence.

This result reflects the structure of the abnormal earnings dynamics, the specification of the pricing kernel and the construction of accounting betas. It is clear from (7) that the role of interest rate risk in valuation can be traced to the relation between the pricing kernel and abnormal earnings, i.e. $cov_t[m_j, y(j)]$. Note from (10) and (11) that m_j is a function of the innovations $v(j)$ and $\xi(j-\tau)$, $\tau \geq 1$. From (12), if $\omega_{11} \neq 0$ then $y(j)$ is a function of the innovations $\varepsilon_1(j-\tau_1)$, $\tau_1 \geq 0$. Thus, generally, $cov_t[m_j, y(j)]$ will depend on $cov_t[v(j), \varepsilon_1(j-\tau_1)]$ and $cov_t[\xi(j-\tau), \varepsilon_1(j-\tau_1)]$, $\tau \geq 1$, $\tau_1 \geq 0$. However, if $\omega_{11} = 0$ then

$y(j)$ is a function only of the contemporaneous innovation $\varepsilon_1(j)$, i.e. $\tau_1 = 0$. By assumption, $cov[\xi(j-\tau), \varepsilon_1(j)] = 0$ for all $\tau \geq 1$. Therefore, when $\omega_{11} = 0$, any dependence between the pricing kernel and abnormal earnings must be attributable to dependence between the two innovations $v(j)$ and $\varepsilon_1(j)$. This dependence is captured by the pricing kernel risk parameter, β_m.[12] In contrast, when $\omega_{11} \neq 0$, the interest rate risk adjustment is also important in valuation.

3.3.2 Abnormal earnings follow a random walk

When abnormal earnings follow a random walk, $\omega_{11} = 1$, $\omega_{12} = \vartheta = 0$ and (14) implies:

$$V(t) = b(t) + B[1, r(t)][y(t) - \sigma_y \sigma_m \beta_m] + \sum_{j=2}^{T-t} B[j, r(t)][y(t) - \sigma_y \sigma_r \beta_r \sum_{l=1}^{j-1} \phi(1, k, l) - \sigma_y \sigma_m \beta_m j]$$

The expected value of abnormal earnings is equal to period t abnormal earnings for all future periods. If interest rates are stochastic, then firm value is equal to book value plus the present value of risk-adjusted expected abnormal earnings, discounted at the risk-free spot rates. For period $t+1$, the risk adjustment depends only on systematic risk. For all later periods, expected abnormal earnings must be adjusted for both systematic and interest rate risks.

If interest rates are deterministic ($\sigma_r = 0$), then the value of firm's equity is equal to book value plus the value of expected abnormal earnings, adjusted by the time-weighted systematic risk premium, again discounted at the pure discount bond rates:

$$V(t) = b(t) + \sum_{j=1}^{T-t} B[j, r(t)][y(t) - j\sigma_y \sigma_m \beta_m]$$

[12] In other words, interest rate risk is irrelevant for valuation when $\omega_{11} = 0 \Rightarrow \beta_r = 0$. In Berk et al (1999), lack of predictability of cash flows eliminates interest rate risk in valuation for precisely this reason.

4 Links to Prior Research

4.1 Ohlson (1995) and Feltham and Ohlson (1995)

Valuation expression (14) nests the abnormal earnings-based valuation models in prior research. The second term in (14), $C_1\overline{y}$, allows for the possibility that abnormal earnings could mean revert to a non-zero value, $\overline{y}$, perhaps because of unrecognized assets due to conservative accounting, or because of monopoly rents. Ohlson (1995) assumes that $\overline{y} = 0$, but it is clear from (14) that the Ohlson model can easily be modified to accommodate non-zero expected long-run abnormal earnings.

If the term structure of interest rates is flat (spot rates are the same for all future periods) then the sum of the first, third and fourth terms is equivalent to the value of the firm in Ohlson (1995) and Proposition 3 in Feltham and Ohlson (1995).[13] Assume that $T \rightarrow \infty$, $\overline{y} = 0$, $\sigma_m = \sigma_r = 0$, $\omega_{12} = 0$ and the interest rate is constant r, with $R = e^r$ and $B[j, r(t)] = R^{-j}$. Then, from (14) we have $C_0(t) = 1$, $C_2(t) \equiv \sum_{j=1}^{\infty} \omega_{11}^j e^{-rj} = \frac{\omega_{11}}{R-\omega_{11}}$,

[13] Similar to Feltham and Ohlson (1995), we could separate operating activities from financial activities.It is straightforward to show that the similar valuation expressions hold based on cashflow and on operating earnings. If we separate financial activities and operating activities as Feltham and Ohlson (1996), then our model would be:

$$V(t) = fa(t) + C_0(t)oa(t) + C_1(t)\overline{ox^a} + C_2(t)ox^a(t) + C_3(t)\vartheta(t) - \sigma_y\sigma_r\Psi_r(t, \beta_r) - \sigma_y\sigma_m\Psi_m(t, \beta_m)$$

where $fa(t)$ denotes net financial assets at time t, $oa(t)$ denotes operating assets at time t, $\overline{ox^a}$ is the long-run mean of abnormal operating earnings, $ox^a(t)$ is abnormal operating earnings for period t. All coefficients in the valuation expression are as set out in Proposition 1.

$C_3(t) \equiv \sum_{j=1}^{\infty} \sum_{i=1}^{j} \omega_{11}^{i-1} \gamma^{j-i} e^{-rj} = \frac{R}{(R-\omega_{11})(R-\gamma)}$ and (14) reduces to:

$$V(t) = b(t) + \frac{\omega_{11}}{R-\omega_{11}} y(t) + \frac{R}{(R-\omega_{11})(R-\gamma)} \vartheta(t)$$

This is identical to the Ohlson (1995) valuation model.

4.2 Feltham and Ohlson (1999)

Feltham and Ohlson (1999, Appendix A, pp.180-1) illustrate a parametric model with priced risk. They assume that the anticipated price risk at date τ is proportional to the expected size of the firm, as represented by the book value of operating assets at date $\tau - 1$, with a factor of proportionality equal to σ_{12}. In our setting this implies that: $\sigma_y \sigma_m \beta_m(t+l) = \sigma_{12} E_t[b(t+l-1)] = \sigma_{12} \omega_{22}^{l-1} b(t)$.[14] Assume that $T \to \infty$, $\overline{y} = 0$, $\sigma_r = 0$, and that interest rates are constant. From (14) we obtain:

$$\begin{aligned} C_0(t) b(t) - \sigma_y \sigma_m \Psi_1(t, \beta_m) &= b(t) + (\omega_{12} - \sigma_{12}) \sum_{j=1}^{\infty} \sum_{i=1}^{j} \omega_{11}^{i-1} {\omega_{22}}^{j-i} e^{-rj} b(t) \\ &= b(t) + \frac{R}{R-\omega_{11}} \frac{\omega_{12} - \sigma_{12}}{R - \omega_{22}} b(t) \end{aligned}$$

Corresponding to this specification of the risk factor, (14) implies that:

$$V(t) - b(t) = \frac{\omega_{11}}{R-\omega_{11}} y(t) + \frac{R}{R-\omega_{11}} \frac{\omega_{12} - \sigma_{12}}{R-\omega_{22}} b(t) + \frac{R}{(R-\omega_{11})(R-\gamma)} \vartheta(t)$$

[14] In FO, the covariance between abnormal earnings and the pricing kernel is defined as $-\sigma_{12}$ times book value. In our model, the dynamics of the pricing kernel (11) is negatively related to the innovation term. That is, a higher beta implies more negative correlation of abnormal earnings with the pricing kernel, and thus, "more risk". Thus, to be consistent with FO $\sigma_y \sigma_m \beta_m(\tau) = \sigma_{12} E_t[b(\tau - 1)]$, rather than $\sigma_y \sigma_m \beta_m(\tau) = -\sigma_{12} E_t[b(\tau - 1)]$. Also, note from the proof of our proposition that if we assume the innovation of book values ε_2 is independent of the innovation of abnormal earnings ε_1 and the innovation of pricing kernel ν, then our valuation model still applies, even if the volatility of abnormal earnings σ_y is a deterministic function of book value.

This is consistent with the first three terms in FO's expression for unrecorded goodwill. Our model could easily be modified to allow the dynamics of book value to include a second "other information" term, as in FO.

4.3 Links to CAPM

The risk parameter, β_m, plays a role analogous to the market beta in the mean-variance CAPM. It summarizes the systematic risk of a firm's abnormal earnings. In the CAPM, β_m is proportional to the market beta and $\frac{m_{t+1}}{m_t}$ is a decreasing, linear function of the market return. Thus, expected stock returns are a linear function of market return. In contrast, in our model the dependence between $\frac{m_{t+1}}{m_t}$ and expected return is a non-linear function of β_m. To illustrate, consider a single period claim with a payoff $y(t+1)$ at end of the period, given by (12). The time t value of this claim is:

$$E_t[\frac{m_{t+1}}{m_t}y(t+1)] = e^{-r(t)}[\overline{y} + \omega_{11}(y(t) - \overline{y}) + \vartheta(t)] - \sigma_y\sigma_m\beta_m$$

The covariance between the return of this claim, RET_{t+1}, and the pricing kernel is:

$$cov_t(\frac{m_{t+1}}{m_t}, RET_{t+1}) \equiv \frac{cov_t[\frac{m_{t+1}}{m_t}, y(t+1)]}{E_t[\frac{m_{t+1}}{m_t}y(t+1)]} = \frac{-\sigma_y\sigma_m\beta_m}{e^{-r(t)}[\overline{y} + \omega_{11}(y(t) - \overline{y}) + \vartheta(t)] - \sigma_y\sigma_m\beta_m}$$

and the expected return is:

$$E_t[RET_{t+1}] = e^{r(t)}\left[1 + \frac{e^{r(t)}\sigma_y\sigma_m\beta_m}{\overline{y} + \omega_{11}(y(t) - \overline{y}) + \vartheta(t) - e^{r(t)}\sigma_y\sigma_m\beta_m}\right]$$

Thus, the expected return is a non-linear function of β_m, as well as other model parameters.

5 Conclusions

In this paper we develop further the theory of risk-adjusted equity valuation based on accounting numbers. We build a partial equilibrium model, based on Feltham and Ohlson (1999). Asset pricing theory techniques allow us to explicitly model the nature of risk adjustment in valuation, when interest rates, the pricing kernel and accounting information dynamics are stochastic. Consistent with asset pricing theory, we show that expected discounted abnormal earnings, but not the discounted value of expected abnormal earnings, are relevant in valuation under the risk neutral probability measure. Firm value is equal to the value obtained assuming risk-neutrality, plus two separable risk adjustment terms depending on accounting betas. The accounting betas reflect the covariances between abnormal earnings and (i) short term interest rates, and (ii) the pricing kernel (or market) risk factor. Our analysis is based on an assumption of a single market risk factor, but the model can easily be extended to a multiple risk factor setting. In this case, there will be an accounting beta and risk adjustment term relating to each systematic risk factor.

The analysis has potential implications for empirical research aimed at modeling observed equity market values and relating estimates of market mispricing, conditional on estimated intrinsic value, to future returns. Recent research estimates intrinsic value by employing a risk-adjusted discount rate in the residual income valuation model (e.g. Frankel and Lee, 1997; Lee, Myers and Swaminathan, 1999) or in the Ohlson (1995) or Feltham and Ohlson (1995) valuation models (e.g. Dechow Hutton and Sloan, 1998; Myers, 1999). Consistent with Feltham and Ohlson (1999), our theoretical results confirm that it is inappropriate to implement risk adjustment in this way. Generally, use of a risk-adjusted discount rate will lead to intrinsic value estimation errors. Our model suggests that the consequences of using risk-adjusted discount rates will be complex and will vary in cross-section. Intrinsic value estimation errors will depend on the risk-adjusted discount rate used, the term structure of interest rates, the parameters of the interest rate and information dynamics processes, the risk factor volatilities and accounting betas. Importantly, to the extent that mispricing estimation error is correlated with the volatility of abnormal earnings and accounting betas, we anticipate such estimates of mispricing will be associated with expected and realized future stock returns, as reported by Frankel and Lee (1997) and Dechow Hutton and Sloan (1998). Future empirical research might seek to examine whether the association between mispricing estimates and future returns is sensitive to the method of risk-adjustment.

6 Appendix

We make use of the following notation:

$$\begin{aligned}
\alpha_1(s) &= \frac{1-k^s}{1-k} \\
\phi_1(s+1) &= \frac{s+1}{2}\sigma_m^2 + [s - k\alpha_1(s)]\overline{r} \\
\eta_r(t,s) &= \sigma_r \sum_{j=1}^{s} k^{s-j}\xi(t+j) \\
\eta_m(t,1) &= \sigma_m v(t+1) \\
\eta_m(t,s+1) &= \sigma_m \sum_{j=1}^{s+1} v(t+j) + \sigma_r \sum_{j=1}^{s} \alpha_1(s+1-j)\xi(t+j) \text{ for } s \geq 1 \\
\eta_y(t,s) &\equiv \sigma_y \sum_{j=1}^{s} \omega_{11}^{s-j}\varepsilon_1(t+j) \\
\eta_b(t,1) &= 0 \\
\eta_b(t,s) &\equiv \sigma_b\omega_{12} \sum_{j=1}^{s-1} \phi(\omega_{11},\omega_{22},s-j)\varepsilon_2(t+j) = \sigma_b\omega_{12} \sum_{j=1}^{s-1}\sum_{i=1}^{s-j} \omega_{11}^{i-1}{\omega_{22}}^{s-j-i}\varepsilon_2(t+j) \\
\eta_v(t,1) &= 0 \\
\eta_v(t,s) &\equiv \sum_{j=1}^{s-1} \phi(\omega_{11},\gamma,s-j)\varepsilon_3(t+j) = \sum_{j=1}^{s-1}\sum_{i=1}^{s-j} \omega_{11}^{i-1}\gamma^{j-i}\varepsilon_3(t+j)
\end{aligned}$$

Let $Z(t) \equiv \begin{bmatrix} y_t \\ b_t \\ \vartheta_t \end{bmatrix}$, $\Omega \equiv \begin{bmatrix} \omega_{11} & \omega_{12} & 1 \\ 0 & \omega_{22} & 0 \\ 0 & 0 & \gamma \end{bmatrix}$, $\overline{Y} = \begin{bmatrix} (1-\omega_{11})\overline{y} \\ 0 \\ 0 \end{bmatrix}$, $\varepsilon(t) \equiv \begin{bmatrix} \sigma_y\varepsilon_1(t) \\ \sigma_b\varepsilon_2(t) \\ \varepsilon_3(t) \end{bmatrix}$,

$\beta_{rm} = \sigma_r\sigma_m cov[\xi(t), v(t)]$,

$\sigma_1^2(1) = \sigma_m^2$, and

$\sigma_1^2(s+1) = (s+1)\sigma_m^2 + \sum_{i=1}^{s}\{\alpha_1^2(i)\sigma_r^2 + 2\alpha_1(i)\beta_{rm}\}$.

Expressions (10) and (11) imply:

$$m_{t+s} = m_t \exp[-\alpha_1(s)r(t) - \phi_1(s) - \eta_m(t,s)]$$

and

$$r(t+s) = k^s r(t) + (1-k^s)\overline{r} + \eta_r(t,s)$$

From LID, $Z(t+1) = \overline{Y} + \Omega Z(t) + \varepsilon(t+1)$. By induction we have, for $s > 1$,

$$Z(t+s) = \sum_{i=0}^{s-1} \Omega^i \overline{Y} + \Omega^s Z(t) + \sum_{j=1}^{s} \Omega^{s-j} \varepsilon(t+j)$$

and

$$\Omega^s = \begin{bmatrix} \omega_{11}^s & \omega_{12} \sum_{i=1}^{s} \omega_{11}^{i-1} \omega_{22}^{s-i} & \sum_{i=1}^{s} \omega_{11}^{i-1} \gamma^{s-i} \\ 0 & \omega_{22}^s & 0 \\ 0 & 0 & \gamma^s \end{bmatrix}$$

Thus (13) follows:

$$\begin{aligned} y(t+s) &= \omega_{11}^s y(t) + (1-\omega_{11}^s)\overline{y} + \omega_{12} \sum_{i=1}^{s} \omega_{11}^{i-1} \omega_{22}^{s-i} b(t) + \sum_{i=1}^{s} \omega_{11}^{i-1} \gamma^{s-i} \vartheta(t) \\ &\quad + \sigma_y \sum_{i=1}^{s} \omega_{11}^{s-j} \varepsilon_1(t+j) + \sigma_b \omega_{12} \sum_{j=1}^{s-1} \sum_{i=1}^{s-j} \omega_{11}^{i-1} \omega_{22}^{s-j-i} \varepsilon_2(t+j) \\ &\quad + \sum_{j=1}^{s-1} \sum_{i=1}^{s-j} \omega_{11}^{i-1} \gamma^{s-j-i} \varepsilon_3(t+j) \end{aligned}$$

Proof of Lemma 1:

Equations (2), (3) and (6) imply:

$$\begin{aligned} V(t) &= \frac{1}{m_t} E_t\{\sum_{j=t+1}^{T} m_j d(j)\} = \frac{1}{m_t} E_t\{\sum_{j=t+1}^{T} m_j [x(j) + b(j-1) - b(j)]\} \\ &= \frac{1}{m_t} E_t[\sum_{j=t+1}^{T} m_j y(j)] - E_t\{\sum_{j=t+1}^{T} \frac{m_j}{m_t} [b(j) - \exp(r(j-1)) b(j-1)]\} \end{aligned} \tag{15}$$

By the property of iterated expectations and given $b(T) = 0$, the last term of (15) can be written as:

$$E_t\{\sum_{j=t+1}^{T} \frac{m_j}{m_t} \exp(r(j-1)) b(j-1)\} = E_t\{\sum_{j=t+1}^{T} \frac{m_{j-1}}{m_t} E_{j-1}[\frac{m_j}{m_{j-1}}] \exp(r(j-1)) b(j-1)\}$$

$$= E_t[\sum_{j=t+1}^{T} \frac{m_{j-1}}{m_t} b(j-1)] = b(t) + E_t[\sum_{j=t+1}^{T} \frac{m_j}{m_t} b(j)]$$

Therefore (15) becomes $V(t) = b(t) + \frac{1}{m_t} E_t[\sum_{j=t+1}^{T} m_j y(j)]$. The second part of the lemma can be shown as in (5).

Lemma 2: If $\{X_i\}_{i=0}^{\infty}$ are serially independent normal distributed with mean μ and variance σ^2, for any constants a_i, $X \equiv \sum_{i=1}^{n} a_i X_i$, then $E[\exp(X)] = \exp(\frac{1}{2}\sum_{i=1}^{n} a_i^2 \sigma^2 + \sum_{i=1}^{n} a_i \mu)$.

Proof: Since $\{X_i\}_{i=0}^{\infty}$ are serially independent normal distributed, for any constants a_i, $X\tilde{\ }N(\sum_{i=1}^{n} a_i\mu, \sum_{i=1}^{n} a_i^2\sigma^2)$. Let $\overline{\mu} \equiv \sum_{i=1}^{n} a_i\mu$, and $\overline{\sigma}^2 \equiv \sum_{i=1}^{n} a_i^2\sigma^2$, then

$$\begin{aligned} E[\exp(X)] &= \int_{-\infty}^{\infty} e^x \frac{1}{\sqrt{2\pi}\overline{\sigma}} e^{-\frac{(x-\overline{\mu})^2}{2\overline{\sigma}^2}} dx \\ &= \frac{1}{\sqrt{2\pi}\overline{\sigma}} \exp[\frac{\overline{\sigma}^2}{2} + \overline{\mu}] \int_{-\infty}^{\infty} e^{-\frac{(x-\overline{\mu}-\overline{\sigma}^2)^2}{2\overline{\sigma}^2}} dx \\ &= \exp[\frac{\overline{\sigma}^2}{2} + \overline{\mu}] \end{aligned}$$

Lemma 3 (Stein's Lemma):[15] If Y and Z are bivariate normal random variables and $f(.)$ is a differentiable function, then

$$cov(Y, f(Z)) = E[f'(Z)]cov(Y, Z)$$

Proof of Proposition 1: From (13) and the price of pure discounted bond $B[s, r(t)] = E_t[\frac{m_{t+s}}{m_t}] = \exp[-\alpha_1(s)r(t) - \phi_1(s) + \frac{1}{2}\sigma_1^2(s)]$, we have:

$$\begin{aligned} \frac{1}{m_t} E_t[\sum_{j=t+1}^{T} m_j y(j)] &= \frac{1}{m_t} E_t\{\sum_{j=t+1}^{T} m_j[\omega_{11}^{j-t} y(t) + (1-\omega_{11}^{j-t})\overline{y} + \phi(\omega_{11}, \gamma, j-t)\vartheta(t) \\ &\quad +\eta_v(t, j-t) + \eta_y(t, j-t)]\} \\ &= \sum_{j=t+1}^{T} [\omega_{11}^{j-t} y(t) + (1-\omega_{11}^{j-t})\overline{y} + \phi(\omega_{11}, \gamma, j-t)\vartheta(t)] B[j-t, r(t)] \end{aligned}$$

[15] See Ingersoll (1987) page13-14.

$$+\sum_{j=t+1}^{T} E_t[\frac{m_j}{m_t}(\eta_v(t,j-t)+\eta_y(t,j-t))] \tag{16}$$

Further,

$$\begin{aligned}
\sum_{j=t+1}^{T} E_t[\frac{m_j}{m_t}\eta_y(t,j-t)] &= \sum_{j=t+1}^{T} E_t\{\exp[-\alpha_1(j-t)r(t)-\phi_1(j-t)-\eta_m(t,j-t)]\eta_y(t,j-t)\} \\
&= \sigma_y \sum_{s=1}^{T-t} E_t\{\exp[-\alpha_1(s)r(t)-\phi_1(s)-\eta_m(t,s)]\sum_{l=1}^{s}\omega_{11}^{s-l}\varepsilon_1(t+l)\} \\
&= \sigma_y \sum_{s=1}^{T-t}\sum_{l=1}^{s}\omega_{11}^{s-l} B[s,r(t)]e^{\frac{-1}{2}\sigma_1^2(s)}E_t[e^{-\eta_m(t,s)}\varepsilon_1(t+l)]
\end{aligned}$$

and

$$\sum_{j=t+1}^{T} E_t[\frac{m_j}{m_t}\eta_v(t,j-t)] = \sum_{s=1}^{T-t} E_t\{\exp[-\alpha_1(s)r(t)-\phi_1(s)-\eta_m(t,s)]\sum_{j=1}^{s-1}\sum_{i=1}^{j}\omega_{11}^{i-1}\gamma^{j-i}\varepsilon_3(t+s-j)\}$$

Substituting into (16)

$$\begin{aligned}
\frac{1}{m_t}E_t[\sum_{j=t+1}^{T} m_j y(j)] &= \overline{y}\sum_{j=1}^{T-t} B[j,r(t)] + (y(t)-\overline{y})\sum_{j=1}^{T-t}\omega_{11}^{j} B[j,r(t)] \\
&+\vartheta(t)\sum_{j=1}^{T-t}\phi(\omega_{11},\gamma,j)B[j,r(t)] \\
&+\sigma_y B[1,r(t)]e^{\frac{-1}{2}\sigma_m^2}E_t[e^{-\sigma_m v(t+1)}\varepsilon_1(t+1)] \\
&+\sigma_y \sum_{j=2}^{T-t}\sum_{l=1}^{j}\omega_{11}^{j-l}B[j,r(t)]e^{\frac{-1}{2}\sigma_1^2(j)}E_t[e^{-\eta_m(t,j)}\varepsilon_1(t+l)] \\
&+\sum_{s=1}^{T-t}\sum_{j=1}^{s-1}\sum_{i=1}^{j}\omega_{11}^{i-1}\gamma^{j-i}B[s,r(t)]e^{\frac{-1}{2}\sigma_1^2(s)}E_t[e^{-\eta_m(t,s)}\varepsilon_3(t+s-j)]
\end{aligned}$$

Lemma 2 and Lemma 3 imply

$$\begin{aligned}
&\sigma_y B[1,r(t)]e^{\frac{-1}{2}\sigma_m^2}E_t[e^{-\sigma_m v(t+1)}\varepsilon_1(t+1)] \\
=\ &-\sigma_y\sigma_m B[1,r(t)]e^{\frac{-1}{2}\sigma_m^2}E_t[e^{-\sigma_m v(t+1)}]cov_t(v(t+1),\varepsilon_1(t+1)) \\
=\ &-\sigma_y\sigma_m B[1,r(t)]\beta_m(t+1)
\end{aligned}$$

Since

$$
\begin{aligned}
var(\eta_m(t,j)) &= E[\sigma_m \sum_{i=1}^{j} v(t+i) + \sigma_r \sum_{i=1}^{j-1} \frac{1-k^{j-i}}{1-k}\xi(t+i)]^2 \\
&= j\sigma_m^2 + \sum_{i=1}^{j-1}(\frac{1-k^i}{1-k})^2\sigma_r^2 + 2\sigma_m\sigma_r E[\sum_{i=1}^{j} v(t+i) \sum_{i=1}^{j-1} \frac{1-k^{j-i}}{1-k}\xi(t+i)] \\
&= j\sigma_m^2 + \sum_{i=1}^{j-1}(\frac{1-k^i}{1-k})^2\sigma_r^2 + 2\sigma_m\sigma_r \sum_{i=1}^{j-1} \frac{1-k^{j-i}}{1-k} cov(v(t+i), \xi(t+i)) \\
&= j\sigma_m^2 + \sum_{i=1}^{j-1}\alpha_1(i)\sigma_r^2 + 2\sum_{i=1}^{j-1}\alpha_1(i)\beta_{rm} \\
&= \sigma_1^2(j)
\end{aligned}
$$

it is clear that $\eta_m \sim N(0, \sigma_1^2)$. When $j > 1$ and $l \geq 1$, by applying Lemma 2 and Lemma 3, we have

$$
\begin{aligned}
E_t[e^{-\eta_m(t,j)}\varepsilon_1(t+l)] &= E_t\{\exp[-\sigma_m \sum_{i=1}^{j} v(t+i) - \sigma_r \sum_{i=1}^{j-1} \frac{1-k^{j-i}}{1-k}\xi(t+i)\ ]\varepsilon_1(t+l)\} \\
&= cov_t(\exp[-\sigma_m \sum_{i=1}^{j} v(t+i) - \sigma_r \sum_{i=1}^{j-1} \frac{1-k^{j-i}}{1-k}\xi(t+i)\ ], \varepsilon_1(t+l)) \\
&= cov_t(e^{-\eta_m(t,j)}, \varepsilon_1(t+l)) = -E_t[e^{-\eta_m(t,j)}]cov_t[\eta_m(t,j), \varepsilon_1(t+l)] \\
&= -\exp[\frac{\sigma_1^2(j)}{2}]cov_t[\sigma_m \sum_{i=1}^{j} v(t+i) + \sigma_r \sum_{i=1}^{j-1} \frac{1-k^{j-i}}{1-k}\xi(t+i), \varepsilon_1(t+l)] \\
&= -\exp[\frac{\sigma_1^2(j)}{2}]\{\sigma_m cov_t[v(t+l), \varepsilon_1(t+l)] \\
&\quad +\sigma_r \sum_{i=1}^{j-1} \frac{1-k^{j-i}}{1-k} cov_t[\xi(t+i), \varepsilon_1(t+l)]\}
\end{aligned}
$$

Since $cov_t[v(t+i), \varepsilon_3(t+l)] = 0$ and $cov_t[\xi(t+i), \varepsilon_3(t+l)] = 0$ for any i, l

$$
\begin{aligned}
& \sigma_y \sum_{j=2}^{T-t}\sum_{l=1}^{j} \omega_{11}^{j-l} B[j, r(t)] e^{\frac{-1}{2}\sigma_1^2(j)} E_t[e^{-\eta_m(t,j)}\varepsilon_1(t+l)] \\
= & -\sigma_y \sum_{j=2}^{T-t}\sum_{l=1}^{j} \omega_{11}^{j-l} B[j, r(t)]\{\sigma_m cov_t[v(t+l), \varepsilon_1(t+l)] \\
& +\sigma_r \sum_{i=1}^{j-1} \frac{1-k^{j-i}}{1-k} cov_t[\xi(t+i), \varepsilon_1(t+l)]\}
\end{aligned}
$$

$$= -\sum_{j=2}^{T-t} B[j,r(t)]\{\sigma_y\sigma_m \sum_{l=1}^{j} \omega_{11}^{j-l}\beta_m(t+l) + \sigma_y\sigma_r \sum_{l=1}^{j-1}[\frac{1-k^{j-l}}{1-k}\omega_{11}^{j-l}\beta_r(t+l)]\}$$

(7) implies:

$$\begin{aligned} V(t) &= b(t) + \frac{1}{m_t}E_t[\sum_{j=t+1}^{T} m_j y(j)] \\ &= b(t) + \overline{y}\sum_{j=1}^{T-t} B[j,r(t)] + (y(t)-\overline{y})\sum_{j=1}^{T-t}\omega_{11}^{j}B[j,r(t)] + \vartheta(t)\sum_{j=1}^{T-t}\phi(\omega_{11},\gamma,j)B[j,r(t)] \\ &\quad -\sigma_y\sigma_m B[1,r(t)]\beta_m(t+1) - \sum_{j=2}^{T-t} B[j,r(t)]\{\sigma_y\sigma_m\sum_{l=1}^{j}\omega_{11}^{j-l}\beta_m(t+l) \\ &\quad +\sigma_y\sigma_r\sum_{l=1}^{j-1}[\frac{1-k^{j-l}}{1-k}\omega_{11}^{j-l}\beta_r(t+l)]\} \end{aligned}$$

Reorganizing, we obtain (14).

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