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Abstract

In this paper we analyze the links between the informational relevance of earnings components for valuation and for forecasting abnormal earnings. We show that forecasting irrelevance implies valuation irrelevance. However, except in the case where aggregate earnings is the sufficient earnings construct for valuation and forecasting, valuation irrelevance does not imply forecasting irrelevance. Additionally, we show that to be irrelevant for valuation and forecasting, an earnings component need not be unpredictable. Our analysis has implications for the design and interpretation of empirical tests of informational relevance. It also has potential implications for practical investment analysis. Generally, value estimates based on aggregate earnings are expected to be less precise than value estimates based on knowledge of earnings components and their relation to other accounting items.

The Ohlson (1995) valuation model is now the natural theoretical framework to underpin capital markets-based financial reporting research. However, many contemporary research issues concern the value relevance of components of earnings. The Ohlson model rules out any informational role for earnings components because it assumes that aggregate abnormal earnings and "other information" are the only variables relevant in forecasting future abnormal earnings. As a direct consequence, the value of the firm depends only on book value, aggregate abnormal earnings and other information. Ohlson (1999) develops a potential informational role for disaggregation of earnings by analyzing the links between "core" and "transitory" earnings components. Informational irrelevance of transitory earnings in forecasting abnormal earnings and their unpredictability result in their irrelevance in valuation. Similarly, Feltham and Ohlson (1995) develop a rationale for separation of earnings and book value into financial and operating components, based on the absence of an information role for net financial assets and income from financial activities in forecasting abnormal earnings. In this paper, we extend this line of research on the informational roles of earnings components for forecasting and the links to valuation.

We analyze a valuation model in which two earnings components, book value and dividends evolve according to a general linear information dynamics structure, based on Stark's (1997) extension of the Ohlson (1989) model. The model allows both earnings components to be predictable. Thus, our discussion is not limited to situations where one earnings component is transitory. We establish the conditions under which one earnings component is informationally irrelevant for valuation and for forecasting abnormal earnings. We adopt Ohlson's (1999) approach to defining informational irrelevance: an earnings component is defined as irrelevant if it may be combined with another accounting item without loss of information. We analyze three forms of irrelevance, depending on the accounting item with which the earnings component may be combined without information loss. We consider how the different concepts of valuation irrelevance and forecasting irrelevance articulate, leading to insights into the properties of the linear information dynamics when earnings components have independent informational roles.

Our analysis leads to the following results. First, when the two earnings components may be combined into an aggregate earnings measure, such that aggregate abnormal earnings and book value are sufficient for valuation, knowledge of aggregate abnormal earnings is also sufficient for forecasting future abnormal earnings. Our results describe the structure of the information dynamics of earnings components (and other accounting items) necessary to ensure no loss of information when earnings components are aggregated. Under these conditions the information dynamics and valuation expressions in Ohlson (1995) apply. Second, we demonstrate that if an earnings component is irrelevant in forecasting *all* future abnormal earnings outcomes, it must also be irrelevant for valuation. However, the converse is not generally true: if an earnings component is valuation irrelevant, this does not necessarily imply irrelevance in forecasting future abnormal earnings outcomes. Third, we show that valuation irrelevance and forecasting irrelevance of an earnings component do not necessarily imply that the earnings component is unpredictable. Thus, an earnings component does not have to satisfy Ohlson's (1999) definition of transitory earnings for it to be irrelevant in valuation and forecasting abnormal earnings.

Our analysis has implications for empirical analysis aimed at establishing the informational relevance of earnings components. First, it suggests general specifications for valuation and abnormal earnings forecasting models that will be applicable when an earnings component is informationally relevant. Second, it emphasizes the importance of considering multi-period forecasting horizons in tests of forecasting relevance. Third, our analysis suggests that tests of valuation- and forecasting relevance may provide deeper insights into the informational role of earnings components if they are based on the three forms of informational irrelevance discussed in the paper.

The analysis also has implications for valuation based on financial statement numbers. We show that the model is consistent with the more parsimonious Ohlson (1995) valuation model when specific restrictions apply to the linear information dynamics. Under more general information dynamics, valuation in our model depends on estimation of many more parameters than in the Ohlson model. Our analysis suggests the factors that will determine the relative precision of value estimates from the two models.

The remainder of the paper is organized as follows: in section 1 we describe the model and the general valuation and abnormal earnings dynamics expressions; in section 2 we present our analysis of the links between concepts of informational irrelevance for valuation and forecasting; in section 3 we discuss the implications of our results; finally, in section 4 we conclude.

1 Model Development

Our model setup is identical to Stark's (1997) extension of the Ohlson (1989) model. Financial statements produced at the end of period t contain the set of accounting items $Z_t = (x_{1t}, x_{2t}, b_t, d_t)$, where x_{1t} and x_{2t} are two earnings components summing to aggregate earnings in period t, x_t ($\equiv x_{1t} + x_{2t}$) ; b_t is book value at the end of period t; and d_t is dividends paid in period t.[1]

1.1 Assumptions

The relation between firm value and accounting numbers is determined by the following two assumptions:

A1. Accounting variables are related by the linear information dynamics specified as:

$$\widetilde{Z}_{t+1} = \Omega Z_t + \varepsilon'_{t+1} \tag{LID}$$

where the constant matrix $\Omega = [\theta_{ij}]_{4\times 4}$, and $\varepsilon'_{t+1} = (\varepsilon'_{1t+1}, ..., \varepsilon'_{4t+1})$ is a vector of noise terms, with each element mean zero and uncorrelated with elements of Z_t. We assume that the maximum characteristic root of Ω is less than R, where $R \equiv 1 + r$ and r is the risk-free interest rate. This regularity condition is sufficient to ensure a unique solution to the model.[2]

A2. The firm is valued in a risk-neutral, arbitrage-free market. This implies that $E_t[\widetilde{P}_{t+1} + \widetilde{d}_{t+1}] = RP_t$, where P_t is the value of the firm at the end of period t.

Assumptions A1 and A2 together imply that the value of a firm is a linear function of the elements of Z_t:

$$P_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 b_t + \beta_4 d_t \tag{VAL-1}$$

where $\beta = (\beta_1, \beta_2, \beta_3, \beta_4) = e_4 \Omega (RI - \Omega)^{-1}$ and $e_4 = (0, 0, 0, 1)$.

Three further assumptions are required, consistent with Ohlson (1989, 1995):

A3. The accounting system satisfies the clean surplus accounting relation:[3]

$$b_t = b_{t-1} + x_{1t} + x_{2t} - d_t \tag{CSR}$$

Similar to Ohlson (1995), we introduce three mathematical restrictions originating in the accounting for owners' equity: (i) $\partial b_t/\partial d_t = -1$; (ii) $\partial x_{1t}/\partial d_t = 0$; and (iii) $\partial x_{2t}/\partial d_t = 0$.[4]

Consistent with Edwards and Bell (1961), Peasnell (1982) and Ohlson (1989, 1995), assumptions A2 and A3 together imply:

$$P_t = b_t + \sum_{s=1}^{\infty} R^{-s} E_t[\tilde{x}^a_{t+s}] \qquad \text{(RIV)}$$

where $x^a_t \equiv x_t - (R-1)b_{t-1}$ is period t "residual income" or "abnormal earnings".

A4. Dividend payment irrelevancy (Miller and Modigliani, 1961) implies that $\partial P_t/\partial d_t = -1$. From A2 and A3 this gives:

$$\beta_4 = \beta_3 - 1 \qquad \text{(MM1)}$$

A5. Consistent with Modigliani and Miller (1958), no arbitrage requires that the reduction of future earnings due to an incremental dollar of dividends, i.e. $\partial E[(x_{1t+1} + x_{2t+1})|Z_t]/\partial d_t$, should equal $R-1$, implying:

$$(\theta_{13} + \theta_{23}) - (\theta_{14} + \theta_{24}) = R - 1 \qquad \text{(MM2)}$$

1.2 General Valuation Expression

Our main analysis ignores "other information" items that might be informationally relevant beyond the accounting system. In section 2.3, we explain that our results are unaffected by the inclusion of a variable reflecting other information. To enable comparison with Ohlson (1995, 1999), we can rewrite expression VAL-1 as follows:

$$P_t = b_t + (\beta_1 + \beta_4)x^a_t + (\beta_2 - \beta_1)x_{2t} + [(R-1)\beta_1 + R\beta_4]b_{t-1} \qquad \text{(VAL-2)}$$

Ohlson (1995) shows that in the absence of other information, current book value and abnormal earnings are sufficient for valuation. VAL-2 indicates that this will only be the case when LID parameter restrictions force $\beta_1 = \beta_2$ and $(R-1)\beta_1 = -R\beta_4$. Our analysis of valuation irrelevance establishes these restrictions.[5]

1.3 General Abnormal Earnings Dynamics

RIV suggests that the informational roles of accounting items in valuation stem from their role in forecasting abnormal earnings. The dynamics of abnormal earnings follow directly from LID and MM2:

$$\tilde{x}^a_{t+1} = (\theta_{11} + \theta_{21})x_{1t} + (\theta_{12} + \theta_{22})x_{2t} + (\theta_{14} + \theta_{24})b_t + (\theta_{14} + \theta_{24})d_t + \varepsilon_{t+1} \qquad \text{(ABED-1)}$$

where $\varepsilon_{t+1} = \varepsilon'_{1t+1} + \varepsilon'_{2t+1}$. There are three "drivers" of abnormal earnings. The first two capture persistence in the two earnings components x_{1t} and x_{2t} and have respective forecasting weights $(\theta_{11} + \theta_{21})$ and $(\theta_{12} + \theta_{22})$; the third is a scale-related driver, captured by the book value of invested capital with weight $(\theta_{14}+\theta_{24})$. The dividend term in ABED-1 has the same weight in forecasting abnormal earnings as book value, reflecting dividend irrelevance. We show that the terms $(\theta_{11} + \theta_{21})$, $(\theta_{12} + \theta_{22})$ and $(\theta_{14} + \theta_{24})$, and their relations to one another, are important in determining relevance in valuation and forecasting.

To facilitate comparison of our model with Ohlson (1995, 1999) it is also useful to rewrite ABED-1 in terms of abnormal earnings:

$$\tilde{x}^a_{t+1} = \omega_{11}x^a_t + \omega_{12}x_{2,t} + \omega_{13}b_{t-1} + \varepsilon_{t+1} \qquad \text{(ABED-2)}$$

where $\omega_{11} = (\theta_{11} + \theta_{21} + \theta_{14} + \theta_{24})$, $\omega_{12} = (\theta_{12} + \theta_{22} - \theta_{11} - \theta_{21})$, and $\omega_{13} = [R(\theta_{14} + \theta_{24}) + (R-1)(\theta_{11} + \theta_{21})]$. ABED-2 indicates that abnormal earnings in period $t+1$ may depend on x_{2t} and b_{t-1}, in addition to period t abnormal earnings. Ohlson's (1995) model assumes that abnormal earnings depend only on lagged abnormal earnings, in addition to "other information", i.e. $\omega_{12} = \omega_{13} = 0$. We show below, in Proposition 1, that the LID parameter restrictions implied by these assumptions are fundamental in determining whether disaggregation of aggregate earnings is informative for valuation.[6]

1.4 Informational Irrelevance

The informational relevance of earnings component x_{2t} can be considered in two contexts: the valuation role and the role in forecasting future abnormal earnings (Ohlson, 1999). The task of identifying the LID conditions under which x_{2t} is informationally relevant for valuation and forecasting is most easily approached by first defining informational irrelevance. Any LID specification not meeting the conditions for irrelevance will then imply informational relevance of x_{2t}. To define irrelevance, consider the information items available from financial statements released at the end of period t. The balance sheet and the income statement in our simple accounting system together contain the set of accounting items Z_t. However, CSR implies that we can infer b_{t-1} from Z_t. Thus, there are five accounting items of potential interest: x_{1t}, x_{2t}, b_t, d_t and b_{t-1}. Following Ohlson (1999), earnings component x_{2t} is defined as irrelevant if it can be combined in any of the four combinations $(x_{1t} + x_{2t})$, $(b_t - x_{2t})$, $(d_t - x_{2t})$ and $(b_{t-1} + x_{2t})$ without loss of information.

Our analysis concentrates on the three combinations $(x_{1t} + x_{2t})$, $(d_t - x_{2t})$ and $(b_{t-1} + x_{2t})$, each related to prior research. First, most traditional empirical tests of informational relevance focus on whether the aggregate earnings combination $(x_{1t} + x_{2t})$ involves loss of information compared to separate knowledge of x_{1t} and x_{2t}. Second, Stark (1997) considers valuation irrelevance defined as $\beta_2 = 0$. This definition implies that x_{2t} can be deleted from the information set without loss of information for valuation. We show that this form of irrelevance is equivalent to the combination discussed by Ohlson (1999) resulting from adjustment of period $t-1$ book value by x_{2t}, i.e. combination $(b_{t-1} + x_{2t})$. Third, Ohlson (1999) considers the case of irrelevance of x_{2t} based on netting off the earnings component against dividends, i.e. $(d_t - x_{2t})$. We show in Appendix A that irrelevance based on the fourth combination of accounting items $(b_t - x_{2t})$, the "reversal of entry" adjustment (Ohlson, 1999), is possible in our model in the valuation context, but not in the context of forecasting abnormal earnings. Since our primary interest is in the links between valuation irrelevance and forecasting irrelevance, we do not consider this case further.

1.4.1 Valuation Irrelevance

We analyze the following three cases of valuation irrelevance (VI):

- VI-1: $\beta_1 = \beta_2$. When x_{2t} is type VI-1 value irrelevant, from VAL-1 firm value can be written as $P_t = \beta_1(x_{1t} + x_{2t}) + \beta_3 b_t + \beta_4 d_t$. Aggregate earnings, $(x_{1t} + x_{2t})$, together with book value and dividends are sufficient for valuation.

- VI-2: $\beta_1 \neq \beta_2 = 0$. When x_{2t} is type VI-2 value-irrelevant, from VAL-1 firm value can be written as $P_t = \beta_1 x_{1t} + \beta_3 b_t + \beta_4 d_t$. Here x_{2t} disappears from the valuation expression and x_{1t}, b_t and d_t are sufficient for valuation. Effectively, x_{2t} is combined with b_{t-1} since by CSR $(b_{t-1} + x_{2t})$ can be used to replace any one of the other three variables used in valuation.[7]

- VI-3: $\beta_1 \neq \beta_2 = -\beta_4$. When x_{2t} is type VI-3 value-irrelevant, from VAL-1 firm value can be written as $P_t = \beta_1 x_{1t} + \beta_3 b_t + \beta_4(d_t - x_{2t})$. Here only one earnings component, x_{1t}, together with b_t and $(x_{2t} - d_t)$ are sufficient for valuation.

In each of these three cases, valuation does not require independent knowledge of x_{2t}. However, only in the case of VI-1 does the valuation irrelevance of earnings component x_{2t} imply that aggregate earnings is the relevant earnings construct for valuation. In the cases of VI-2 and VI-3, earnings component x_{1t} must be known, or be capable of being inferred from other accounting items using CSR.

1.4.2 Forecasting Irrelevance

Analogous to our definitions of valuation irrelevance, we consider three cases of forecasting irrelevance. We define x_{2t} as forecasting irrelevant if x_{2t} has no independent role in forecasting all future abnormal earnings realizations, after controlling for an information set Z_t^a in which x_{2t} has been combined with another accounting item. Formally, x_{2t} is forecasting irrelevant if, *for all future periods* $t+s$, $s \geq 1$ we have $E[x_{t+s}^a|Z_t] = E[x_{t+s}^a|Z_t^a]$. We examine three specific cases of forecasting irrelevance (FI):

- FI-1: $E[\tilde{x}_{t+s}^a|Z_t] = E[\tilde{x}_{t+s}^a|(x_{1t}+x_{2t}), b_t, d_t]$. When x_{2t} is type FI-1 forecasting irrelevant, $(x_{1t} + x_{2t})$, b_t and d_t are sufficient to forecast all future abnormal earnings realizations.

- FI-2: $E[\tilde{x}_{t+s}^a|Z_t] = E[\tilde{x}_{t+s}^a|x_{1t}, b_t, d_t]$. When x_{2t} is type FI-2 forecasting irrelevant, x_{1t}, b_t, d_t are sufficient to forecast all future abnormal earnings realizations. Effectively x_{2t} is combined with the fifth accounting item, b_{t-1} and $(x_{2t} + b_{t-1})$ can be inferred from the other items by CSR.[8]

- FI-3: $E[\tilde{x}^a_{t+s}|Z_t] = E[\tilde{x}^a_{t+s}|x_{1t}, b_t, (d_t - x_{2t})]$. When x_{2t} is type FI-3 forecasting irrelevant it is netted off against d_t and x_{1t}, b_t and $(x_{2t} - d_t)$ are sufficient to forecast all future abnormal earnings realizations.

Our analysis begins by examining the conditions under which VI-1 and FI-1 hold. We show that VI-1 and FI-1 are equivalent. We then examine the links between VI-2 and FI-2. We show that FI-2 implies VI-2, but the reverse is not true. Results for FI-3 and VI-3 are similar.

2 Analysis

2.1 When is Earnings Disaggregation Uninformative?

This section is concerned with the informational irrelevance cases VI-1 and FI-1, aggregate earnings $(x_{1t} + x_{2t})$ is the sufficient earnings construct for valuation and forecasting. First we consider valuation, i.e. irrelevance type VI-1. The necessary and sufficient conditions consistent with $\beta_1 = \beta_2$ are established in Appendix B and are presented in the form of the following Lemma:

Lemma 1: *Assume that A1 to A5 hold. The necessary and sufficient conditions for* $\beta_1 = \beta_2$ *are as follows:*

$$\theta_{12} + \theta_{22} = \theta_{11} + \theta_{21} = \frac{R}{1-R}(\theta_{14} + \theta_{24})$$

Proof: See Appendix B.

Lemma 1 states the LID parameter restrictions consistent with two earnings components attracting identical valuation weights. The respective pairs of linear information dynamics parameters for the two earnings components (θ_{i1} and θ_{i2}, $i = 1, 2$) are complements, summing to the same constant $\frac{R}{1-R}(\theta_{14} + \theta_{24})$.[9] Thus, earnings components need not follow identical processes in order to attract equal weights in valuation. However, notice that under VI-1, the abnormal earnings drivers $(\theta_{11} + \theta_{21})$, $(\theta_{12} + \theta_{22})$ and $(\theta_{14} + \theta_{24})$ are closely related. Both earnings components drivers have identical impacts on future earnings and, in turn, they are related the book value driver by the capitalization factor.

We now explore the implications of Lemma 1 for valuation and abnormal earnings dynamics. Under the conditions specified in Lemma 1, it is straightforward to find a parsimonious valuation expression, as follows:

Corollary: *When $\beta_1 = \beta_2$, current book value, b_t, and aggregate abnormal earnings, x_t^a, are sufficient for valuation and $P_t = b_t + (\beta_1 + \beta_3 - 1)x_t^a$.*

Proof: See Appendix B.

The abnormal earnings dynamics are also very simple under Lemma 1. The first equality in Lemma 1, i.e. $\theta_{12} + \theta_{22} = \theta_{11} + \theta_{21}$, is equivalent to $\omega_{12} = 0$. It can also be shown that if $\omega_{12} = 0$ then the second equality in Lemma 1, i.e. $\theta_{11} + \theta_{21} = \frac{R}{1-R}(\theta_{14} + \theta_{24})$, must hold, in which case $\omega_{13} = 0$.[10] From ABED-2, type FI-1 forecasting irrelevance occurs when $\omega_{12} = \omega_{13} = 0$. Thus FI-1 holds when Lemma 1 is satisfied and FI-1$\Leftrightarrow$VI-1. We summarize this result as follows:

Proposition 1: *If x_{2t} is type FI-1 forecasting irrelevant then x_{2t} is type VI-1 valuation irrelevant (i.e., $\beta_1 = \beta_2$)* and *aggregate abnormal earnings and book value are sufficient for valuation. Conversely, if x_{2t} is type VI-1 valuation irrelevant then x_{2t} is type FI-1 forecasting irrelevant and aggregate abnormal earnings are sufficient for forecasting abnormal earnings.*

Proposition 1 establishes the link between the earnings components model and the Ohlson

(1995) model. Ignoring "other information", the analysis shows that when $\beta_1 = \beta_2$ the information for forecasting and valuation in the four-dimensional LID system is equivalent to the information in Ohlson's one-dimensional LID. The abnormal earnings dynamics is given by:

$$\tilde{x}^a_{t+1} = \omega_{11} x^a_t + \varepsilon_{t+1}$$

where $\omega_{11} = (\theta_{11}+\theta_{21})/R$. It is also straightforward to show that when $\beta_1 = \beta_2$, the valuation parameters in the components model and the Ohlson (1995) are equivalent. Specifically, when $\beta_1 = \beta_2 = -\frac{R(\theta_{11}+\theta_{21})}{(\theta_{11}+\theta_{21})-R^2}$ then $\beta_3 = 1 + \frac{1-R}{R}\beta_1$ and the valuation equation VAL-1 can be written as:

$$P_t = k[\varphi(x_{1t} + x_{2t}) - d_t] + (1-k)b_t$$

where $\varphi \equiv \frac{R}{R-1}$ and $k\varphi = \beta_1 = \frac{R\omega_{11}}{R-\omega_{11}}$. Equivalently, we can write:

$$P_t = b_t + \alpha x^a_t$$

where $\alpha = \frac{\omega_{11}}{R-\omega_{11}} = \frac{\beta_1}{R} = \beta_1 + \beta_3 - 1$.[11]

The two extreme cases of valuation considered in Ohlson (1995) are special cases of Lemma 1. First, when $\theta_{11} + \theta_{21} = \theta_{12} + \theta_{22} = \theta_{14} + \theta_{24} = 0$, $\tilde{x}^a_{t+1}$ is pure noise and $\beta_1 = \beta_2 = \beta_4 = 0$, $\beta_3 = 1$, and the valuation expression relation reduces to $P_t = b_t$. Second, when $\theta_{11} + \theta_{21} = \theta_{12} + \theta_{22} = R$ and $\theta_{14} + \theta_{24} = 1 - R$, $\tilde{x}^a_{t+1}$ is a random walk, $\beta_1 = \beta_2 = \varphi$, $\beta_3 = 0$ and $\beta_4 = -1$ and the valuation expression is $P_t = \varphi(x_{1t} + x_{2t}) - d_t$.

2.2 Informational Irrelevance of One Earnings Component

We now consider the other irrelevance cases where separate knowledge of x_{1t} is required in both valuation and forecasting, but where x_{2t} may be combined with b_{t-1} and d_t respectively, without loss of information. Our focus is on the links between the respective valuation irrelevance and forecasting irrelevance conditions VI-2/FI-2 and VI-3/FI-3. Note that under both VI-2 and VI-3, $\beta_1 \neq \beta_2$ and at least one of the equalities in Lemma 1 cannot hold.

2.2.1 VI-2 and FI-2

Under valuation irrelevance VI-2, $\beta_2 = 0$ and valuation reduces to three dimensions. Our interest lies in whether the dimensions of the accounting system for forecasting abnormal earnings also reduces to the same three dimensions.

We focus this part of the analysis on the abnormal earnings dynamics ABED-1. However, note that ABED-1 could be restated by replacing one of x_{1t}, b_t and d_t by $(b_{t-1}+x_{2t})$, without affecting the coefficient on x_{2t}. In each such case the analysis of forecasting irrelevance is identical to the discussion that follows.

If x_{2t} is irrelevant in forecasting abnormal earnings *over a one period forecast horizon*, i.e. $E_t[\widetilde{x}^a_{t+1}|Z_t] = E[\widetilde{x}^a_{t+1}|x_{1t}, b_t, d_t]$, then ABED-1 indicates that the earnings driver associated with earnings component x_{2t} must be zero, i.e. $\theta_{12} + \theta_{22} = 0$. Note from Lemma 1 that $(\theta_{11}+\theta_{21}) = (\theta_{12}+\theta_{22}) = (\theta_{14}+\theta_{24}) = 0$ is ruled out because under VI-2 $\beta_1 \neq \beta_2$. Therefore, at least one of the accounting items x_{1t+1}, b_{t+1} and d_{t+1} must predict x^a_{t+2}. In turn, x_{1t+1}, b_{t+1} and d_{t+1} could be related to x_{2t}. Therefore, additional LID restrictions are required to rule out an indirect role for x_{2t} in forecasting abnormal earnings beyond period $t+1$. These restrictions ensure either that accounting items predicted by x_{2t} are themselves irrelevant in forecasting abnormal earnings; or that when x_{2t} forecasts other accounting items, the effects of the forecasts exactly offset in aggregation to forecasted abnormal earnings. The full set of LID restrictions for FI-2 is detailed in Appendix B.

Analysis indicates that when $\beta_1 \neq \beta_2 = 0$, the LID restrictions for FI-2 are not necessarily satisfied. Proposition 2 establishes the links between valuation irrelevance VI-2 and forecasting irrelevance FI-2. It also contains the additional constraints that must be imposed on LID, given VI-2, to satisfy FI-2:

Proposition 2. *If x_{2t} is type FI-2 forecasting irrelevant then x_{2t} is type VI-2 valuation irrelevant (i.e., $\beta_1 \neq \beta_2 = 0$). However, if x_{2t} is type VI-2 valuation irrelevant this does not imply that x_{2t} is type FI-2 forecasting irrelevant. Given VI-2, FI-2 will also hold if either (i) $\theta_{12} + \theta_{22} = 0$ and $\theta_{32} = 0$; or (ii) $\theta_{12} + \theta_{22} = 0$ and $\theta_{14} = \theta_{24} = 0$.*

Proof: See Appendix B.

The first statement in Proposition 2 is related to Stark (1997), who shows that irrelevance of x_{2t} in forecasting all accounting items is sufficient for $\beta_2 = 0$. Proposition 2 identifies a weaker sufficient condition for $\beta_2 = 0$, namely that x_{2t} is type FI-2 irrelevant for forecasting abnormal earnings. For forecasting irrelevance FI-2, x_{2t} may have a role in predicting individual accounting items, as long as in aggregation to abnormal earnings the predictions offset exactly.

The second part of Proposition 2 indicates that forecasting irrelevance does not necessarily follow from valuation irrelevance. The Proposition details the additional minimum LID restrictions required to obtain FI-2, given VI-2. Both sets of restrictions (i) and (ii) indicate that VI-2 will be consistent with FI-2 as long as the abnormal earnings driver associated with x_{2t} is zero, i.e. $\theta_{12}+\theta_{22} = 0$. This is not implied by VI-2. Note that x_{2t} may predict one period ahead earnings components, as long as the effects on the two earnings components exactly offset ($\theta_{12} = -\theta_{22} \neq 0$).

The additional conditions in restrictions (i) and (ii) ensure multiperiod forecasting irrelevance, given VI-2. Restriction (i) states that VI-2 is consistent with FI-2 if one-period ahead book value is independent of x_{2t} (i.e. $\theta_{32} = 0$). Again, this is not implied by VI-2. It can also be shown that under restriction (i), x_{2t} has no predictive role for one-period ahead dividends, because from CSR and $\theta_{12} + \theta_{22} = 0$ it follows that $\theta_{32} = 0 \Leftrightarrow \theta_{42} = 0$.

Under restriction (ii), VI-2 is consistent with FI-2 when neither earnings component individually depends on dividends ($\theta_{14} = \theta_{24} = 0$). Under these circumstances, from MM2 the book value driver of abnormal earnings is equal to zero. It follows that when VI-2 and restriction (ii) hold, x_{1t} is the only relevant driver of abnormal earnings, i.e. $E[\widetilde{x}^a_{t+s}|Z_t] = E[\widetilde{x}^a_{t+s}|x_{1t}, b_t, d_t] = E[\widetilde{x}^a_{t+s}|x_{1t}]$ for all $s \geq 1$ and the abnormal earnings dynamics is given by:

$$\widetilde{x}^a_{t+1} = (\theta_{11} + \theta_{21})x_{1t} + \varepsilon_1$$

It can also be shown that $E[\widetilde{x}_{1t+1}|Z_t] = E[\widetilde{x}_{1t+1}|x_{1t}]$, i.e. $\widetilde{x}_{1t+1} = \theta_{11}x_{1t} + \varepsilon^{'}_1$. Thus, in this case earnings component x_{1t} evolves independently of book value.

Proposition 2 shows that VI-2 can be consistent with FI-2 even when x_{2t} has forecasting ability for other accounting items. Restriction (ii) indicates that x_{2t} may have a role in predicting book value and dividends. By CSR, when $\theta_{12} + \theta_{22} = 0$ restriction (ii) implies that $\theta_{32} + \theta_{42} = 0$ but not $\theta_{32} = \theta_{42} = 0$. As long as the effects of x_{2t} on b_{t+1} and d_{t+1} are exactly offsetting, i.e. $\theta_{32} = -\theta_{42}$, x_{2t} may still be irrelevant in forecasting abnormal earnings.

Finally, Proposition 2 also indicates that informational irrelevance of x_{2t} in valuation and forecasting does not depend on x_{2t} being unpredictable. Under restriction (ii), x_{2t} has the following dynamics:

$$\widetilde{x}_{2t+1} = \theta_{21}x_{1t} + (R-1)b_t + \varepsilon^{'}_2$$

Here, x_{2t+1} may depend on x_{1t} and also contains the capital charge component of abnormal earnings in period $t+1$. This case contrasts with Ohlson's (1999) analysis of transitory earnings, where valuation irrelevance and forecasting irrelevance imply lack of predictability for x_{2t}. This is a special case of VI-3 and FI-3 discussed in the next subsection.

2.2.2 VI-3 and FI-3

Under VI-3, $\beta_2 = -\beta_4$ and valuation requires knowledge only of x_{1t}, b_t and $(d_t - x_{2t})$. Valuation again reduces to three dimensions. Our interest lies in whether the accounting system reduces to the same three dimensions for forecasting abnormal earnings. We examine the conditions under which FI-3 is consistent with VI-3.

The abnormal earnings dynamics ABED-1 can be rewritten as:

$$\begin{aligned}\widetilde{x}^a_{t+1} &= (\theta_{11}+\theta_{21})x_{1t} + (\theta_{12}+\theta_{22}+\theta_{14}+\theta_{24})x_{2t} + (\theta_{14}+\theta_{24})b_t \\ &\quad + (\theta_{14}+\theta_{24})(d_t - x_{2t}) + \varepsilon_{t+1} \qquad \text{(ABED-3)}\end{aligned}$$

When $(\theta_{12}+\theta_{22}+\theta_{14}+\theta_{24}) = 0$ (i.e., $\omega_{11}+\omega_{12} = 0$) earnings component x_{2t} is irrelevant for forecasting abnormal earnings *over a one-period horizon*, i.e. $E_t[\widetilde{x}^a_{t+1}|Z_t] = E[\widetilde{x}^a_{t+1}|x_{1t}, b_t, x_{2t} - d_t]$. As before, forecasting irrelevance requires further LID parameter restrictions to ensure that the individual components x_{2t} and d_t are redundant for multiperiod forecasts of abnormal earnings. The general LID restrictions sufficient to ensure type FI-3 forecasting irrelevance are detailed in Appendix B.

Proposition 3 articulates the links between valuation irrelevance VI-3 and forecasting irrelevance FI-3. It also contains the additional constraints that must be imposed on LID, given VI-3, to obtain FI-3:

Proposition 3. *If* x_{2t} *is type FI-3 forecasting irrelevant then* x_{2t} *is type VI-3 valuation irrelevant. However, if* x_{2t} *is type VI-3 valuation irrelevant this does not imply that* x_{2t} *is type FI-3 forecasting irrelevant. Given VI-3, FI-3 will also hold if either (i)* $\theta_{11} + \theta_{21} = \frac{R}{1-R}(\theta_{14}+\theta_{24})$, $\theta_{12}+\theta_{14} = 0$ and $\theta_{22} = \frac{R-1}{R}\theta_{21} = -\theta_{23}$; *or (ii)* $\theta_{12}+\theta_{14} = 0$ *and* $\theta_{32}+\theta_{34} = 0$.

Proof: See Appendix B.

Proposition 3 states that FI-3 implies VI-3, but that the reverse is not true. One of the two sets of restrictions (i) and (ii) must hold in addition to VI-3 to ensure type FI-3 forecasting irrelevance.

Proposition 3 is related to Ohlson's (1999) analysis of transitory earnings. Ohlson demonstrates that any two of the following three properties imply the third:

(a) valuation irrelevance $\beta_2 = -\beta_4$ (i.e. VI-3);

(b) one-period forecasting irrelevance of x_{2t} for abnormal earnings, i.e. $\omega_{11} + \omega_{12} = 0$; and

(c) lack of predictability of x_{2t} (i.e. $\theta_{2i} = 0, i = 1, ..., 4$).

Proposition 3 contrasts with Ohlson's results in two important respects. First, the Proposition states that forecasting irrelevance FI-3 implies valuation relevance VI-3. In contrast, Ohlson shows that *one-period* forecasting irrelevance is not sufficient for valuation irrelevance. Additional restrictions on the information dynamics are required for valuation irrelevance, and these are provided by the assumption in Ohlson's model setup that $\omega_{13} = 0$ and the restriction on the dynamics of x_{2t}, i.e. $\theta_{2i} = 0$, $i = 1, ..., 4$. Together these two assumptions effectively ensure multiperiod forecasting irrelevance FI-3. In the proof of Proposition 3 we show that VI-3 and $\theta_{2i} = 0$ implies $\theta_{12} + \theta_{14} = 0$.

The second point of divergence between our results and those in Ohlson (1999) concerns the dynamics of x_{2t}. Under the information dynamics structure assumed by Ohlson, valuation irrelevance and one period forecasting irrelevance imply that x_{2t} is unpredictable. In contrast, Proposition 3 suggests that valuation irrelevance and multiperiod forecasting irrelevance are consistent with more general information dynamics in which x_{2t} may be predictable. To see this, we examine each set of restrictions in Proposition 3 in turn.

Restriction (i) generates a case that is directly comparable with the Ohlson (1999) model setup. The first condition in (i), i.e. $\theta_{11} + \theta_{21} = \frac{R}{1-R}(\theta_{14} + \theta_{24})$, is also the second equality in Lemma 1. It eliminates b_{t-1} as a relevant variable in forecasting abnormal earnings in ABED-2, i.e. $\omega_{13} = 0$. Additionally, Ohlson's model assumes that b_{t-1} is irrelevant in valuation, i.e. $(R-1)\beta_1 + R\beta_4 = 0$ in VAL-2 . We show in the Appendix B that these restrictions, combined with one-period forecasting irrelevance (i.e. $\omega_{11} + \omega_{12} = 0$) and VI-3, give:

$$\widetilde{x}^a_{t+1} = \left(\frac{\theta_{22} - \theta_{13}}{R-1} - 1\right) x^a_{1t} + \varepsilon_{1t+1} \qquad \text{(ABED-4)}$$

where, following Ohlson (1999) $x^a_{1t} \equiv x_{1t} - (R-1)b_{t-1}$ is labelled "core abnormal earnings". Note that θ_{22} plays a role in the abnormal earnings dynamics. The second and third conditions in (i) give the following dynamics for x_{2t}:

$$\widetilde{x}_{2t+1} = \frac{\theta_{22}}{R-1} x^a_{1t} + \varepsilon'_{2t+1}$$

Therefore, x_{2t} will be predictable if $\theta_{22} \neq 0$. Unpredictability of x_{2t} in Ohlson's analysis is due to the information dynamics assumption $\widetilde{x}_{2t+1} = \theta_{22}x_{2t} + \varepsilon'_{2t+1}$, i.e. $\theta_{21} = \theta_{23} = \theta_{24} = 0$. Consistency with the second condition in (i) requires that $\theta_{22} = 0$ and, hence, that x_{2t} must be unpredictable.

Restriction (ii) leads to more general information dynamics combining with VI-3 to give FI-3. It is considerably weaker than case (i) because it allows $\omega_{13} \neq 0$ in ABED-2. In turn, this creates potential informational roles for b_t and $(x_{2t} - d_t)$ in forecasting one-period ahead abnormal earnings from ABED-2. The following are true under restriction (ii):

$$E[\tilde{x}_{1t+1}|Z_t] = E[\tilde{x}_{1t+1}|x_{1t}, b_t, x_{2t} - d_t]$$

$$E[\tilde{y}_{t+1}|Z_t] = E[\tilde{y}_{t+1}|x_{1t}, b_t, x_{2t} - d_t]$$

$$E[\tilde{x}_{2t+1} - \tilde{d}_{t+1}|Z_t] = E[\tilde{x}_{2t+1} - \tilde{d}_{t+1}|x_{1t}, b_t, x_{2t} - d_t]$$

Thus, earnings component x_{2t} is irrelevant for forecasting each of x_{1t+1}, b_{t+1}, and $(x_{2t+1} - d_{t+1})$, in the sense that x_{2t} can be netted off against d_t without loss of information. Together with one-period forecasting irrelevance of x_{2t} for abnormal earnings (i.e., $\omega_{11} + \omega_{12} = 0$), this renders x_{2t} irrelevant for forecasting all future abnormal earnings realizations, even though b_t and $x_{2t} - d_t$ are relevant. It is also clear that under restriction (ii), x_{2t} is again predictable because there is no requirement that θ_{2i} equal zero for all i.

2.3 Other Information

Our analysis has been conducted by ignoring "other information". We conjecture that this is innocuous for the main results of the paper. Following the approach of Ohlson (1999), we could easily allow for an additional variable reflecting "other information", as long as it is contemporaneously uncorrelated with the other variables included in the model. The coefficients matrix Ω would now equal $[\theta_{ij}]_{5\times 5}$ and the value of the firm $P_t = \beta Z_t$, where $\beta = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_v)$ and $Z_t = (x_{1t}, x_{2t}, b_t, d_t, v_t)$. The clean surplus relation can be written as: $(0, 0, 1, 0, 0)Z_t = E_t((-1, -1, 1, 1, 0)Z_{t+1})$. In particular, $x_{2t+1} = \theta_{21}x_{1t} + \theta_{22}x_{2t} + \theta_{23}b_t +$

$\theta_{24}d_t + \theta_{25}v_t$ and the general abnormal earnings dynamics ABED-2 would be modified as follows:

$$x^a_{t+1} = \omega_{11}x^a_t + \omega_{12}x_{2t} + \omega_{13}b_{t-1} + (\theta_{15} + \theta_{25})v_t + \varepsilon_{t+1}$$

Under these circumstances the general valuation expression would be:

$$P_t = (\beta_1 + \beta_3 - 1)x^a_t + (\beta_2 - \beta_1)x_{2t} + b_t + [(R-1)\beta_1 + R(\beta_3 - 1)]b_{t-1} + \beta_v v_t$$

As in Ohlson (1995, 1999), our main results would be unaffected if they were to be developed in such a framework with one additional dimension.

3 Discussion

In this section we discuss the implications of our results. First, we examine the implications for empirical research concerned with the informational relevance of earnings components. We then discuss the implications of our model for practical valuation.

3.1 Testing informational relevance of earnings components

Tests of the informational relevance of earnings components may be based on estimates of the relation between market value and accounting items (valuation studies)[12] and on the forecasting ability of earnings components for abnormal earnings and other accounting items (forecasting relevance studies). Our results have implications for the choice of testing methodology and the design and interpretation of such tests.

3.1.1 Valuation studies

Typical tests of earnings component valuation relevance examine the null hypothesis VI-1, i.e. $\beta_1 = \beta_2$. Rejection of VI-1 is interpreted as evidence of the informational relevance of earnings components. Our analysis suggests that valuation expressions VAL-1 and VAL-2 provide appropriate frameworks for such tests. However, restricted versions of VAL-2 may also be suggested by alternative models. For example, the Ohlson (1999) model suggests a valuation expression similar to VAL-2, but with the b_{t-1} term omitted.[13] Our results suggest that unless specific LID restrictions hold, b_{t-1} will capture relevant information. Generally, under LID, b_{t-1} will be correlated with the other variables in VAL-2. Therefore omission of b_{t-1} in estimated valuation equations based on VAL-2 could affect inferences concerning the valuation relevance of earnings components.

Subject to appropriate test specification, it is clear that rejection of VI-1 indicates aggregate earnings is not a sufficient earnings construct for valuation. The additional definitions of valuation irrelevance suggested by Ohlson (1999), and analyzed in this paper, suggest that additional insights into valuation relevance can be obtained by supplementing tests that reject VI-1 by tests of VI-2, i.e. $\beta_1 \neq \beta_2 = 0$ and VI-3, i.e. $\beta_1 \neq \beta_2 = -\beta_4$. Rejection of VI-1, VI-2 and VI-3 would suggest that separate knowledge of x_{1t} and x_{2t} is relevant for valuation. Rejection of VI-1 and failure to reject VI-2 would suggest that a dirty surplus accounting system that reports x_{1t} but omits to report x_{2t} (or aggregates x_{2t} with b_{t-1} as a prior year adjustment) is no less useful in valuation than a system that reports both earnings components. Rejection of VI-1 and failure to reject VI-3 would suggest that a dirty surplus accounting system that nets off x_{2t} against d_t will be no less useful in valuation than a system that reports both earnings components.

3.1.2 Forecasting relevance studies

Tests of informational relevance based on the relevance of earnings components in forecasting abnormal earnings and other accounting items are potentially attractive. They do not require knowledge of market values and they do not require the assumption of stock market efficiency. Potentially, conclusions concerning the relevance of accounting items in valuation can be developed based on evidence of forecasting relevance and a theoretical valuation model providing a link between firm value and the target variable for forecasting (e.g. future abnormal earnings). Our analysis suggests the need for caution in the design and interpretation of forecasting relevance tests.

The analysis indicates that forecasting irrelevance for all future abnormal earnings realizations (multi-period forecasting irrelevance) is sufficient for valuation irrelevance. Specifically, Propositions 1 to 3 indicate that FI-1 (FI-2) (FI-3) implies VI-1 (VI-2) (VI-3). The analysis distinguishes between one period forecasting irrelevance and multiperiod forecasting irrelevance. A test of single period forecasting irrelevance based on ABED-1 (ABED-2) (ABED-3) is not sufficient to demonstrate forecasting irrelevance over longer horizons. However, if the first order structure of LID is descriptive, one-period irrelevance and two-period forecasting irrelevance will imply multi-period forecasting irrelevance. Two-period forecasting irrelevance can be tested by replacing x^a_{t+1} with x^a_{t+2} in the relevant abnormal earnings dynamics model.

Our analysis also shows that valuation irrelevance type VI-2 (VI-3) does not imply forecasting irrelevance type FI-2 (FI-3). Thus, rejection of FI-2 or FI-3 cannot be taken as a reliable signal that an earnings component will be valuation relevant. Only in the case of FI-1 is rejection of valuation irrelevance type VI-1 necessarily implied.

Notwithstanding the lack of direct correspondence between informational irrelevance for valuation and for forecasting, our analysis suggests general forms for one period abnormal earnings dynamics to underpin tests of FI-1, FI-2 and FI-3. For example, ABED-2 indicates that b_{t-1} may capture forecasting relevant information beyond abnormal earnings and earnings components. Since b_{t-1} is expected to be correlated with x_t^a and x_{2t}, its omission from the abnormal earnings forecasting equation could lead to incorrect inferences regarding the forecasting irrelevance of x_{2t}.

The analysis also indicates the dangers of attempting to draw inferences concerning potential valuation relevance from the information dynamics of individual earnings components. Lemma 1 provides precise restrictions on the joint dynamics of x_{1t} and x_{2t} leading to valuation irrelevance VI-1. However, VI-2 or VI-3 could be found under a wide range of LID parameter values. In particular, it is not necessary for x_{2t} to be unpredictable for it to be valuation- and forecasting irrelevant.

3.2 Informational Requirements for Firm Valuation

A major attraction of using the Ohlson (1995) model as a basis for valuation is its relative parsimony and low data and computational demands. However, our analysis suggests that model parsimony could result in loss of precision in value estimates. In both the Ohlson (1995) model and the components model, the estimated value of the firm is equal to book value plus the discounted value of expected future abnormal earnings. Thus, the relative precision of value estimates depends on the relative precision of the implicit abnormal earnings forecasts embedded within each model.

Proposition 1 indicates that when VI-1/FI-1 hold, value estimates based on Ohlson's modified autoregressive abnormal earnings dynamics and valuation expressions will be identical to valuation under the components model.[14] If VI-1/FI-1 does not hold, the Ohlson abnormal earnings dynamics are expected give unbiased forecasts of abnormal earnings, if empirical estimation of the model includes a constant term capturing the average effect of the omitted variables. However, the forecasts will have higher error variance than forecasts based on ABED-1 (or equivalently ABED-2 or ABED-3). Thus, value estimates obtained from the Ohlson model will be noisier than estimates obtained from the full information components model. Differences in value estimates obtained from the two models will depend on the additional accounting items entering ABED-2 and VAL-2, i.e. x_{2t} and b_{t-1}.

Improved precision in valuation estimates based on the components model can only be achieved at the price of higher data and computational demands. If the VI-1 holds the model requires the estimation of just one parameter, ω_{11} and valuation based on the Ohlson model applies. However, when VI-1 does not hold, identification of the valuation weights $\beta_1..\beta_4$ involves application of Cramer's Rule to the equation system (1)-(4) described in Appendix B. This requires a much larger set of information dynamics parameters to be estimated. If VI-2 or VI-3 hold, the relevant financial statement information has three dimensions and valuation requires estimation of nine LID parameters. If no valuation irrelevance condition holds, the financial statement information has four dimensions and valuation requires estimation of sixteen LID parameters. In both cases, parameter estimation is relatively demanding in computational terms.

4 Conclusions

In this paper we analyze the links between the informational relevance of earnings components for valuation and for forecasting abnormal earnings. We consider three definitions of informational irrelevance suggested by Ohlson (1999), reflecting different ways in which an earnings component may be combined with another accounting item without loss of information. We show that forecasting irrelevance implies valuation irrelevance. However, except in the case where aggregate earnings is the sufficient earnings construct for valuation and forecasting, valuation irrelevance does not imply forecasting irrelevance. Additionally, we show that to be irrelevant for valuation and forecasting, an earnings component need not be unpredictable.

Our analysis has implications for the design and interpretation of empirical tests of informational relevance. It suggests appropriate specifications for general valuation and abnormal earnings dynamics expressions. Our results also suggest that forecasting relevance tests are not good substitutes for direct tests of valuation relevance. Specifically, it is possible for an accounting item to be relevant in forecasting abnormal earnings in combination with other items, but for it to be irrelevant in valuation.

Finally, our analysis has potential implications for practical investment analysis. Generally, value estimates based on aggregate earnings are expected to be less precise than value estimates based on knowledge of earnings components and their relation to other accounting items. Investment analysis techniques that lead to better understanding of the links between earnings components, aggregate earnings, book value and dividends can be expected to improve the precision of value estimates obtained within an abnormal earnings-based valuation context. The model provides a framework with potential for interpreting practical techniques such as the analysis of trends and persistence of earnings components, and methodologies used by some analysts to undo GAAP, for example by separating out less persistent earnings components from aggregate earnings.

5 Appendix A

We show that the forecasting irrelevance of x_{2t} based on the combination $(b_t - x_{2t})$ is generally infeasible. ABED-1 may be rewritten as follows:

$$\begin{aligned}\widetilde{x}^a_{t+1} &= (\theta_{11}+\theta_{21})x_{1t} + (\theta_{12}+\theta_{14}+\theta_{22}+\theta_{24})x_{2t} + (\theta_{14}+\theta_{24})d_t \\ &\quad +(\theta_{14}+\theta_{24})(b_t - x_{2t}) + \varepsilon_{t+1}\end{aligned}$$

If we net off x_{2t} against b_t, LID reduces to:

$$\begin{aligned}x_{1t+1} &= \theta_{11}x_{1t} + (\theta_{12}+\theta_{13})x_{2t} + \theta_{14}d_t + \theta_{13}(b_t - x_{2t}) + \varepsilon'_1 \\ d_{t+1} &= \theta_{41}x_{1t} + (\theta_{42}+\theta_{43})x_{2t} + \theta_{44}d_t + \theta_{43}(b_t - x_{2t}) + \varepsilon'_4 \\ (b_{t+1} - x_{2t+1}) &= (\theta_{31}-\theta_{21})x_{1t} + (\theta_{32}+\theta_{33}-\theta_{22}-\theta_{23})x_{2t} + (\theta_{34}-\theta_{24})d_t \\ &\quad +(\theta_{33}-\theta_{23})(b_t - x_{2t}) + \varepsilon'_3 - \varepsilon'_2\end{aligned}$$

If x_{2t} is irrelevant in forecasting $\widetilde{x}^a_{t+s}$, $s > 1$, then x_{2t} must not, generally, predict any of the three accounting items x_{1t+1}, d_{t+1} and $(b_{1t+1} - x_{2t+1})$. Thus, (i) $\theta_{12}+\theta_{13} = 0$, (ii) $\theta_{42}+\theta_{43} = 0$, and (iii) $\theta_{32}+\theta_{33} = \theta_{22}+\theta_{23}$. Note that CSR implies the following LID parameter restrictions:

$$\theta_{11}+\theta_{21}-\theta_{31}-\theta_{41} = 0$$

$$\begin{aligned}\theta_{12}+\theta_{22}-\theta_{32}-\theta_{42} &= 0\\ \theta_{13}+\theta_{23}-\theta_{33}-\theta_{43} &= -1\\ \theta_{14}+\theta_{24}-\theta_{34}-\theta_{44} &= 0\end{aligned}$$

(i), (iii) and the second and third LID restrictions from CSR imply that $\theta_{42}+\theta_{43}=-1$. This is inconsistent with (ii). Therefore, this type of forecasting irrelevance is infeasible.

6 Appendix B

From A2 and LID:

$$E_t[(\beta_1,\beta_2,\beta_3,\beta_4+1)\widetilde{Z}_{t+1}]=RP_t$$

and

$$(\beta_1,\beta_2,\beta_3,\beta_4+1)\Omega Z_t=R(\beta_1,\beta_2,\beta_3,\beta_4)Z_t$$

give

$$(\beta_1,\beta_2,\beta_3,\beta_4+1)\Omega-R(\beta_1,\beta_2,\beta_3,\beta_4)=0$$

This is equivalent to the following four equation system:

$$(\theta_{11}-R)\beta_1+\theta_{21}\beta_2+\theta_{31}\beta_3+\theta_{41}\beta_4 = -\theta_{41} \tag{1}$$

$$\theta_{12}\beta_1+(\theta_{22}-R)\beta_2+\theta_{32}\beta_3+\theta_{42}\beta_4 = -\theta_{42} \tag{2}$$

$$\theta_{13}\beta_1+\theta_{23}\beta_2+(\theta_{33}-R)\beta_3+\theta_{43}\beta_4 = -\theta_{43} \tag{3}$$

$$\theta_{14}\beta_1+\theta_{24}\beta_2+\theta_{34}\beta_3+(\theta_{44}-R)\beta_4 = -\theta_{44} \tag{4}$$

From (1), (2), (3), (4) and $\beta_4=\beta_3-1$ (i.e. MM1) we have:

$$(\theta_{11}-R)\beta_1+\theta_{21}\beta_2+(\theta_{11}+\theta_{21})\beta_3 = 0 \tag{5}$$

$$\theta_{12}\beta_1 + (\theta_{22} - R)\beta_2 + (\theta_{12} + \theta_{22})\beta_3 = 0 \tag{6}$$

$$\theta_{13}\beta_1 + \theta_{23}\beta_2 + (\theta_{13} + \theta_{23} + 1 - R)\beta_3 = 0 \tag{7}$$

$$\theta_{14}\beta_1 + \theta_{24}\beta_2 + (\theta_{14} + \theta_{24} - R)\beta_3 + R = 0 \tag{8}$$

Proof of Lemma 1. If $\beta_1 = \beta_2$, (5) and (6) imply:

$$(\theta_{11} + \theta_{21} - R)\beta_2 + (\theta_{11} + \theta_{21})\beta_3 = 0 \tag{9}$$

$$(\theta_{12} + \theta_{22} - R)\beta_2 + (\theta_{12} + \theta_{22})\beta_3 = 0$$

$$(\theta_{11} + \theta_{21} - \theta_{12} - \theta_{22})(\beta_2 + \beta_3) = 0$$

Under VI-1, $\beta_2 = -\beta_3$ is impossible, otherwise, $\beta_1 = \beta_2 = \beta_3 = 0 \Rightarrow \beta_4 = -1$. However, $\beta_4 = -1$ does not satisfy (4), hence:

$$\theta_{11} + \theta_{21} = \theta_{12} + \theta_{22} \tag{10}$$

(7) implies:

$$(\theta_{14} + \theta_{24} + R - 1)\beta_2 + (\theta_{14} + \theta_{24})\beta_3 = 0$$

This, together with (9), implies:

$$\left(\frac{\theta_{14} + \theta_{24}}{R - 1} + \frac{\theta_{11} + \theta_{21}}{R}\right)(\beta_2 + \beta_3) = 0$$

and, since $\beta_2 \neq -\beta_3$

$$\frac{\theta_{14} + \theta_{24}}{R - 1} + \frac{\theta_{11} + \theta_{21}}{R} = 0 \tag{11}$$

If (10) and (11) hold, (5) and (6) imply:

$$(\theta_{11} - \theta_{12} - R)(\beta_1 - \beta_2) = 0$$

The regularity condition in A1 implies that $\theta_{11} - \theta_{12} - R \neq 0$. Therefore $\beta_1 = \beta_2$.

Proof of Corollary: We show that if $\beta_1 = \beta_2$, then $(R-1)\beta_1 + R(\beta_3 - 1) = 0$. If $\beta_1 = \beta_2$, then (5) and (8) imply that $(\theta_{11} + \theta_{21})(\beta_1 + \beta_3) = R\beta_1$ and $(\theta_{14} + \theta_{24})(\beta_1 + \beta_3) = R(\beta_3 - 1)$. If we rule out the trivial case where $\beta_1 = \beta_2 = \beta_4 = 0$ and $\beta_3 = 1$, then $\theta_{11} + \theta_{21} \neq 0$. (11) further implies $(R-1)\beta_1 + R(\beta_3 - 1) = 0$. From VAL-2, it follows that current book value and aggregate abnormal earnings are sufficient for valuation and VAL-2 reduces to the expression in the Corollary.

Proof of Proposition 2: The first part of Proposition 2 states that FI-2 implies VI-2. From VAL-1, A2 and A3, $\beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 b_t + \beta_4 d_t = b_t + \sum_{s=1}^{\infty} R^{-s} E_t[\tilde{x}^a_{t+s}]$. It is clear that any accounting item that is irrelevant in predicting x^a_{t+s} will be irrelevant in valuation. If FI-2 holds, $E[\tilde{x}^a_{t+s}|Z_t] = E[\tilde{x}^a_{t+s}|x_{1t}, b_t, d_t]$ for all $s \geq 1$ and VI-2 follows, i.e. $\beta_1 \neq \beta_2 = 0$. This can be confirmed by considering the LID restrictions implied by FI-2, assuming x_{1t} is not forecasting irrelevant, i.e. $\theta_{11} + \theta_{21} \neq 0$:

(a) From ABED-1, one period ahead forecasting irrelevance of x_{2t} for abnormal earnings requires $\theta_{12} + \theta_{22} = 0$.

(b) FI-2 also requires x_{2t} to be irrelevant in multi-period forecasting of abnormal earnings. There are two possible sets of LID restrictions that rule out a multi-period forecasting role for x_{2t}:

(b-1) $\theta_{12} = \theta_{22} = \theta_{32} = \theta_{42} = 0$: If $\theta_{11} + \theta_{21} \neq 0$ and $\theta_{14} + \theta_{24} \neq 0$ then from ABED-1 and the definition of FI-2 we require that x_{1t+1}, b_{t+1} and d_{t+1} should be independent of x_{2t}, i.e. $\theta_{12} = \theta_{32} = \theta_{42} = 0$. In this case, from (a) above we also have $\theta_{22} = 0$.

In other words, x_{2t} has no forecasting ability for other accounting items. This is the forecasting irrelevance case discussed in Stark (1997).

(b-2) $\theta_{12} = \theta_{13} = \theta_{14} = \theta_{24} = 0$: If $\theta_{11} + \theta_{21} \neq 0$ and $\theta_{14} + \theta_{24} = 0$ then, from ABED-1 and the definition of FI-2 we require that x_{1t+1} should be independent of all other accounting items, i.e. $\theta_{12} = \theta_{13} = \theta_{14} = 0$.

If FI-2 holds as a result of (a) and (b-1) then VI-2 ($\beta_1 \neq \beta_2 = 0$) follows from (6).

If FI-2 holds as a result of (a) and (b-2) then VI-2 ($\beta_1 \neq \beta_2 = 0$) follows from (7) and MM-2.

The second part of Proposition 2 identifies the LID restrictions additional to those implied by $\beta_2 = 0$ necessary to ensure FI-2. The two cases in the Proposition correspond, respectively, to the LID forecasting irrelevance conditions (a) and (b-1) and (a) and (b-2) above, as follows:

Restriction (i): If $\beta_2 = 0$, and $\theta_{12} + \theta_{22} = 0$, then the second equation (6) implies $\theta_{12}\beta_1 = 0$. Since $\beta_1 \neq 0$ under VI-2, $\theta_{12} = \theta_{22} = 0$. CSR implies that $\theta_{32}+\theta_{42} = \theta_{12}+\theta_{22} = 0$. Thus, $\theta_{32} = 0 \Leftrightarrow \theta_{42} = 0$. That is, VI-2 and restriction (i) in Proposition 2 together ensure consistency with FI-2. However, VI-2 alone does not imply FI-2.

Restriction (ii): The proof of $\theta_{12} = \theta_{22} = 0$ is identical to restriction (i). From $\theta_{14} = \theta_{24} = 0$, MM2 and (7) we obtain $\theta_{13} = 0$, because $\beta_1 \neq 0$. That is, VI-2 and restriction (ii) in Proposition 2 together ensure consistency with FI-2. However, VI-2 alone does not imply FI-2.

Proof of Proposition 3: The first part of Proposition 3 is similar to the proof of the first part of Proposition 2. If FI-3 holds, $E[\tilde{x}^a_{t+s}|Z_t] = E[\tilde{x}^a_{t+s}|x_{1t}, b_t, d_t - x_{2t}]$ for all $s \geq 1$. d_t and x_{2t} have exactly equal and opposite information in forecasting abnormal earnings. Thus

VI-3 follows, i.e. $\beta_1 \neq \beta_2 = -\beta_4$. This can be confirmed by considering the LID restrictions implied by FI-2, assuming x_{1t} is not forecasting irrelevant, i.e. $\theta_{11} + \theta_{21} \neq 0$:

(c) From ABED-3, one period ahead forecasting irrelevance of x_{2t} for abnormal earnings requires $\theta_{12} + \theta_{22} + \theta_{14} + \theta_{24} = 0$.

(d) FI-3 also requires x_{2t} to be irrelevant in multi-period forecasting of abnormal earnings. There are two possible sets of LID restrictions that rule out a multi-period forecasting role for x_{2t}:

(d-1) $\theta_{11} + \theta_{21} = \frac{R}{1-R}(\theta_{14} + \theta_{24})$ and $\theta_{22} = \frac{R-1}{R}\theta_{21} = -\theta_{23} = -\theta_{24}$: When (c) holds we can rewrite ABED-2 as

$$\tilde{x}^a_{t+1} = (\theta_{11} + \theta_{21} + \theta_{14} + \theta_{24})x^a_{1t} + [R(\theta_{14} + \theta_{24}) + (R-1)(\theta_{11} + \theta_{21})]b_{t-1} + \varepsilon_{t+1}$$

where $x^a_{1t} = x_{1t} - (R-1)b_{t-1}$. If we assume that $\theta_{11} + \theta_{21} = \frac{R}{1-R}(\theta_{14} + \theta_{24})$ (otherwise case (d-2) below applies) the definition of FI-3 requires $\theta_{22} = \frac{R-1}{R}\theta_{21} = -\theta_{23} = -\theta_{24}$ and $x_{2t+1} = \frac{\theta_{22}}{R-1}x^a_{1t} + \varepsilon'_{2t+1}$. Specifically this is consistent with Ohlson(1999) if x_{2t+1} is unpredictable, i.e. $\theta_{2i} = 0$ $(i = 1-4)$. Thus $\tilde{x}^a_{1t+1} = \frac{\theta_{11}+\theta_{21}}{R}x^a_{1t} + \varepsilon^{'}_{1t+1}$ and $\tilde{x}^a_{t+1} = \frac{\theta_{11}+\theta_{21}}{R}x^a_{1t} + \varepsilon_{t+1}$. Therefore, $\tilde{x}^a_{t+1}$ is independent of x_{2t}.

(d-2) $\theta_{12} + \theta_{14} = \theta_{22} + \theta_{24} = \theta_{32} + \theta_{34} = \theta_{42} + \theta_{44} = 0$: If $\theta_{11} + \theta_{21} \neq 0$ then from ABED-3 and the definition of FI-3 we require that x_{1t}, b_t and $d_t - x_{2t}$ should be independent of x_{2t}, i.e. $\theta_{12} + \theta_{14} = \theta_{32} + \theta_{34} = 0$ and $\theta_{42} + \theta_{44} = \theta_{22} + \theta_{24}$. From the LID restrictions due to CSR described in Appendix A, we know $\theta_{42} + \theta_{44} = 0$.

VI-3 ($\beta_2 + \beta_4 = 0$) follows directly from (6) and (8) under both (c) and (d-1) and (c) and (d-2).

The second part of Proposition 2 identifies the LID restrictions additional to those implied by $\beta_2+\beta_4 = 0$ necessary to ensure FI-3. If $\beta_2+\beta_4 = 0$ and using MM1, the equation system (5)-(8)becomes

$$(\theta_{11} - R)\beta_1 + \theta_{11}\beta_4 + \theta_{11} + \theta_{21} = 0 \quad (12)$$

$$\theta_{12}\beta_1 + (R + \theta_{12})\beta_4 + \theta_{12} + \theta_{22} = 0 \quad (13)$$

$$\theta_{13}\beta_1 + (\theta_{13} + 1 - R)\beta_4 + \theta_{14} + \theta_{24} = 0 \quad (14)$$

$$\theta_{14}\beta_1 + (\theta_{14} - R)\beta_4 + \theta_{14} + \theta_{24} = 0 \quad (15)$$

The two cases in Proposition 3 correspond, respectively, to the LID forecasting irrelevance conditions (c) and (d-1) and (c) and (d-2) above, and can be analyzed as follows:

Restriction (i): If $\theta_{12} + \theta_{14} = 0$, adding (13) and (15) we obtain $\theta_{22} + \theta_{24} = 0$. Thus condition (c) and (d-1) hold given restriction (i). Note that if $\theta_{2i} = 0$, ($i = 1...4$), adding (13) and (15) we obtain $(\theta_{12} + \theta_{14})(\beta_1 + \beta_4 + 1) = 0$. Adding (12) and (15) we obtain $(\theta_{11} + \theta_{14} - R)(\beta_1 + \beta_4 + 1) = -R$. These together imply $\beta_1 + \beta_4 + 1 \neq 0$ and $\theta_{12} + \theta_{14} = 0$.

Restriction (ii): If $\theta_{12}+\theta_{14} = 0$, we know $\theta_{22}+\theta_{24} = 0$. Further the LID restrictions from CSR in Appendix A imply $\theta_{32} + \theta_{34} = 0 \Leftrightarrow \theta_{42} + \theta_{44} = 0$. Thus (c) and (d-2) hold and x_{2t} is type FI-3 forecasting irrelevant.

Proof of ABED-4: As in Ohlson (1999), assume $\omega_{11} + \omega_{12} = \omega_{13} = 0$ and that the coefficient on b_{t-1} in VAL-2 is zero (i.e. $(R - 1)\beta_1 + R\beta_4 = 0$). VAL-2 implies:

$$P_t = b_t + (\beta_1 - \beta_2)x^a_{1t}$$

From $\omega_{11} + \omega_{12} = 0$, (13), (15) imply $(\theta_{12} + \theta_{14})(\beta_1 + \beta_4) = 0$. Combining this with $(R-1)\beta_1 + R\beta_4 = 0$ we obtain $(\theta_{12} + \theta_{14})\beta_1 = 0$. If $\beta_1 \neq 0$ (otherwise $P_t = b_t$) then $\theta_{12} + \theta_{14} = 0$ and $\theta_{22} + \theta_{24} = 0$.

From $\omega_{13} = 0$, (12) and (13) we obtain $[(R-1)\theta_{11} - R\theta_{12} - R^2]\beta_1 + R^2\beta_1 = 0$. If $\beta_1 \neq 0$ then $(R-1)\theta_{11} - R\theta_{12} = 0$ and $(R-1)\theta_{21} - R\theta_{22} = 0$.

From $(R-1)\beta_1 + R\beta_4 = 0$, (14) and (15) imply $(\theta_{13} - \theta_{14} + 1 - R)\beta_1 = 0$. If $\beta_1 \neq 0$ then $\theta_{13} - \theta_{14} + 1 - R = 0$. Together with MM2, we have $\theta_{23} = \theta_{24}$.

Therefore, the dynamics followed by x_t^a and x_{2t} are as follows:

$$\tilde{x}_{t+1}^a = \frac{\theta_{22} - \theta_{13} - R + 1}{R-1} x_{1t}^a + \varepsilon_{1t+1}$$

$$\tilde{x}_{2t+1} = \frac{\theta_{22}}{R-1} x_t^a - \frac{\theta_{22}}{R-1} x_{2t} + \varepsilon_{2t+1}^{'} = \frac{\theta_{22}}{R-1} x_{1t}^a + \varepsilon_{2t+1}^{'}$$

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Notes

1. In order to relate our results to Ohlson (1999), we can label the two components as "core earnings" (x_{1t}) and "transitory earnings" (x_{2t}) in some of the analysis that follows.

2. An alternative analysis, involving the derivation of three sets of sufficient conditions consistent with the other model assumptions and each leading to a unique model solution is available from the authors on request. All the results presented below are unaffected by the use of this more restrictive regularity condition.

3. Equivalently, the clean surplus relation can be expressed as $(1, 1, -1, -1)\Omega = (0, 0, -1, 0)$.

4. Under certain circumstances, including when $\beta_1 = \beta_2$, we need only impose the weaker restriction $\partial(x_{1t} + x_{2t})/\partial d_t = 0$ in place of $\partial x_{1t}/\partial d_t = 0$ and $\partial x_{2t}/\partial d_t = 0$.

5. VAL-2 may seem to imply that $x_{2,t}$ and b_{t-1} can be independently relevant in valuation. However, as noted by Ohlson (1999), CSR allows any one element of $(x_{1t}, x_{2t}, b_t, b_{t-1}, d_t)$ to be inferred from the other four. This redundancy means that one accounting variable in VAL-2, such as b_{t-1}, can be replaced by a combination of the other four elements.

6. Again, ABED-2 may seem to imply that $x_{2,t}$ and b_{t-1} can be independently relevant in forecasting abnormal earnings. But redundancy due to CSR means that any one accounting variable in ABED-2, such as b_{t-1}, can be replaced by a combination of the other four elements.

7. Therefore we could also restate the valuation relation under VI-2 as: $P_t = (\beta_1+\beta_4)x_{1t}+ b_t+\beta_4(b_{t-1}+x_{2t}) = -\beta_1(b_{t-1}+x_{2t})+(\beta_1+\beta_3)b_t+(\beta_1+\beta_4)d_t = (\beta_1+\beta_3)x_{1t}-d_t+\beta_3(b_{t-1}+x_{2t})$

8. In forecasting abnormal earnings, $(b_{t-1} + x_{2t})$ could be introduced to replace any one of the other three accounting items. Therefore we could also restate FI-2 as $E[\tilde{x}^a_{t+s}|Z_t] = E[\tilde{x}^a_{t+s}|x_{1t}, b_t, (b_{t-1} + x_{2t})] = E[\tilde{x}^a_{t+s}|b_t, d_t, (b_{t-1} + x_{2t})] = E[\tilde{x}^a_{t+s}|x_{1t}, d_t, (b_{t-1} + x_{2t})]$.

9. Note that from MM2 the right hand term in Lemma 1 is also equal to $R + \frac{R}{1-R}(\theta_{13} + \theta_{23})$. This result is also obtained in Stark's (1997) Proposition 1 and Lemma 1.

10. Existence conditions for a unique model solution and $\omega_{12} = 0$ imply that $\omega_{13} = 0$. Proposition 1 in Stark (1997) suggests that $\omega_{12} = 0$ is necessary and sufficient for $\beta_1 = \beta_2$. The additional insight here concerning the equivalence of FI-1 and VI-1 is possible because of consideration of the internal consistency of the model.

11. Details of the derivation are available on request.

12. Tests of valuation relevance may be based on the relation between the level of market value and accounting items (levels studies) or the relation between stock returns and accounting items (returns studies). It is possible to derive expressions for returns from our model. We discuss the implications of our analysis for levels studies. Generally, they will also apply to returns studies.

13. See Barth, Beaver, Hand and Landsman (1999) for a recent example of an empirical study based on Ohlson (1999).

14. Consistent with the earlier analysis, this discussion ignores the role of "other information". When VI-1 holds, financial statement information for valuation has one dimension and the modified autoregressive model of abnormal earnings produces the same forecasts of abnormal earnings as more general abnormal earnings dynamics such as ABED-2.

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