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Sustained Competitive Advantage**

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AN EARNINGS-BASED VALUATION MODEL IN THE PRESENCE OF SUSTAINED
COMPETITIVE ADVANTAGE

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An earnings-based valuation model in the presence of sustained competitive advantage

Abstract

In this paper, the process which generates a company's economic value and its accounting numbers is represented in terms of the company's investment in, and utilisation of, competitive advantage. Within this representation, it is shown that a company which earns normal economic returns might plausibly generate perpetual exponential growth in positive net present value projects, in unrecorded goodwill and in residual income. Since exponential growth in residual income may make it impracticable to construct earnings-based valuation models which employ the time-series properties of unscaled residual income (or of unscaled earnings), it is argued that earnings-based valuation models should employ the time-series properties of scaled residual income (or of scaled earnings). A model which incorporates such properties is then derived. In a certainty setting in which there are no shocks to the economic return series, economic value is a function of normal profitability and of normal book value growth; in a setting in which shocks to the economic return series occur, it is necessary to add a term which reflects transitory abnormal profitability and a term which reflects transitory abnormal book value growth. The importance of the abnormal profitability term is determined by persistence in abnormal profitability; the importance of the abnormal book value growth term is determined by the normal market-to-book ratio.

1. Introduction

This paper represents the joint evolution of a company's economic value and accounting numbers in terms of the company's investment in, and utilisation of, competitive advantage. In such a setting, the company might plausibly experience persistent exponential growth in positive net present value (NPV) projects, in unrecorded goodwill and in residual income. Since such growth in residual income may make it impracticable to construct earnings-based valuation models which employ the time-series properties of unscaled residual income (or of unscaled earnings), it is argued that earnings-based valuation models should employ the time-series properties of scaled residual income (or of scaled earnings). A model which incorporates such properties is then derived.

It is well known that economic value can be expressed as the sum of accounting book value and the present values of all expected future residual incomes (Edwards and Bell, 1961; Edey, 1962; Peasnell, 1982): the residual income component of the expression represents unrecorded goodwill. Ohlson (1995) provided an important impetus to the modelling of the links between economic value and accounting numbers by developing this residual income-based valuation relationship. Making the assumptions that accounting book value is an unbiased estimator of economic value and that residual income is generated by a zero-mean stationary time-series process, he derived a model in which economic value is partly expressed as a weighted average of (i) book value and (ii) an ex-div earnings multiple. The model provided an important illustration of how a theoretically supported accounting-based valuation model could incorporate knowledge of the time-series properties of earnings. The unrealistic assumption of unbiased accounting that was made in Ohlson (1995) was relaxed in Feltham and Ohlson (1995) and in Feltham and Ohlson (1996). Feltham and Ohlson (1996) allowed persistent unrecorded goodwill to arise from two sources which accountants might recognise intuitively: (i) depreciation errors and (ii) the existence of a persistently growing stream of positive NPV projects, of which the full value is not immediately captured by the balance sheet. Accounting-based valuation models, such as that derived in Feltham and Ohlson (1996), which allow for the observable phenomenon of persistent exponential growth in unrecorded goodwill to result from the observable phenomenon of

persistent streams of positive NPV projects, are attractive as potential bases for the practical task of valuation and for the empirical research designs of market-based accounting researchers. However, there is an apparent problem in modelling persistent growth in unrecorded goodwill in terms of persistent growth in positive NPV projects: it is not clear how it can be expected that a persistently growing stream of positive NPV projects will be generated in a competitive environment. Rappaport (1986) made the point that competition should eventually eliminate the availability of positive NPV projects. This point was echoed in studies by Bernard (1993) and by Ou and Penman (1993) which suggested that competition is likely to eliminate positive NPV projects within a finite horizon and that, therefore, one of the main potential causes of unrecorded goodwill and of positive residual income is likely to be eliminated within such a horizon.

This paper addresses this issue by demonstrating that, even in a setting in which a company does not earn abnormal economic returns, it is plausible that the company might be generating persistently growing streams of positive NPV projects, of unrecorded goodwill and of residual income. This demonstration rests upon a representation of the economic value and accounting numbers of a going concern company as outputs of a process in which the company continually invests in, and utilises, competitive advantage. The accounting depreciation is 'correct' in the sense that the rate matches the rate of decline in project cash flows but it is 'wrong' in the sense that it ignores the value of the competitive advantage that is embedded within projects. This representation provides indications as to the impact of the acquisition and utilisation of competitive advantage upon the normal level of profitability and upon the normal level of the market-to-book ratio. This representation is then developed in order to derive an earnings-based valuation model which incorporates the time-series properties of scaled earnings that might be observed in the setting described. Initially, the derivation is effected in a certainty setting: the resultant model is similar to one which has appeared in the shareholder value analysis literature, in which economic value is expressed in terms of normal profitability and the normal book value growth rate. Subsequently, shocks to the economic return series are incorporated: in the resultant model, economic value is expressed in terms of normal profitability,

the normal book value growth rate, transitory abnormal profitability and the transitory abnormal book value growth rate.

The model, together with the underlying analysis, has a number of interesting features. First, it expresses the central roles of the prediction of profitability and of the prediction of growth in the task of fundamental analysis.¹ Second, it suggests a focus for literature that is concerned with the finite-horizon properties of accounting numbers: it suggests that abnormal profitability and the abnormal book value growth rate might plausibly be expected to approximate to zero within a finite horizon. Third, although the analysis in this paper does not employ the technology of options theory, the representation developed here provides some insight into the impact of real options (Dixit and Pindyck, 1994; Trigeorgis, 1996) on items, such as profitability and the market-to-book ratio, which are of interest to financial statement analysts. Fourth, the analysis gives indications of the effect of the acquisition and utilisation of competitive advantage in determining the normal level of profitability. This is relevant for those concerned with the use of managerial performance measures based on variants of residual income such as Economic Profit (Copeland, Koller and Murrin, 1995; McTaggart, Kontes and Mankins, 1994) and Economic Value Added (EVA[®])² (Stern, Stewart and Chew, 1995).

The remainder of this paper is organised as follows: section 2 shows that, within a setting in which the company continually invests in competitive advantage, it is possible to accommodate a persistent stream of positive NPV projects, with consequent exponential growth in unrecorded goodwill and in residual income, without requiring that the company as a whole should earn abnormal economic returns; section 3 derives an earnings-based valuation model which allows for such growth in unrecorded goodwill and in residual income; section 4 concludes the paper.

2. Positive net present value projects, unrecorded goodwill, accounting rate of return and residual income where payoffs from projects arise partly in the form of competitive advantage

¹ The importance attached by analysts to the prediction of profitability and growth is evidenced by an account of the attitudes of successful analysts which was published by the London *Sunday Times* on 1 June 1997 (page 3 of the 'Money' section).

² EVA[®] is a service mark of Stern Stewart & Co. in the United States, the United Kingdom, and other countries of the world.

The analysis in this section of the paper is carried out in a no-arbitrage certainty setting in which the company as a whole earns normal economic returns. Unrecorded goodwill, the accounting rate of return (ARR) and residual income are represented as outputs of a process in which the going concern company continually makes investments in competitive advantage which is embedded within projects. The company makes an initial investment in a zero NPV first generation project of which the payoffs accrue partly in the form of direct cash receipts and partly in the form of competitive advantage. This competitive advantage is realised as positive NPV second generation projects. Investment in these second generation projects itself results in subsequent payoffs which accrue partly as direct cash receipts and partly as positive NPV third generation projects. This process continues indefinitely, with the company continually 'cashing in' previously acquired competitive advantage through investment in positive NPV projects which themselves deliver competitive advantage which will bring about subsequent positive NPV projects. The accounting system depreciates the competitive advantage along with the projects in which it is embedded.

It is shown that, within the framework represented here, persistent exponential growth in positive NPV investments, in unrecorded goodwill and in residual income might plausibly arise even in the absence of abnormal economic returns for the company as a whole. It is also shown that the asymptotic levels of ARR and of the market-to-book ratio are functions of parameters which capture (i) the proportion of total project value that is represented by investment in competitive advantage and (ii) the proportion of the cost of new projects that is represented by the 'cashing in' of previously acquired competitive advantage. The following assumptions are made regarding the analysis in this section:

- i. At incorporation (time 0), the company makes an initial issue of equity capital which is wholly invested in the company's first generation project. The amount of equity capital raised is the opening book value of the company (y_0). Abnormal economic returns for the company are precluded from this part of the analysis, so the first generation project has zero NPV: the economic value of the company at time 0 (P_0) is equal to y_0 . (If the first generation project is allowed to have a non-zero NPV, this does not change the asymptotic results derived below.)

- ii. The periodic payoffs from the first generation zero NPV project each accrue partly in the form of cash receipts directly attributable to that project and partly in the form of positive NPV second generation projects which result from competitive advantage acquired through investment in that first generation project. These second generation projects partly consist of an investment in competitive advantage. The payoffs from the second generation projects therefore also accrue partly in the form of cash receipts and partly in the form of third generation positive NPV projects: the process continues indefinitely. This process is characteristic of a company which is in the going concern phase of its existence. That part of the total period t payoff of the company which accrues in the form of cash at period t is denoted by C_t ; that part of the period t payoff which accrues in the form of positive NPV projects at period t is denoted by N_t . The proportion of total payoffs which accrues in the form of cash receipts is denoted by F . F is assumed to be constant across all projects and across time. Some, but not all, of project payoffs are assumed to arise as cash receipts: therefore, $0 < F < 1$. For all t ,

$$F = C_t / (C_t + N_t). \quad (1)$$

The proportion of total project payoffs which accrues as positive NPV projects arising from earlier investment in competitive advantage is $(1-F)$. Thus, $(1-F)$ represents the proportion of the value of projects which is made up of the investment in competitive advantage.

- iii. The total cash cost of buying into all of the second and subsequent generation projects that arise at period t is denoted by I_t . The ratio of cash cost to present value of such new projects is denoted by H . As with F , H is assumed to be constant across all projects and across time. All second and subsequent generation projects are assumed to be positive NPV projects with an initial cash cost of greater than 0: therefore, $0 < H < 1$. $(1-H)$ represents the ratio of NPV to present value of second generation and subsequent projects. $H/(1-H)$ therefore represents the ratio of cash cost to NPV of such projects. Consequently for all t :

$$I_t = N_t \left(\frac{H}{1-H} \right). \quad (2)$$

Where the company as a whole is earning normal economic returns, $(1-H)$ represents the proportion of the total value of second generation and subsequent projects that is contributed by the utilisation of competitive advantage embedded within earlier projects.

- iv. Each project generates a stream of total periodic payoffs (i.e. C plus N) which accrues in the form of a declining perpetuity, where the rate of decline is B per period. $0 < B \leq 1$. The use of this assumed payoff pattern is convenient because it produces a relatively simple analysis. However, there is empirical evidence to support the use of such a pattern. As will become apparent later, this declining payoff pattern gives rise to a profitability persistence parameter in the form of the autoregressive coefficient of an autoregressive process of order 1 (AR(1) process): empirical evidence in O'Hanlon (1996) suggests that, of various standard time-series generating processes, AR(1) is the one which best characterises ARR series in the U.K.
- v. There are no financial assets, financial liabilities or working capital at any period end. The dividend for period t , denoted by d_t , is the excess of the cash receipts from projects for that period over the cash cost of investment in new projects for that period:

$$d_t = C_t - I_t . \tag{3}$$

- vi. Each project is recorded in the balance sheet at its historic cash cost less accumulated accounting depreciation. The accounting depreciation rate for all projects is determined by the rate of decline in the projects' cash payoff stream. The rate is therefore B per period on a declining balance basis. The depreciation is 'correct' in that the rate matches the rate of decline in project cash flows but is 'wrong' in that it ignores the value of the competitive advantage embedded within projects:³ the investment in this competitive advantage is depreciated as part of the projects within which it is embedded, rather than being capitalised in anticipation of the subsequent positive NPV projects to which it will give rise. This non-recognition ('over-depreciation') by the accounting system of the valuable competitive advantage gives rise to a market-to-book premium.

³ A further complication could be introduced by allowing the depreciation rate to differ from the rate of decline in cash flows: this complication is not introduced here.

vii. Accounting obeys the clean surplus relationship:

$$y_t = y_{t-1} + x_t - d_t , \quad (4)$$

where y_t (y_{t-1}) is the accounting book value at period end t ($t-1$) and x_t is accounting earnings for period t , where $x_t = C_t - y_{t-1}B$.

Before proceeding with the analysis, two points are made. First, given the no-arbitrage certainty setting, the dollar return is as follows for all t :

$$(R - 1)P_{t-1} = P_t - P_{t-1} + d_t = P_t - P_{t-1} + C_t - I_t , \quad (5)$$

where R is one plus the cost of equity, which is assumed to be constant, and P_t (P_{t-1}) is the economic value of equity capital at period-end t ($t-1$). P_t can be written as the present value at period-end t of the projects which made up P_{t-1} plus the present value of new projects arising at period t :

$$P_t = P_{t-1} (1 - B) + N_t + I_t . \quad (6)$$

Combining (5) and (6), the dollar return for period t can be written as the period t total payoff (C_t plus N_t) less the decline during period t in the present value of the projects which made up P_{t-1} :

$$(R - 1)P_{t-1} = N_t + C_t - P_{t-1}B . \quad (7)$$

Second, it is important to comment on the term ‘positive NPV’ which is used above and in the following analysis. If the definition of ‘cost’ is amplified to include the value of the previously acquired competitive advantage that is being ‘cashed in’ to provide positive NPV projects, these projects are not positive NPV at all: they just appear to be positive NPV because standard capital budgeting procedures ignore the value of the competitive advantage that is being utilised. The impression that these projects have positive NPV is reinforced by the accounting depreciation procedure which ‘over-depreciates’ the investment in competitive advantage, with the consequence that the book value of second and subsequent generation projects is less than their present value. In this setting, the standard capital budgeting procedures and the standard accounting depreciation procedures conspire together to make projects look as though their present value exceeds their cost. Such a process is likely to be present in the return-earnings generating process of any going concern business and is at the heart of the

subsequent exposition. For the sake of terminological convenience, and to be consistent with other studies which allow for such processes (e.g. Feltham and Ohlson (1996)), I will continue to use the term ‘positive NPV’ in the exposition.

In order to facilitate understanding of the setting described above, Table 1 presents a numerical example of the evolution of economic value and accounting numbers in that setting. In this example, $R = 1.20$, $F = 0.70$, $H = 0.667$ and $B =$ of 1.00 . Making B equal to 1.00 , gives a particularly simple example. Table 1 contains the following notation not previously defined in the text: $A_t (= x_t/y_{t-1})$ denotes the ARR for period t , $x_t^a (= x_t - (R-1)y_{t-1})$ denotes residual income for period t and χ_t^a denotes the rate of residual income for period t ($= x_t^a / y_{t-1}$).

(INSERT TABLE 1 ABOUT HERE)

The subsequent analysis within this section is presented in the form of three propositions. Proposition 1 suggests that, in the setting described above, economic value and investment in positive NPV projects might plausibly grow exponentially. It is shown that the rate of growth is a function of R , F , B and H . Proposition 2 states that the asymptotic rate of growth in book value and in unrecorded goodwill is the same as the rate of growth in economic value defined in Proposition 1. It is shown that the asymptotic market-to-book ratio is the reciprocal of H . Finally, Proposition 3 states that unscaled residual income grows exponentially at the rate of growth in book value defined in Proposition 2. On the basis of these three propositions, it is argued that earnings-based valuation models incorporating time-series properties of earnings should reflect the exponential growth property of accounting numbers that is likely to be observed in the presence of continual investment in competitive advantage.

Proposition 1: In the absence of abnormal economic returns, the rate of growth in economic value and in new investment in positive NPV projects is a weighted average of (R-1) and (-B), where the weights reflect the relative proximity of F to 1 and H respectively. If dividends are positive, positive exponential growth in economic value and in new investment in positive NPV projects occurs if (R-1)(1-F)/(F-H) exceeds B. Such growth does not depend upon the presence of abnormal economic returns.

Proof. From (2) and (3), the dividend yield at period t (= d_t/P_{t-1}), denoted by D_t , is

$$D_t = \frac{C_t - N_t(H / (1 - H))}{P_{t-1}}. \quad (8)$$

Substitution of (1) into (7) gives

$$\begin{aligned} C_t &= P_{t-1} (R - 1 + B) F \\ N_t &= P_{t-1} (R - 1 + B) (1 - F). \end{aligned} \quad (9)$$

Substitution of (9) into (8) gives

$$\begin{aligned} D_t &= \frac{P_{t-1} (R - 1 + B) F - P_{t-1} (R - 1 + B) (1 - F) (H / (1 - H))}{P_{t-1}} \\ &= (R - 1 + B) \left(\frac{F - H}{1 - H} \right). \end{aligned} \quad (10)$$

R, B, F and H are constant, so D_t (hereafter D) is constant. (Note here that positive dividends require $F > H$.) The rate of growth in economic value, denoted by \bar{g} , is constant at

$$\bar{g} = (R - 1) - D = (R - 1) \left(\frac{1 - F}{1 - H} \right) - B \left(\frac{F - H}{1 - H} \right). \quad (11)$$

From (11), \bar{g} is a weighted average of (R-1) and (-B), where the weights reflect the relative proximity of F to 1 and H respectively.⁴ If dividends are positive (i.e. if $F > H$), positive exponential growth in P occurs if $(R-1)(1-F)/(F-H) > B$. From (9), N_t is proportional to P_{t-1} . Therefore, such growth in N also occurs if $(R-1)(1-F)/(F-H) > B$. Positive exponential growth in P and in N does not depend upon the existence of abnormal economic returns. ■

⁴ Note that as F approaches 1, little of the payoff from projects accrues as competitive advantage, retentions approach zero and \bar{g} approaches -B; as F falls towards H, the proportion of payoffs that accrues in the form of cash falls towards the proportion of new project value that is made up of cash cost: payout approaches zero and \bar{g} approaches (R-1).

Proposition 2: The asymptotic rate of growth in book value and in unrecorded goodwill is equal to the rate of growth in economic value, \bar{g} .

Proof: Subtraction of accounting depreciation ($y_{t-1}B$) from the expression for cash payoffs for period t gives accounting earnings for period t , as

$$x_t = C_t - y_{t-1}B = P_{t-1}(R-1+B)F - y_{t-1}B. \quad (12)$$

The period t earnings which is retained, denoted by x_t^r , is

$$x_t^r = P_{t-1}((R-1+B)F - D) - y_{t-1}B.$$

Using this expression for x_t^r , book value evolves as follows (recall that $P_0 = y_0$):

$$\begin{aligned} y_1 &= P_0((1-B) + (R-1+B)F - D) \\ y_2 &= P_0((1-B)^2 + ((R-1+B)F - D)(1-B) + ((R-D)(R-1+B)F - D)) \\ y_t &= P_0 \left((1-B)^t + \frac{((R-1+B)F - D)((R-D)^t - (1-B)^t)}{(R-D) - (1-B)} \right). \end{aligned}$$

Since $(1-B) < 1$, as $t \rightarrow \infty$

$$y_t \rightarrow P_0 \left(\frac{((R-1+B)F - D)(R-D)^t}{(R-D) - (1-B)} \right). \quad (13)$$

The constant rate to which the book value growth rate asymptotes is therefore $\bar{g} = (R-D-1)$.

Since P_t is equal to $P_0(R-D)^t$ for all t , the asymptotic market-to-book ratio, denoted by \bar{M} , is

$$\bar{M} = \frac{(R-D) - (1-B)}{(R-1+B)F - D},$$

which by substitution of (10) is

$$\bar{M} = \frac{(R-1+B) - (R-1+B)(F-H)/(1-H)}{(R-1+B)F - (R-1+B)(F-H)/(1-H)} = \frac{1}{H}. \quad (14)$$

The market-to-book ratio therefore asymptotes to the constant value of $1/H$. At the asymptote, for $H < 1$, unrecorded goodwill ($= P_t - y_t = (\bar{M} - 1)y_{t-1}$) is positive and grows exponentially at the rate of \bar{g} per period. ■

Proposition 3: For $(R-1) > 0$ and $B > 0$ and $F \neq H$, the asymptotic rate of residual income differs from zero and the sign and magnitude of the difference depend upon the sign and magnitude of the difference between F and H . The asymptotic rate of growth in unscaled residual income is equal to the rate of growth in book value, \bar{g} .

Proof: Using (12), ARR, for period t , denoted by A_t , is

$$A_t = \frac{x_t}{y_{t-1}} = \frac{P_{t-1}(R-1+B)F - y_{t-1}B}{y_{t-1}}.$$

As $t \rightarrow \infty$, since the market-to-book ratio $\rightarrow 1/H$,

$$\begin{aligned} A_t &\rightarrow \frac{P_{t-1}(R-1+B)F - P_{t-1}HB}{P_{t-1}H} = \frac{F}{H}(R-1+B) - B \\ &= \frac{F}{H}(R-1) + B\left(\frac{F-H}{H}\right) = \bar{A}, \end{aligned} \quad (15)$$

where \bar{A} is the asymptotic ARR. Residual income for period t , denoted by x_t^a , is

$$x_t^a = x_t - (R-1)y_{t-1}. \quad (16)$$

The rate of residual income (i.e. ARR less the cost of equity) is denoted by χ_t^a , and is defined as:

$$\chi_t^a = \frac{x_t^a}{y_{t-1}} = \frac{x_t - (R-1)y_{t-1}}{y_{t-1}} = A_t - (R-1). \quad (17)$$

The asymptotic rate of residual income, denoted by $\bar{\chi}^a$, is

$$\begin{aligned} \bar{\chi}^a &= \bar{A} - (R-1) = \frac{F}{H}(R-1) + B\left(\frac{F-H}{H}\right) - (R-1) \\ &= (R-1+B)\left(\frac{F-H}{H}\right). \end{aligned} \quad (18)$$

For $(R-1) > 0$ and $B > 0$ and $F \neq H$, $\bar{\chi}^a$ differs from zero and the sign and magnitude of the difference depend upon the size and magnitude of the difference between F and H . (Note that, if dividends are positive, $F > H$ and $\bar{\chi}^a > 0$.) At the asymptote, unscaled residual income ($x_t^a (= \bar{\chi}^a y_{t-1})$) grows exponentially at the rate of growth in book value, \bar{g} . ■

The propositions that have been developed in this section suggest how residual income might be expected to evolve through time in the case of a going concern company which is continually investing in competitive advantage. Even in a situation in which the company as a whole is earning normal economic returns only, one might expect to observe exponential growth in economic value, in positive NPV projects, in book value, in unrecorded goodwill and in residual income. The process by which investment in one project generates competitive advantage that can be exploited through investment in subsequent projects is a fundamental determinant of the way in which going concern companies evolve. (“Learning by doing” is an example of a phenomenon which results in investment in competitive advantage being bundled up with other outlays and expensed in the going concern company.) This process is therefore likely to be a fundamental determinant of the joint evolution of economic value, book value and accounting earnings. Attempts to analyse the joint evolution of economic value, book value and accounting earnings might therefore benefit from an accounting-based valuation model that explicitly allows for the behaviour of accounting numbers that is likely to be observed in the presence of continual investment in competitive advantage. The following section develops an earnings-based valuation model which allows for the exponential growth in residual income that might be observed in the presence of such sustained competitive advantage.

Before proceeding further with the analysis, it is instructive to dwell briefly on some of the implications of expressions (15) and (18). These expressions suggest that, in the presence of sustained competitive advantage, where accounting depreciation does not separately recognise investment in that competitive advantage, normal profitability and ‘normal excess profitability’ are a function of the investment in, and utilisation of, competitive advantage: for $F > H$, the normal excess of profitability over the cost of equity is a positive function of R , B and F and is a negative function of H . This result has implications in two related contexts. First, although the analysis in this paper does not employ the technology of options theory, the acquisition and utilisation of competitive advantage is essentially about the acquisition and utilisation of real options. The representation that is developed here provides an indication of how the acquisition and utilisation of real options (Dixit and Pindyck, 1994; Trigeorgis,

1996) might impact on items, such as profitability and the market-to-book ratio, which are of interest in financial statement analysis. Second, the analysis is relevant for those concerned with the implementation of managerial performance measurement systems based on variants of residual income. Such variants include Economic Profit (Copeland, Koller and Murrin, 1995; McTaggart, Kontes and Mankins, 1994) and Economic Value Added (EVA[®]) (Stern, Stewart and Chew, 1995). Economic Profit and EVA[®] are variants of residual income in which certain ‘accounting distortions’ are corrected: the intention is that the resulting adjusted residual income measure can be used such that positive (negative) residual income indicates superior (inferior) economic performance. As can be seen from expressions (15) and (18), the interaction of F and H can produce a normal rate of residual income of greater than zero. To the extent that the accounting corrections do not deal with the phenomenon represented here, positive residual income may not necessarily be an indicator of superior economic performance: it may be incorrect to regard a positive or negative value for residual income as an indication of positive or negative abnormal performance.⁵

3. An earnings-based valuation model where payoffs from projects accrue partly in the form of competitive advantage

Section 2 shows that unscaled residual income might plausibly grow exponentially at the book value growth rate. Standard autoregressive, integrated moving average (ARIMA) time-series modelling techniques involve the transformation through differencing of a series to a stationary series and then the estimation of the autoregressive and moving average parameters for the appropriately differenced series. If the series exhibits exponential growth, differencing may fail to achieve stationarity. Attempts to use ARIMA-based approaches for the estimation of residual income persistence (or, for similar reasons, for the estimation of earnings persistence) on the basis of unscaled data may fail or may result in the identification of generating processes with unnecessarily high orders of differencing. For reasons related

⁵ The need to set benchmarks of other than zero for residual income and its variants is recognised by Stern Stewart & Co., who market the EVA[®] variant of residual income. In setting EVA[®] benchmarks, the difference between market value and (adjusted) book value is sometimes used as a basis for imputing expectations of future (non-zero) EVA[®]. I am grateful to Joel Stern for this information.

to those outlined in Section 2, stock price changes are also likely to exhibit exponential growth. However, whilst the Finance literature has long acknowledged that work involving the time-series properties of stock price changes should focus on scaled changes, it is not widely acknowledged that work on the time-series properties of accounting flows should focus on the time-series properties of the scaled flows. On the basis of the analysis in section 2, it is argued that theoretical or empirical earnings-based valuation models which incorporate the time-series properties of residual income should focus on scaled residual income and that, since the driver of growth in residual income is book value, the appropriate scaling item is book value. The remainder of this section is concerned with the construction of a model based on residual income scaled by book value.

Peasnell (1982) showed that, if accounting obeys the clean surplus relationship, the dividend capitalisation model,

$$P_t = \sum_{\tau=1}^{\infty} E_t (d_{t+\tau}) R^{-\tau}, \quad (19)$$

where $E(\cdot)$ is an expectations operator, can be re-written as

$$P_t = y_t + \sum_{\tau=1}^{\infty} E_t (x_{t+\tau}^a) R^{-\tau}. \quad (20)$$

Appendix 1 shows that, in a certainty setting where there have been no shocks to the economic rate of return series and where x^a and y will grow at the constant rate of $(R-D-1) = \bar{g}$, (20) can be re-written as

$$P_t = y_{t-1} \left(\frac{\bar{A} - \bar{g}}{\bar{\gamma} - 1} \right), \quad (M.1)$$

where $\bar{\gamma} = R / (1 + \bar{g})$. This is a no-shock accounting-based valuation model, containing a normal ARR term and a normal book value growth rate term. (M.1) can also be written as

$$P_t = y_t \left(\frac{\bar{A} - \bar{g}}{(R-1) - \bar{g}} \right),$$

which is described as the ‘equity spread’ model in a shareholder value analysis text by McTaggart, Kontes and Mankins (1994).

The effect of past and current shocks to the economic rate of return series is now overlaid upon the no-shock accounting-based valuation model, (M.1). (M.1) is written in terms of normal profitability and the normal book value growth rate: allowance for shocks requires the addition of terms which deal with transitory abnormal profitability and the transitory abnormal book value growth rate. In permitting shocks to the economic rate of return series, the analysis is moving from a certainty setting to an uncertainty setting. In such a setting, the values of the future payoffs (C plus N) from projects are expected values rather than certain values. The investment of I_t at period t becomes an investment in the cash payoffs from a group of projects plus an investment in a basket of options embedded within that group of projects. The present value at period-end t of the expected cash payoffs from that group of projects and of the expected expiration values of the embedded options, taken together, is I_t / H (i.e. the cash cost of the projects times the ratio of present value to cost).

In developing (M.1) to allow for present and past shocks, it is assumed that the company has reached its asymptotic values of ARR (\bar{A}), market-to-book ratio (\bar{M}) and book value growth rate (\bar{g}) and has previously encountered no shocks to its economic return series. It then experiences a shock to its economic return series which represents the effect of a revision concerning the total payoffs (i.e. C plus N) expected to accrue from the company's existing projects. The changes to the expected payoffs which represent the shock are capitalised as a declining perpetuity. Such a shock causes the economic rate of return to diverge from $(R-1)$ in the period in which the shock arises. Market efficiency is assumed: consequently, the economic rate of return is expected to be $(R-1)$ in all periods subsequent to the period in which the shock arises. The parameters, F , B and H , are assumed to remain constant in the aftermath of the shock, as is the cost of equity, $(R-1)$. Furthermore, it is assumed that the accounting depreciation policy does not change in response to the shock: consequently, it takes time for the impact of the economic income shock to be captured by the accounting system and there results time-series dependence (persistence) in the ARR series. Accounting continues to obey the clean surplus relationship. In Appendix 2 it is shown that this set of circumstances gives rise to the following accounting-based valuation model:

$$P_t = y_{t-1} \left(\frac{\bar{A} - \bar{g}}{\bar{\gamma} - 1} + (A_t - \bar{A}) \frac{\bar{\omega}}{\bar{\gamma} - \bar{\omega}} + (g_t - \bar{g}) \frac{\bar{\gamma} \bar{M} - \bar{\omega}}{\bar{\gamma} - \bar{\omega}} \right), \quad (\text{M.2})$$

where $(A_t - \bar{A})$ is the transitory abnormal ARR, \bar{g} is now defined to be the normal book value growth rate, $(g_t - \bar{g})$ is the transitory abnormal book value growth rate and $\bar{\omega}$ is the asymptotic persistence parameter for abnormal ARR, for the abnormal rate of residual income and for the abnormal book value growth rate. As stated in Appendix 2, the persistence parameter, $\bar{\omega}$, takes the form of the autoregressive coefficient that would be estimated in an autoregressive model of order 1 (AR(1) model) for each of these items. The definition of the persistence parameter in terms of an AR(1) coefficient results from the underlying assumption that payoffs take the form of a declining perpetuity.⁶ (M.2) is consistent with a version of equation (3.a) from Feltham and Ohlson (1996) in which cash flow decline is set equal to the reducing balance depreciation rate.⁷

According to (M.2), the value of equity is obtained by adjusting the steady state model (M.1), which contains a normal ARR term and a normal book value growth rate term, by a term reflecting temporary divergence from normal ARR and by a term reflecting temporary divergence from the normal book value growth rate. The multiplier on the abnormal ARR term comprises the growth adjusted discount factor and the asymptotic ARR persistence parameter, $\bar{\omega}$: as $\bar{\omega}$ approaches zero, the multiplier on abnormal ARR approaches zero. The multiplier on the abnormal book value growth rate term also includes these two terms but the key component of this multiplier is the asymptotic market-to-book ratio \bar{M} ($=1/H$, where $H < 1$). As H ($=$ the ratio of cost to present value of new projects) falls, the multiplier on the abnormal book value growth rate diverges positively from one.

(M.2) does not include an ‘other information’ term, such as that included in the Ohlson (1995) analysis, which captures the effect on the economic value of the company of that information which has not yet impacted on accounting numbers and which is not captured by persistence multipliers applied to

⁶ As was mentioned earlier, empirical evidence in O’Hanlon (1996) suggests that, of various standard time series processes, AR(1) is the one which best characterises ARR series in the U.K..

⁷ The lengthy but straightforward demonstration of the consistency is not reproduced here but is available on request from the author.

current accounting numbers. In an empirical setting, this consideration may be important but no attempt is made here to include such a term. Such a term would be compound function of the stochastic properties of accounting numbers and of the stochastic properties of ‘other information’:⁸ it is not clear that the precise form of such a term would add significantly to the insights afforded by this analysis and, consequently, it is omitted.

As well as suggesting how the time-series properties of earnings might be incorporated into earnings-based valuation models in the setting described, (M.2) has a number of additional interesting features. First, it expresses the central roles of the prediction of profitability and of the prediction of growth in the task of fundamental analysis. Second, it suggests a focus for literature concerned with the finite-horizon properties of accounting numbers. Examples of such literature include Bernard (1993) and Ou and Penman (1993). Under the conditions described in this paper, where persistent streams of positive NPV projects are generated, it is plausible to expect that unscaled residual income will not approximate to zero within a finite horizon. However, where the normal characteristics of the company's projects (denoted here by R, B, F, H) do not change, it is plausible to expect that the abnormal ARR and the abnormal book value growth rate will approximate to zero over such a horizon. Therefore, the development of models based on finite horizon properties of accounting might usefully focus on abnormal ARR and on the abnormal book value growth rate. Furthermore, the underlying analysis provides an indication of the impact of the acquisition and utilisation of competitive advantage on normal profitability levels. This is of potential relevance both for those concerned with the impact of real options on financial statement items and for those concerned with the use of residual income, and its variants, as measures of managerial performance.

4. Conclusion

This paper represents the joint evolution of economic value, accounting book value and accounting earnings as the product of a process in which investments by companies include investment in competitive advantage which gives rise to future positive NPV projects. The depreciation is ‘correct’ in

⁸ See Ohlson (1995) for an example of the form of such an ‘other information’ term.

the sense that the rate matches the rate of decline in project cash flows but is ‘wrong’ in the sense that it ignores the value of the competitive advantage that is embedded within projects. This process may give rise to perpetual exponential growth in the generation of positive NPV projects, in unrecorded goodwill and in residual income, even in the absence of abnormal economic returns for the company as a whole. Since such growth would make it difficult to exploit the time-series properties of unscaled residual income (or of unscaled earnings) in earnings-based valuation models, it is argued that earnings-based valuation models should employ the time-series properties of residual income scaled by book value (or of total earnings scaled by book value). The paper then derives such a valuation model. The model incorporates normal profitability, transitory abnormal profitability, the normal book value growth rate and the transitory abnormal book value growth rate. The multiplier on abnormal profitability is driven by persistence in abnormal profitability; the multiplier on the abnormal book value growth rate is driven by the normal market-to-book ratio. The model expresses the central roles of the prediction of profitability and of the prediction of growth in the task of fundamental analysis and suggests a focus for literature that is concerned with the finite-horizon properties of accounting numbers. Furthermore, the underlying analysis provides an indication of the impact of the acquisition and utilisation of competitive advantage on normal profitability levels and on normal market-to-book levels.

The analysis in this paper contains a number of simplifying features which indicate directions in which this type of analysis might be developed both empirically and theoretically. First, it is assumed that the required return and payoff characteristics (R , F , B and H) of all projects are constant both across projects and through time: allowance for cross-sectional and time-series variation in these parameters might prove to be important in an empirical setting. Second, it is assumed that the payoffs from projects take the form of a declining perpetuity. Although there is some empirical justification for this simplifying assumption, it may not hold in all settings: different payoff patterns would give rise to different multipliers on the terms in the model. Third, it could be argued that the model presented in this paper should be augmented by an ‘other information’ term, such as that included in the Ohlson (1995) analysis. In an empirical setting, this consideration may be important but no attempt is made here to

include such a term. Fourth, although this paper affords some insights as to the impact of real options on items such as profitability and the market-to-book ratio, it does not employ the technology of options theory to represent the process: there is much scope for the exploration of the impact of real options on the properties of financial statement items which are used for valuation purposes.

Appendix 1: Derivation of (M.1)

If accounting obeys the clean surplus relationship,

$$P_t = y_t + \sum_{\tau=1}^{\infty} E_t(x_{t+\tau}^a) R^{-\tau}, \quad (20)$$

where $E_t(\cdot)$ denotes expectations at time t and the remaining terms are as defined earlier. In a certainty setting where there have been no shocks to the economic rate of return series and where x^a and y will grow at the constant rate of $(R-D-1) = \bar{g}$, (20) can be re-written as

$$P_t = y_t + \sum_{\tau=1}^{\infty} x_t^a \left(\frac{R}{1+\bar{g}} \right)^{-\tau}. \quad (A1.1)$$

The growth deflated discount factor, $\bar{\gamma}$, is now defined as $\bar{\gamma} = R / (1 + \bar{g})$. From (17), $x_t^a = \chi_t^a y_{t-1}$.

Substitution of these two expressions into (A1.1) gives

$$P_t = y_t + \sum_{\tau=1}^{\infty} \chi_t^a y_{t-1} \bar{\gamma}^{-\tau}. \quad (A1.2)$$

From (18), the asymptotic rate of residual income is defined as $\bar{\chi}^a = \bar{A} - (R - 1)$. At the asymptote, in a certainty setting,

$$\frac{x_t^a}{y_{t-1}} = \chi_t^a = \bar{\chi}^a$$

for all t . In such a setting, (A1.2) can be re-written as

$$P_t = y_t + \sum_{\tau=1}^{\infty} \bar{\chi}^a y_{t-1} \bar{\gamma}^{-\tau} = y_t + \left(\frac{\bar{\chi}^a}{\bar{\gamma} - 1} \right) y_{t-1}. \quad (A1.3)$$

Substitution into (A1.3) of $y_t = y_{t-1} (1 + \bar{g}) = y_{t-1} \left(\frac{R - 1 - \bar{g}}{\bar{\gamma} - 1} \right)$ and of $\bar{\chi}^a = \bar{A} - (R - 1)$ gives:

$$P_t = y_{t-1} \left(\frac{\bar{A} - \bar{g}}{\bar{\gamma} - 1} \right). \quad (M.1)$$

Appendix 2: Derivation of (M.2)

(M.1) is a no-shock accounting-based valuation model, containing an asymptotic ARR term and an asymptotic book value growth rate term:

$$P_t = y_{t-1} \left(\frac{\bar{A} - \bar{g}}{\bar{\gamma} - 1} \right). \quad (\text{M.1})$$

The further development of (M.1) is effected through the analysis of the impact of a single shock to the economic return series at period s , where the shock generates an abnormal economic return of Z in period s . The dollar value of the shock is ZP_{s-1} . The impact on (unscaled) earnings in period s of the shock can be written as follows:

$$(A_s - \bar{A}) y_{s-1} = ZP_{s-1} \left(\frac{1}{1 + \frac{1-B}{R-1+B}} \right) F = ZP_{s-1} \left(\frac{R-1+B}{R} \right) F, \quad (\text{A2.1})$$

where A_s is the ARR for period s and \bar{A} , y , Z , P , B , R and F are as defined earlier. The term in brackets is due to the declining perpetuity nature of the expected payoffs which are capitalised into the shock. This term needs to be multiplied by F because only the proportion F of the period s payoffs accrues as cash flows in period s . The impact of the shock on the (unscaled) change in book value in period s is

$$(g_s - \bar{g}) y_{s-1} = ZP_{s-1} \left(\frac{R-1+B}{R} \right) \left(\frac{(1-F)H}{1-H} \right), \quad (\text{A2.2})$$

where g_s is the book value growth rate for period s and \bar{g} is now defined to be the normal book value growth rate. In (A2.2), $(1-F)$ is the proportion of the time s payoff which arises in the form of positive NPV projects. $H/(1-H)$ is the ratio of cost to NPV of these positive NPV projects. Therefore, $H/(1-H)$ represents the effect on period s book value of each unit of positive NPV arising in period s . Since accounting is assumed to obey the clean surplus relationship, the impact of the shock on the (unscaled) dividend in period s is as follows, being the difference between (A2.1) and (A2.2):

$$((A_t - \bar{A}) - (g_s - \bar{g})) y_{s-1} = ZP_{s-1} \left(\frac{R-1+B}{R} \right) \left(\frac{F-H}{1-H} \right). \quad (\text{A2.3})$$

The question now arises as to what valuation multiples should be applied in the earnings-based valuation model to these disturbances to the various series. Since each of (i) earnings, (ii) change in book value and (iii) dividend can be deduced from the other two items, the valuation impact can be captured by focusing on two items only. For the purpose of this analysis, the two items that are chosen are the impact of the shock on the change in book value, given by (A2.2), and the impact of the shock on the dividend, given by (A2.3). In addition to what would have been expected to occur in the absence of the shock, the following is now expected to occur:

- i. a declining perpetuity of changes to the expected stream of investment in projects, where the first change occurs at time s and where the rate of decline is B . Given that all new projects have a ratio of cost to present value of H , the present value at time s of this declining perpetuity of changes to the expected investment stream is:

$$(g_s - \bar{g}) y_{s-1} \left(1 + \frac{1-B}{R-(1-B)} \right) \left(\frac{1}{H} \right) = (g_s - \bar{g}) y_{s-1} \left(\frac{R}{R-1+B} \right) \left(\frac{1}{H} \right), \quad (\text{A2.4})$$

which by substitution of (A2.2) is

$$ZP_{s-1} \left(\frac{R-1+B}{R} \right) \left(\frac{(1-F)H}{1-H} \right) \left(\frac{R}{R-1+B} \right) \left(\frac{1}{H} \right) = ZP_{s-1} \left(\frac{1-F}{1-H} \right). \quad (\text{A2.4a})$$

- ii. a declining perpetuity of changes to the expected dividend, where the first change occurs at time s and where the rate of decline is B . Since the economic value at time s is stated ex-div, the capitalisation multiple applied to abnormal dividends in the earnings-based valuation model reflects the abnormal dividends for period $s+1$ and subsequent periods only:

$$((A_s - \bar{A}) - (g_s - \bar{g})) y_{s-1} \left(\frac{1-B}{R-1+B} \right), \quad (\text{A2.5})$$

which by substitution of (A2.3) is

$$ZP_{s-1} \left(\frac{R-1+B}{R} \right) \left(\frac{F-H}{1-H} \right) \left(\frac{1-B}{R-1+B} \right) = ZP_{s-1} \left(\frac{1-B}{R} \right) \left(\frac{F-H}{1-H} \right). \quad (\text{A2.5.a})$$

As a check on the correctness of the above decomposition of the shock to the economic return series, it can be verified that the aggregate of the capitalised abnormal changes in book value for time $s+1$

onwards (equation (A2.4a)), the capitalised abnormal dividends for time $s+1$ onwards (equation (A2.5a)) and the time s abnormal dividend (equation (A2.3)), is equal to the time s shock:

$$ZP_{s-1} \left(\frac{1-F}{1-H} \right) + ZP_{s-1} \left(\frac{1-B}{R} \right) \left(\frac{F-H}{1-H} \right) + ZP_{s-1} \left(\frac{R-1+B}{R} \right) \left(\frac{F-H}{1-H} \right) = ZP_{s-1} .$$

The capitalisation factors that are applied to the abnormal change in book value and the abnormal dividend are now re-cast in terms of a parameter which measures persistence in the abnormal book value growth rate, in abnormal ARR and in the abnormal dividend scaled by opening book value. (Hereinafter, dividend scaled by opening book value is termed “the dividend rate”.) Since the parameters R , B , F and H remain unchanged, each of the period $s+k$ abnormal investments which result from the ‘positive NPV project’ component of the period s shock is expected to generate normal changes in book value, normal earnings and normal dividends after it has been made. Therefore, the abnormal change in book value for period $s+k$ will result from the total payoffs (C plus N) arising from the period s shock which accrue in period $s+k$. Successive abnormal changes in book value are related to each other as follows:

$$\begin{aligned} (g_{s+k} - \bar{g})y_{s+k-1} &= (g_s - \bar{g})y_{s-1}(1-B)^k \\ (g_{s+k-1} - \bar{g})y_{s+k-2} &= (g_s - \bar{g})y_{s-1}(1-B)^{k-1} . \end{aligned} \tag{A2.6}$$

Since

$$\frac{y_{s+k-1}}{y_{s+k-2}} = 1 + g_{s+k-1} ,$$

successive deviations from the normal book value growth rate are related to each other as follows:

$$\frac{(g_{s+k} - \bar{g})}{(g_{s+k-1} - \bar{g})} = \frac{1-B}{1+g_{s+k-1}} .$$

Because of their declining perpetuity form, the abnormal changes in book value are expected to become small relative to the normal changes as k grows large. Therefore as $k \rightarrow \infty$,

$$\frac{(g_{s+k} - \bar{g})}{(g_{s+k-1} - \bar{g})} \rightarrow \frac{1-B}{1+\bar{g}} = \bar{\omega} , \tag{A2.7}$$

where $\bar{\omega}$ is the asymptotic persistence parameter for the abnormal book value growth rate. Successive abnormal earnings are related to each other in a similar fashion to the abnormal changes in book value:

$$\begin{aligned} (A_{s+k} - \bar{A})y_{s+k-1} &= (A_s - \bar{A})y_{s-1}(1-B)^k \\ (A_{s+k-1} - \bar{A})y_{s+k-2} &= (A_s - \bar{A})y_{s-1}(1-B)^{k-1} . \end{aligned} \quad (\text{A2.8})$$

Therefore, $\bar{\omega}$ is also the asymptotic persistence parameter for the abnormal ARR and for the abnormal dividend rate. Since the rate of residual income differs from ARR by a constant, it is also the persistence parameter for the rate of residual income. This persistence parameter, $\bar{\omega}$, is a measure of the decline thorough time in deviations from the norm for ARR, for the rate of residual income, for the book value growth rate and for the dividend rate. It therefore takes the form of the autoregressive coefficient that would be estimated in an autoregressive model of order 1 (AR(1) model) for each of these items. The definition of the persistence parameter in terms of an AR(1) coefficient results from the underlying assumption that payoffs take the form of a declining perpetuity.

Now, the multipliers on the abnormal book value growth rate and on the abnormal dividend rate can be re-expressed in terms of $\bar{\omega}$ and $\bar{\gamma}$ ($= R/(1+\bar{g})$) and, in the case of the abnormal book value growth rate, in terms of the normal market-to-book ratio ($\bar{M} = 1/H$). The multiplier that is applied to the abnormal book value growth in (A2.4) can now be re-expressed as follows:

$$\left(\frac{R}{R-1+B} \right) \frac{1}{H} = \frac{\bar{\gamma} \bar{M}}{\bar{\gamma} - \bar{\omega}} ,$$

allowing (A2.4) to be re-written as

$$(g_s - \bar{g})y_{s-1} \left(\frac{R}{R-1+B} \right) \frac{1}{H} = (g_s - \bar{g})y_{s-1} \frac{\bar{\gamma} \bar{M}}{\bar{\gamma} - \bar{\omega}} . \quad (\text{A2.9})$$

Similarly, the multiplier that is applied to the abnormal dividend in (A2.5) can be re-expressed as follows:

$$\frac{1-B}{R-1+B} = \frac{\bar{\omega}}{\bar{\gamma} - \bar{\omega}} ,$$

allowing (A2.4) to be re-written as

$$\begin{aligned}
& ((A_s - \bar{A}) - (g_s - \bar{g}))y_{s-1} \left(\frac{1-B}{R-1+B} \right) \\
& = ((A_s - \bar{A}) - (g_s - \bar{g}))y_{s-1} \frac{\bar{\omega}}{\bar{\gamma} - \bar{\omega}}.
\end{aligned} \tag{A2.10}$$

Adding (A2.9) and (A2.10) to the no-shock valuation model (M.1) gives

$$P_s = y_{s-1} \left(\frac{\bar{A} - \bar{g}}{\bar{\gamma} - 1} + ((A_s - \bar{A}) - (g_s - \bar{g})) \frac{\bar{\omega}}{\bar{\gamma} - \bar{\omega}} + (g_s - \bar{g}) \frac{\bar{\gamma} \bar{M}}{\bar{\gamma} - \bar{\omega}} \right).$$

Collecting the growth rate terms together gives an expression in terms of abnormal profitability and the abnormal book value growth rate:

$$P_s = y_{s-1} \left(\frac{\bar{A} - \bar{g}}{\bar{\gamma} - 1} + (A_s - \bar{A}) \frac{\bar{\omega}}{\bar{\gamma} - \bar{\omega}} + (g_s - \bar{g}) \frac{\bar{\gamma} \bar{M} - \bar{\omega}}{\bar{\gamma} - \bar{\omega}} \right). \tag{A2.11}$$

The impact of the shock on abnormal changes in book value, abnormal dividends and abnormal earnings is, in each case, a constantly declining perpetuity. Therefore, the multipliers that need to be applied to the deviation from normal ARR and to the deviation from the normal book value growth rate in adjusting (M.1) are the same regardless of whether the shock giving rise to the deviations occurred in the current period or in a previous period. Therefore, the multipliers can be applied to abnormal ARR in aggregate and to the abnormal book value growth rate in aggregate. This allows (A2.10) to be generalised to an expression for P_t :

$$P_t = y_{t-1} \left(\frac{\bar{A} - \bar{g}}{\bar{\gamma} - 1} + (A_t - \bar{A}) \frac{\bar{\omega}}{\bar{\gamma} - \bar{\omega}} + (g_t - \bar{g}) \frac{\bar{\gamma} \bar{M} - \bar{\omega}}{\bar{\gamma} - \bar{\omega}} \right). \tag{M.2}$$

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Table 1: Numerical example of movement in economic value and in accounting book value in the setting described in Section 2 (for B=1).

Panel A: Movement in economic value:

	<i>Period</i>	<i>Opening economic value</i>	<i>Payoff</i>			<i>Investment in new projects</i>			<i>Dividend</i>	<i>Closing economic value</i>	<i>Economic rate of return (%)</i>
			<i>Total</i>	<i>Cash</i>	<i>NPV of Positive NPV projects</i>	<i>Cash cost</i>	<i>NPV</i>	<i>Present value</i>			
<i>Notation for item:</i>		P_{t-1}	$C_t + N_t$	C_t	N_t	I_t	N_t	$I_t + N_t$	d_t	P_t	$(R-1)$
<i>Composition of item:</i>			$= P_{t-1} * 1.20$	$= (C_t + N_t) * 0.70$	$= (C_t + N_t) * (1-0.70)$	$= (N_t * 0.667) / (1-0.667)$	$= (C_t + N_t) * (1-0.70)$	$= I_t + N_t$	$= C_t - I_t$	$= I_t + N_t$	$= (P_t + d_t - P_{t-1}) / P_{t-1}$
	1	100.00	120.00	84.00	36.00	72.00	36.00	108.00	12.00	108.00	20%
	2	108.00	129.60	90.72	38.88	77.76	38.88	116.64	12.96	116.64	20%
	3	116.64	139.97	97.98	41.99	83.98	41.99	125.97	14.00	125.97	20%

Panel B: Movement in accounting book value:

	<i>Period</i>	<i>Opening book value</i>	<i>Cash payoff</i>	<i>Depreciation</i>	<i>Accounting earnings</i>	<i>Dividend</i>	<i>Closing book value</i>	<i>Accounting rate of return (%)</i>	<i>Residual income</i>	<i>Rate of residual income (%)</i>
<i>Notation for item:</i>		y_{t-1}	C_t	$-y_{t-1}B$	x_t	d_t	y_t	A_t	x_t^a	χ_t^a
<i>Composition of item:</i>			$= (C_t + N_t) * 0.70$	$-y_{t-1}$	$= C_t - y_{t-1}B$	$= C_t - I_t$	$= y_{t-1} + x_t - d_t$	$= x_t / y_{t-1}$	$= x_t - (y_{t-1} * 0.20)$	$= x_t^a / y_{t-1}$
	1	100.00	84.00	-100.00	-16.00	12.00	72.00	-16%	-36.00	-36%
	2	72.00	90.72	-72.00	18.72	12.96	77.76	26%	4.32	6%
	3	77.76	97.98	-77.76	20.22	14.00	83.98	26%	4.67	6%

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Table 1: Numerical example of movement in economic value and in accounting book value in the setting described in Section 2 (for B=1) (continued).

Notes:

1. In this example, the parameters of the setting described in Section 2 take the following values:
- | | |
|--|-------|
| R (= one plus the required rate of return on equity): | 1.20 |
| F (= the proportion of project payoffs that accrues in the form of net cash receipts): | 0.70 |
| H (= the ratio of cash cost to present value of new projects): | 0.667 |
| B (= the rate of decline in the payoffs from projects) | 1.00 |
- (The use of B = 1 produces a particularly simple example.)

The initial economic value and the initial accounting book value are each set at 100.

2. Note that economic value and the other items in Panel A grow at 8%. This can be computed from expression (11):

$$\bar{g} = (R - 1) - D = (R - 1) \left(\frac{1 - F}{1 - H} \right) - B \left(\frac{F - H}{1 - H} \right),$$

which, in this example, gives

$$0.08 = 0.20 - 0.12 = (0.20) \left(\frac{1 - 0.70}{1 - 0.667} \right) - 1 \left(\frac{0.70 - 0.667}{1 - 0.667} \right).$$