Strategic Quality Choice under Uncertainty: A Real Options Approach*

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Abstract

This paper studies the effects of demand uncertainty and imperfect competition on market entry and product quality choice. We develop a dynamic duopoly model allowing for either a fixed or a flexible quality choice. We find that under the fixed quality choice the follower chooses a higher quality provision. The quality provision is shown to generally increase with the level of demand volatility. The strategic choice of quality provision by the follower may result in the exit of the first mover. Furthermore, flexibility in quality choice of the pioneering firm can constitute a strategic disadvantage. Finally, our results show that the degree of horizontal differentiation between the supplied goods plays a pivotal role in determining the market structure in the long-run.

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1 Introduction

Product quality is one of the most important strategic variables of a firm operating in an imperfectly competitive market. The quality choice is the outcome of a trade-off between the cost of an incremental quality provision and a *ceteris paribus* higher demand for a superior product. Higher quality allows the growth potential of the market to be captured, whereas lower quality often enables the firm to reduce potential losses in bad states of demand. For example, the options available to the subscribers of a Japanese operator NTT DoCoMo via the *i*-mode and related third generation (3G) services in the first stage of their implementation were scaled down compared to initial plans. This was due to the fact that demand, in relation to the associated costs, turned out to be lower than expected. At the time of launching the new product, the subscribers were not able to videoconference or receive video clips, and what remained in the package offered to them was accessing e-mail, downloading news and weather reports, and calling up location-specific information. Adding new services was postponed until the demand was sufficiently high. Also, looming competitive entries of KDDI and, subsequently, Vodafone undoubtedly influenced both the timing and quality decisions of the incumbent.[1]

The last two decades have yielded a number of contributions concerning the quality choice of firms. Motta (1993) analyzes the magnitude of vertical differentiation under Bertrand and Cournot competition, whereas Aoki and Prusa (1996) and Lehmann-Grube (1997) show that the firm providing higher quality earns a higher profit. Foros and Hansen (2002) incorporate network externalities, Hoppe and Lehmann-Grube (2001) show the existence of a second-mover advantage, while Dubey and Wu (2002) observe an inverted U-shaped relationship between the number of firms and quality provision. More recently, Lambertini and Tedeschi (2007) obtain that the leader achieves higher profit by offering a lower quality product. These contributions constitute a static approach to the problem of optimal

[1] See *The Economist*, October 13th, 2001, 'The Mobile Internet: A Survey' and Nov 16th, 2004, 'Geeing up 3G'.

quality provision under imperfect competition.

A changing economic environment results in firms maximizing their values not only by selecting product characteristics, such as *quality*, but also by choosing the *timing* of a product launch. By definition, this timing aspect is not taken into account by the existing static models.[2] Moreover, given the fact that entry is (at least partially) irreversible, both quality and timing are also likely to be dependent on the magnitude of economic uncertainty. To incorporate market dynamics and the underlying uncertainty, we build upon the literature of real option games, which deals with uncertain irreversible investment in the presence of strategic interactions in product markets. A far from complete list of references includes Smets (1991), Grenadier (1996, 2000), Huisman (2001), Hoppe (2002), Lambrecht and Perraudin (2003), and Mason and Weeds (2005).

To study the quality choice and timing decision we employ a simple duopoly model in which the firms' products are imperfect substitutes. Horizontal differentiation results from the fact that some of the products' features cannot be directly compared in terms of their contribution to the consumers' utility. For example, Internet Service Providers operating via a cable TV will have different features than the ones using a telephone connection. A similar difference exists between wireless and cable telecommunication services (cf. Foros and Hansen (2002) for a discussion). We show that, in general, the follower firm supplies a higher quality. This result contradicts some of the past contributions (cf. Motta (1993), Aoki and Prusa (1996), and Lehmann-Grube (1997)) and is supported by an anecdotal historical evidence (see, for example, the cases of Edison and Westinghouse in the electricity generation, De Havilland Comet and Boeing 707 in the commercial jet aircraft, and VisiCalc and Lotus 1-2-3 in the spreadsheet markets, in which a pioneering firm offered an inferior quality and lost a considerable part of the market share to the second entrant). Such quality choices indicate the trade-off that firms potentially face in the dynamic case: the leader will operate

[2]The notable exceptions are Hoppe and Lehmann-Grube (2001) and Pennings (2004), who model the quality choice as a (deterministic and stochastic, respectively) timing game.

for a longer period but it is the follower who will have the ultimate quality advantage.

Finally, we also consider a situation in which the pioneering firm has the ability to continuously adjust quality. In order to determine the additional value of flexibility in quality choice, we compare the case of flexible quality to the fixed quality technology. Flexible quality is associated with the presence of sufficient know-how within the firm, the use of a more advanced technology or contractual flexibility (e.g. via a flexible agreement with content providers in the case of a 3G mobile operator).

As mentioned before, if the fixed quality technology is used, the quality chosen by the leader is lower than that of the follower. This is due to the fact that it invests at a lower level of consumer demand, which, in turn, implies a lower marginal return on investment in quality.

We also show that both under fixed and flexible quality cases, the follower can drive the pioneering firm out of the market. This is caused by the fact that the follower invests when demand is high, which implies a higher optimal quality provision. This effect is stronger for the leader's flexible quality. In this case, the leader is not able commit to the initial quality level, which makes it less costly for the follower to become a monopolist in the market.

Using a different set-up, Pennings (2004) also employs a real option approach to analyze the optimal quality choice. However, his framework is restricted to perfect substitutes and a fixed quality choice. One of Pennings' main results confirms ours in the sense that under high uncertainty the follower will wait and enter the market later with a higher quality product. Still, he obtains that under low uncertainty the leader offers the higher quality product, which contradicts our result in the fixed quality framework. The latter result follows from the fact that the present value of the monopolistic profit would be insufficient to compensate for the disadvantage of having to compete with a lower quality good after the followers' entry when demand is fairly predictable.

The paper is organized as follows. In Section 2 we present the setup of the general model. In Section 3 the game with a fixed quality technology is considered. The analysis of a flexible quality choice is presented in Section 4. Section 5 concludes. Proofs and derivations are included in the appendix.

2 The Model

Consider a situation in which a firm (Firm 1) has an investment opportunity to launch a new product (or a service) in an uncertain market. It chooses the optimal timing of investment and the quality of the product taking into account the possibility of entry of another firm (Firm 2). Denote the degree of substitution between the firms' products by $\rho \in (0,1)$. Parameter ρ is assumed to be exogenous. For ρ close to unity, the products are close substitutes, whereas ρ equal to zero is equivalent to Firms 1 and 2 operating in separate markets.

All consumers are identical and share the following utility function (cf. Singh and Vives (1984) and Häckner (2000), see also Dixit (1979)):

$$U(n_1,n_2) = q_1 n_1 + q_2 n_2 - \frac{1}{2}\left(\gamma_1 n_1^2 + 2\rho n_1 n_2 + \gamma_2 n_2^2\right), \tag{1}$$

where q_i is the quality of good $i, i \in \{1,2\}$, n_i is its quantity per consumer and γ_1, γ_2 are constants. Following Häckner (2000), it is assumed that $\gamma_1 = \gamma_2 = 1$.[3] Consumers maximize $U(n_1,n_2) - p_1 n_1 - p_2 n_2$, where p_i is the unit price of good i. This maximization problem results in demand functions

$$p_i = q_i - n_i - \rho n_j \text{ for } i, j \in \{1,2\} \text{ and } i \neq j. \tag{3}$$

[3]A possible introduction of network externalities would translate into multiplying the third component of (1) by $(1-a)$, where a reflects the intensity of network externalities. The demand function can in such a case be written as

$$p_i = q_i - n_i - \rho n_j + a\left(n_i + \rho n_j\right). \tag{2}$$

The interpretation of network externalities is restricted by the model specification here and is related to the quantity of the goods consumed by a single consumer. However, it is unrelated to the total mass of consumers, which enters the model in a multiplicative way.

The cost of delivering one unit of output equals c_i. Then, the (instantaneous) profit per consumer of firm i equals in equilibrium:

$$\pi_i(q_i,q_j) = \frac{1}{(1-\rho^2)^{\theta}}\left[\frac{(2-\theta\rho^2)(q_i-c_i)-\rho(q_j-c_j)}{4-\rho^2}\right]^2, \tag{4}$$

where θ is an indicator function equal to 1 for Betrand competition and 0 if firms compete à la Cournot (cf. Singh and Vives (1984)). Unless the following set of inequalities is satisfied

$$\frac{1}{\Delta}(q_j-c_j) > q_i - c_i > \Delta(q_j-c_j), \tag{5}$$

where

$$\Delta = \frac{\rho}{2-\theta\rho^2}, \tag{6}$$

one of the firms does not generate a positive profit and leaves the market. Throughout the analysis, we assume that the leader-follower roles of the firms are predetermined so that Firm 2 can only enter after Firm 1 has already done so. The quality selected by the follower (Firm 2) can be obtained analytically (by directly maximizing its value function). The fixed quality choice of the leader (Firm 1) takes into account the expected time it operates as a monopolist and as a duopolist, and is obtained numerically. Once the quality is selected, it cannot be changed. (Later, we discuss the implications of the possibility of the leader to change the quality level at the moment the follower enters.)

The uncertainty in the model is incorporated by allowing the number of (identical) consumers at time t, x_t, to follow a geometric Brownian motion (cf. Pennings (2004)):

$$dx_t = \alpha x dt + \sigma x dw_t. \tag{7}$$

Here α denotes the deterministic drift rate and σ is the instantaneous volatility of the process. In the remainder of the paper x_t is used as a variable reflecting the underlying economic uncertainty. The initial realization of the process x_t, denoted by x_0, is assumed to be sufficiently low which means that the market is too small for an immediate entry to be optimal. Both firms are risk neutral and the riskless interest rate is r.

To enter the market, Firm $i \in \{1,2\}$ has to incur a sunk investment cost $I(q_i) > 0$. The cost of quality provision is defined as

$$I(q_i) = I_0 q_i^{\varepsilon}, \tag{8}$$

where $I_0 > 0$ can be interpreted as an efficiency parameter (so higher values of I_0 correspond to a lower efficiency in R&D). The elasticity parameter ε is assumed to satisfy $\varepsilon > 2\beta/(\beta - 1)$, where β is defined in Appendix B (see (B.5)). Otherwise, the marginal net benefit of increasing quality is positive for all quality levels. The decision of Firm i is to choose the optimal quality, q_i^*, and the timing of entry, x_i^*, in order to maximize the value of its investment opportunity, given the strategy selected by the competitor.

3 Fixed Quality

In this section we assume that a once chosen quality cannot be changed. The idea of a fixed quality choice is therefore similar to Ueng (1997), who considers an infinitely repeated oligopoly game in which the qualities are chosen before the first period. A fixed quality typically occurs when the production technology is provided by an external vendor, or when the firm's R&D department is of a relatively small size.

The game is solved backwards in time. First, Firm 2's reaction function, consisting of its entry threshold and quality choice, is determined. Subsequently, given Firm 2's reaction function, the optimal strategy of Firm 1 is derived. Define T_2 as the (random) time at which x_t hits Firm 2's optimal entry threshold for the first time. Then the value of Firm 2's investment opportunity at $t \leq T_2$ can be written as

$$F_2(x_t) = \max_{q_2, T_2} E\left[\int_{T_2}^{\infty} \pi_2(q_2, q_1)\, x_s e^{-r(s-t)} ds - I(q_2) e^{-r(T_2 - t)}\right], \tag{9}$$

where q_1 is the quality selected by Firm 1. The value of Firm 2's investment opportunity equals the present value of its profits net of investment cost, maximized with respect to the quality and the timing of entry. Using standard dynamic programming techniques (see Appendix B) allows us to derive Firm 2's optimal threshold, x_2^*:

$$x_2^*(q_2, q_1) = \frac{\beta}{\beta - 1} \frac{I(q_2)(r - \alpha)}{\pi_2(q_2, q_1)}. \tag{10}$$

From (10) it can be concluded that Firm 2 invests at the Marshallian trigger multiplied by the mark-up $\beta/(\beta - 1) > 1$. This mark-up, reflecting revenue uncertainty and irreversibility of entry, is positively related to σ and α, and negatively related to r.

The value of Firm 2's investment opportunity is equal to

$$F_2(x_t) = \max_{q_2} \left(\frac{\pi_2(q_2, q_1)\, x_2^*}{r - \alpha} - I(q_2) \right) \left(\frac{x_t}{x_2^*} \right)^{\beta}, \tag{11}$$

which corresponds to the product of the projects' net present value at the time of entry and the stochastic discount factor corresponding to the moment of x_t hitting x_2^* for the first time.[4] Substituting (10) into (11), and calculating the first-order condition yields the equation for the optimal quality of Firm 2:[5]

$$\beta\pi_2'(q_2, q_1)I(q_2) - (\beta - 1)\pi_2(q_2, q_1)I'(q_2) = 0. \tag{12}$$

Using (4), we subsequently obtain

$$q_2^* = c_2\Gamma + \xi(q_1 - c_1)\Gamma\Delta, \tag{13}$$

where

$$\Gamma = \frac{(\beta - 1)\varepsilon}{(\beta - 1)\varepsilon - 2\beta}, \tag{14}$$

Δ is defined by (6) and ξ is an indicator function equal 1 if Firm 1 stays in the market following the entry of Firm 2 and 0 otherwise. From (13) it is obtained that as long as Firm 1 stays in the market following the entry of Firm 2, q_2 is a strategic complement of q_1, so the quality chosen by Firm 2 is positively related to the quality choice made by Firm 1.

[4]The stochastic discount factor is just the price of Arrow-Debreu security associated with event $\{t = T_2\}$, where $E\left[e^{-rT_2}\right] = (x_0/x_2^*)^{\beta}$ (see, e.g., Dixit and Pindyck (1994)).

[5]The second-order condition of (11) is satisfied at q_2^*.

The complete characterization of Firm 2's reaction function to the quality choice of Firm 1 requires considering an additional case, in which q_2 is not chosen according to the first-order condition (12). This additional case corresponds to the lowest quality level that can induce the exit of Firm 1. In fact, Firm 2 may find it optimal to set the quality level just marginally above the level at which Firm 1 will be indifferent between staying and leaving the market (that is, the instantaneous profit of Firm 1 at that level will equal 0.). This "strategic" monopolistic quality level of Firm 2 is lower than the optimal duopolistic level but higher than the unconstrained monopolistic one. The three quality regimes of Firm 2 are described in the following proposition.

Proposition 1 *The optimal level of quality selected by Firm 2 equals*

$$q_2^*(q_1) = \begin{cases} c_2\Gamma & \textit{for} \quad q_1 < q_1', \\ c_2 + \frac{q_1 - c_1}{\Delta} & \textit{for} \quad q_1 \in [q_1', q_1''), \\ c_2\Gamma + \xi(q_1 - c_1)\Gamma\Delta & \textit{for} \quad q_1 \geq q_1'', \end{cases} \tag{15}$$

where

$$q_1' = c_1 + c_2(\Gamma - 1)\Delta, \textit{ and} \tag{16}$$

$$q_1'' = c_1 + c_2(\Gamma - 1)\frac{\Delta}{1 - \Gamma\Delta^2}. \tag{17}$$

Regime $q_1 < q_1'$ corresponds to a(n) (unconstrained) monopoly of Firm 2 following its entry, whereas for $q_1 \geq q_1''$ a duopoly is the prevailing market structure. For $q_1 \in [q_1', q_1'')$ a monopolistic outcome occurs following the strategic quality choice by Firm 2.

Proof. See Appendix A. ■

From Proposition 1 it follows that q_2^* is a piecewise linear function of q_1 (see Figure 1). Quality level q_2^* (weakly) increases with parameter Γ, which can be associated with good investment conditions (it is positively related to the market growth rate, α, and volatility, σ, and negatively to the discount rate, r, and the exponent of the cost function, ε). In the second regime, q_2^* is negatively related to parameter Δ, which positively depends both on Bertrand competition indicator, θ, as well as on the level of the products' substitutability, ρ. This negative relationship can be explained by the fact that for its given quality choice, Firm 1 is less likely to remain in the market which is more competitive (Bertrand competition combined with high substitutability of firms' products). Finally, in the duopolistic outcome, for a given quality of Firm 1, the quality selected by Firm 2 is *higher* under Bertrand competition than in the Cournot framework, and when goods offered by firms are closer substitutes (as $\partial q_2^*/\partial\Delta > 0$ in this case). Furthermore, if $\Gamma\Delta^2 \geq 1$, that is, when good investment conditions are combined with firms being close competitors, duopoly never prevails following the entry of Firm 2.

[Please insert Figure 1 about here]

To describe the optimal reaction of Firm 2 to the changes in the quality level of Firm 1, we formulate the following proposition.

Proposition 2 *Firm 2 responds optimally to an increased quality of Firm 1 not only by raising its own quality but also by delaying its timing of entry, that is, the following inequalities hold*

$$\frac{dq_2^*}{dq_1} > 0, \textit{ and} \tag{18}$$

$$\frac{dx_2^*}{dq_1} > 0. \tag{19}$$

Proof. See Appendix A. ■

The choice of a higher quality level by Firm 1 results therefore in a higher cost of entry of Firm 2, due to its higher optimal response q_2^*. Moreover, it induces a later entry of Firm 2.

Having determined the reaction function (the optimal quality and the resulting entry threshold) of Firm 2, we are in a position to analyze the decision of Firm 1. First, we note that the value of Firm 1 before its entry is given by

$$
\begin{aligned}
F_1(x_t) \quad &= \quad \max_{q_1(q_2),T_1} \left\{ E\left[\int_{T_1}^{T_2} \pi_1(q_1)\, x_s e^{-r(s-t)} ds - I(q_1) e^{-r(T_1-t)}\right] \right. \\
&\qquad \left. +\xi E\left[\int_{T_2}^{\infty} \pi_1(q_1,q_2)\, x_s e^{-r(s-t)} ds\right]\right\}. \qquad (20)
\end{aligned}
$$

Working out the expectations yields

$$
F_1(x_t) = \max_{q_1(q_2)} \left\{ \underbrace{\left[\frac{\pi(q_1)\,x_1^*}{r-\alpha} - I(q_1)\right]\left(\frac{x_t}{x_1}\right)^{\beta_1}}_{\text{Monopolistic value}} + \underbrace{\frac{[\xi\pi_1(q_1,q_2) - \pi(q_1)]\,x_2^*}{r-\alpha}\left(\frac{x_t}{x_2^*}\right)^{\beta_1}}_{\text{Value lost due to the competitive entry}} \right\}, \qquad (21)
$$

where $\pi_i(q_i)$ is defined as a special case of (4) with $\rho = 0$ (monopolistic market). Equation (21) implies that as soon as competitive entry becomes very remote, that is, when $x_2^* \to \infty$, the value of the investment opportunity reduces to the valuation of a monopolistic firm.

The optimal entry threshold, x_1^*, is found by applying standard dynamic programming techniques, and equals

$$
x_1^*(q_1) = \frac{\beta_1}{\beta_1 - 1}\frac{I(q_1)(r-\alpha)}{\pi_1(q_1)}. \qquad (22)
$$

The optimal investment timing of Firm 1 does not *explicitly* depend on the choices made by Firm 2 regarding its timing of entry and quality. This outcome results from the fact that the roles of the firms (leader and follower) are pre-determined.[6] However, the outcome of our model still differs from the standard result from the real options theory concerning the irrelevance of the follower's timing of entry for the decision of the leader. The reason is that Firm 1's timing decision is affected by its own choice of quality, q_1, which, in turn, depends on Firm 2's both entry threshold and quality choice.

The resulting dependence of Firm 1's investment threshold on the behavior of Firm 2 is therefore caused by the fact that Firm 1 has two decision variables (the timing of entry and quality) as opposed to a single variable in standard real option models. As competitive entry

[6]If the roles of the firms are pre-determined, the optimal timing of the leader equals the one of a monopolistic firm in a standard real-option game (see, e.g., Reinganum (1981) and Huisman (2001), p. 170).

changes the optimal quality choice q_1^* (which is no longer equal to that of an uncontested monopolist), the monopolistic timing of entry is, in general, not optimal either.

The value of the investment opportunity is determined by maximizing the RHS of (21). The derivative of F_1 with respect to q_1 can be easily computed since x_1^*, x_2^* and q_2^* are known functions of q_1. However, due to the non-linearity of the resulting relationship, the root of the derivative is determined numerically.

Numerical calculations (based on maximizing the value of the investment opportunity of Firm 1) indicate that the presence of a potential competitor tends to increase the quality provision of Firm 1 for low levels of uncertainty, σ, and to reduce it in the opposite case (see Table 1). Higher (lower) quality q_1 results, in turn, in the optimal entry threshold of Firm 1 being higher (lower) than the monopolistic counterpart.

Moreover, our simulations show that the quality provision of Firm 1 increases with the quality of its competitor. In particular, this can be seen under those scenarios in which Firm 2 selects the quality level that results in the exit of Firm 1 ($REG^k = MS$ or M in Table 1). This finding, together with Proposition 2, implies that firms' qualities are strategic complements.

[Please insert Table 1 about here]

From a consumer surplus perspective, industry concentration has two opposing effects. First, it increases the provision of (average) quality, which benefits consumers. Second, it can delay investment of the pioneering firm (Firm 1) so the utility derived by the consumers has to be discounted more heavily. Therefore, the overall impact of industry concentration on consumer surplus is ambiguous.

Higher demand uncertainty generally leads to a higher quality provision. This result, consistent with Pennings (2004), is due to the convexity of the value of the firm's investment opportunity in the underlying stochastic variable. This convexity implies a higher optimal level of investment in quality (cf. Bar-Ilan and Strange (1999), who obtain a positive relationship between uncertainty and the optimal scale of investment). Moreover, the dispersion of the firms' quality levels is an increasing function of uncertainty. However, the positive relationship may not hold if higher uncertainty leads to a different market regime following Firm 2's entry (in particular, a monopoly – as monopolistic quantities are generally lower than duopolistic ones).

As qualities are strategic complements, q_1 is generally higher under Bertrand competition and for horizontally less differentiated products. Our numerical simulations also indicate that the follower's quality is higher than that of the leader. In fact, for the cost parameters being equal across the firms, the sufficient condition for the follower's quality to be always higher is

$$\Gamma\Delta > 1. \tag{23}$$

Inequality (23) implies that the follower's quality exceeds that of the leader for sufficiently small cost elasticity parameter ε, sufficiently large market uncertainty (captured by the reciprocal of β) and for a low degree of product (horizontal) differentiation. In general, this relationship is due to the fact that the follower invests later than the leader (that is, at a larger market size – captured by x) and its marginal return on investment in quality is higher.

Upon analyzing Table 1 it can be concluded that the relative gap between Firm 1's and Firm 2's entry thresholds widens with uncertainty. This implies that higher uncertainty hampers competition in the sense of the follower optimally having to wait for a higher level of demand before committing to entry.

The nature of the market structure implies that the leader can be completely ousted from the market by the entrant offering a higher quality product. This will happen if the quality selected by the leader satisfies the following condition:

$$q_1 < c_1 + c_2(\Gamma - 1)\frac{\Delta}{1 - \Gamma\Delta^2}. \tag{24}$$

When (24) is satisfied, the optimal quality q_2 selected by the follower results in a negative instantaneous duopoly profit of the leader and, ultimately, in its exit. A closer inspection of (24) indicates that for a given level of market-specific parameters (captured by β) and the degree of product differentiation, the leader is more likely to leave under Bertrand competition than under Cournot.[7]

Non-monotonicity of quality choice and entry timing in σ and ρ can also be observed. This non-monotonicity is a result of the switching across the three quality regimes of Firm 2 in the response to the varying model parameters. In particular, when the resulting market outcome after Firm 2's entry is a monopoly with Firm 2 having chosen quality strategically ($FMA^k = MS$), the quality selected by Firm 1 exceed that corresponding to a duopoly ($FMA^k = D$). By increasing quality Firm 1 simply delays the entry of its competitor, who is going to capture the entire market. On the other hand, when the resulting outcome is monopoly with Firm 2 selecting the first-best (monopolistic) quality level, there is no scope for Firm 1 to increase quality – Firm 2 does not have to engage in quality competition as it induces the exit of Firm 1 by just choosing the (low) monopolistic quality level.

The direct effect of the degree of product differentiation on the chosen quality levels and the optimal timing of entry is not so clear-cut. However, product differentiation has a fundamental role in determining the market structure after the entry of the follower (Firm 2). As high ρ leads generally more often to monopolistic outcomes, it makes the quality choices and entry thresholds more extreme. In particular, those quality levels and investment thresholds are expected to be higher if the monopoly is triggered by the strategic quality choice of Firm 2 $FMA^k = MS$) and lower if Firm 2 can afford to select the first-best monopolist level $FMA^k = M$).

Finally, the presence of the first mover advantage is generally associated with a duopolistic outcome following Firm 2's entry. This result is consistent with Lambertini and Tedeschi (2007), who also obtain that leader achieves a higher payoff despite supplying the lower quality good. In our model, the first-mover advantage is completely eroded (and assumes a

[7]It is easy to show that this condition is always satisfied under Bertrand competition for ρ sufficiently close to 1. Under Cournot competition, investment cost elasticity ε has to be lower than $8\beta/(3(\beta-1))$ for (24) to always be satisfied for ρ sufficiently close to 1.

negative value) if the pioneering firm is forced by the new entrant to leave the market.

4 Flexible Quality

In this section the pioneering firm (Firm 1) is assumed to have sufficient know-how that enables it for a single quality upgrade following the entry of Firm 2. The ability to change quality could result from, among other factors, Firm 1's technology being the outcome of its own R&D process.[8]

The cost of upgrading quality from q_0 to q_1, $I(q_1,q_0)$, is equal to $I(q_1)-I(q_0)$. In other words, investing in a given level of quality in two stages is not associated with any additional costs relative to a single-stage investment.

As in Section 3, the optimal initial quality of Firm 1, q_0, is found by maximizing the value of Firm 1's investment opportunity ignoring the possibility of the future entry of Firm 2. The entry threat of Firm 2 can be ignored since the quality chosen initially of Firm 1 can be instantaneously changed following the entry of the competitor and the total cost of the reaching the new quality level is not path dependent. As a result, the optimal quality of Firm 1 operating in the initially uncontested market is given by

$$q_m = c_1\Gamma. \tag{25}$$

The cost structure of investment in quality implies that the solution to the original game following the entry of Firm 1 is equivalent to a simpler game in which *i*) Firm 2 chooses the timing on entry, x_2, and its quality, q_2, and *ii*) Firm 1 has to respond by immediately entering with quality q_1 or not entering at all. The latter game is solved in three steps: identifying the reaction function of Firm 1, $q_1(q_2)$, for an arbitrary level of x_2, finding the optimal quality choice of Firm 2 given the reaction function $q_1(q_2)$ for an arbitrary x_2, and selecting x_2 that

[8] An alternative way of modeling flexible quality choice would be via allowing for marginal increases of quality. However, the analysis of a differential game arising as a result of such a modeling approach is outside the scope of this paper.

maximizes the value of Firm 2's investment opportunity. The reaction function of Firm 1 is given by the solution to

$$\max_{q_1} \frac{\pi_1(q_1, q_2)\, x_2}{r - \alpha} - I(q_1, q_0), \tag{26}$$

whereas Firm 2's follower's maximization problem is given by

$$F_2(x_t) = \max_{x_2} \left(\frac{\pi_2(q_2(x_2), q_1(q_2(x_2), x_2))\, x_2}{r - \alpha} - I(q_2) \right) \left(\frac{x_t}{x_2} \right)^{\beta}, \tag{27}$$

Maximization (27) is done numerically as the explicit form of $q_2(q_1)$ is unknown.

As in previous section, the value of the investment opportunity of Firm 1, F_1, consists of two components. One reflects the present value of the monopolistic profit flow, whereas the other reflects the value of future revenues lost due to the competitive entry (and, possibly, Firm 1's exit).

Table 2 illustrates the effect of the possibility of making the quality adjustment by Firm 1 upon Firm 2's entry on a the firm's strategic variables and the magnitude of the first-mover advantage. As the entry threshold for Firm 1 does not depend on a future competitive entry and is identical to the monopolistic threshold, only the threshold of Firm 2 is reported in a conjunction with the qualities supplied by both firms.[9]

[Please insert Table 2 about here]

One of the most clear findings following from the analysis of Table 2 is the effect of Firm 1's flexibility to adjust quality on the resulting market structure.[10] For a wide range of parameters, Firm 2 sets it quality at such a level that makes it unattractive for Firm 1 to stay in the market ($REG = M$). Only for a combination of relatively large demand volatility and highly differentiated products, Firm 1 stays in the market following the entry of Firm

[9] As the results do not differ significantly between Bertrand and Cournot competition, we only report those of the latter.

[10] To maintain the consistency of the concept of "Firm 1's flexibility", its exit is associated here with a negative quality investment, that is, with incurring cash inflow of $I(q_0)$.

2. The intuition for the monopoly being the prevailing market outcome following Firm 2's entry is as follows. By being effectively the leader in the quality game played at x_2, Firm 2 sets quality at such a level that makes exit the optimal strategy for Firm 1. Therefore, this outcome can be interpreted as a negative value of flexibility in a strategic setting. In fact, this is the absence of the commitment to the initial quality level that puts Firm 2 is such an unfavorable strategic position.

A comparison of Tables 1 and 2 leads to the observation that the flexible quality technology of Firm 1 generally lowers the optimal quality of Firm 2. As a consequence, the entry threshold of Firm 2 becomes lower as well. Again, higher uncertainty is associated with a higher quality provision.

Contrary to the fixed quality case, the value of Firm 1's investment opportunity is lower than the one of Firm 2 for a wide range of parameter values (cf. FMA in Table 2). This finding contradicts Aoki and Prusa (1996) and Pennings (2004), where a higher value of the leader is obtained. However, in those contributions the follower does not have a strategic advantage of effectively being the first-mover in the quality setting game. Upon inspecting Table 2 we conclude that the first-mover advantage of Firm 1 can prevail only in two situations. First, the value of the leader can be higher if demand volatility is low and the entry of Firm 2 leading to Firm 1's exit – remote (in the discounted probability-weighted terms). Second, for relatively highly differentiated products, Firm 1 can enjoy both the the earlier arrival of the revenue stream and the market presence also following the entry of Firm 2.

To summarize, the relative value of being the leader is generally much lower when quality is flexible rather than fixed. Such an outcome results from the fact that Firm's 1 flexible quality choice is associated with its inability to commit to the initial quality level when confronted with the entry of Firm 2.

5 Conclusions

Extending the strategic quality choice literature by allowing for dynamics and uncertainty leads to a number of interesting results. First, we study a duopoly game where both firms can once make a fixed quality choice. We show that the quality chosen by the leader is generally higher than in the monopolistic case for low levels of uncertainty and lower when the demand level is less predictable. Moreover, higher uncertainty generally widens the wedge between the firms' qualities and raises the time interval elapsing between their entries. Furthermore, the first-mover advantage exists as long as a duopoly is a resulting market outcome following the second mover's entry. However, we show that for certain configurations of market parameters the entrant finds it optimal to set quality strategically to induce the exit of the pioneering firm. It is also possible that the first-best monopolistic quality level of the entrant results in the leader's exit. In such cases the first-mover advantage is unlikely to prevail.

Second, we modify the game to allow the first firm to change its quality following the entry of the second mover. By being effectively the leader in the new quality game played at the point of its entry, the second mover sets in most scenarios its quality at a level that induces the leader's exit. It is the inability of the leader to commit to the initial quality level that allows Firm 2 to obtain the strategic advantage. A comparison of firms' values under two alternative technologies leads to conclusion that the relative value of the leader is generally much lower when the quality of the pioneering firm is flexible. Moreover, the flexible quality of the first mover has implications for the average quality provision. As the follower generally invests in such a case sooner, its marginal return of quality is lower, so very high quality levels are unlikely to occur.

A Proofs of Propositions

Proof of Proposition 1. The proof is straightforward and follows from the definition of the instantaneous profit function of Firm 1 (cf. equation (4)). In regimes $q_1 < q_1'$ and $q_1 \geq q_1''$ Firm 2 chooses its quality response by solving the first-order condition of the value maximization problem (11). For $q_1 \in [q_1', q_1'')$ Firm 1 would find it optimal to stay in the market if Firm 2 chose its (unconstrained) monopolistic quality level. However, Firm 2 is better off by setting a second-best quality level that triggers Firm 1's exit than by selecting the first-best duopolistic quality level. As a result, for $q_1 \in [q_1', q_1'')$ the monopoly of Firm 2 results. ■

Proof of Proposition 2. The sign of derivative dq_2^*/dq_1 immediately follows from (13) and Proposition 1. In order to evaluate the sign of dx_2^*/dq_1^*, we first decompose it into

$$\frac{dx_2^*}{dq_1} = \frac{\partial x_2^*}{\partial q_1} + \frac{\partial x_2^*}{\partial q_2^*}\frac{dq_2^*}{dq_1}. \tag{A.1}$$

The positive sign of $\partial x_2^*/\partial q_1$ and dq_2^*/dq_1 follows directly from the relevant definitions. The sign of $\partial x_2^*/\partial q_2^*$ can be evaluated by writing first

$$x_2^* = \frac{\beta I_0(r-\alpha)(4-\xi\rho^2)^2(1-\theta\rho^2)}{\beta-1}\left[\frac{(q_2^*)^{\frac{\varepsilon}{2}}}{(2-\xi\theta\rho^2)(q_2^*-c_2)-\xi\rho(q_1-c_1)}\right]^2. \tag{A.2}$$

The derivative of the expression in the square bracket equals

$$\frac{\varepsilon}{2}(q_2^*)^{\frac{\varepsilon}{2}-1}\left[(2-\xi\theta\rho^2)((1-2/\varepsilon)q_2^*-c_2)-\xi\rho(q_1-c_1)\right]. \tag{A.3}$$

By substituting the expression for q_2^* (cf. 13), we obtain that the sign of (A.3) is indeed positive:

$$\varepsilon(q_2^*)^{\frac{\varepsilon}{2}-1}\frac{c_2(2-\xi\theta\rho^2)+\xi\rho(q_1-c_1)}{(\beta-1)\varepsilon-2\beta} > 0. \tag{A.4}$$

For intermediate regime (that is, for $q_1 \in [q_1', q_1'')$), the inequality holds as well. This can be shown by observing that the positive sign of derivative (A.3) for q_2^* under regime $q_1 < q_1'$ implies a positive sign for q_2^* under regime $q_1 \in [q_1', q_1'')$, as under the latter q_2^* is greater and (A.3) has a unique root. ■

B Derivation of Firm 2's value using dynamic programming

The basic problem is to find the optimal timing of entry, given that the consumer mass, which drives the instantaneous revenue from the project, follows a geometric Brownian motion (GBM):

$$dx_t = \alpha x_t + \sigma x_t dw_t. \tag{B.1}$$

First, we determine the value of the firm after its entry, V_2. Since the immediate payoffs to the owner of the firm equals its instantaneous profit, π_2, the corresponding Bellman equation can be written as

$$rV_2 dt = E\left[dV_2\left(x\right)\right] + \pi_2(q_2, q_1)x dt. \tag{B.2}$$

Equation (B.2) means that for a risk-neutral firm, the expected rate of change in its value over the time interval dt plus immediate payoffs equals the riskless rate. Applying Itô's lemma to the RHS of (B.2), and dividing both sides of the equation by dt results in the following ordinary differential equation (ODE):

$$rV_2 = \alpha x \frac{\partial V_2}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V_2}{\partial x^2} + \pi_2(q_2, q_1)x. \tag{B.3}$$

The general solution to (B.3) has the following form:

$$V_2\left(x\right) = A_1 x^{\beta} + A_2 x^{\lambda} + \frac{\pi_2(q_2, q_1)x}{r - \alpha}, \tag{B.4}$$

where A_1 and A_2 are constants, and

$$\beta, \lambda = -\frac{\alpha}{\sigma^2} + \frac{1}{2} \pm \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{B.5}$$

Moreover, it holds that $\beta > 1$ and $\lambda < 0$. In order to find the value of Firm 2, $V_2\left(x\right)$, the following boundary conditions are applied to (B.4):

$$\lim_{x \to \infty} V_2\left(x\right) = \frac{\pi_2(q_2, q_1)x}{r - \alpha}, \tag{B.6}$$

$$V_2\left(0\right) = 0. \tag{B.7}$$

Conditions (B.6) and (B.7) implies that $A_1 = A_2 = 0$, which is associated with no optionalities held by Firm 2 after entry.

The value of Firm 2's investment opportunity satisfies a differential equation similar to (B.3). Since the opportunity is associated with no immediate payoffs, the corresponding equation does not include the non-homogenous part. Therefore, its general solution is

$$F_2(x) = A_{F1}x^{\beta} + A_{F2}x^{\lambda}, \tag{B.8}$$

where A_{F1} and A_{F2} are constants. In order to find the value of the investment opportunity, $F_2(x)$, and the entry threshold, x_2^*, the following boundary conditions are applied to (B.8):

$$\begin{aligned} F_2(x_2^*) &= V_2(x_2^*) - I(q_2), && \text{(B.9)} \\ F_2'(x_2^*) &= V_2'(x_2^*) - I'(q_2), && \text{(B.10)} \\ F_2(0) &= 0. && \text{(B.11)} \end{aligned}$$

Conditions (B.9) and (B.10) are called the value-matching and the smooth-pasting conditions, respectively, and ensure continuity and differentiability of the value function at the investment threshold. Condition (B.11) ensures that the investment option is worthless at the absorbing barrier $x = 0$. Consequently, it implies that $A_{F2} = 0$. Substitution of (B.8) into (B.9)–(B.11) and some algebraic manipulation yield the value of the optimal entry threshold (10) and the value of investment opportunity (11).

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Optimal Quality Response of Firm 2

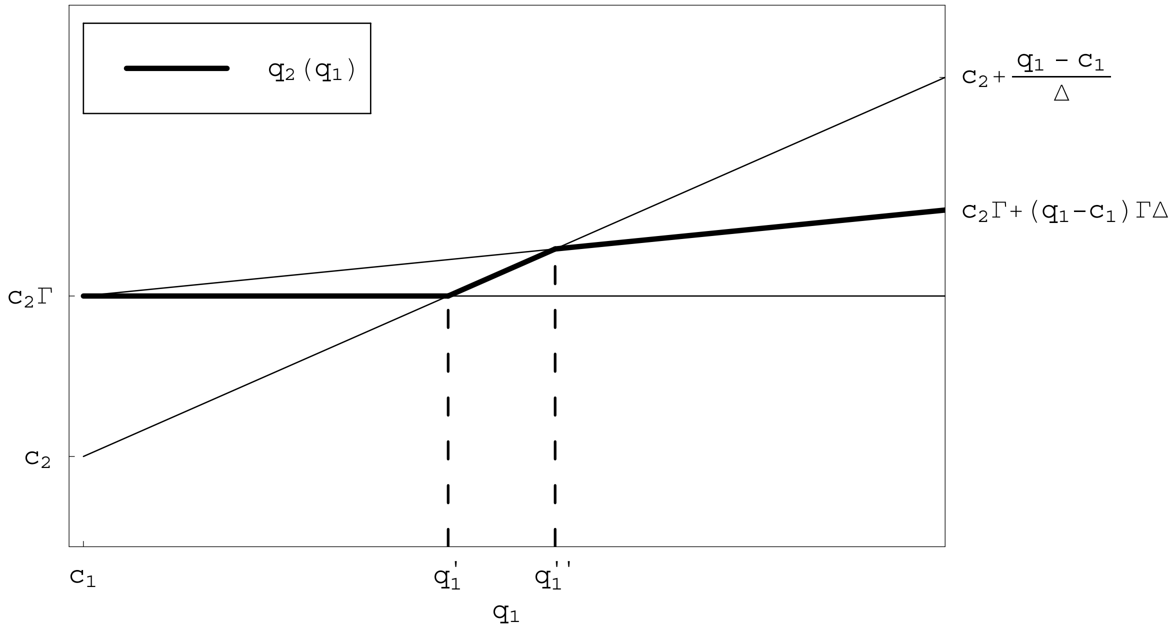

Figure 1: Quality choice of Firm 2, q_2, as the function of the quality selected by Firm 1, $q_1 \in (c_1, \infty)$. For $q_1 < q_1'$, Firm 2 optimally selects the monopolistic quality, at which Firm 1 leaves the market. For $q_1 \in [q_1', q_1'')$, Firm 1 selects quality strategically to induce Firm 1's exit. For $q_1 \geq q_1'$ Firm 2 optimally selects the duopolistic quality level.

Quality Choice Policies and Optimal Timing of Entry

σ	ρ	q_1^c	q_1^b	q_2^c	q_2^b	x_1^c	x_1^b	x_2^c	x_2^b	FMA^c	FMA^b	q_m	x_m	REG^c	REG^b
	0.1	0.943	0.943	0.982	0.982	0.089	0.089	0.101	0.101	0.254	0.256			*D*	*D*
0.05	0.5	0.972	0.972	1.163	1.194	0.091	0.091	0.147	0.157	7.566	10.845	0.941	0.089	*D*	*D*
	0.9	0.972	1.385	1.340	1.670	0.091	0.152	0.163	0.222	7.547	0.279			*D*	*MS*
	0.1	1.326	1.326	1.504	1.504	0.192	0.192	0.264	0.265	0.024	0.024			*D*	*D*
0.15	0.5	1.415	1.415	2.024	2.115	0.217	0.217	0.569	0.636	0.476	0.617	1.389	0.209	*D*	*D*
	0.9	1.444	3.598	2.569	4.596	0.225	2.010	0.842	3.910	0.327	-0.791			*D*	*MS*
	0.1	3.303	3.303	4.743	4.748	2.163	2.163	6.136	6.156	0.016	0.016			*D*	*D*
0.25	0.5	3.450	3.450	9.169	9.950	2.429	2.429	39.162	49.019	0.121	0.154	3.705	2.939	*D*	*D*
	0.9	4.571	1.680	9.547	3.705	5.208	0.416	41.907	2.939	-0.159	-0.523			*MS*	*M*

Table 1: Comparative statics results concerning debt choice policies, q_i^k, optimal timing of entry, x_i^k, the degree of the firm mover advantage, FMA^k, benchmark monopoly choices, q_m and x_m, and market regimes following Firm 2's entry, REG^k, for different levels of the volatility of consumer mass, σ, and product (horizontal) differentiation, ρ. Remaining parameter values are: $c_1 = c_2 = 0.5$, $\varepsilon = 5$, $r = 0.05$, $\alpha = 0.0$ and $I_0 = 0.1$. FMA^k is defined as $F_1^k/F_2^k - 1$. Subscript $k = c$ ($k = b$) indicates Cournot (Bertrand) competition. *D* denotes duopoly, *MS* – monopoly with strategically selected quality, *M* – monopoly (with the first-best quality level of Firm 2).

Effect of Quality Adjustment of Firm 1 on Quality Choices and Timing of Entry

σ	ρ	q_1	q_2	x_2	FMA	REG
0.05	0.1	–	1.063	0.109	-0.274	M
	0.5	–	1.188	0.141	1.384	M
	0.9	–	1.249	0.172	4.952	M
0.15	0.1	–	1.156	0.148	-0.287	M
	0.5	–	1.227	0.184	-0.881	M
	0.9	–	1.613	0.322	-0.754	M
0.25	0.1	3.882	3.817	3.438	0.121	D
	0.5	–	3.641	2.750	-0.996	M
	0.9	–	3.679	2.931	-1.000	M

Table 2: Comparative statics results concerning debt choice policies, q_i, optimal timing of entry of the follower, x_2, the degree of the firm mover advantage, FMA, and market regimes following Firm 2's entry, REG, for different levels of the volatility of consumer mass, σ, and product (horizontal) differentiation, ρ. The leader's quality is adjustable upon the follower's entry. Remaining parameter values are: $c_1 = c_2 = 0.5$, $\theta = 0$, $\varepsilon = 5$, $r = 0.05$, $\alpha = 0.0$ and $I_0 = 0.1$. FMA is defined as $F_1/F_2 - 1$. D denotes duopoly, M – monopoly (with the first-best quality level of Firm 2).