Vortex Nucleation in Ultradilute Superfluid ^{3}He/^{4}He Solutions

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A detailed investigation of vortex nucleation in He II containing traces of ^{3}He isotopic impurity is reported. Systematic measurements of the nucleation rate due to negative ions were made at 23 bars. A model is proposed which, when fitted to the data, implies that a single ^{3}He atom trapped on the surface of the ion has the dual effect of reducing the critical velocity for vortex nucleation by circa 4 m s^{-1}, while simultaneously increasing the corresponding rate constant by a factor of circa 10^3.

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Almost a quarter of a century has now passed since the discovery[1] of the quantum of circulation in He II. Yet, notwithstanding the rapid advances currently being made towards a detailed theory of the growth and decay processes governing quantized turbulence,[2,3] the mechanism through which vortex lines are created in the superfluid in the first place still remains a mystery. The seemingly intractable nature of this long-standing problem arises in large part from the difficulty (perhaps impossibility, in practical terms) of preparing bulk He II in a vortex-free state: Recent work[4] by Awschalom and Schwarz has confirmed the presence of remanent vorticity[5] at a characteristic density that is apparently independent of the liquid's prior history. An important consequence of this result has been to vindicate the widespread suspicion that threshold critical velocities observed for many years in flow and thermal counterflow experiments[2] refer to the physical conditions needed to expand preexisting vortex lines, and not to those required for the creation of a vortex *ab initio.* It follows that studies of the formation of charged quantized vortex rings in He II by ions[6] (where, by virtue of the small physical size[7] of these probes, the influence of remanent vorticity on the transition is expected to be negligible) quite probably represent the only available technique through which the intrinsic nucleation mechanism may be investigated. Detailed measurements of the rate at which negative ions nucleate vortex rings in isotopically pure superfluid ^{4}He have already been published.[6] In this Letter we report the principal results of the first detailed study to be made of the astonishingly potent influence[8,9] exerted on the nucleation process by tiny traces of ^{3}He isotopic impurity.

It is well established[10] that the ^{3}He atoms in ^{3}He-^{4}He solutions have a tendency to condense on the free surface[11] provided by the negative ion, and it has long been suspected[12] that this phenomenon plays a vital role in the unknown mechanism through which the ^{3}He influences the nucleation of vortices. Only recently, however, has it become apparent[9] that the nucleation rate ν can still be drastically modified by ^{3}He even when present at such low concentrations that the average number of trapped ^{3}He atoms per ion is considerably less than unity.

In the present work, we have measured ν under a pressure of $P = 23$ bars for a wide range of electric fields E and temperatures T in isotopically purified ^{4}He and in eight extremeley dilute solutions, the ^{3}He/^{4}He ratio x_3 in the *strongest* solution being approximately the same as that found in naturally occurring (gas well) helium. Experimental values of ν are plotted against x_3 for various values of E and T in Fig. 1. It is found that, while $\nu(x_3)$ is markedly nonlinear for small values of E, the measurements for larger values follow straight lines within experimental error. In what follows, we will attribute the (quite unexpected) upturn in $\nu(x_3)$ shown in Fig. 1(a) to the onset of conditions such that a significant fraction of ions in the ensemble possess two (or more) trapped ^{3}He atoms; whereas, in the linear regime of Fig. 1(b), the probability of there being more than one trapped ^{3}He on an ion remains negligible. In order (initially) to treat the relatively straightforward case where each ion can be assumed to have either one trapped ^{3}He atom, or none, we have fitted the data acquired for each permutation of E and T by the

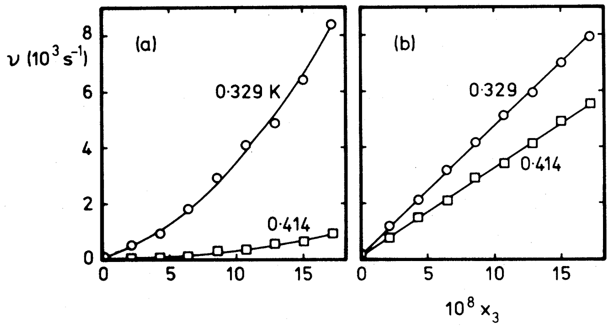

FIG. 1. The rate ν at which negative ions nucleate vortex rings in dilute ^{3}He-^{4}He solutions, measured as a function of the ^{3}He/^{4}He ratio x_3 for two temperatures with an electric field of (a) 1.27×10^4 V m^{-1} and (b) 9.50×10^4 V m^{-1}. The full curves are guides to the eye.

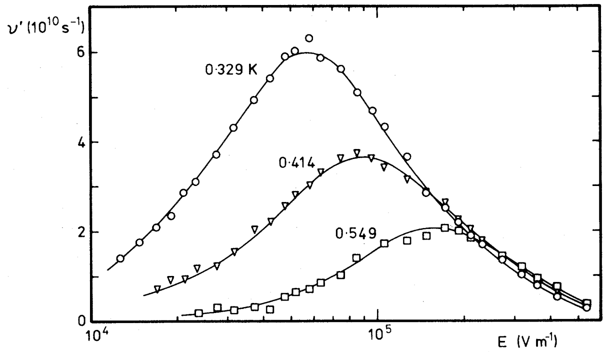

FIG. 2. Experimental values of the linear coefficient ν' from Eq. (1), plotted as a function of electric field E for three temperatures. The full curves represent fits to the data of the model described in the text.

power-law expansion

$$\nu(x_3) = \nu_0 + \nu' x_3 + \nu'' x_3^2, \qquad (1)$$

where ν_0 represents the value of ν in pure ^{4}He. In doing so, we have found that, for the present parameter range, there is no statistically significant advantage in considering terms beyond the quadratic one.

The physical significance of the coefficient ν' is that it represents the ^{3}He contribution to the nucleation rate (per unit concentration) that would be measured in the limiting case of very low concentrations. Some experimental values of $\nu'(E)$ and $\nu'(T)$ are shown by the data points of Figs. 2 and 3. It may be noted, first, that $\nu'(E)$ displays a well defined temperature-dependent maximum; secondly, that ν' tends towards zero for both small and large E; and, thirdly, that ν' is almost temperature independent for large values of E.

These results can be accounted for in considerable detail on the basis of a simple model in which we postulate the existence of two different nucleation rates: ν_1, characterizing ions that have one trapped ^{3}He atom; and the rate ν_0 for bare ions that has already been determined through experiments[6] in pure ^{4}He. The measured rate in a dilute solution can therefore be written

$$\nu = (1 - n_B)\nu_0 + n_B \nu_1.$$

The complicated variation of ν with E and T is then due in large measure to changes in n_B, the proportion of ions having a ^{3}He atom bound in one of the angular momentum states[13] of energy

$$\epsilon_l = E_B + gl(l+1),$$

where $g = \hbar^2/2m_s R_i^2$, m_s is the ^{3}He effective mass of the surface state, and R_i is the radius of the ion. For low fields, n_B is determined by normal thermal equilibrium processes: ^{3}He absorption may occur, with emission of a phonon, and vice versa. Thus n_B, and consequently also ν', will be highly temperature dependent, as observed. In stronger fields it becomes energetically possible[14] for the ^{3}He to be emitted together with a pair of rotons,[15] and n_B consequently falls below its thermal equilibrium value. For strong enough fields, the phonon-driven ^{3}He emission rate becomes negligible compared to the rate R of emission with rotons, and so ν' will become almost independent of temperature, which again is in accord with experiment. Taking account of the $2(2l+1)$ possible states,[13] n_B is

$$n_B = \sum_{l=0}^{L} 2(2l+1) n_l,$$

where L is the maximum value of l for which ϵ_l

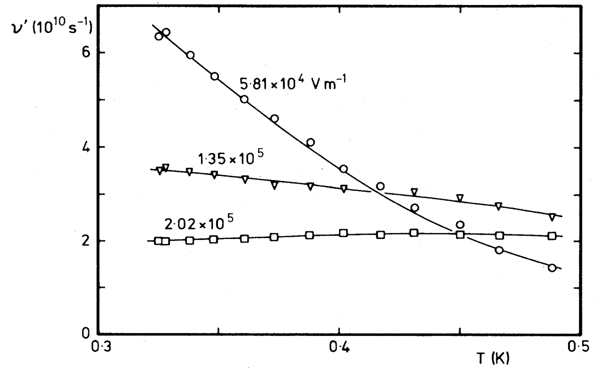

FIG. 3. Experimental values of the linear coefficient ν' from Eq. (1) plotted as a function of temperature T for three electric fields. The full curves represent fits to the data of the model described in the text.

is negative. It is straightforward to show that the occupation n_l of the lth state is

$$n_l = \frac{n_l^{\mathrm{eq}}}{1 + n_l^{\mathrm{eq}} R'/n_3 K},$$

where

$$n_l^{\mathrm{eq}} = [1 + \exp\beta(\epsilon_l - \mu)]^{-1}$$

is the value of n_B in thermal equilibrium; $R' = R + \nu_1 - \nu_0$; and the chemical potential is

$$\mu = -k_B T \ln\left[\frac{2}{n_3}\left(\frac{m_3^* k_B T}{2\pi\hbar^2}\right)^{3/2}\right].$$

The rate K of ^{3}He absorption per unit ^{3}He number density n_3 can be written

$$K = \alpha \bar{v}_{\mathrm{rel}} p(T) \sigma_g,$$

where σ_g is the geometrical cross section of the ion and α is a constant representing the probability that a ^{3}He reaching the surface of the ion will in fact be trapped. The rate will be weakly temperature dependent because of the change with T of the thermal velocity distribution of ^{3}He quasiparticles in the liquid. This affects K in two separate ways. First, the average relative velocity $\bar{v}_{\mathrm{rel}}$ between ion and ^{3}He will decrease slightly with decreasing T. Secondly, the proportion $p(T)$ of ^{3}He quasiparticles with energy sufficient to surmount the small potential barrier[16] that exists close to the ion, and thus reach its surface, will also decrease as T becomes smaller. The slight decrease in ν' with decreasing T observed in relatively strong electric fields (Fig. 3) may thus be attributed to the temperature dependence of K.

We have found that the simple model outlined above can be made to fit the data closely over a very wide parameter range. In carrying out the fitting procedure (which will be described in detail elsewhere), we have treated E_B, ν', and R'/α as adjustable parameters, with the latter two quantities being functions of E but not of T. The full curves of Figs. 2 and 3 represent examples of fits to the data generated in this way. We find that $E_B/k_B = -2.83 \pm 0.17$ K and that $\nu_1(E)$ takes the values shown in Fig. 4. We have also ascertained the effect of fitting the data by a more sophisticated variant of the model (model B) in which $R(E)$ is required to vary with l so as to make explicit allowance for the (expected) variation with l of the critical velocity for ^{3}He emission with rotons. No additional adjustable parameters are thereby introduced, but the quality of the the fit is slightly improved. We then find that $E_B/k_B = -2.52 \pm 0.09$ K, and that $\nu_1(E)$ takes the same form as shown in Fig. 4 but scaled up by a factor of 1.3. Although the derived value of E_B is to some extent model dependent, it is reassuring to note that, as might have been anticipated, both of the values obtained are comparable with, but somewhat larger than, the value of -2.22 ± 0.03 K deduced[17] for ^{3}He atoms binding to a (plane) He II surface at zero pressure.

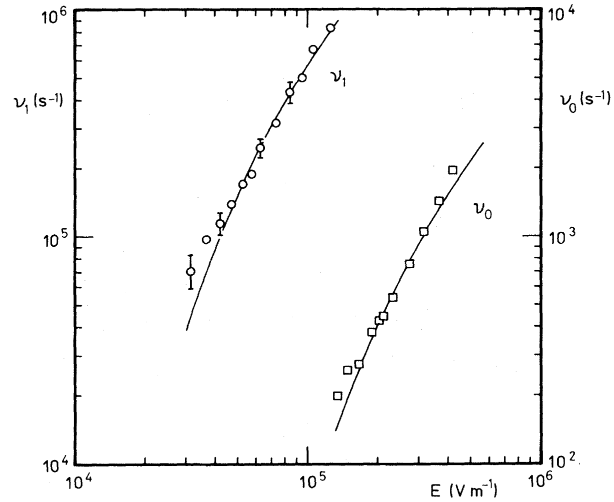

FIG. 4. Values of ν_1, the vortex nucleation rate for ions with one bound ^{3}He atom, plotted as a function of electric field E. For comparison, the rate ν_0 for bare ions is also shown (with a different ordinate scale). The full curves represent fits to $\nu_1(E)$ and $\nu_0(E)$ of the model proposed in Refs. 6 and 14.

The excellent agreement between the model and the data (Figs. 2 and 3), taken in conjunction with the fact that the fitted value of E_B is reasonable, can be regarded as strong support for the main premises of the model. We have also found that, with a small extension, the model is able to account satisfactorily for the observed form of $\nu''(E,T)$. Full details of the calculations, together with a very much more extensive set of experimental data, will be presented elsewhere.

The fitted values of $\nu_1(E)$ are, as expected, considerably larger than $\nu_0(E)$ measured for the bare ion, which are also shown for comparison in Fig. 4. The functional forms of $\nu_1(E)$ and $\nu_0(E)$ are, however, evidently rather similar. A detailed analysis of $\nu_1(E)$ for weak E, closely following our earlier treatment[6,14] of $\nu_0(E)$, leads to the conclusion that the critical velocity for vortex nucleation by an ion with one trapped ^{3}He atom is circa 56.5 m s^{-1} (56.9 m s^{-1} for the simple model and 56.1 m s^{-1} for model B); which is to be compared to 60.7 m s^{-1} previously determined[6] for a bare ion. The corresponding rate constant is found to be larger, by a factor of circa 10^3, than that for a bare ion.

It is interesting to note that our results are consistent, at least qualitatively, with the viewpoint that vortex nucleation occurs through a form of quantum tunneling[16,18]: If the trapped ^{3}He atom were to be incorporated into the core of the nascent vortex, then one would expect a corresponding reduction[19] in the height of the energy barrier[5] that impedes the nucleation process, thereby reducing the critical velocity and increasing the rate constant. Further calculations are now required to establish whether or not such an effect could account quantitatively for the 4 m s^{-1} reduction in the critical velocity deduced from the present experiments.

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