Moss *et al.* Respond: We differ from Brand in the level of description on which the analogy between bifurcations in nonequilibrium systems and phase transitions in equilibrium systems is established. If the stationary probability density of a nonequilibrium system, usually obtained from a Fokker-Planck equation or a Master equation, is written as

$$p = N \exp(-F), \tag{1}$$

where N is the normalization constant, then F can be interpreted as a type of thermodyamic potential, as first suggested by DeGiorgio and Scully,[1] and by Graham and Haken.[2] This viewpoint has been subsequently developed by various authors and has led to useful analogies between transitions in far-from-equilibrium systems and the Landau theory of equilibrium phase transitions. For a detailed presentation of this analogy see the work of Haken.[3] If f is interpreted as somewhat analogous to a thermodynamic free energy, then the state of the nonequilibrium system will be described by the minima of F, i.e., the maxima of p. This seems to us to be a useful viewpoint, since the most probable values of p, i.e., the peaks of the distribution, are the values most likely to be observed in a trail experiment.[4] In experiments on optical bistability and bistability in chemical systems a hystersis phenomenon is observed. This corresponds to the observation of the most probable values of p, since the moments of p are unique and do not show any hysteresis.

The above viewpoint on the analogy between nonequilibirum bifurcations and equilibrium phase transitions has been most succintly stated by Mandel, Roy, and Singh[5] in the context of quantum optics: "If we regard the laser field as a thermodynamic system that undergoes a phase transition as the pump parameter is varied, we may look on the most probable value of the field—or of the light intensity I—as an order parameter for the phase transition.[1,3] [. . .] Moreover, if we substitute the most probable value of the light intensity I_m for I in the potential F, we obtain a quantity, the most probable potential F_m, that is to some extent analogous to the free energy of a system in thermodynamic equilibrium."

Indeed, the choice of the most probable value of p fits the definition of an order parameter: "A mechanical variable which is undetermined, namely the density ρ in the liquid-gas case and the magnetization m in the ferromagnetic case, will be referred to as the order parameter.[6]" In the system studied by Smythe, Moss, and McClintock[7] a Landau-type description is obtained if the most probable value of p is interpreted as the analog of the order parameter for an equilibrium phase transition: Classical critical exponents are found as well as critical slowing down.[8] Thus on the level of the description first introduced in Refs. 1 and 2, the system of Ref. 7 looks like the Landau description of a second-order phase transition. Therefore, it seems useful to us to establish an analogy with equilibrium phase transitions on the level of the most probable values of p. This is in accordance with current usage in the literature, see for instance Refs. 1–3 and 5. We feel therefore that the name "noise-induced phase transition" is as justified and as useful as the name "nonequilibrium phase transition" for the laser and other far-from-equilibrium systems.

Finally, it should be noted that the transition described in Ref. 7 corresponds to a change from a single-humped to a double-humped probability density. A transition to bistable behavior is hardly a subtle detail. The macroscopic physical properties of the system are distinctly different for the two cases.

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Received 28 November, 1983
PACS numbers: 64.60.Fr, 05.40.+j

[1]V. DeGiorgio and M. O. Scully, Phys. Rev. A **2**, 1170 (1970).

[2]R. Graham and H. Haken, Z. Phys. **237**, 31 (1970).

[3]H. Haken, *Synergetics, An Introduction* (Springer, Berlin, 1978), 2nd ed.

[4]A. Schenzle and H. Brand, Phys. Rev. A **20**, 1628 (1979).

[5]L. Mandel, R. Roy, and S. Singh, in *Optical Bistability,* edited by C. M. Bowden, M. Ciftan, and H. R. Robl (Plenum, New York, 1981).

[6]Shang-Keng Ma, *Modern Theory of Critical Phenomena* (Benjamin, Reading, Mass., 1976), p. 4.

[7]J. Smythe, F. Moss, and V. E. McClintock, Phys. Rev. A **51**, 1062 (1983).

[8]W. Horsthemke, and R. Lefever, *Noise-Induced Transitions* (Springer, Berlin, 1984).

