Comment on "Stochastic Resonance in Bistable Systems"

In a recent paper,[1] Gammaitoni *et al.* described an important experimental and theoretical study of stochastic resonance[2] (SR) in a system simulating a Brownian particle which moves in a symmetric double-well potential. In particular, the remarkable enhancement by additive white noise of the signal-to-noise ratio R for weak periodic forcing of the system was investigated.

We would like to comment that a qualitative and quantitative understanding of the latter phenomenon, and of the dependence of R on relevant parameters, can be gained through an analysis of the power spectral density $Q(\omega)$ of the system in the *absence* of periodic forcing, which is already known (see Ref. 3, and references therein). SR is connected directly with a pronounced narrow peak in $Q(\omega)$ at $\omega=0$ resulting from noise-induced interwell transitions and with the corresponding peak in the susceptibility of the system $\chi(\omega)$. Such peaks are a universal feature[4] of bistable systems in the range of parameters where stationary populations of the stable states are close in magnitude.

In the presence of the weak periodic force $A\cos(\Omega t)$, the averaged coordinate of the symmetric system[1] $\langle x(t)\rangle = A\,\mathrm{Re}[\chi(\Omega)\exp(-i\Omega t)]$. Therefore in the power spectrum there arises a δ-shaped spike at frequency Ω [because, according to the principle of the decay of correlations, $\langle x(t)x(0)\rangle \to \langle x(t)\rangle\langle x(0)\rangle$ as $t\to\infty$; see also Ref. 5]. If R is defined[6] as the ratio of the strength of this spike to $Q(\Omega)$, then

$$R=\tfrac{1}{4}A^2|\chi(\Omega)|^2/Q(\Omega),\quad \chi(\omega)\equiv\chi'(\omega)+i\chi''(\omega) \tag{1}$$

[where $R(A,D)$ of Ref. 1 is equal to $R/\pi\Delta\nu_{\rm expt}$]. For the "quasithermal" system under consideration, $\chi(\omega)$ can be expressed easily via the fluctuation-dissipation theorem in terms of $Q(\omega)$ and of the noise-intensity parameter D corresponding[1] to the temperature of the Brownian motion:

$$\chi'(\omega)=\frac{2}{D}\int_0^\infty d\omega_1[\omega_1^2/(\omega_1^2-\omega^2)]Q(\omega_1)\,,$$
$$\chi''(\omega)=(\pi\omega/D)Q(\omega)\,. \tag{2}$$

When only the peak caused by interwell transitions is taken into account in $Q(\omega)$, Eqs. (1) and (2) and Eq. (22) of Ref. 3 result in $R=\frac{1}{2}\pi A^2x_m^2\mu_K/D^2$, where x_m is the equilibrium value of the coordinate and the transition probability $\mu_K\propto\exp(-\Delta V/D)$ (ΔV is the height of the potential barrier; we suppose that $\Delta V\gg D$). This expression coincides with Eq. (5) of Ref. 1 for small A and $D/\Delta V$, and it gives explicitly the dependence of R on D. We stress, however, that within the domain of linear-response theory it is, in principle, valid for *arbitrary* Ω/μ_K, not only for small Ω/μ_K as implied in Ref. 1. In practice, the limitations on the range of applicability come from the other contributions to $Q(\omega)$ discussed in Ref. 3 and it is the latter that give rise to the deviations in R observed in Ref. 1 for $\Omega/2\pi=500$ Hz.

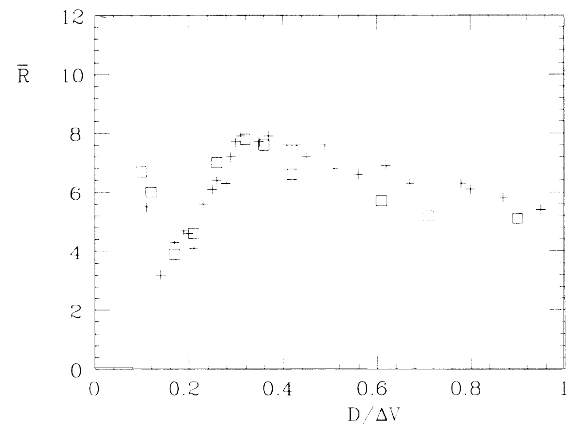

FIG. 1. Comparison of experimental (squares) and calculated (crosses) values of $\bar{R}=1.54\times10^3R$, as a function of reduced noise intensity $D/\Delta V$, for an electronic model of the system $\ddot{x}=-\gamma\dot{x}+x-x^3+A\cos(\Omega t)+V(t)$, with $\Omega=0.06952$, $A=0.1$, $\gamma=0.25$, $\langle V(t)\rangle=0$, and $\langle V(0)V(t)\rangle=2\gamma D\delta(t)$.

Measured values of R (squares) are compared with those calculated from $Q(\omega)$ (crosses) in Fig. 1. The calculation contains no adjustable parameters.

We acknowledge valuable discussions with C. J. Lambert and R. J. Watts-Tobin.

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Received 16 October 1989
PACS numbers: 05.40.+j, 02.50.+s

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