Journal of Physics: Conference Series **150** (2009) 032056 doi:10.1088/1742-6596/150/3/032056

Energy cascades and rogue waves in superfluid ^{4}He

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Abstract. We report the results of recent experiments on second sound in a high Q resonator. Nonlinear wave interactions give rise to a form of acoustic turbulence in which there is a Kolmogorov-like cascade of energy through the frequency scales from the frequency at which it is injected towards higher frequencies until, eventually, dissipative processes come to dominate and terminate the energy flux. We report that that, under some circumstances, an inverse energy cascade occurs, whereby there is a steady flux of energy towards *lower* frequencies, a wholly unexpected result that bears on the suggestion that rogue waves on the ocean may arise through nonlinear interactions between conventional wind-blown waves.

1. Introduction

The phenomenon of *wave turbulence* (WT) crops up in a diversity of physical contexts, including e.g. phonon turbulence in solids [1], turbulence of sound waves in oceanic waveguides [2], shock waves in the solar wind and their coupling with Earth's magnetosphere [3], and magnetic turbulence in interstellar gases [4], as well as surface waves on liquid hydrogen [5] and second sound in superfluid ^{4}He [6]. In each case, the system is in a highly excited state with many degrees of freedom – just like the more familiar vortex turbulence. The generally accepted picture is that, just as in the case of vortex turbulence, there is then a directed flux of energy through rising frequency scales due to nonlinear wave interactions [7]. These result in a cascade of energy towards ever-shorter wavelengths until, eventually, it can be dissipated by viscous effects. But there have been few opportunities to test these ideas in a laboratory setting.

In this paper we discuss the use of He II as a modeling medium for the study of WT and report our most recent results, some of which have been unexpected and serve to show that the conventional Kolmogorov scenario [7] is incomplete. The particular advantages of using second sound (temperature-entropy) waves for modeling WT are its large nonlinearity coefficient α and the fact that the magnitude and sign of α can conveniently be adjusted by changing the temperature T [8]. The velocity u_2 of second sound is given to a good approximation by

$$u_2 = u_{20}(1 + \alpha\delta T), \tag{1}$$

where u_{20} is the second sound velocity at negligibly small amplitude. The nonlinear wave interactions leading to WT are expected to become important at large amplitudes, and these are readily attained by use of a resonant cavity. Given the small bulk attenuation of second sound, and the correspondingly large values of the quality Q-factor for cavity resonance, high second sound amplitudes can be created for relatively modest heat inputs to the He II sample.

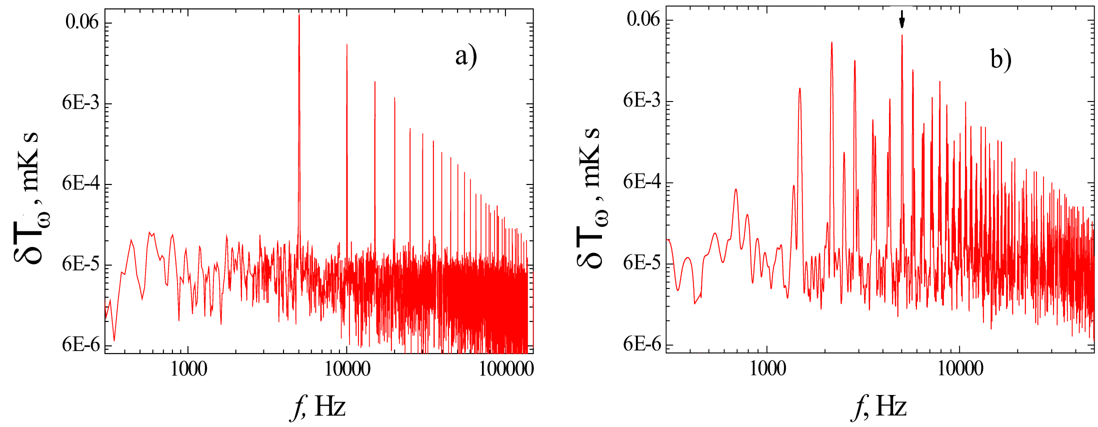

Figure 1. Instantaneous second sound spectra measured (a) 1 s after the drive was applied at a frequency slightly detuned ($\delta\omega_d/\omega_d \sim 10^{-4}$) from the 51st resonance, and (b) 70 s later. The helium sample temperature was close to 2.08 K. The AC heat flux density was $W = 20$ mW/cm^2. The drive frequency corresponds to the highest peak in (a) and is arrowed in (b).

2. Experiments on second sound acoustic turbulence

Our experimental cell has been described elsewhere [6] but, in essence, it consists of a quartz cylinder of length $L = 7\,\text{cm}$ capped by two endplates carrying a thin film heater and a superconducting bolometer, respectively. The heater was driven from a harmonic voltage generator within the frequency range $0.1 < \frac{\omega_g}{2\pi} < 100\,\text{kHz}$. The amplitude δT of the resultant standing wave of second sound (at $\omega_d = 2\omega_g$) could be varied from 0.05 mK up to a few mK. The Q factor of the resonator, determined from the resonant width in the linear regime at small amplitude, was $Q \sim 3000$ for resonance numbers between 30 and 100, but fell to $Q \sim 1000$ for resonance numbers below 10.

When the cavity is excited at sufficiently high amplitude close to a resonance, second sound acoustic turbulence is created as described previously [6], and there is then a flux of energy from ω_d towards higher frequencies. We now discuss what happens if the cell is detuned slightly away from resonance, a typical detuning being $\delta\omega_d/\omega_d \sim 10^{-4}$. There are two main results to report. First, in addition to the forward energy cascade to frequencies $\omega \geq \omega_d$, we observe the onset of a flux of energy from ω_d towards *lower* frequencies. Secondly, this inverse flux does not appear immediately, but seems to take a finite time to become established. Fig. 1 shows some typical results: in (a) one second after application of the drive, the forward energy flux is well-established and looks very similar to the results reported earlier [6]; but a minute later the situation looks very different, as shown in (b). In the latter case, a series of peaks have appeared at frequencies lower than ω_d. There is strong evidence that these represent a continuous flux of energy corresponding to the formation of an inverse energy cascade. Fig. 2 (left) shows what happens to the energy in the high-frequency ($\omega \geq \omega_d$) and low-frequency ($\omega < \omega_d$) regimes following a small change in driving frequency at $t = 19\,\text{s}$ to populate the low frequency spectra: there is a clear reduction in the flux of energy to the right, implying a sharing of energy between two fluxes, and thus the onset of a flux to the left. Fig. 2 (right) shows a time domain signal and how it evolves: isolated rogue waves (of amplitude larger than nearby waves) appear, as shown more clearly in the inset.

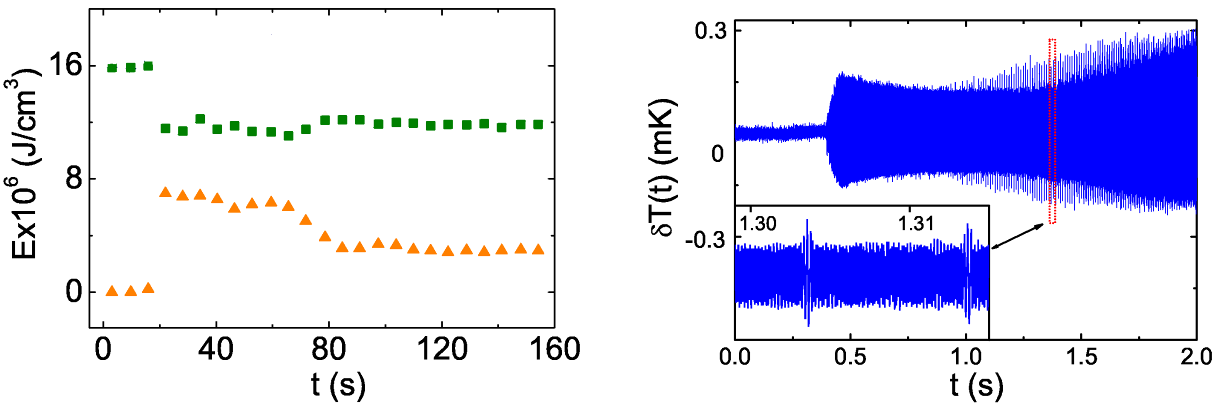

Figure 2. Left: dependence on time of the wave energy contained in the low-frequency (triangles) and high-frequency (squares) spectral domains after a change in the driving frequency at $t = 19\,\mathrm{s}$. Right: evolution of a signal after switch-on at $t = 0.4\,\mathrm{s}$; the inset is a blow-up to illustrate a pair of isolated rogue waves.

3. Discussion

It would appear that, under certain conditions, the second sound wave undergoes a nonlinear decay into two waves of lower frequency governed by the conservation law [9]

$$\omega_1 = \omega_2 + \omega_3. \tag{2}$$

Here $\omega_1 = u_{20}k_1$ is the frequency of a linear wave of wave vector k_1. This instability results in the generation of subharmonics which, for sufficient amplitude, can give rise to the inverse cascade. We have tested this idea by direct numerical integration of the 2-fluid thermohydrodynamical equations [10], expanded up to quadratic terms in the wave amplitude. We use a Hamiltonian representation of the second sound waves [6, 11] similar to that described earlier [12]. However, we take explicit account of wave damping at all frequencies, which turns out to be essential for a correct description of the generation of subharmonics. To correspond with the experimental conditions, we have assumed full reflection of the waves from the resonator boundaries, rather than periodic boundary conditions [12]. The results in Fig. 3(a) show that, as the instability develops, isolated rogue waves appear in the signal (cf. Fig. 2 right). As time evolves, they appear more frequently. At the final stage (Fig. 3(b)), the initially harmonic wave has become grossly distorted (inset) and its period has increased substantially owing to subharmonic generation (main figure) with several spectral peaks appearing at frequencies below that of the driving force (arrowed). Such decay instabilities (especially threshold and near-threshold behaviour) have also been studied for e.g. spin waves [13, 14, 15], magnetohydrodynamic waves in plasma [16], and interacting first and second sound waves in superfluid helium near the superfluid transition [17]. A quite similar parametric process, but caused by 4-wave scattering (modulation instability), is thought to be responsible for the generation of large wind-driven ocean waves [18]. In this sense, our demonstration of leakage of energy towards lower frequencies can be regarded as evidence in favour of the suggestion by Dyachenko and Zakharov that rogue waves on the ocean build up through nonlinear wave interactions between ordinary wind-blown waves [18], which is an alternative to the recent suggestion [19] based on optical solitons that rogue waves may arise from scattering of nonlinear waves on a continuous noisy background.

We note that there are other contexts in which inverse energy cascades have been considered, including 2-dimensional incompressible liquids and Bose gases [20], and quantized vortices in He II [21, 22]. To our knowledge, however, the present work represents the first experimental observation of an inverse energy cascade in acoustic wave turbulence.

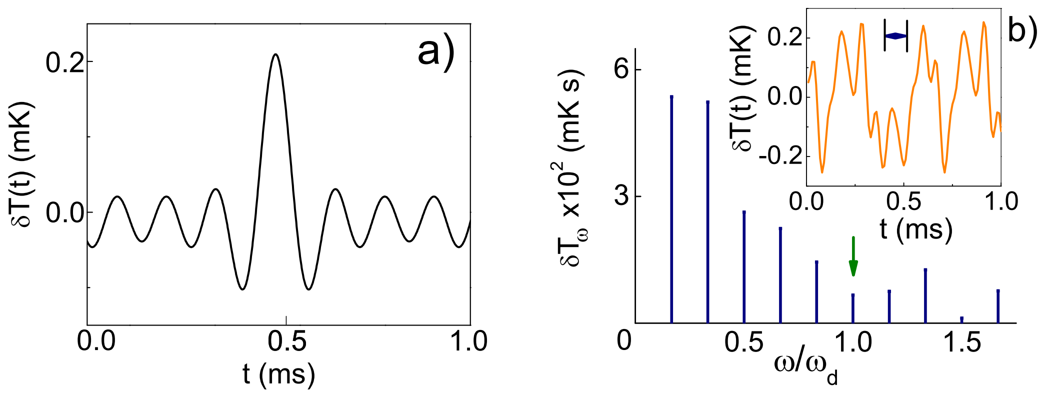

Figure 3. (a) Numerical result showing a single rogue wave formed on the second-sound wave profile $\delta T(t)$ in a high Q resonator during development of the instability for $W \sim 40$ mW/cm^2, i.e. above a critical value. (b) Calculated steady-state second-sound power spectrum δT_ω established after the instability develops. The vertical arrow labels the fundamental peak at $\omega = \omega_d$. Inset: the calculated wave shape corresponding to the spectrum shown in the main plot. The bars indicate the fundamental period of the wave.

4. Conclusion

In summary, we have exploited the special features of second sound to demonstrate that an inverse energy cascade can exist in acoustic wave turbulence, carrying energy towards lower frequencies.

The work was supported by Engineering and Physical Sciences Research Council (UK), the Russian Foundation for Basic Research, project No. 07-02-00728, and by the Presidium of the Russian Academy of Sciences under the programs "Quantum Macrophysics" and "Mathematical Methods in Nonlinear Dynamics".

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