

# Transparency and Simplification of Rule-Based Models for *on-line* Adaptation

**R. A. Buswell**

Dept. Civil and Building Eng.,  
Loughborough University,  
Loughborough, Leicestershire,  
LE11 3TU, UK.

e-mail: r.a.buswell@lboro.ac.uk

**P. P. Angelov**

Dept. Civil and Building Eng.,  
Loughborough University,  
Loughborough, Leicestershire,  
LE11 3TU, UK.

e-mail: p.p.angelov@lboro.ac.uk

**J. A. Wright**

Dept. Civil and Building Eng.,  
Loughborough University,  
Loughborough, Leicestershire,  
LE11 3TU, UK.

□e-mail: j.a.wright@lboro.ac.uk

## Abstract

One of the principal advantages of fuzzy rule based models over *black-box* approaches such as Neural Networks or polynomial models is transparency. The linguistic concept associated with the membership functions related to measured variables results in rules that are 'readable'. This quality is useful in analysing the functionality of processes through the model generated by data mining techniques. The greater the number of rules and the less descriptive the linguistic terms, the less transparent the model. The fewer rules, however, inevitably reduces the model precision with respect to the modelled process. This paper investigates the properties of Takagi-Sugeno models with either a linear function or singleton consequent with respect to model precision and transparency. The study is focused on a 'steady-state' heat-exchanger model applied to the air-cooling process commonly found in heating, ventilating and air-conditioning (HVAC) equipment. The similarity measures are suitable to application to the *on-line* generation of these models.

**Keywords:** FRB and TS models, identification, HVAC, modelling.

## 1 Introduction

The last few decades have marked an intensive development of alternative modelling techniques such as fuzzy rule-based (FRB) models, neural networks (NN) and hybrid versions of these approaches. A principal driving force for these efforts has been the desire to develop improved control of highly non-linear processes, where classical approaches [1] do not perform well. Several of these methods gained wide acceptance

and amongst them, the so-called Takagi-Sugeno (TS) FRB models [3].

TS models are constructed of many linear sub-models and have a predisposition for generating smooth function surfaces. This approach combines the flexibility to enable the representation of complex non-linear systems with simple identification procedures [4-7].

The identification of TS models can be solved by initially partitioning the input space using a clustering technique [5,6]. The parameters of the output sub-models can then be estimated by linear least squares when linear functions of the inputs describe the output space [4-6]. When the output variable is described using singletons, clustering can be used. Techniques such as gradient-based back-propagation and genetic algorithms (GA) [8] have been applied for simultaneous identification of both model structure and parameters. This approach has the advantage of a higher resultant model precision, but it is computationally more demanding.

These approaches are often called 'data-driven' or 'knowledge extraction' modelling techniques. Expert knowledge plays an insignificant, if any, role in the model generation process. An important issue is whether such automated techniques produce models that are readily interpretable. This paper considers the issue of transparency in conjunction with the precision of model predictions. TS models using both a singleton and a linear function consequent are investigated. The focus is on the modelling of a heat-exchange process. Emphasis is placed on minimising the number of rules and parameters in a model through model simplification. The simplification measures described are set in the context of continuing work based on a recently introduced approach for recursive *on-line* identification of TS models [4].

## 2 Parameter and structure adaptation of TS models

FRB models of TS type are considered [3],

$$R_i: \text{IF } (x_1 \text{ is } V_{i1}) \dots \text{AND } (x_n \text{ is } V_{in}) \text{ THEN } (y_i = \rho_i),$$

where  $i=1, 2, \dots, N$ .  $R_i$  denotes the  $i^{\text{th}}$  fuzzy rule of which there are  $N$  number.  $V_{ij}$  denotes  $j^{\text{th}}$  linguistic variable of the antecedent part for the  $i^{\text{th}}$  fuzzy rule ( $j=1, 2, \dots, n$ ),  $y_i$  is the output of the  $i^{\text{th}}$  rule;  $\rho_i$  represents the output function. This can either be a singleton (a constant) or linear function of the input variables.  $x$  is the input vector  $x=[x_1, x_2, \dots, x_n]^T$ . The model output is calculated by aggregating individual rules' contribution and applying centre of area defuzzification operator [5]. This model can be formulated such that it is able to adapt to changes in the modelled object [4]. The model can be described as self-learning. It is desirable to learn new features of the process rather than simply retrain a model at given intervals for reasons of computational efficiency. TS models are a promising candidate for the solution of this problem. A procedure for *on-line* recursive identification of TS models has been developed in [4]; The procedure consists of:

- calculation of the potential of new data points to form a new or replacement rule;
- recursive up-date of the potential measure of the existing membership functions centres;
- recursive up-date of the reference potential that controls the rule update process;
- recursive estimation of parameters of the consequent part of the rules.

The FRB model is generated and is used as an initial estimation of the non-linear mapping between inputs and the output(s). The precision of the model can be improved by the application of a GA.

### 3 Model structure simplification

Generating rule bases using subtractive clustering [6] can lead to many similar membership functions being generated in the data space. This is not always adequately controlled by the adjustment of the clustering neighbourhood parameter. The existence of effectively redundant membership functions reduces the transparency of the model. Since the membership functions (derived using this technique) are of the same type and have the same spread, the similarity can be measured using the proximity of the membership function centres. One parameter,  $\gamma$ , (a decimal percentage of the input space,  $\gamma = 0.1$  here) describes the similarity. Membership functions that are 'close' to others are removed in accordance with this criterion. The rule base is not altered, but

the references to the membership functions in affected rules, are rewritten accordingly.

One drawback with the linear function mapping of the output space is that there is one function per rule. This cannot be reduced and so there is an inevitable loss of transparency. Initially, there are also the same number of output singletons as rules. Since these are single values, however, the similarity measure applied to the inputs can be applied here. This can increase the transparency significantly, although it requires an additional similarity parameter,  $\lambda$ , (expressed as a decimal percentage of the output space,  $\lambda = 0.01$  here). In general, acceptable precision in the model predictions will require more 'membership functions' to describe the output variable than is needed to describe the input variables, and so  $\lambda < \gamma$ .

### 4 The Heat-Exchanger Model

Figure 1 demonstrates the critical fluid quantities and properties associated with the air to water heat exchanger commonly found in HVAC systems. Cooling and dehumidification of the air approaching the coil is an important process in terms of the comfort of the occupants of air-conditioned spaces.

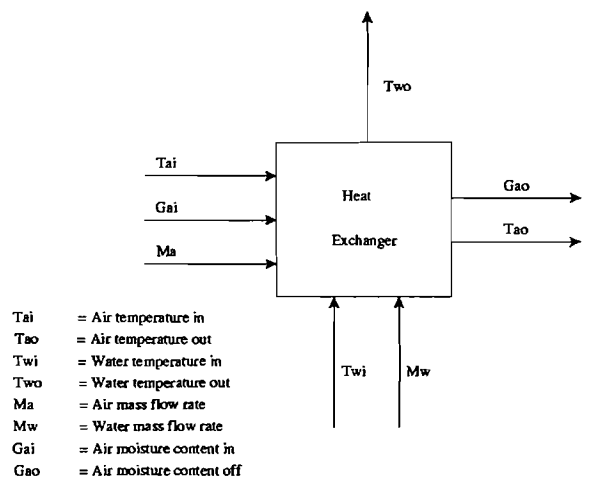


Figure 1: The heat exchanger process.

One widely used, first principles based, steady-state model of this process is based on the  $N_w$  method first introduced by Carrier [2]. The model estimates the heat transferred between the water and air based on the velocities of the fluids and the resistance of the heat-exchanger to heat transfer. The mass transfer that occurs as the coil dehumidifies the air is also modelled. This model has been used to generate the data on which to investigate the two types of TS model. The data used for training the models and for

validation, were generated by stepping each of the input variables over predefined ranges. Figure 2 details the training data used.

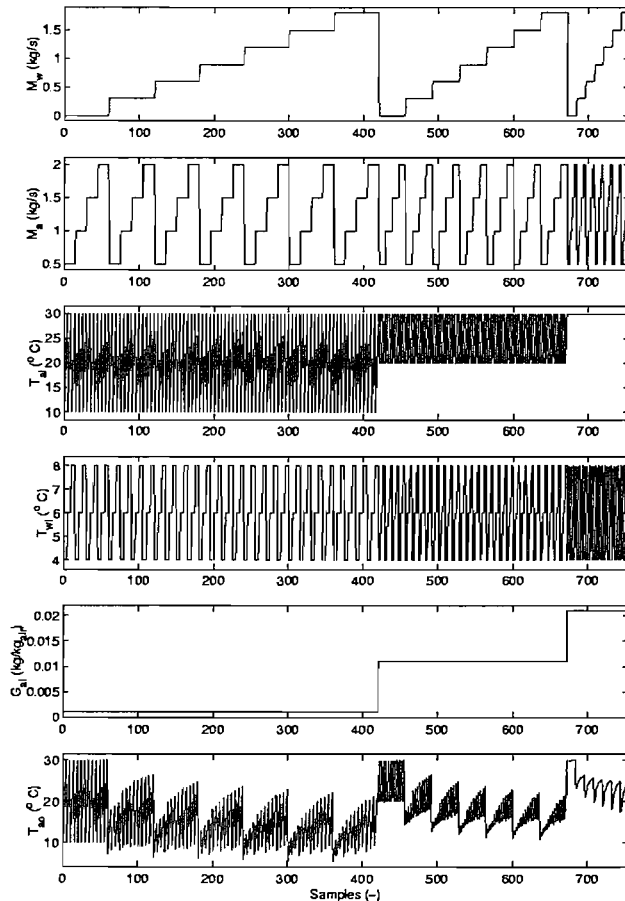


Figure 2: Data used for model characterisation.

#### 4.1 Labelling Membership Functions

The cooling coil shown in Figure 1 has five input variables, water mass flow rate,  $m_w$  (kg/s), air mass flow rate,  $m_a$  (kg/s), air temperature onto the coil,  $T_{ai}$  (°C), humidity of the air onto the coil,  $G_{ai}$  (kg/kg<sub>air</sub>) and the temperature of the water into the coil,  $T_{wi}$  (°C). The controlled variable in the real process is usually the air temperature off the coil,  $T_{ao}$  (°C). This has been selected as the model output.

In order to evaluate the transparency of the models, it is necessary to predefine the linguistic terms associated with the input and output variables. Once the centres of the membership functions have been found, the parameter value is compared to the predetermined range and the appropriate linguistic term is ascribed. The decision as to which term to use is on the basis of crisp rules defining the mid-way point between consecutive values. Table 1 gives a summarised version of the ranges applicable to the results in this work. The linguistic labels; ‘u’, ‘l’, ‘f’,

‘q’, ‘r’ and ‘v’ refer to, ‘ultra’, ‘little’, ‘fairly’, ‘quite’, ‘really’ and ‘very’, respectively; and ‘L’, ‘H’ and ‘M’ refer to ‘low’, ‘high’ and ‘medium’.

Table 1: Variable ranges and linguistic terms.

$m$	0.0	0.4	0.6	1.0	1.2	1.6	1.8	2.0
	No	vL	L	IH	fH	QH	rH	vH
$T_a$	0.6	0.9	1.2	1.5	1.8	2.1	2.4	3.0
$\times 10$	VL	L	M	IH	fH	H	qH	vH
$T_w$	4.0	6.0	8.0		$G_{ai}$	0.5	1.0	2.0
	VL	L	M		$\times 10^{-2}$	uL	L	qH

The key point is that the inlet and outlet temperatures are labelled on the same scale (as are the mass flow rates). These values are therefore directly comparable in the analysis of the rules.

#### 5 Model structure analysis

In principle, it is desirable to model the process with as few rules as possible. This decreases the computational demands, reduces the number of parameters required and makes the model more comprehensible (increases transparency). Figure 3 shows the effect that the number of rules has on model precision for the linear function and singleton models (based on the data in Figure 2). The figure demonstrates the increased precision produced by the former approach. The improvement of precision significantly reduces after about 14 rules.

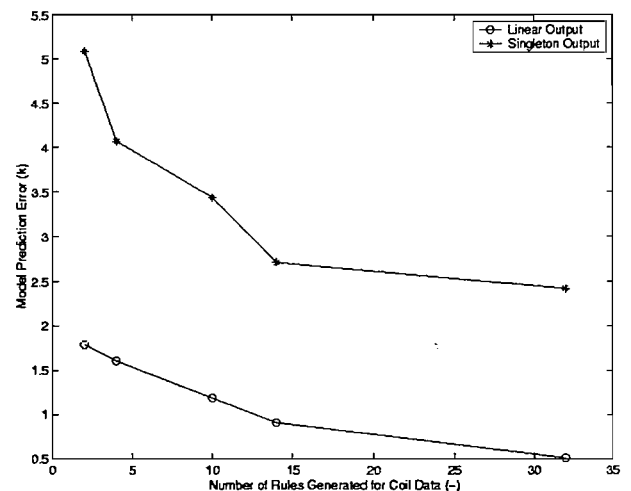


Figure 3: Number of rules and model precision.

Figure 4 demonstrates the prediction errors generated by both models with this number of rules. Significantly, the prediction errors in validation demonstrate that the singleton models give comparable performance. It should also be noted that the number of rules in the linear output function

version cannot be increased passed ~30 rules with this data because of numerical problems in the least squares estimation of the output parameters.

The simplified singleton model only requires 35 parameters, a reduction of 58% on the initial model configuration. The initial singleton model generated an RMSE of 1.82K in validation. The simplified singleton model demonstrates a 67% reduction in the number of parameters compared to the linear output function model. In addition, the singleton model is less prone to over fitting and does not have the problems associated with least squares estimation when the number of data are small compared to the number of parameters that require estimation.

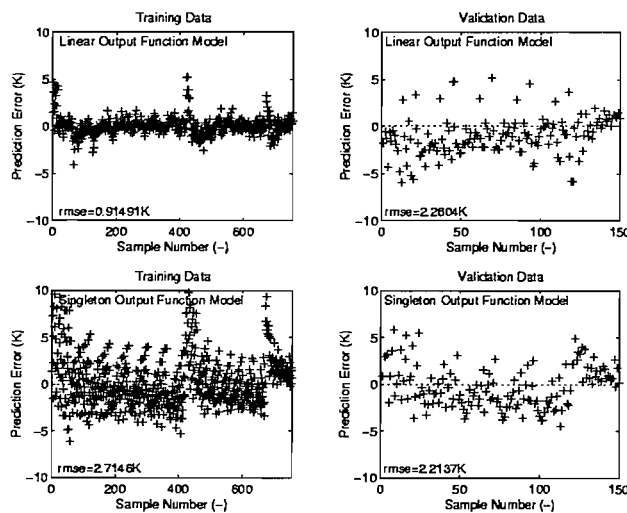


Figure 4: Model prediction errors.

The output of the linear output function is given by,

$$T_{ao} = a_i m_a + b_i m_w + c_i T_{ai} + d_i T_{wi} + e_i G_{ai} + f_i,$$

where  $a \rightarrow f$  are the estimated coefficients for each rule. For more than 2-dimensional input space this becomes difficult to readily understand. Figure 5 gives the rules for the singleton model used in Figure 4.

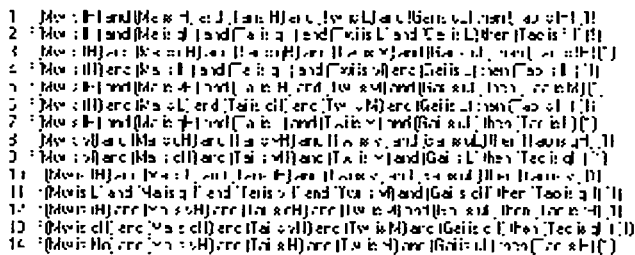


Figure 5: The singleton model rules.

All the rules in Figure 5 correctly describe the process. In particular: rule 14 demonstrates that

there is no change in temperature over the coil when there is no chilled water flowing through the coil; rules 10 and 5 show that a larger change in temperature across the coil occurs at lower air flow rates (due to the process characteristics, the difference in the chilled water mass flow rates are negligible and the other variables are nominally the same).

## 6 Conclusions

This paper investigates Takagi-Sugeno models with either a linear function or singleton consequent with respect to model precision and transparency. The study is focused on a the simplification of a 'steady-state' heat-exchanger model applied to the air-cooling process commonly found in HVAC equipment. The simplification method is suitable for application to TS models generated *on-line*.

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