Coherent radiation of ultra-short electron-bunches from linear-acceleration.

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We consider the radiation emitted by a purely longitudinally accelerated relativistic electron bunch and find contrary to usual expectation of negligible emission that for ultrashort compact bunches within reach of current experiments a significant and measurable radiation pulse is produced. The underlying mechanism for the enhanced emission is coherence in the single-electron emission over $\mathcal{O}(10^8)$ electrons. We derive analytic descriptions of the emission spectrum and energy for acceleration fields and bunch conditions encountered in RF-driven particle accelerators, and in the extreme high-gradient conditions of plasma-driven acceleration. Both bunch duration and the duration of the acceleration have a role in describing a phase-matching limitation to the coherence, with the latter dependent on the acceleration field-strength and the initial energy of the electrons.

In extremely compact bunches, just beyond that currently realised in the laboratory, the radiation losses approach the Larmor radiation for a massively-charged particle ($Q \sim 10^9 e$). In such conditions the implied energy of the longitudinal acceleration radiation may approach the energy available from the electric field driving the acceleration, raising questions of a connection to mechanisms of field depletion.

I. INTRODUCTION

The radiation emitted by charge during accelerated motion is a feature of electrodynamics that has greatly influenced the field of particle acceleration since its discovery[1–3]; synchrotron radiation in particular has been an extremely important research theme as both a major cause of energy loss or as a brilliant light source[4]. The interplay between radiation and intra-bunch electron energy gain or loss is integral to Free electron laser process, while incoherent and coherent SR have limiting effects in accelerator facility capabilities.

In these cases the emission is driven by transverse acceleration of the electron beam. The emission from longitudinal acceleration is rarely a subject of study , with emission expected to be measurably and physically negligible, by many orders of magnitude[3, 5]. In 2007, Geloni et al.[6] and Bosch [7] considered the back-reaction on a bunch due to the emission of transition radiation in the limit of instantaneous acceleration, and found that significant bunch energy losses are induced. Geloni also calculated the total radiated energy in this case.

In this article, it is shown that for highly compact femtosecond duration electron bunches the radiation released during purely linear acceleration may be significant and measurable due to a coherence in the single-electron emission [8, 9]. Sufficiently compact bunches are within reach of contemporary novel acceleration methods such as laser- or beam-driven plasma acceleration [10] [11–13], and in forefront x-ray free electron laser facilities. For conditions encountered in plasma accelerators, with bunch charges in the region of 10 pC - 100 pC, femtosecond duration, and micron transverse dimensions, radiative losses of 100's μJ may be produced. For conditions associated with RF accelerators with femtosecond-duration electron bunches, broadband coherent radiation with approximately 100nJ pulse energies is predicted.

In setting the scale of the potential coherent radiation effects, we consider Larmor [3, 5] power loss for a single electron, enhanced by N coherent emitters from an extremely compact bunch. The power radiated relative to the power supplied by the acceleration electric field $E_{\rm ext}$ will be, in the absence of a short-range field depletion,

$$\frac{P}{dU/dt} = \frac{Ne^3 E_{\rm ext}}{6\pi\epsilon_0 m_e^2 c^4} = 3.7\times 10^{-21}\left[\frac{\rm m}{\rm V}\right]\cdot N E_{\rm ext} \tag{1}$$

It follows from this (overly) simplistic calculation that for a bunch of charge 500 pC ($N = 3\times 10^9$) electrons and fields of $E_{\rm ext} \sim 100$ GVm^{-1}, the radiated energy would exceed that extracted from the accelerating field; inclusion of a collective 'radiation reaction' would be required for a self-consistent energy-conserving description.

Here, a comprehensive analysis of the coherent, far-field radiation spectrum of a linearly accelerated charge distribution is developed. The effects of radiation-electron velocity slippage, finite acceleration duration of acceleration, and the bunch 'form factor' are included in the analysis. Ultra-relativistic assumptions of electron velocity $v_e = c$ are avoided, even for highly relativistic electrons. We account for the change in energy and the changing radiation slippage over the finite duration of the acceleration, in contrast to [6],[7] whom only consider the transition radiation of charge exiting the accelerator.

The article is organised as follows; in section (II) the model of the linear accelerated charge distribution undergoing relativistic, hyperbolic motion and its radiation field is established. A derivation of the coherent power spectrum is given. Then in section (III) the generic features and scaling of the resulting radiation pulse are discussed. This is followed by numerical evaluation in the situations of extremely high field gradient acceleration such as in plasma accelerators (IV A), and acceleration of highly compact femtosecond bunches in RF (IV B) linear accelerators.

II. THEORETICAL DESCRIPTION

We consider a continuous charge distribution representing a bunch of total charge eN that is linearly accelerated between two times t_1 and t_2 by an externally applied field E_{ext} in a lab-frame. The bunch undergoes hyperbolic motion during linear acceleration[14, 15] and is assumed to be mono-energetic at the onset of acceleration. As all charge follows the same hyperbolic motion, the initial spatial distribution is maintained throughout the acceleration (this would not be the case if instead a finite spatial region for acceleration was assumed). The charge distribution may therefore be defined as $\rho(\mathbf{r}, t) \equiv \rho(\mathbf{r} - \mathbf{r}_0(t), \mathbf{r}_0(t)) = \rho(\mathbf{s}, \mathbf{r}_0(t))$ where $\mathbf{r}_0(t)$ is the trajectory of an arbitrary reference particle and $\mathbf{s} \equiv \mathbf{r} - \mathbf{r}_0(t)$ is independent of time.

Assuming only the acceleration term of the Lienard Weichert fields contributes to the far-field radiation, the radiation field can be written as the integration of the charge distribution over

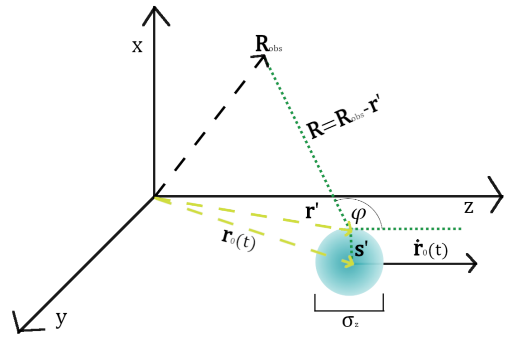


Figure 1. Definitions of the coordinate labels used throughout.

the retarded single-particle field[16].

$$\mathbf{E}(\mathbf{R}_{\text{obs}}, t) = \int d^3\mathbf{r}' \frac{\rho(\mathbf{r}' - \mathbf{r}_0(t^{\text{ret}}))}{4\pi\epsilon_0} \left[\frac{\hat{\mathbf{n}} \times (\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}}{c(1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta})^2 R} \right]^{\text{ret}} \tag{2}$$

where $\hat{\mathbf{n}}$ is the unit vector from the source to the observer, $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$ are the relativistic velocity and acceleration respectively, and R is the distance from the source to the observer $R = |\mathbf{R}_{\text{obs}} - \mathbf{r}'|$ and the 'ret' superscript denotes variables evaluated at the retarded time $t^{\text{ret}} = t - \frac{R}{c}$. The non-retarded time variable t here is the time as measured in the lab-frame by the observer. The integration over $d^3\mathbf{r}'$ is over the volume of non-zero charge density.

The far-field power spectrum in frequency space detected by an observer is [16] as

$$\frac{d^2U}{d\omega d\Omega} = \frac{\epsilon_0 c}{\pi} R^2 |\tilde{\mathbf{E}}_{\text{rad}}(\mathbf{R}, \omega)|^2 \tag{3}$$

$\tilde{\mathbf{E}}_{\text{rad}}(\mathbf{R}, \omega)$ is the Fourier transform of the electric component of the radiation field. If one considers a discrete collection of N charges distributed according to ρ then (3) would acquire a term accounting for the incoherent single particle emission spectrum[17]. Consequently, incoherent effects are encoded by the single particle spectrum. This term, which is proportional to N, is neglected because it is negligible in comparison to the coherent N^2 term in the high-charge ultra-short bunch duration regime.

In the Fourier domain, the radiation field becomes

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = \frac{1}{4\pi\epsilon_0}\int_{-\infty}^{\infty} dt^{\text{ret}} \int d^3\mathbf{r}'\rho(\mathbf{r}'-\mathbf{r}_0(t^{\text{ret}}))\cdot \left[\frac{\hat{\mathbf{n}}\times(\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}}{c(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^2 R}\right]^{\text{ret}} e^{i\omega\left(t^{\text{ret}}+\frac{R}{c}\right)} \tag{4}$$

where we have transformed to integration over the retarded time. We apply the usual far-field approximations, with $R \approx r + \hat{\mathbf{n}}\cdot\mathbf{r}'$ and $\hat{\mathbf{n}} \approx \mathbf{r}/r$ in the exponential exponent, while $R \approx r$ in the denominator, so that

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = \frac{e^{i\omega\frac{r}{c}}}{4\pi\epsilon_0 r}\int_{-\infty}^{\infty} dt^{\text{ret}} \int d^3\mathbf{r}'\rho(\mathbf{r}'-\mathbf{r}_0(t^{\text{ret}})) e^{i\omega\frac{\hat{\mathbf{n}}\cdot\vec{r}'}{c}} \left[\frac{\hat{\mathbf{n}}\times(\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}}{c(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^2}\right]^{\text{ret}} e^{i\omega t^{\text{ret}}} \tag{5}$$

We now apply the specific model conditions of longitudinal acceleration over a finite interval, with compact but finite bunch dimensions to the radiation spectrum of (5). For motion along the z-axis such that $\mathbf{r}_0(t) = (0, 0, z_0(t))$, the vector products in the square brackets reduce as follows:

$$\hat{\mathbf{n}}\cdot\boldsymbol{\beta} = \cos(\varphi)\beta \tag{6}$$

$$\begin{aligned}\hat{\mathbf{n}}\times(\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}} &= \hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\dot{\boldsymbol{\beta}}) \\ &= \dot{\beta}\cos(\varphi)\hat{\mathbf{n}} - \dot{\boldsymbol{\beta}}\end{aligned} \tag{7}$$

where φ is the angle between the path of a point in the distribution and the observer. The longitudinal acceleration, $\dot{\beta}$ is determined by the equation of motion, i.e. the relativistic Lorentz force for the external field E_{ext} acting on a point charge between times t_1 and t_2. For a time τ

$$\frac{d\beta}{d\tau} = \frac{eE_{\text{ext}}}{\gamma^3 mc} H(\tau - t_1)H(t_2 - \tau), \tag{8}$$

with the Heaviside functions $H(t)$ restricting the acceleration to the interval $t_1 \leq \tau < t_2$. To distinguish the time of interest from the observer time t we have introduced τ as a general lab-frame time coordinate. Solving the equation of motion for a point particle initially at $\mathbf{r}_0(0)$ with initial energy $\gamma_1 mc^2$ yields a solution that is hyperbolic in time, which we cast in the compact form

$$\begin{aligned}\mathbf{r}_0(\tau) &= (0, 0, \nu\left(\gamma(\tau) - \gamma_1\right)) + \mathbf{r}_0(0) \\ \gamma(\tau) &= \sqrt{1 + \left(\frac{c}{\nu}[\tau - t_1] + \beta_1\gamma_1\right)^2} \\ \beta(\tau) &= \frac{\left(\frac{c}{\nu}[\tau - t_1] + \beta_1\gamma_1\right)}{\sqrt{1 + \left(\frac{c}{\nu}[\tau - t_1] + \beta_1\gamma_1\right)^2}}\end{aligned} \tag{9}$$

for times $t_1 < \tau \leq t_2$. The bunch is drifting with constant energy for $\tau > t_2$. The parameter ν, defined as

$$\nu = \frac{mc^2}{eE_{\text{ext}}} \tag{10}$$

is a characteristic length scale of the acceleration. The length scale for typical acceleration gradients is shown in figure 2. We later consider the RF accelerator and high field-gradient plasma accelerator regimes, with length scales of a few millimeters and tens of microns, respectively.

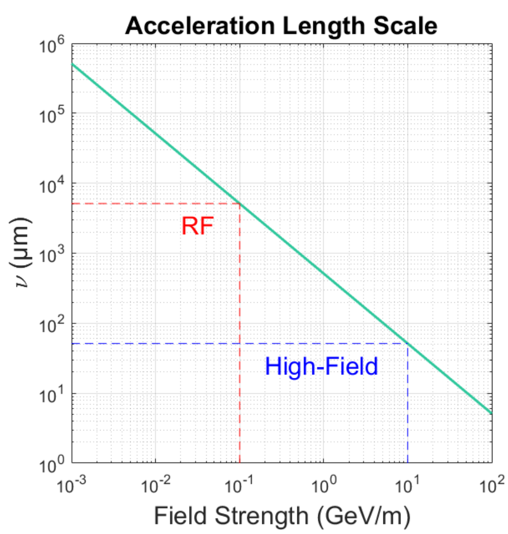


Figure 2. Characteristic acceleration length ν, the distance over which charge gains its rest mass-energy. Examples with acceleration gradients typical of RF and high-gradient plasma accelerators are considered.

Equations,(5)-(9) yield the far-field electric-field Fourier spectrum

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = \frac{ce^{i\frac{\omega r}{c}}}{4\pi\epsilon_0 \nu r}\int_{t_1}^{t_2} dt^{\text{ret}} \int d^3\mathbf{r}'\rho(\mathbf{r}'-\mathbf{r}_0(t^{\text{ret}})) e^{i\omega\frac{\hat{\mathbf{n}}\cdot\vec{r}'}{c}} \left[\frac{\cos(\varphi)\hat{\mathbf{n}} - \hat{\mathbf{z}}}{c(1-\cos(\varphi)\beta)^2\gamma^3}\right]^{\text{ret}} e^{i\omega t^{\text{ret}}} \tag{11}$$

where $\mathbf{r}_0$, γ and β are evaluated at the retarded time. The integrals within (11) can be separated by a transformation to a retarded "co-moving" coordinate system $\mathbf{s} = \mathbf{r}' - \mathbf{r}_0(t^{\mathrm{ret}})$. The resulting power spectrum can then be expressed as

$$\begin{aligned}\frac{d^2U}{d\omega d\Omega} =& \frac{c^2\sin^2(\varphi)}{16\pi^3\epsilon_0 c\nu^2}\left|\int d^3s'\rho(\mathbf{s}')e^{-i\frac{\omega\hat{\mathbf{n}}\cdot\mathbf{s}'}{c}}\right|^2 \cdot \\ & \left|\int_{t_1}^{t_2} d\tau \frac{e^{i\omega\left(\tau-\frac{\cos(\varphi)z_0(\tau)}{c}\right)}}{(1-\beta\cos(\varphi))^2\gamma^3}\right|^2 \\ =& \frac{c^2\sin^2(\varphi)}{16\pi^3\epsilon_0 c\nu^2}|\hat{F}(\varphi,\omega)|^2|\hat{S}(\varphi,\omega)|^2\end{aligned} \quad (12)$$

where in the final line we have introduced the complex form factors $\hat{F}$ and $\hat{S}$,

$$\hat{S}(\varphi,\omega) = \int_{t_1}^{t_2} d\tau \frac{e^{i\omega\left(\tau-\frac{\cos(\varphi)z_0(\tau)}{c}\right)}}{(1-\beta\cos(\varphi))^2\gamma^3} \quad (13)$$

$$\hat{F}(\varphi,\omega) = \int d^3s'\rho(\mathbf{s}')e^{-i\frac{\omega\hat{\mathbf{n}}\cdot\mathbf{s}'}{c}}. \quad (14)$$

The longitudinal form factor $\hat{F}$ describes the effect of coherent enhancement that occurs for a finite bunch length, with emission from different locations within the bunch (different $\mathbf{s}'$) collectively enhanced; the same form factor is encountered in descriptions of CSR and CTR emission [17–19].

The phase-slippage form factor $\hat{S}$ describes the coherent addition (or absence of it) of emission from a specific local part of the bunch occurring at different stages in the particles' trajectory; radiation emitted early in the acceleration region will advance in phase compared to radiation emitted from the same bunch location (same $\mathbf{s}'$) at a later region of acceleration. Phase slippage effects on bandwidth are similarly encountered in non-linear optics (for example in harmonic generation), and in free-electron laser resonance.

III. FEATURES OF RADIATION

A. Slippage Bandwidth

The power spectrum (12) describes the spectrum of coherent radiation emitted by charge undergoing truncated hyperbolic motion, with the slippage factor $\hat{S}$ encoding the spectrum of a point-charge, and form factor $\hat{F}$ accounting for the distribution of charge on the overall radiation spectrum. It follows then to proceed by first studying the factor $\hat{S}$ that describes the point-charge case. The integration in (13) cannot be carried out analytically, green such that we resort to numerical evaluation in III B. It is nonetheless possible to extract features of the radiation spectrum via approximation of the phase of the exponential in the integrand.

In the integrand of (13), the exponential describes the emission of waves emitted by a point charge at time τ, such that the emitted radiation at a given frequency is given by the superposition of waves of various phases $\phi(\omega,\varphi,\tau) = \omega(\tau - \cos(\varphi)z_0(\tau)/c)$ emitted at various times along the trajectory of the particle. If the frequency ω is sufficiently large, then the exponential oscillates rapidly within the limits $t \in [t_1, t_2]$ and the integral will become negligible, effectively setting a cutoff frequency for coherent emission. An estimate for the bandwidth ω_s can be obtained by averaging over phases over the trajectory, predicting an angle dependent bandwidth[16]

$$\omega_s(\varphi) \approx \frac{4\pi\gamma_{\mathrm{avg}}^2}{\Delta t[1+2\gamma_{\mathrm{avg}}^2(1-\cos\varphi)]} \quad (15)$$

where $\gamma_{\mathrm{avg}} \approx (\gamma_2+\gamma_1)/2$ is the average Lorentz factor. It can be supposed that the peak of the spectrum occurs around small angle $\varphi \sim 1/2\gamma_{\mathrm{avg}}$. Expanding $\cos\varphi \approx 1 - 1/8\gamma_{\mathrm{avg}}^2$,

$$\omega_s \approx \frac{16\pi\gamma_{\mathrm{avg}}^2}{5\Delta t} \quad (16)$$

Expressing γ_{avg} in terms of acceleration duration Δt and field strength E_{ext} reveals the scaling of the bandwidth

$$\omega_s \approx \frac{16\pi}{5}\left[\frac{\gamma_1^2}{\Delta t} + \frac{eE_{\mathrm{ext}}\gamma_1}{mc} + \frac{e^2E_{\mathrm{ext}}^2}{4m^2c^2}\Delta t\right] \quad (17)$$

From equation 17 we observe that the cut-off frequency arising from phase-slippage will initially decrease with acceleration time, followed by an increase for longer periods of acceleration. The latter increase in cut-off frequency is a consequence of the energy gain and a γ^{-2} dependence of the radiation-particle velocity mismatch.

B. Spatial Form Factor, $\hat{F}$.

For the form factor $\hat{F}$, which accounts for the spatial extent of the charge distribution, we con-

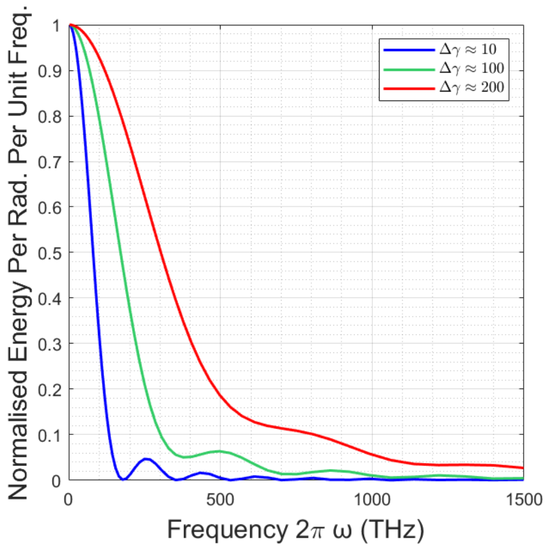


Figure 3. Spectra for point charge emission for low- and high-energy gain ($\Delta\gamma << \gamma_1$ and $\gamma_2 \sim 2\gamma_1$, respectively), at the peak emission angle $\varphi \sim 1/2\gamma_{\rm avg}$. The charge is accelerated over 3cm by field strengths of $E_{\rm ext} = 200$MeV/m and 2 GeV/m and 4 GeV/m respectively.

sider the example of a Gaussian distribution symmetric about the axis of motion

$$\rho(x,y,s) = \frac{eN}{(\sqrt{2\pi})^3\sigma_z\sigma_\perp^2}\exp\left(-\frac{s^2}{2\sigma_z^2} - \frac{(x^2+y^2)}{2\sigma_\perp^2}\right) \tag{18}$$

The form factor (14) in this case is

$$\hat{F}(\varphi,\omega) = eN\exp\left(-\frac{\omega^2\left(\sigma_z^2\cos^2\varphi + \sigma_\perp^2\sin^2\varphi\right)}{2c^2}\right) \tag{19}$$

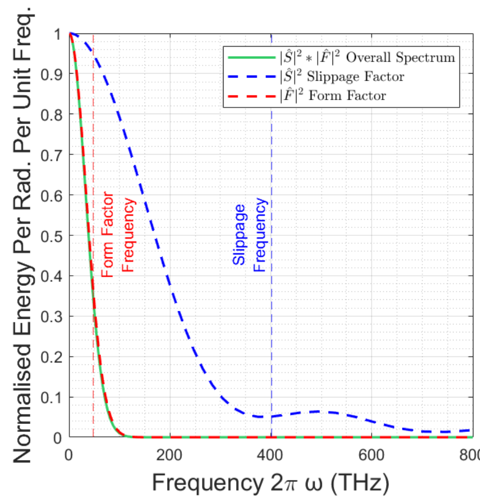
In the relativistic case, where emission will be at an angle $\varphi \ll 1$, the longitudinal contribution dominates and the charge distribution is able to be approximated as 1-Dimensional (i.e. a "line charge") and the form factor is simply a Gaussian function of the frequency,

$$\hat{F}(\varphi,\omega) \approx eN\exp\left(-\frac{\omega^2\sigma_z^2\cos^2\varphi}{2c^2}\right) \tag{20}$$

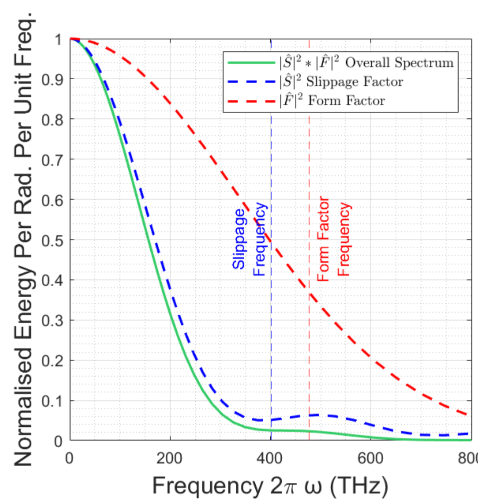
The form factor essentially acts as a cut-off for the coherent radiation spectrum with a characteristic bandwidth inversely proportional to the bunch length,

$$\omega_F = c/\sigma_z \tag{21}$$

The product of the form factor and slippage factor yields the overall spectrum of the accelerated bunch, with the characteristic frequencies of both factors determining the overall bandwidth of the emitted spectrum.



(a)



(b)

Figure 4. Normalised far-field emission spectra for bunch lengths of (a) 1μm, and (b) 0.1μm. for a 2GeV/m accelerating field acting over 3 cm, as well as an initial electron energy of 51 MeV, accelerating to 110MeV. The critical frequency ω_s associated with the phase slippage and is fixed in both cases.

In Figure 4 we show the relative influence of the longitudinal form factor and the slippage factor for two bunches with $\sigma_z = 1\,\mu$m and $\sigma_z = 0.1\,\mu$m

(3 fs and 0.3 fs duration, respectively).

From the overall frequency spectrum (Eqn. (12)), we observe that depending on the bunch length σ_z the coherence cut-off frequency may be determined by either the slippage factor $\hat{S}$ or the spatial form factor $\hat{F}$. We find that for

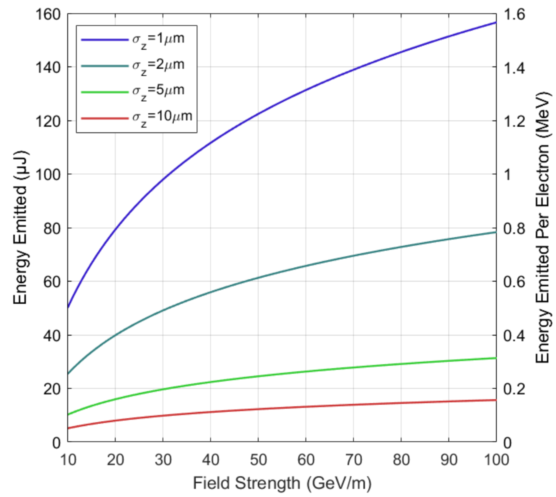
$$\sigma_z < \frac{5L}{8\gamma_{\mathrm{avg}}^2} \tag{22}$$

where L is the length of the accelerator, the emission is limited by the phase-slippage and the spectrum approximates that expected for a 'macro-particle' carrying charge eN.

IV. NUMERICAL ANALYSIS

A. High Field-gradient acceleration

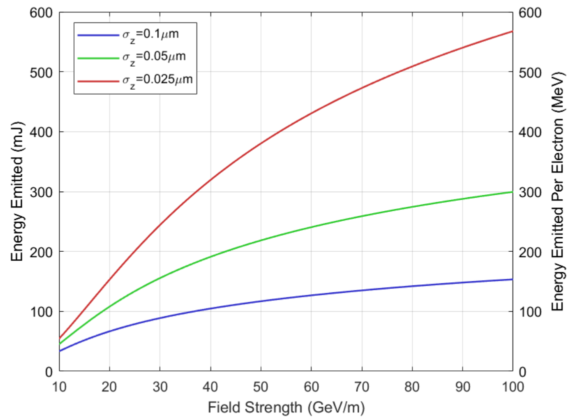
In the high field-gradient case, the radiation emitted for a variety of field-strengths $E_{\mathrm{ext}} \sim \mathcal{O}\,(10\mathrm{GeV/m})$ in an accelerator of length $L = 3\mathrm{cm}$ is considered, a parameter regime relevant to the current state of high performance plasma and laser accelerators[12, 13]. The emitted energy of a range of ultra-short bunch lengths ranging from $1-10\mu$m (duration 3 fs-33 fs) with charge 100pC is shown in Fig. 5.



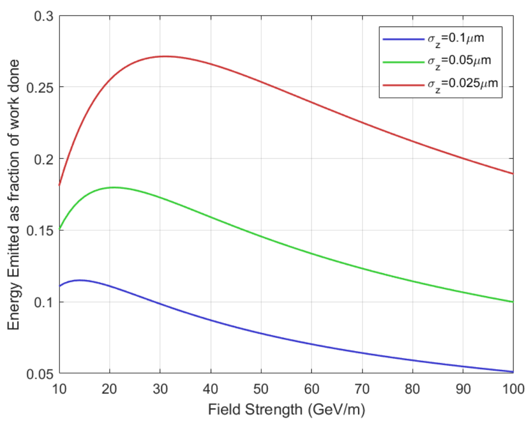
(a)

Figure 5. Radiative energy loss of ultra-short, 100pC electron bunches accelerated over an interval of 100 ps (3 cm). The initial electron energy was 51MeV ($\gamma_1 = 100$).

In this regime the scaling of the emitted energy with respect to bunch length is very sensitive, with the 1μm length bunch emitting an order of magnitude more energy than a bunch of length 10μm. Notably, the emissions are of the order $\mathcal{O}(10-100\mu\mathrm{J})$, an energy that is readily within reach of experimental observation. The emission scaling is far below the E_{ext}^2 quadratic scaling of Larmor point-charge emission, with the high frequencies of the power-spectrum are significantly curtailed by the form factor $\hat{F}$ and the $\omega_{\hat{F}} = c/\sigma_z$ cutoff. For shorter bunches it is expected that the emitted energy not only increases, but that the scaling with the externally applied field should approach Larmor's point-charge limit.



(a)



(b)

Figure 6. Radiative Emissions of sub-femto second duration, 1 nC bunch (a) Absolute value of losses overall and per-electron (b) Ratio of losses compared to work done. As the bunch shrinks in duration it begins to emit far more coherently and emits away $> 10\%$ of its kinetic energy at $\sigma_z < 0.1\mu$m, $E_{\rm ext} < 20$GeV/m.

The same spread of field-gradient values as the previous case is considered for acceleration of a very dense, charge 1 nC bunch ($N = 3.125 \times 10^{10}$). In this extreme regime it is observed (Fig. 6) that the emission is comparable to that delivered to the bunch from the external field. Such a situation points to a breakdown in the model assumption of a rigid bunch, and the necessity to consider the back-reaction of the emission process in the near-field. Indeed, in this near-field region the effects of space-charge in a high acceleration regime will lead to a modification in the effective field driving the acceleration.

B. RF acceleration gradients.

In an RF accelerator significantly less radiation is emitted overall compared to the High Field-Gradient case due to the reduced acceleration field strength, but the lower overall energy of the process leads to a slippage factor dominated spectral cut-off (Eqn. (22)), and the potential for observing the emission approaching the Lamour limit of coherent emission for a macro-particle. Figure 7 shows the emission energy as a function of bunch length and injection energy. The accelerating field is fixed at 100MeV/m and the length of the accelerator was $L = 3$cm. The charge of the bunch is 100pc.

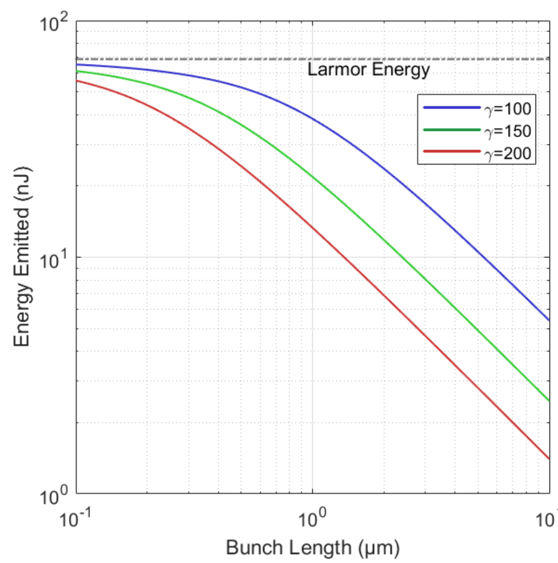


Figure 7. Scaling of emission bunch length in the RF regime. The emitted energy approaches the Larmor's limit of a charged macro-particle for the most compact bunches.

For the ultra-short bunch in the energy range considered, 10's of nJ are emitted, a measurable quantity of energy. The energy emitted strongly depends on the injection energy, with a significantly reduced emission for an injection energy > 100MeV ($\gamma = 200$).

V. CONCLUSION

We have shown that in the regime of high field-gradient acceleration relevant to plasma acceleration, and also in the high-performance RF acceleration regime, potentially measurable and significant electromagnetic radiation is emitted by ultra-short compact bunches. This emission challenges the commonly held assumption that emission from linearly accelerated is negligible. Ultra-short bunches of widths $1 - 10\mu$m are predicted to emit coherent radiation of order $\mathcal{O}(10\text{nJ})$ and $\mathcal{O}(10\mu\text{J})$ when accelerated with fields of strengths $E_{\text{ext}} \sim \mathcal{O}(100\text{MeV/m})$ and $E_{\text{ext}} \sim \mathcal{O}(10\text{GeV/m})$ respectively.

In extremely compact bunches, only just past that currently achievable and demonstrated, the calculated radiated energy approaches or even exceeds the available work-done of the accelerating field. The apparent violation of energy conservation illustrates the need for a fully self-consistent model of acceleration and radiation from charged particles, where the emitted energy is associated with a depletion of the accelerating field. While this work deals with emission to the far-field, the corresponding collective Lienard Weichert fields in the near-field region will perturb the acceleration process, or act on the bunch post-acceleration in a manner similar to that of CSR[18, 20, 21] longitudinal wakes. Purely longitudinal acceleration-induced wakes have been considered previously [6, 7, 22] for non-relativistic beams, or under assumptions of instantaneous energy change. An extension of the work here, which includes the acceleration and post-acceleration intervals and relativistic beams in the near field is in progress and will be published elsewhere.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this article are openly available in the Lancaster University Research Repository at `https://doi.org/10.17635/lancaster/researchdata/573`

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