

Advances in Portfolio Selection and Asset Pricing in Honor of Harry Markowitz

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Abstract

This paper introduces the Special Issue of *The European Journal of Finance* that seeks to honor the seminal contributions of Harry Markowitz to modern portfolio theory. Reflecting the fact that Markowitz's work is particularly influential, the articles in this special issue tackle a wide range of research questions. In summarizing these articles, we group them into three broad topics: (1) estimation of optimal portfolios; (2) portfolio selection beyond the mean-variance model; and (3) asset pricing implications of equilibrium models of portfolio selection. We also provide a brief discussion of possible directions for future research.

KEYWORDS: Portfolio selection, mean-variance model; asset pricing; estimation risk; transaction costs; behavioral finance

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1. Introduction

Harry Markowitz is widely recognized as the father of modern portfolio theory in light of his seminal work on portfolio selection (Markowitz, 1952a). He was awarded the 1990 Alfred Nobel Memorial Prize in Economic Sciences jointly with Merton Miller and William Sharpe for their pioneering work in the theory of financial economics. Markowitz assumes that investors prefer the portfolio with the *smallest* variance among portfolios with the same mean. Hence, investors select portfolios on the *mean-variance frontier*, where there exists no portfolio with a smaller variance for a given mean. Following Markowitz, modern portfolio theory often considers investors who have an objective function defined over the means and variances of portfolio returns and parameterized by a risk aversion coefficient.

Importantly, Markowitz (1952b) is also a pioneer in incorporating certain aspects of behavioral portfolio theory by considering investors who display both risk-seeking behavior and risk aversion. In Das et al. (2010), Markowitz and three co-authors develop a portfolio selection model that also incorporates aspects of behavioral portfolio theory. They consider investors who divide wealth into two or more mental accounts, use threshold returns and probabilities to parameterize goals for these accounts, and select possibly different optimal portfolios within different accounts.

Reflecting the fact that Markowitz's work is a key cornerstone of modern finance, the articles in this special issue tackle a wide range of research questions. For convenience, we group these articles into three broad topics: (1) estimation of optimal portfolios; (2) portfolio selection beyond the mean-variance model; and (3) asset pricing implications of equilibrium models of portfolio selection. In order, we next summarize the articles that cover each of these topics.

2. Estimation of Optimal Portfolios

In this section, we summarize the seven articles in the special issue that address the estimation of optimal portfolios.

Bodnar et al. (2025b) highlight that estimation risk is an important consideration when the mean-variance model of Markowitz (1952a) is implemented in practice. Specifically, the use of sample estimates of the mean vector and variance-covariance matrix of risky assets result in in-sample optimal portfolios that yield sub-par out-of-sample performance. Accordingly, they discuss: (1) shrinkage estimators for the mean vector and variance-covariance matrix of risky asset returns; (2) a test for the optimality of a vector or portfolio weights; and (3) the use of an R package named HDSHOP to implement modern portfolio theory in practice. Additionally, they empirically illustrate the use of this package by using returns of the stocks in the S&P 500 during the sample period of 2005-2025. In doing so, they find that the use of shrinkage estimators is beneficial particularly during the COVID-19 Pandemic.

Zhou et al. (2025) cope with a similar estimation risk problem by proposing an integrated framework to select portfolios, which they refer to as *Multi-Factor-Based Direct Reinforcement Learning*. This framework integrates: (i) a non-linear neural network that uses financial, fundamental, and technical factors to estimate the expected return vector and variance-covariance matrix for the risky assets that are available; (ii) a mean-variance portfolio optimization that considers transaction costs and linear constraints on portfolio weights; and (iii) a direct reinforcement learning approach that updates the network parameters based on a reward function that combines a mean-variance quadratic function of realized portfolio returns and a penalty

function of return prediction errors of the model. In the empirical part of the paper, the authors apply their framework to a problem of wealth allocation among six S&P sector indices using ten factors encompassing variables such as GDP growth, interest rates, inflation, price-to-earnings ratio, and trading volume. The paper finds that the portfolios obtained in their framework perform well relative to standard benchmarks (e.g., equally-weighted portfolio) when using various performance metrics (e.g., Sharpe ratio).

Bodnar et al. (2025a) investigate the asymptotic properties of sample estimators of three quantities that characterize the parabola representing portfolios on the mean-variance frontier in mean-variance space: (1) the expected return of the global minimum-variance portfolio; (2) the variance of this portfolio; and (3) the asymptotic slope of the top part of the aforementioned parabola as the variance of portfolios on the mean-variance frontier tends to infinity. In doing so, they focus on the case where both the number of assets and the number of sample vectors of asset returns converge to infinity and the ratio of these two numbers converges to a constant strictly between zero and one. Their paper theoretically shows that the sample estimator of the first quantity is consistent, whereas the sample estimators of the second and third quantities are not. Moreover, the paper presents consistent estimators for the three quantities. The empirical part of the paper compares these consistent estimators to other commonly used estimators by using intraday returns of stocks in the S&P 500 during the period of 2017-2022.

Golosnoy et al. (2025) develop an estimation approach for the weights of the global minimum-variance portfolio (GMVP) that combines two or more prediction rules for such weights (e.g., equally-weighted portfolio and weights based on a set of realized variance-covariance matrices). The empirical part of the paper implements this approach by using a sample of 265 stocks in the S&P 500, the sample period of 2005-2022, and six prediction rules for the estimation

of the weights of the GMVP. The paper finds that an appropriate *combination* of prediction rules to estimate the GMVP leads to a *reduction* of out-of-sample portfolio variance relative to the use of a single prediction rule.

Fletcher, Marshall, and O’Connell (2025) examine the out-of-sample performance benefits arising from using the best factor models from a Bayesian model scan in portfolio selection with the mean-variance model. The authors consider the problem of allocating wealth among a risk-free asset as well as 13 European regional factors and two currency factors. In assessing out-of-sample performance of the best factor models from the Bayesian model, they consider two measures (Sharpe ratio and the certainty-equivalent return), two benchmarks (an equally-weighted portfolio of factors and a European regional market index), and six alternative factor models (ranging from one to eight factors). The authors find that the portfolios selected when using the best factor models from the Bayesian model scan have better out-of-sample performance than the benchmarks as well as the portfolios selected when using the six alternative factor models.

Bašić et al. (2025) examine a portfolio optimization model to allocate wealth among individual US stocks in the presence of transaction costs and transaction cost risk. Their model considers three objective functions: (1) a *standard* one defined over the mean and variance of portfolio returns; (2) an *expanded* one defined over the mean and variance of portfolio returns as well as the mean of transaction costs; and (3) an *even more expanded* one defined over the mean and variance of portfolio returns as well as the mean and variance of transaction costs. Using a proprietary dataset of real trading data for 123 stocks in the S&P 500 from a large institutional asset manager, the authors show how to estimate the variance-covariance matrix of the stocks’ transaction costs effectively and find that portfolios that maximize objective function (3) tend to outperform portfolios that maximize objective function (2), which in turn tend to outperform

portfolios that maximize objective function (1). Here, performance is measured by the out-of-sample Sharpe ratio net of transaction costs.

Damjanovic et al. (2025) examine whether market capitalization- or price-weighted index weighing is superior with regard to diversification from the perspective of an investor who uses passive strategies (e.g., invest in an index fund). The authors use data from the period of 2006-2024 along with both *absolute* and *relative* measures of diversification (the relative ones equal the absolute ones *divided* by the number of portfolio constituents). They find that *absolute* diversification measures are higher for the (market capitalization-based) S&P 500 than the (price-weighted) DJIA due to the S&P 500 having a larger number of constituents. However, they also find that the *relative* diversification measures are higher for the DJIA than for the S&P 500 due to almost all DJIA constituents contributing to diversification.

3. Portfolio Selection Beyond the Mean-Variance Model

In this section, we summarize the five articles in the special issue that investigate portfolio selection beyond the mean-variance model.

Baptista (2025) derives analytical results on: (1) the loss in the certainty-equivalent return (CER) arising from misspecifying a risk aversion coefficient in a mean-variance objective function; (2) the impact of using thresholds instead of a risk aversion coefficient to specify investor objectives on portfolio selection; and (3) the risk aversion coefficients of investors in the model of Markowitz (1952a) who select the same portfolios as an investor in the model of Das et al. (2010) selects within accounts. These results have two key practical implications. First, in implementing Markowitz's model, practitioners should be aware that the loss in CER arising from underestimating the risk aversion coefficient of an investor by some amount *exceeds* the loss in

CER arising from overestimating the coefficient by the same amount. Second, in implementing Das et al.'s model, practitioners should be aware that optimal portfolios within accounts and the risk aversion coefficients implied by such portfolios are *very* sensitive to the thresholds of an investor for relatively large coefficients but notably less so for relatively small coefficients.

Şensoy, Özekici, and Sak (2025) theoretically examine the optimal portfolio of an investor who has an exponential value function separately defined over losses and gains when: (i) a risk-free asset is present; (ii) risky asset returns are assumed to have a multivariate Normal distribution; and (iii) the probability distortion part of prospect theory is absent. They find that the optimal portfolio is proportional to the optimal portfolio in the mean-variance model of Markowitz. The authors empirically find that their portfolio optimization approach results in higher (lower) subsequent average returns for groups of US stocks with high (low) prospect theory values.

Kofina, Psaradellis, and Topaloglou (2025) examine whether four major stock indices (S&P 500, FTSE 100, DAX 40, and Nikkei 225) as well as ETFs that track such indices are *optimal* from perspective of investors with objective functions that reflect risk-seeking behavior for gains and risk aversion for losses. In doing so, the authors employ tests of Markowitz stochastic dominance. The authors find that these indices and ETFs are systematically *suboptimal* for such investors. Moreover, they find that optimal portfolios of index constituents (i.e., properly designed active strategies) have notably better out-of-sample performance for these investors than the aforementioned indices and ETFs (i.e., passive strategies). In assessing out-of-sample performance, they use Sharpe, downside Sharpe, and upside-potential ratios.

Loperfido and Shushi (2025) extend the mean-variance model of Markowitz by considering an investor who has an objective function defined over the mean and variance of portfolio returns as well as on a coefficient of risk aversion that depends on the distribution of

portfolio returns. Assuming that asset returns have a multivariate skew-elliptical distribution, the authors establish a method for finding the solution to the investor's portfolio selection problem. They show that the solution retains the structure of a parabolic efficient frontier as in the mean-variance model of Markowitz and that the solution moves along the frontier as the risk aversion coefficient changes.

Landsman and Shushi (2025) examine the problem of maximization of an objective function that depends on one or more linear forms as well as a quadratic form for selecting portfolios. The authors characterize the solution to this problem which can be viewed as an extension of the single-period mean-variance model (with a single linear form) to a multi-period model (with multiple linear forms). In the empirical part of the paper, the authors apply the model to stocks of five large firms listed in Nasdaq using weekly returns and use eight linear forms corresponding to the mean vectors of these stocks in the eight quarters in the period of 2023-2024. They find that the stock weights when using these eight mean vectors (i.e., a multi-period model) differ notably from the stock weights when using a single mean vector (i.e., a single-period model).

4. Asset Pricing Implications of Equilibrium Models of Portfolio Selection

In this section, we summarize the two articles in the special issue that explore the asset pricing implications of equilibrium models of portfolio selection.

Levy and Levy (2025) develop a portfolio theory-based equilibrium model to derive the true marginal cost of capital for an expanding firm. They compare: (1) the marginal cost of capital (MCC) of a firm that takes on a new project that scales up operations; and (2) the average cost of capital (ACC) of the firm. A key theoretical result in their model is that MCC *exceeds* ACC. This result holds because the aforementioned project forces investors to *increase* the fraction of wealth

invested in the firm and investors do so only if the firm's expected return *increases* in an equilibrium. Levy and Levy highlight an important implication of such a result for the stock price reactions to Seasoned Equity Offerings (SEOs). The erroneous use of ACC (instead of the higher MCC) to discount the cash-flows of an expansion financed with an SEO may result in the acceptance of a *negative* NPV project and a corresponding *negative* stock price reaction following the expansion consistent with empirical evidence. Levy and Levy similarly highlight implications of their theoretical result for Initial Public Offerings (IPOs) and stock repurchases. Specifically, their result is consistent with empirical evidence of *positive* short-term stock returns following IPOs and *positive* stock price reactions to stock repurchases.

Siman and Siman (2025) propose an equilibrium model where investors have preferences represented by a function that reflects risk aversion along with investor- and asset-specific ambiguity. They derive a pricing equation where the expected return of a risky asset in excess of the risk-free rate depends on *three* factors: (1) beta (similar to the original CAPM of Sharpe (1964)); (2) size as measured by the weight of the asset in the market portfolio in excess of asset-specific ambiguity aggregated over all investors; and (3) an interaction between size and the ratio of ambiguity aversion to risk aversion of investors. Using simulation, the authors find a positive risk premium for both the beta and size factors. The result that the risk premium on the size factor is positive contrasts with the small firm effect of Banz (1981).

5. Summary and Possible Directions for Future Research

Markowitz's work has been influential in *both* academia and practice for more than seven decades. Inspired by his work, researchers have developed an extensive body of research that has

yielded notable improvements to the areas of portfolio selection and asset pricing. The articles in this special issue highlight (and add to) this body of research.

Possible directions for future research promising further improvements in the areas of portfolio selection and asset pricing are as follows. First, it would be useful to develop approaches to more accurately infer the preferences of investors and properly identify their corresponding ‘optimal’ portfolios. Second, it would be beneficial to investigate whether the latest advances in Artificial Intelligence (AI) help improve the precision of estimators of the optimization inputs for the practical implementation of the mean-variance model. Third, it would be valuable to assess whether automated portfolio management systems can be designed to mitigate the behavioral biases of investors to the fullest extent possible. Fourth, it would be helpful to examine the asset pricing implications arising from investors using the most recent AI models and automated portfolio management systems in portfolio selection.

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No potential conflict of interest was reported by the authors.

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