

Modelling Quintessential Inflation with Branes

Konstantinos Dimopoulos^{1,2}

¹*Physics Department, Lancaster University, Lancaster LA1 4YB, UK*

²*Department of Physics, University of Oxford, Keble Road, Oxford OX1 3RH, UK*

Abstract. I discuss why quintessential inflation model-building is more natural in the context of brane cosmology and study a particular model as an example.

1 Introduction

To minimize its fine-tuning problems, such as initial conditions, there have been attempts to unify quintessence with the inflaton field in a single scalar [3, 9, 12, 13]. This way, introducing yet again another unobserved scalar field is avoided. Also, a common theoretical framework can be used to describe both inflation and quintessence. For quintessential inflation one needs a sterile inflaton, with only minimal gravitational coupling to the standard model, because the field should survive until today. The Universe reheats through gravitational particle production [5]. The minimum of the potential (assumed zero) is typically placed at infinity, because it should not have yet been reached. This results in the so-called quintessential tail of the potential. Candidates for the quintessential–inflation scalar field are moduli fields or the radion field.

2 Dynamics

The Universe is modeled as a collection of perfect fluids; the background fluid with density ρ_B (comprised by matter and radiation) and the scalar field ϕ with density $\rho_\phi = \rho_{\text{kin}} + V$ and pressure $p_\phi = \rho_{\text{kin}} - V$, where $\rho_{\text{kin}} \equiv \frac{1}{2}\dot{\phi}^2$ is the kinetic density of ϕ and the dot denotes derivative w.r.t. the cosmic time t . For every component one defines the barotropic parameter as $w_i \equiv p_i/\rho_i$. The Universe expansion accelerates when $\rho_B < \rho_\phi$ and $w_\phi < -\frac{1}{3}$. Energy-momentum conservation demands $d(a^3\rho) = -p d(a^3)$, which, for decoupled fluids, gives $\rho_i \propto a^{-3(1+w_i)}$, where a is the scale factor. To study the dynamics of the Universe, one also needs the Friedman equation and the ϕ field equation: $\dot{\rho}_{\text{kin}} + 6H\rho_{\text{kin}} + \dot{V} = 0$, where $H \equiv \dot{a}/a$ is the Hubble parameter. In spatially-flat, FRW cosmology the Friedman equation is $H^2 = \rho/3m_P^2$, with m_P being the reduced Planck mass. This results in the evolution equations:

$$H = \frac{2t^{-1}}{3(1+w)} \quad a \propto t^{\frac{2}{3(1+w)}} \quad \rho = \frac{4}{3(1+w)^2} \left(\frac{m_P}{t}\right)^2 \quad (1)$$

where w corresponds to the dominant fluid component. In brane cosmology the Friedman equation is modified for density above the string tension λ [11].

Assuming a 5th dimension, the Friedman equation becomes $H \simeq \rho/\sqrt{6\lambda} m_P$, where $\lambda = \frac{3}{4\pi}(M_5^6/m_P^2)$, with M_5 being the fundamental, 5-dim Planck mass. Then the evolution equations are:

$$H = \frac{t^{-1}}{3(1+w)} \quad a \propto t^{\frac{1}{3(1+w)}} \quad \rho = \frac{\sqrt{6\lambda}}{3(1+w)} \left(\frac{m_P}{t} \right) \quad (2)$$

The modified dynamics affect the inflationary era due to excessive friction on the roll of the scalar field, which allows inflation with steep potential. Then, the slow-roll parameters become: $\epsilon \simeq 2\lambda m_P^2 (V')^2/V^3$ and $\eta \simeq 2\lambda m_P^2 V''/V^2$, where the prime denotes derivative w.r.t. ϕ . The modified slow-roll changes the amplitude and spectral index of the generated density perturbations [11]:

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{2\sqrt{6}\pi} \frac{V^3}{\lambda^{3/2}|V'|m_P^3} \quad \text{and} \quad n_s - 1 \simeq -4m_P^2 \frac{\lambda}{V} \left[3 \left(\frac{V'}{V} \right)^2 - \frac{V''}{V} \right] \quad (3)$$

Gravitational reheating creates a thermal bath of temperature $T_{\text{reh}} = \frac{\alpha}{2\pi} H_{\text{end}}$ [5], where α is the reheating efficiency and the subscript 'end' denotes the end of inflation. Due to the inefficiency of reheating, after the end of inflation the Universe becomes dominated by ρ_{kin} , i.e. $\rho \simeq \rho_{\text{kin}} \propto a^{-6}$. As long as $\rho_{\text{kin}} > 2\lambda$ we have $a \propto t^{1/6}$ and $\phi(t) = \phi_{\text{end}} + \frac{2}{\sqrt{3}} \sqrt{\lambda/V_{\text{end}}} \left(\sqrt{t/t_{\text{end}}} - 1 \right) m_P$. However, when $\rho_{\text{kin}} < 2\lambda$ the brane regime ends and one has $a \propto t^{1/3}$ and $\phi(t) = \phi_{\text{end}} + \frac{1}{\sqrt{6}} [2 + \ln(t/t_\lambda)] m_P$, where $t_\lambda = \frac{1}{2\sqrt{6}} m_P/\sqrt{\lambda}$ is the crossover time for which $\phi_\lambda = \phi_{\text{end}} + \sqrt{2/3} m_P$. The thermal bath eventually dominates the density of the Universe and the Hot Big Bang begins at the temperature:

$$T_* = \frac{\alpha^3}{96\pi^2} \sqrt{\frac{g_*}{15}} \frac{V_{\text{end}}^{5/2}}{\lambda^{3/2} m_P^3} \quad (4)$$

where $g_* \sim 10^2$ is the number of relativistic degrees of freedom. In order not to affect Nucleosynthesis (BBN) we require $T_* > 1$ MeV. After the onset of the Hot Big Bang ρ_{kin} rapidly decreases and the field freezes at the value:

$$\phi_F = \phi_{\text{end}} + \frac{m_P}{\sqrt{6}} \left[14 + \frac{3}{2} \ln \left(\frac{30\pi^2}{g_*\alpha^4} \right) - 4 \ln \left(\frac{m_P^4}{\lambda} \right) + 5 \ln \left(\frac{m_P^4}{V_{\text{end}}} \right) \right] \quad (5)$$

3 Why Branes

Standard-cosmology quintessential inflation needs a potential with two flat regions: the inflationary plateau and the quintessential tail [3]. BBN and coincidence demand that the density scale of the flat regions differs by $\sim 10^{100}$! To prepare for this abysmal dive, V is strongly curved near the end of inflation. As a result, the slow-roll parameter η is large, leading to spectral index n_s far from unity. This makes it difficult to construct single-branch models¹

¹Multi-branch models such as [9, 12, 13] are disfavoured in our minimalistic approach.

for quintessential inflation, although solutions do exist [3]. In contrast, this η -problem does not appear when considering brane cosmology because the modified Friedman equation allows for steep inflation [2, 8, 10, 11, 14]. Hence, one may obtain successful quintessential inflation with a simple potential.

4 The exponential tail

String theory disfavors eternal acceleration [4, 7]. This immediately rules out quintessential tails milder than exponential and also frozen quintessence (in which $\phi = \phi_F$ at present). In any case, coincidence is hard to achieve with mild tails (e.g. inverse power-law type [8]) and frozen quintessence is virtually indistinguishable from the cosmological constant alternative. On the other hand steeper-than-exponential tails have disastrous attractors [3]. Thus, the most reasonable approach is the exponential tail, $V \simeq V_0 e^{-b\phi/m_P}$, where b is a positive constant. Then, the ϕ -field equation gives an exact attractor solution:

$$\phi_{\text{attr}} = \frac{2m_P}{b} \ln \left[\sqrt{\frac{V_0}{2} \left(\frac{1+w}{1-w} \right) \frac{bt}{m_P}} \right] \quad \text{and} \quad V_{\text{attr}} = \frac{2}{b^2} \left(\frac{1-w}{1+w} \right) \left(\frac{m_P}{t} \right)^2 \quad (6)$$

If $\rho_\phi < \rho_B$ then $w = w_B$ and $\rho_\phi/\rho_B = 3(1+w_B)/b^2 = \text{const.}$, which means $w_\phi = w_B \geq 0$ and $b^2 > 3(1+w_B)$. If, however, $\rho_\phi > \rho_B$ then $w = w_\phi$ and $\rho = \rho_\phi \propto a^{-3(1+w_\phi)}$. This time $b^2 = 3(1+w_\phi)$. Therefore, *dark energy dominates without eternal acceleration when* $2 < b^2 < 3(1+w_B)$. Brief acceleration is possible at the time when ϕ unfreezes to follow the attractor, due to superfreezing [3]. In fact, superfreezing enlarges the range to $2 < b^2 < 24$ [1].

5 A toy-model example

Consider the potential (also studied in [6]):

$$V(\phi) = \frac{M^4}{\cosh(b\phi/m_P) - 1} \quad \Rightarrow \quad \begin{cases} V \simeq 2M^4 e^{-b\phi/m_P} & \phi \gg m_P/b \\ V \simeq 2M^4 (m_P/b\phi)^2 & \phi \ll m_P/b \end{cases} \quad (7)$$

The slow-roll parameters are: $\eta = 2A[\cosh(b\phi/m_P) + 2] = \epsilon + 2A$. Thus, inflation ends when $\eta(\phi_{\text{end}}) \equiv 1$, where $A \equiv b^2(\lambda/M^4)$. The spectral index is

$$n_s - 1 = -4A \frac{3 + \left(\frac{1-6A}{1-2A} \right) \exp(-4N_{\text{dec}}A)}{1 - \left(\frac{1-6A}{1-2A} \right) \exp(-4N_{\text{dec}}A)} \quad (8)$$

where $N_{\text{dec}} \simeq 69$ is the number of e-folds of remaining inflation corresponding to the scale of the horizon at decoupling. From the observations $|n_s - 1| \leq 0.1$, which demands the constraint $A < \frac{1}{148}$. Using $A \ll 1$ the above becomes

$n_s - 1 \simeq -\frac{4}{N_{\text{dec}}+1}$, which gives $n_s \simeq 0.94$. The density contrast with $A \ll 1$ is:

$$\frac{\delta\rho}{\rho} \simeq \frac{2b^2}{\sqrt{6}\pi} \left(\frac{M}{m_P}\right)^2 \sqrt{A} (N_{\text{dec}} + 1)^2 \quad (9)$$

Similarly, for the inflationary scale we find $V_{\text{end}} \simeq 2AM^4$ or, equivalently,

$$V_{\text{end}} \simeq \frac{3\pi^2}{b^4} \left(\frac{\delta\rho}{\rho}\right)^2 \frac{m_P^4}{(N_{\text{dec}} + 1)^4} \quad (10)$$

The field freezes at $\phi_F = \phi_{\text{end}} + \frac{1}{\sqrt{6}}(61.7 - 4 \ln b)m_P$. Then coincidence demands: $b \simeq 14.5$, which is too large for brief acceleration. Further, Eq.(10) gives $V_{\text{end}}^{1/4} \simeq 2 \times 10^{13} \text{GeV}$. Using $V_{\text{end}} \simeq 2b^2\lambda$ one finds $\lambda^{1/4} \simeq 6 \times 10^{12} \text{GeV}$ and so $M_5 \simeq 5 \times 10^{14} \text{GeV}$. The bound on A suggests: $M > 7 \times 10^{13} \text{GeV}$, which enables the identification $M = M_5$. The Hot Big Bang begins at temperature:

$$T_* = \frac{\alpha^3}{16b(N_{\text{dec}} + 1)^4} \sqrt{\frac{2g_*}{15}} \left(\frac{\delta\rho}{\rho}\right)^2 m_P \quad (11)$$

which satisfies the BBN constraint if $\alpha \gtrsim 0.1$. With $\alpha \sim 0.1$, the modified Friedman equation gives $T_{\text{reh}} \sim 10^7 \text{GeV}$, which satisfies the gravitino bound.

6 Conclusions

Quintessential-inflation model-building is easier in brane-cosmology because the η -problem is overcome by considering steep inflation [2]. The dynamics of the Universe from inflation through to the present have been analysed and employed in the toy-model presented. This model incorporates two natural mass-scales (m_P and M_5) and leads to a spectral index within observational bounds. However, it fails to provide late-time accelerated expansion.

Acknowledgements. Work supported by the EU network: HPRN-CT-00-00152.

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